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Heat and mass transfer of a second grade magnetohydrodynamic fluid over a convectively heated stretching sheet

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Abstract

The present work is concerned with heat and mass transfer of an electrically conducting second grade MHD fluid past a semi-infinite stretching sheet with convective surface heat flux. The analysis accounts for thermophoresis and thermal radiation. A similarity transformations is used to reduce the governing equations into a dimensionless form. The local similarity equations are derived and solved using Nachtsheim-Swigert shooting iteration technique together with Runge–Kutta sixth order integration scheme. Results for various flow characteristics are presented through graphs and tables delineating the effect of various parameters characterizing the flow. Our analysis explores that the rate of heat transfer enhances with increasing the values of the surface convection parameter. Also the fluid velocity and temperature in the boundary layer region rise significantly for increasing the values of thermal radiation parameter.

Keywords: Thermophoresis; Thermal radiation; Convective boundary condition; Second grade fluid

1. Introduction

The study of heat and mass transfer of non-Newtonian fluid has been increased due to their applications in many branches of science and engineering, such as metallurgical process, polymer extrusion, glass blowing, crystal growing and so on. The boundary layer flow of a non-Newtonian viscous fluid has drawn the attention of many researchers [1–5]. Pal and Mondal [6] discussed MHD non-Darcian mixed convection heat and mass transfer over a non-linear stretching sheet. Heat and mass transfer past a stretching surface in a MHD micropolar fluid through a porous medium was studied by Pal and Chatterjee [7]. Pal and Mondal [8] extended their work [6] by considering Soret and Dufour effects on MHD non-Darcian in presence of non-uniform heat source/sink. Kandelousi [9] investigated the effect of spatially variable magnetic field on ferrofluid flow. Sheikholeslami et al. [10] discussed the impact of non-uniform magnetic field on forced convection heat transfer of Fe_3O_4–water nanofluid. Sheikholeslami and Rashidi [11] developed the work of Sheikholeslami et al. [10] by considering the space dependent magnetic field. Effect of electric field on hydro-thermal behavior of nanofluid in a complex geometry was investigated by Sheikholeslami et al. [12]. Sheikholeslami et al. [13] worked on forced convection heat transfer in a semi annulus under the influence of a variable magnetic field.

MHD flow problem in presence thermal radiation has become more important in industry at high temperature. So the knowledge of the radiation heat transfer becomes very important. Cogley et al. [14] observed that in the optically thin limit the fluid does not absorb its own emitted radiation but absorb radiation emitted by the boundaries. The effect of thermal radiation on heat transfer problems have studied by Makinde [15], Ibrahim et al. [16] Pal and Chatterjee [17], Olajuwon [18] and Zheng et al. [19]. Pal and

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Mondal [20] examined the effect of chemical reaction and thermal radiation on mixed convection heat and mass transfer over a stretching sheet in Darcian porous medium. Sheikholeslami et al. [21] examined numerically MHD free convection of Al_2O_3–water nanofluid in presence of thermal radiation. Ferrofluid flow and heat transfer in a semi annulus enclosure in the presence of thermal radiation was studied by Sheikholeslami et al. [22]. Sheikholeslami et al. [23] considered the effect of thermal radiation on two phase model of nanofluid flow and heat transfer.

Thermophoresis, a physical phenomenon in which aerosol particles move from hot surface to cold surface, has attracted considerable attention for collection of sub-micrometer and nanometer particles. The force experienced by the suspended particles due to the temperature gradient is termed as thermophoretic force which is used in commercial precipitators. In this occurrence, the repulsion of particles from hot objects takes place and so a layer is obtained around hot bodies which is particle free (Goldsmith and May [24]). This phenomenon has many applications: to remove small particles from gas particle trajectories from combustion devices and to study the particulate material deposition turbine blades. The effect of thermophoresis particle deposition on boundary layer flow under different situation was discussed by many researchers (Selim et al. [25], Chamkha and Pop [26], Chamkha et al. [27] , Zucco et al. [28]). Pal and Mondal [29] discussed the effect of thermophoresis on magnetohydrodynamic heat and mass transfer over a non-Isothermal wedge. KKL correlation for simulation of nanofluid flow and heat transfer in a permeable channel was examined by Sheikholeslam [30]. Sheikholeslami and Ganji [31] studied nanofluid flow and heat transfer between parallel plates using DTM.

In the study of boundary layer flow problems, the boundary conditions are either a specified surface temperature or a specified surface heat flux [32]. But there are many problems in which surface heat transfer depends on the surface temperature. Newtonian heating arises in the situation where the heat is supplied to the convective fluid through a bounding surface with a finite heat capacity. Recently, boundary layer heat transfer problems concerning with a convective boundary condition were investigated by Makinde and Aziz [33], Ishak [34] and Rahman [35]. Recently Das [36] studied the effect of chemical reaction on MHD mixed convection second grade fluid flow passing through a semi-infinite stretching sheet.

Motivated by the above investigations present paper deals with the second grade fluid flow passing through a semi-infinite stretching sheet with convective surface heat flux. The impact of thermophoresis and thermal radiation on heat and mass transfer are included in the present model. The Nachtsheim and Swigert shooting iteration technique together with Runge-Kutta sixth-order integration scheme is used to solve the problem numerically.

2. Mathematical formulation of the problem

Consider the steady boundary layer flow of an incompressible and electrically conducting second grade fluid over a stretching sheet coinciding with the plane $y = 0$ and the flow being confined to $y > 0$ in the presence of viscous dissipation and joule heating as depicted in Fig. 1. The flow is generated, due to the stretching of the sheet caused by the simultaneous action of two equal and opposite forces along the x-axis. The sheet is then stretched with a velocity $u_w(x) = ax$, where a is a constant and x is the coordinate measured along the stretching surface from the slit. The thermal radiation is taking place in the flow and the effect of thermophoresis is being taken into account to help in understanding of the mass deposition variation on the surface. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. The applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall effect are negligible. It is assumed that there is no applied voltage which implies the absence of an electric field. The stretching surface is maintained at constant temperature T_w higher than the constant temperature T_∞ of the ambient fluid. Due to the boundary layer behavior the temperature gradient along y-direction is much more than that along x-direction and hence only the thermophoretic velocity component which is normal to the surface is of importance.

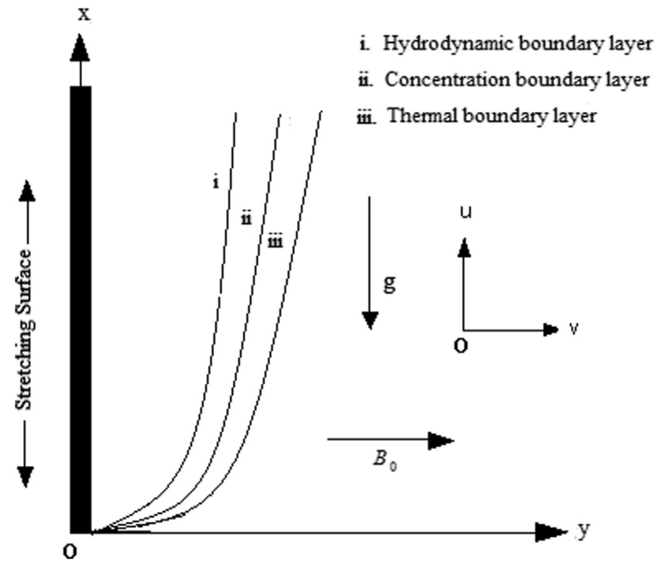

Fig. 1. Physical model and coordinate system.

Under these assumptions, the governing boundary layer equations for a second grade fluid flow can be written as [18,36]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right] - \frac{\sigma B_0^2 u}{\rho} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_1\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\right] + \sigma {B_0}^2 u^2, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}-\frac{\partial(V_T C)}{\partial y}-k_r(C-C_\infty), \tag{4}$$

Here the velocity components along x, y-axis are u, v respectively, g is the acceleration due to gravity, ρ is the fluid density, ν is the kinematic viscosity, σ is the electrical conductivity of the fluid, α_1 is the material parameter, β is the thermal expansion coefficient, β^* is the volumetric expansion coefficient, T, T_∞ is the temperature of the fluid within the boundary layer and the fluid temperature in the free stream respectively while C, C_∞ are the corresponding species concentrations of the fluid, κ is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure p, q_r is the radiative heat flux, D_m is the molecular diffusivity, K_T is the thermal diffusion ratio, T_m is the mean fluid temperature, k_r is the rate of chemical reaction, $k_r>0$ represents for destructive reaction, $k_r<0$ represents for generative reaction and $k_r=0$ represents for no reaction, $V_T=-\frac{k\nu}{T_r}\frac{\partial T}{\partial y}$ is the thermophoretic velocity, where k is the thermophoretic coefficient and T_r is some reference temperature.

We assume the bottom surface of the plate is heated by convection from a hot fluid at temperature T_w which provides a heat transfer coefficient h_w. The boundary conditions of the present model are [18,34,36]

$$\left.\begin{array}{llll} u=u_w, & v=v_0, & -\kappa\frac{\partial T}{\partial y}=h_w(T_w-T), & C=C_w \text{ at } y=0, \\ u\to 0, & \frac{\partial u}{\partial x}\to 0, & T\to T_\infty, & C\to C_\infty \text{ as } y\to\infty \end{array}\right\} \tag{5}$$

where v_0 is the suction/injection velocity.

Using the Rosseland approximation, the radiative heat flux term is given by [34]

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* is the Stefan Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, we expand T^4 in Taylor's series about T_∞ and neglecting higher order terms, we get [34]

$$T^4=4T_\infty^3 T-3T_\infty^4, \tag{7}$$

Thus we have

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient.

Following the lines of Olajuwon [18], the similarity transformations as given below are introduced:

$$\text{u}=\text{cx}f'(\eta), \text{v}=-(c\nu)^{1/2}f(\eta), \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \tag{9}$$

where

$$\eta=\left(\frac{c}{\nu}\right)^{1/2}y, \tag{10}$$

Eq. (1) is automatically satisfied. Using Eq. (8) in (3) and applying transformation (9), Eqs. (2)–(4) reduce to the ordinary differential equations:

$$f'^2-ff''-f'''-\lambda_1(2f'''f'-ff^{iv}-f''^2)+Mf'-Gr\theta-Gm\phi=0, \tag{11}$$

$$(1+Nr)\theta''+Prf\,\theta'+PrEc[f''^2+Mf'^2+\lambda_1 f''(f'f''-ff''')]=0, \tag{12}$$

$$\phi''+Sc(f-\tau\theta')\phi'+Sc(Sr-\tau\phi)\theta''-ScKr\phi=0. \tag{13}$$

The boundary conditions (5) then turn into

$$\begin{array}{l} f=f_w, f'=1, \theta'=-\gamma(1-\theta), \phi=1 \text{ at } \eta=0 \\ f'\to 0, f''\to 0, \theta\to 0, \phi\to 0 \text{ as } \eta\to\infty \end{array} \tag{14}$$

Here prime denotes differentiation with respect to η, $\lambda_1=\alpha_1 a/\rho\nu$ is the second grade fluid parameter, $M=\sigma B_0^2/\rho a$ is the magnetic field parameter, $f_w=v_0/(a\nu)^{1/2}$ is the suction parameter, $Nr=16T_\infty^3\sigma^*/3k^*\kappa$ is the thermal radiation parameter, $Pr=\mu c_p/\kappa$ is the Prandtl number, $Ec=u_w^2/c_p(T_w-T_p)$ is the Eckert number, $\tau=-k(T_w-T_\infty)/T_r$ is the thermophoretic parameter, $Sc=\nu/D_m$ is the Schmidt number, $Sr=D_m K_T(T_w-T_\infty)/T_m\nu(C_w-C_\infty)$ is the Soret number, $Gr=g\beta(T_w-T_\infty)/a$ is the thermal Grashof number, $Gm=g\beta^*(C_w-C_\infty)/a$ is the solutal Grashof number, $Kr=k_r\nu/aD_m$ is the chemical reaction parameter and $\gamma=h_w/\kappa(\nu/a)^{1/2}$ is the surface convection parameter.

The physical quantities of practical and engineering primary interest are the skin friction coefficient, Nusselt number and Sherwood number. The equation defining the wall shear stress is

$$\tau_w=\left[\mu\frac{\partial u}{\partial y}+\rho\alpha_1\left(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+u\frac{\partial^2 u}{\partial x\partial y}\right)\right]_{y=0} \tag{15}$$

The local dimensionless skin friction coefficient is given by

$$C_f=2Re^{-1/2}[1+3\lambda_1 f'(0)]f''(0) \tag{16}$$

or,

$$C_f^*=[1+3\lambda_1 f'(0)]f''(0) \text{ where } C_f^*=\frac{1}{2}Re^{1/2}C_f \tag{17}$$

Knowing the temperature field, it is interesting to study the effect of the free convection and thermal radiation on the rate of heat transfer q_w, is given by

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}-\frac{16T_\infty^3\sigma^*}{3k^*}\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=0} \tag{18}$$

So the rate of heat transfer in terms of dimensionless Nusselt number is defined as follows:

$$Nu=-Re^{1/2}(1+Nr)\theta'(0) \tag{19}$$

or,

$$Nu^*=-(1+Nr)\theta'(0) \text{ where } Nu^*=Re^{-1/2}Nu \tag{20}$$

Similarly the rate of mass transfer in terms of dimensionless Sherwood number Sh is given by

$$Sh = -Re^{1/2}\phi'(0) \tag{21}$$

or,

$$Sh^* = -\phi'(0) \text{ where } Sh^* = Re^{-1/2}Sh \tag{22}$$

3. Numerical method for solution

The set of Eqs. (11)–(13) under the boundary conditions (14) are solved numerically by applying the Nachtsheim and Swigert shooting iteration technique together with Runge-Kutta sixth-order integration scheme. The initial conditions for $f''(0)$, $\theta(0)$ and $\phi(0)$ are presumed and then integrated numerically as an initial value problem. A step size of $\Delta\eta = 0.005$ is used to obtain the numerical solutions with $\eta_{\max} = 5$. The procedure is repeated until we get the results up to the desired degree of accuracy, namely 10^{-6}. It should be noted that the Nachtsheim and Swigert shooting iteration technique is well established and has been successfully implemented to study a variety of non-linear fluid flow problems.

The following algorithm is used to solve the non-linear ordinary differential Eqs. (11)–(13) along with the boundary conditions (14):

Step 1. Reduce Eqs. (11)–(13) to a system of first order equations.
Step 2. Set the boundary conditions and initial values.
Step 3. Use Nachtsheim and Swigert shooting technique to guess initial values.
Step 4. Repeat the step 3 until far filled boundary conditions are satisfied.
Step 5. Solve the reduced system of first order equations by Runge-Kutta method.
Step 6. Repeat these steps until the convergence criterion of 10^{-6} holds good.

To check the validity of the numerical code, the values of $-\theta'(0)$ have been calculated for $\tau = Ec = Kr = \gamma = 0$ and for different values of thermal radiation parameter Nr in Table 1. It has been observed from the Table 1 that the results obtained by the present code and those of Olajuwon [10] and Das [20] show excellent agreement. Thus the present numerical code used for current model is justified.

4. Numerical results and discussion

The numerical computations are performed using the method described in the previous section for various values of parameters that describe the flow characteristics of a second grade fluid over a convectively heated stretching sheet. The results are illustrated graphically in Figs. 2–9 and in Table 2. There are many parameters involved in the final form of the model. The problem can be protracted on many directions, but the first one seems to consider the effects of surface convection parameter, radiation parameter, second grade parameter and thermophoretic parameter. The default values of material parameters are considered in the simulation as $M = 4.0$, $Sr = 0.5$, $Sc = 0.64$, $\lambda_1 = 1.5$, $Kr = 0.2$, $\gamma = 0.1$, $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Nr = 0.4$, $Ec = 0.02$ and $\tau = 0.2$ unless otherwise specified.

Figs. 2 and 3 display the behavior of the velocity and temperature distribution for various values of thermal radiation parameter Nr. Fig. 2 shows that an increase in radiation parameter tends to increase the fluid velocity in the boundary layer region. The physics behind the results is that the thermal radiation increases the thickness of momentum boundary layer, which ultimately enhances the velocity. From Fig. 3, it is also observed that the temperature distribution increases uniformly

Table 1
Comparison of $-\theta'(0)$ for various values of Nr.

Nr	$-\theta'(0)$		
	Present results	Das [36]	Olajuwon [18]
0.2	1.595721	1.58882	1.59570
0.5	1.170500	1.72067	1.17050
0.7	0.373516	0.37381	0.37350
2.0	2.675600	2.59830	2.67560
5.0	2.354533	2.34861	2.35450

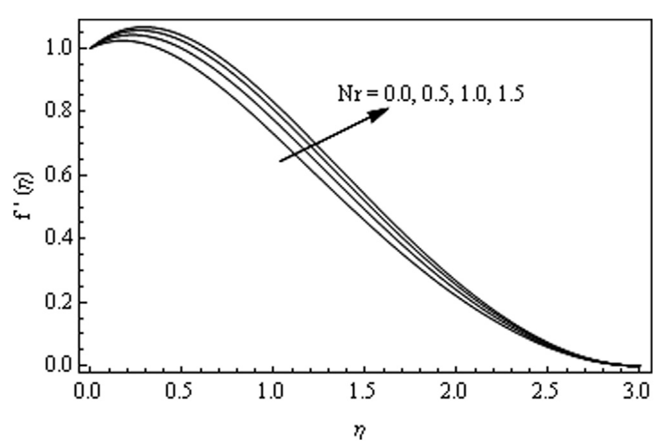

Fig. 2. Velocity profiles for various values of Nr.

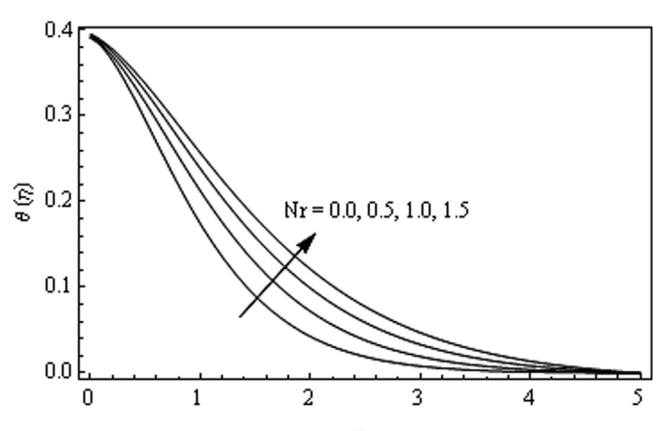

Fig. 3. Temperature profiles for various values of Nr.

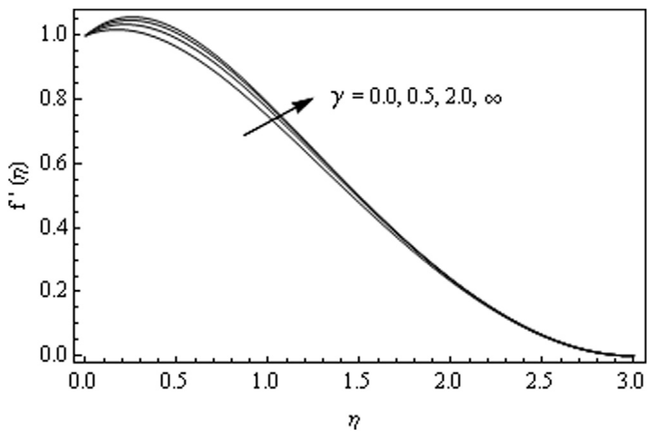

Fig. 4. Velocity profiles for various values of γ.

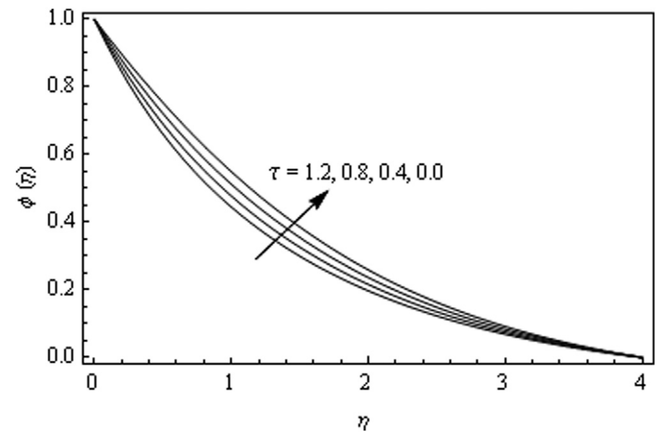

Fig. 7. Concentration profiles for various values of τ.

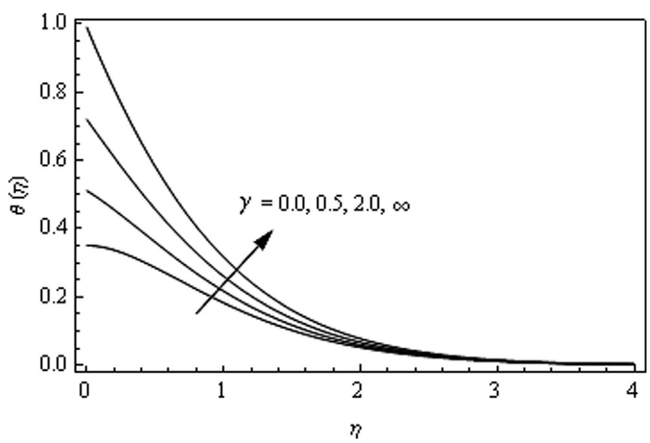

Fig. 5. Temperature profiles for various values of γ.

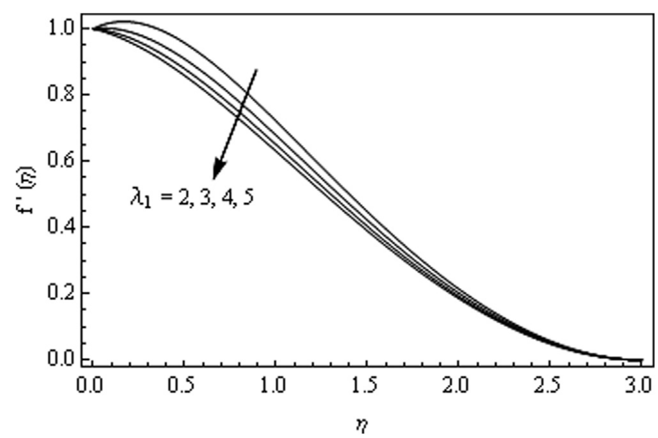

Fig. 8. Velocity profiles for various values of λ_1.

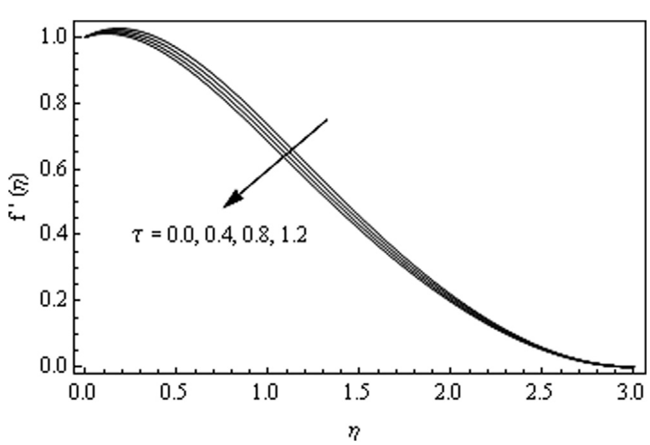

Fig. 6. Velocity profiles for various values of τ.

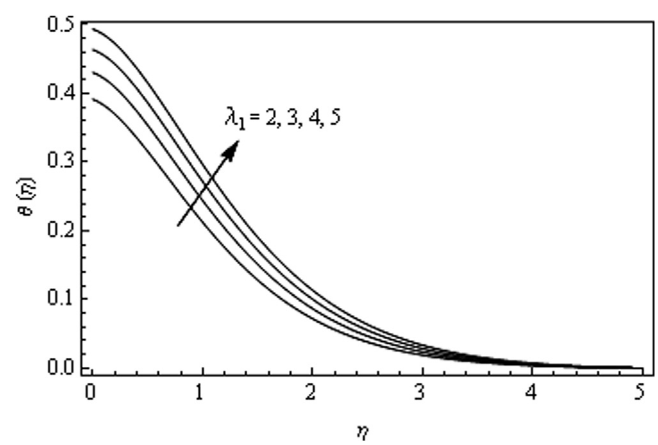

Fig. 9. Temperature profiles for various values of λ_1.

with increasing thermal radiation parameter *Nr*. Thus, by escalating *Nr*, thermal boundary layer thickness enhances. It can be seen from Table 2 that the magnitude of the reduced skin friction coefficient decreases with increase in the radiation parameter *Nr* whereas the thermal radiation increases the rate of heat transfer.

The effect of surface convection parameter γ on the stream wise velocity component is shown in Fig. 4. As the value of γ increases, the flow rate enhances and thereby giving rise to an increase in the velocity profiles as depicted in Fig. 4. The impact of surface convection parameter γ on fluid temperature in presence of thermal radiation is demonstrated in Fig. 5. It is

Table 2
Effects of various parameters on C_f^*, Nu^*, Sh^*.

Nr	γ	τ	λ_1	C_f^*	Nu^*	Sh^*
0.0	0.1	0.3	2.0	−5.62433	0.0601334	0.735132
0.4				−5.49046	0.0852837	–
0.8				−5.41339	0.1110960	–
0.2	0.0			−5.62588	0.00000	0.738316
	0.5			−5.31722	0.313210	–
	2.0			−4.92010	0.705406	–
	∞			−4.40139	1.12814	–
	0.1	0.0		−5.53923	0.0789732	0.735515
		0.6		−5.56781	–	0.746494
		1.2		−5.59621	–	0.767568
		0.3	2.0	−5.55354	0.0789168	0.735831
			4.0	−9.14144	0.0662915	–
			6.0	−12.61510	0.0547938	–

observed from the figure that the fluid temperature increases with increase in γ in the boundary layer region. For large values of γ i.e. $\gamma \to \infty$, the solution reduces to the solution for constant surface temperature. From the boundary condition (14), it can be seen that $\theta(0)=1$ as $\gamma \to \infty$ which sustenance the numerical results obtained in the present study. From Table 2, it is observed that the heat transfer rate at the plate increases with increasing the values of γ whereas the effect is opposite for the wall shear stress (in magnitude) at the plate i.e. the reduced skin friction coefficient (in magnitude) decreases with increasing the surface convection parameter.

Figs. 6 and 7 illustrate the variation of the velocity distribution and temperature distribution for various values of thermophoretic parameter τ. It is observed from Fig. 7 that the fluid velocity decreases with increase in the thermophoretic parameter and so the momentum boundary layer thickness decreases. In the boundary layer region, the concentration of the fluid decreases with increasing the values of thermophoretic parameter τ as presented in Fig. 7. So, thermophoretic parameter is expected to alter the concentration boundary layer. It is found from Table 2 that an increase in τ leads to an increase in both the values of the wall shear stress (in magnitude) and Sherwood number.

The effect of second grade parameter λ_1 on the fluid velocity and temperature distribution is illustrated in Figs. 8 and 9. One may see from Fig. 8 that the velocity component across the boundary layer reduces with an increase in the second grade parameter and also decreases asymptotically to zero at the edge of the hydrodynamic boundary layer. From Fig. 9 it is observed that with increase in the second grade parameter, the temperature profiles increase and hence thickness of thermal boundary layer increases. From Table 2, it is observed that with increasing the values of λ_1, the reduced skin friction coefficient (in absolute sense) increases whereas the reduced Nusselt number diminishes.

5. Conclusions

The effect of thermal radiation on MHD boundary layer flow of a second grade fluid past a stretching sheet with convective surface heat flux in the presence of thermophoresis have been studied in the present study. The governing equations are transformed into a system of ordinary differential equations by applying suitable similarity transformations and then solved numerically. The results are presented through graphs and tables to illustrate the details of the flow characteristics and their dependence on material parameters. The conclusions can be made from the present work as follows:

(1) The fluid velocity in the boundary layer region increases for increasing the values of thermal radiation parameter and surface convection parameter but the effect is reverse for thermophoretic parameter and second grade parameter.
(2) The temperature profile enhances with increase in the thermal radiation parameter, second grade parameter and surface convection parameter.
(3) The chemical species concentration decreases in presence of thermophoresis. Consequently, the rate of mass transfer increases as thermophoretic parameter
(4) The skin friction coefficient (in magnitude) decreases with increase of thermal radiation parameter and surface convection parameter but effect is reverse for thermophoretic parameter and second grade parameter.
(5) The rate of heat transfer increases for increasing the values of the surface convection parameter and thermal radiation parameter while it decreases with increase in the values of second grade parameter.

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