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Analysis of Two-tier Supply Chains with Multiplicative Random Yields

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ABSTRACT
We consider a two-tier supply chain with multiplicative random yield. We focus on the supply chain performance with respect to the control scheme of determining the production lot size. The profit loss due to distributed control is analyzed to give an insight for devising efficient supply contracts.

Keywords: Single-Period Inventory Control, Random Yield, Supply Chain

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1. INTRODUCTION

The newsvendor model (see the survey by Khouja, 1999) is used to analyze the single-period inventory control problem with demand uncertainty and is further extended to analyze multi-echelon supply chains. However, many researches usually focus on the demand-side uncertainty, ignoring the possibility of the supplier being unable to complete the order. There are many cases where the uncertainty in the supply side can be significant. The uncertainty in the supply process may be due to randomness in the production yield (see Yano and Lee, 1995) and/or randomness in the production process (see Ciarallo *et al*., 1994).

In this paper, we consider a two-tier supply chain composed of a single producer (supplier) and a single buyer (distributor or retailer). We assume that the producer's yield is random but the demand is fixed and known in advance. As Keren (2009) noted, this type of supply chain is common in agriculture, the chemical industry producing tailor-made chemical, and the electronic industry producing specialized processors. We model the supply-side uncertainty as multiplicative random yield, where the variance of the yield is proportional to the production lot size. An example of the multiplicative random yield in agriculture is the rain at harvest times (Newbery and Stiglitz, 1981).

Recently, Keren (2009) addressed such a two-tier supply chain with random yield (in particular, uniform yield) and presented an analytic solution of the optimal production lot size and the buyer's order quantity. Li *et al*. (2012) extended the results and presented a more comprehensive analysis assuming that the yield is a general continuous random variable. The results given in these papers are general but mathematically complicated, making it difficult to use them when devising efficient supply contracts. In this paper, we focus on the supply chain performance with respect to the inventory control schemes used: centralized control vs. distributed control. Assuming general random yield, we present the result on the profit loss due to distributed control in a unified and more thorough way. The results are mathematically simple and easy to understand, so they can give managerial insights when devising supply contracts to improve supply chain performance. Moreover, the results can be extended to cover discrete random yields and additional results on the supply chain performance are given.

There is a large body of literature dealing with single-stage inventory control under random production yield, including Shih (1980), Noori and Keller (1986), Herhardt and Taube (1987), Gerchak *et al*. (1988), Henig and Gerchak (1990), Anupindi and Akella (1993), Parlar and Wang (1993), Erdem and Ozekici (2002), Inderfurth

(2004), Gupta and Cooper (2005), and Rekik *et al*. (2007). In addition, for a review on supply contracts see Cachon (2003).

The rest of the paper is organized as follows. In section 2, we give models of optimally deciding the producer's lot size and the buyer's order quantity. Section 3 analyzes the problem when the producer and the buyer are integrated. Section 4 deals with the distributed control case where the producer and the buyer optimize their profit independently of each other. The comparison between the centralized and distributed control cases is presented in Section 5. Finally, concluding remarks are given in Section 6.

2. MODEL DESCRIPTION

Consider a two-tier supply chain composed of a single producer and a single buyer. The buyer knows the exact market demand and places an order with the producer. The producer's yield is random multiplicative with respect to the production lot size. Upon receiving the buyer's order, the producer determines his production lot size and starts production. When the production is completed, the units are delivered to the buyer, which are used to meet the market demand. Note that due to the randomness of the production yield, the number of units delivered may be less than the buyer's order size.

To derive mathematical models of the related problems, we need some notations. Let d be the market demand. The numbers c, w, and r denote unit production cost, wholesale price, and retail price, respectively, where $0<c<w<r$. There can be other parameters influencing the profits such as holding and penalty cost. Theses parameters are ignored to simplify the analysis and to clarify the results. However, most of the results can be extended to cover such cases. The production yield is xU, where x is the production lot size and U is a random variable with its support [0, 1]. Let F, f, and μ denote its distribution function, continuous density function, and mean, respectively. To avoid the trivial solution of not producing anything, it is assumed $w\mu > c$. The buyer's order quantity is denoted as y.

When the buyer's order size is y, the producer's expected profit is

$$\Pi_P(x;\, y) = w\mathbf{E}\min(y,\, xU) - cx. \tag{1}$$

It can be easily seen that for $y>0$, $\Pi_P(x;\, y) = y\Pi_P(x/y; 1)$. Also note that $\mathbf{E}\min(1,\, xU) = 1-\mathbf{E}(1-xU)_+$, where $a_+ = \max(0,\, a)$, for any real number a. Therefore, the producer's problem of determining the lot size to maximize his expected profit is

$$\max_{x\ge 0}\{\Pi_P(x) = w - cx - w\mathbf{E}(1-xU)_+\}. \tag{2}$$

The function $\Pi_P(x)$ is concave and so, there is an optimal solution to the above problem (2). Let s be its optimal solution. Then when the buyer's order size is y, the optimal production lot size is sy and the maximum expected profit is $y\Pi_P^*$, where $\Pi_P^* = \Pi_P(s)$.

Now consider the buyer's problem of determining his order size. Let us define a random variable V as $\min(1,\, sU)$, which is the number of units received when the buyer's order size is 1. Note that when the buyer orders y units from the producer, the producer's lot size is sy. So the number of units received is $\min(y,\, ysU) = yV$. If the market demand is d, the buyer's expected profit is

$$\Pi_B(y;\, d) = r\mathbf{E}\min(d,\, yV) - wy\mathbf{E}V. \tag{3}$$

Also in this case, we have $\Pi_B(y;\, d) = d\Pi_B(y/d; 1)$. So in the following, without loss of generality, we assume the market demand d is 1. Then the buyer's problem of deciding his order size is

$$\max_{y\ge 0}\{\Pi_B(y) = r - wy\mathbf{E}V - r\mathbf{E}(1-yV)_+\}. \tag{4}$$

It is interesting to note that the buyer's problem (4) is similar to the producer's problem (2). In fact, the buyer's problem (4) corresponds to a producer's problem with production yield V, production cost $w\mathbf{E}V$, and wholesale price r. Since $\Pi_B(y)$ is concave in y, there exists an optimal order size that is denoted as λ. Also let the buyer's maximum expected profit be $\Pi_B^* = \Pi_B(\lambda)$.

From the above, it is easy to see that the producer's optimal lot size is $s\lambda$, where s is determined by the producer while λ is determined by the buyer. The supply chain's total profit Π_{SC}^* is defined as the sum of the producer's maximum expected profit and buyer's one, that is, $\Pi_{SC}^* = \lambda\Pi_P^* + \Pi_B^*$. It is the expected profit resulted when the decisions are made optimally but independently. This corresponds to *distributed control*. The other case is *integrated* (*centralized*) *control*, where the production lot size is determined to maximize the supply chain's total profit. For instance, this is the case when the producer and the buyer belong to the same company. In the next sections, we will analyze the two cases and make a comparison between them.

3. CENTRALIZED CONTROL

When the producer and the buyer belong to the same company, the wholesale price is meaningless and the problem becomes

$$\max_{x\ge 0}\{\Pi_I(x) \equiv r - cx - r\mathbf{E}(1-xU)_+\}. \tag{5}$$

Since $\mathbf{E}(1-xU)_+ = \int_0^{1/x}(1-x\xi)\, f(\xi)\, d\xi$,

$$\frac{d}{dx}\Pi_I(x) = -c + r\int_0^{1/x}\xi\, f(\xi)\, d\xi, \tag{6}$$

which is clearly decreasing in *x*. Since $\lim_{x\downarrow 1}\frac{d}{dx}\Pi_I(x)=\frac{d}{dx}\Pi_I(1)=-c+r\mu>0$ (note $c<w\mu$ and $w<r$) and $\lim_{x\to\infty}\frac{d}{dx}\Pi_I(x)=-c<0$, there is unique optimal lot size $S>1$. The optimal lot size *S* is characterized in the following Proposition 1.

Proposition 1: In the centralized control, the unique optimal lot size is $S>1$ which satisfies

$$\int_0^{1/S}\xi f(\xi)d\xi=c/r. \tag{7}$$

The corresponding maximum expected profit is stated in the following Proposition 2.

Proposition 2: In the centralized control, the maximum expected profit is

$$\Pi_I^*=\Pi_I(S)=r[1-F(1/S)]. \tag{8}$$

Proof: By (7), $\mathbf{E}(1-SU)_+=\int_0^{1/S}(1-S\xi)f(\xi)d\xi=F(1/S)-(c/r)S$, which is substituted into $\Pi_I(S)=r-cS-r\mathbf{E}(1--SU)_+$, yielding the above result (8). □

4. DISTRIBUTED CONTROL

4.1 Producer's Problem

In the distributed control, the producer's problem (2) is essentially the same as the problem (5) with *r* replaced by *w*. When the buyer's order size is *y*, the producer's optimal lot size and its maximum expected profit can be summarized as follows.

Proposition 3: In the distributed control, let *y* be the buyer's order quantity. Then the optimal lot size is *sy* where $s>1$ satisfies

$$\int_0^{1/s}\xi f(\xi)d\xi=c/w. \tag{9}$$

In addition, the producer's maximum expected pro-fit is $y\Pi_P^*$, where

$$\Pi_P^*=w[1-F(1/s)]. \tag{10}$$

4.2 Buyer's Problem

First, we will show that the buyer's optimal order size λ cannot be less than 1. To show that, suppose $y<1$. Then since $V=\min(1,sU)\le 1$, $\mathbf{E}(1-yV)_+=\mathbf{E}(1-yV)$. Therefore, the buyer's expected profit is $\Pi_B(y)=(r-w)y\mathbf{E}V$, which is increasing in *y*. So we can conclude that $\lambda\ge 1$. By using this result, we can characterize the buyer's optimal order size.

Proposition 4: Let *s* be chosen by the producer's optimality condition (9). In the distributed control, the buyer's optimal order size is $\lambda=1$ if

$$\int_0^{1/s}\xi f(\xi)d\xi=c/w\le(w/rs)\mathbf{E}V. \tag{11}$$

Otherwise, the optimal order size is $\lambda>1$ which satisfies

$$\int_0^{1/s\lambda}\xi f(\xi)d\xi=(w/rs)\mathbf{E}V=(c/r)+(w/r)(1/s)(1-F(1/s)). \tag{12}$$

Proof: Note that by using the producer's optimality condition (9), we have

$$\mathbf{E}V=\mathbf{E}\min(1,sU)=s\int_0^{1/s}\xi f(\xi)d\xi+(1-F(1/s))=s(c/w)+(1-F(1/s)).$$

For $y\ge 1$, $\mathbf{E}(1-yV)_+=\mathbf{E}(1-syU)_+=\int_0^{1/sy}(1-sy\xi)f(\xi)d\xi$. So

$$\frac{d}{dy}\Pi_B(y)=-w\mathbf{E}V+rs\int_0^{1/sy}\xi f(\xi)d\xi, \tag{13}$$

which is clearly decreasing in *y*. Hence if $\lim_{y\downarrow 1}\frac{d}{dy}\Pi_B(y)=-w\mathbf{E}V+rs\int_0^{1/s}\xi f(\xi)d\xi\le 0$, then the optimal order size is $\lambda=1$. Otherwise, since $\lim_{y\to\infty}\frac{d}{dx}\Pi_B(y)=-w\mathbf{E}V<0$, there is unique optimal order size $\lambda>1$ which satisfies (12). □

The buyer's maximum expected profit can also be characterized in a similar way as the producer's maximum expected profit.

Proposition 5. In the distributed control, the buyer's maximum expected profit is

$\Pi_B{}^*=(r-w)\mathbf{E}V$, if $\lambda=1$.
Otherwise if $\lambda>1$,
$\Pi_B{}^*=r(1-F(1/s\lambda))$.

5. COMPARISON BETWEEN THE CENTRALIZED CONTROL AND DISTRIBUTED CONTROL

In this section, first, we will show that the supply chain's profit under distributed control cannot exceed that under centralized control. The following proposition states the result.

Proposition 6: The supply chain's maximum expected profit under distributed control cannot exceed that under centralized control.

Proof: In the distributed control, when the buyer's order size is *y*, the buyer's expected profit is $\Pi_B(y) = r - wy\mathbf{E}V - r\mathbf{E}(1-yV)_+$. Since $y \geq 1$, $\mathbf{E}(1-yV)_+ = \mathbf{E}(1-syU)_+$. Also in the proof of the Proposition 4, $\mathbf{E}V = s(c/w) + (1-F(1/2))$. Therefore, $wy\mathbf{E}V = csy + y\Pi_P^*$ (see (10)). Using these results, we can get $\Pi_B(y) = r - csy - y\Pi_P^* - r\mathbf{E}(1-syU)_+$. Hence $y\Pi_P^* + \Pi_B(y) = r - csy - r\mathbf{E}(1-syU)_+ = \Pi_I(sy)$, for all $y \geq 1$ (see (5)). Therefore, we have $\Pi_{SC}^* = \lambda\Pi_P^* + \Pi_B^* = \Pi_I(s\lambda) \leq \Pi_I(S) = \Pi_I^*$. □

Besides the expected profit, the in-stock ratio (one of the most widely used service level in the inventory control literature) in the centralized control is always better than that in the distributed control.

Proposition 7: The in-stock ratio (the probability of meeting all the demand) in the centralized control is always better than that in the distributed control.

Proof. In the distributed control, the in-stock ratio is $\Pr[s\lambda U > 1] = 1 - F(1/s\lambda)$ and that in the centralized control is $\Pr[SU > 1] = 1 - F(1/S)$. First consider the case $\lambda = 1$. Then since $\int_0^{1/S} \xi f(\xi) d\xi = c/r < c/w = \int_0^{1/s} \xi f(\xi) d\xi$, we have $1/S < 1/s$, so the result follows. Now suppose $\lambda > 1$. Then by (12), $\int_0^{1/S} \xi f(\xi) d\xi = c/r \leq \int_0^{1/s\lambda} \xi f(\xi) d\xi$, and so $1/S < 1/s\lambda$. This completes the proof. □

6. CONCLUDING REMARKS

This paper gives a unified analysis of two-tier supply chains with certain demand but with random yield. After analyzing the optimality conditions of the production lot size and the buyer's order quantity, the profit loss due to distributed control is analyzed. The supply chain's profit as well as the in-stock probability is shown to deteriorate when distributed control is used.

An interesting issue that is not addressed in this paper is the information asymmetry. Note that the producer is always benefited from the increase in the buyer's order quantity. Moreover, the producer can reduce the production cost by improving the yield quality (for example, by reducing the variance of the yield). Therefore, the producer has an incentive to hide the detailed information on the production yield characteristics from the buyer to take advantage of the increased order quantity. In this respect, further research is required to find out an effective mechanism to enable information sharing among the supply chain participants.

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