한국경영과학회지
제41권 제2호
2016년 5월
http://dx.doi.org/10.7737/JKORMS.2016.41.2.001

두 대체품에 대한 수익관리 모형 연구*

김 상 원†
울산대학교 경영대학 경영학부

Yield Management Models for Two Substitutable Products

Sang-Won Kim
College of Business Administration, University of Ulsan

■ Abstract ■

Yield management, which originated from the U.S. service industry, uses pricing techniques and information systems to make demand management decisions. Demand uncertainty is an important factor in the area of demand management. A key strategy to reduce the effects of demand uncertainty is substitution. The most generally known type of substitution is inventory-driven substitution, in which consumers substitute an out-of-stock product by buying a similar or other type of product. Another type of substitution is the price-driven substitution, which occurs as a result of price changes. In this research, we consider two market segments that have unique perishable products. We develop yield management optimization models with stochastic demand based on the newsvendor model where inventory-driven and price-driven substitutions are allowed between products in the two market segments. The most significant contribution of this research is that it develops analytical procedures to determine optimal solutions and considers both types of substitution. We also provide detailed theoretical analysis and numerical examples.

Keywords : Pricing, Substitution, Service Industry, Yield Management

논문접수일 : 2015년 11월 17일 논문게재확정일 : 2015년 12월 01일
* 이 논문은 2016년 울산대학교 연구비지원에 의하여 연구되었음.
† 교신저자, studyksw@ulsan.ac.kr

1. Introduction

Yield management (YM) employs the application of "information systems and pricing strategies to allocate the right capacity to the right customer at the right place at the right time." YM set prices for the forecasted demands, and price sensitive consumers will buy the same product at different prices at off-peak or peak times [5]; therefore, it is very sensitive to demand uncertainty. Yield management techniques have been developed for service industries for a long time to overcome demand uncertainty. Perishable products are common object in service industry, and food stores are good examples for perishability. Managers of a food store (for example, coffee & donut stores) decide the number of donuts to order or bake every morning. Also, perishable products in retailing such as fashion goods have multiple demand classes. In retailing area, product variety management has grown significantly. Many researchers pointed out that product variety increases product operations costs because of demand uncertainty. Many strategies to reduce the impact of demand uncertainty have been developed. One of the most important well-known strategies is substitution. Different variations of the same product (e.g. different color, flavors etc.) are good substitutable product each other. Substitution plays a key role in determining the exact demands under demand uncertainty.

Newsvendor problems have been used widespread for optimization of perishable products in service industry. The classical newsvendor problem focuses only on deciding the optimal ordering level without considering demand movement to maximize his expected profit [24, 25]. Customers will purchase initially requested products as long as the price compares favorably to the prices of similar substitutable products; however, if the price differences between the similar products becomes large, demand movement occurs. The cross elasticity of demand measures the responsiveness of the quantity demanded of a product to a change in the price of another product. Setting a lower price for a product may increase potential customers and setting a higher price for the same product increase profit margin but may lead movement of potential customers to buying different products. This type of demand movement is called price-driven substitution. The most generally known type of demand movement, which is called inventory-driven substitution, occurs when consumers substitute for a product that is out of stock by buying a similar substitutable product [1, 2, 5, 6, 11, 15, 17, 19, 20, 7].

YM is concerned with demand-management decisions and the methodology required making them, and it uses the three basic demand-management decisions : structural decisions, pricing decisions and quantity decisions. Firms have to adjust price and capacity level decisions by advertising prices or capacity decisions in advance. If the decisions affecting demand aspects (e.g. types of customers, different type of products to sell, time, purchase behavior, etc.) are independent the decision making problem is very simple. But demand-management is far more difficult. Demand decisions for different products and customers are closely linked to the information by the firm. Customer purchase behavior depends on customers' heterogeneity. Customers in the service industry certainly exhibit these characteristics. For example, a customer who prefers a higher quality product may choose to buy a lesser quality product because of its lower price. Customer buying behavior is also dependent on other factors such as stock-outs. Demand functions might consist of buyers' different behavior components. The substitution effect for a product with temporary stock-outs is one of the elements that affect the demand function significantly. Customer response to the stock-outs is divided among these possibilities: substitute the item they sought, delay the purchase, or leave without purchasing. Stock-outs may negatively affect the image of the brand or the store so that understanding customers' response to stock-outs will lead to better policies for pricing and production decisions. This kind of substitution can also be found in the retail market as well as airlines. Business class seats on airplanes can be substitutes for economy seats. Customers who initially intend to buy economy seats but cannot buy them may buy business seats. Likewise, low-capacity microchip customers may buy a high-capacity microchip if they cannot buy a low-capacity microchip that is cheaper than the high-capacity one.

Most research papers in operations research and management science, operations management have considered inventory-driven substitution models and rarely considered price-driven substitution. However, understanding the potential substitution effects on pricing and production decisions is very important in making production plan. Inventory-driven substitution research has been studied by many authors. Examples of this research include Karaesmen and van Ryzin [10], Rao et al. [17], Dong et al. [3], Dutta, and Chakraborty [4], Shumsky, and Zhang [19], Wang and Kapuscinski [23], Zhao et al. [27], Karakul and Chan [9]. Kuyumcu and Popescu [14] studied joint pricing and inventory control by considering deterministic optimization models for multiple substitutable products, and they showed that the optimization problem could be reduced to a pure pricing problem. Karakul and Chan [3] considered stochastic optimization problems, which are a joint pricing and inventory decision, for an existing product and a new improved production decisions. The authors developed an in-depth mathematical procedure for finding optimal solutions for the stochastic problem with inventory-driven substitution. Research on optimization models with price-driven substitution has been done by a few researchers e.g. Kocabiykoglu and Popescu [13], Kim [11], Lus and Muriel [15], Tang and Yin [22] and Yu [26]. Lee and Kim [7] developed an optimal pricing and production decision model with price-driven substitution alone, where a part of the demand lost in one market moves to the other market. Lus and Muriel [15] considered deterministic pricing/quantity decision problems with two substitutable products and compared two alternative measures of product substitutability for linear demand functions. Tang and Yin [22] explored joint pricing and ordering policies for two substitutable products with deterministic price-dependent demand. They considered price-driven substitution alone. Yu [26] studied joint pricing and inventory decisions with price-driven substitution for two substitutable products, where there was a resource constraint for the two market segments.

In this research, we study demand-management decisions in YM. We develop a single-period model for deciding optimal prices and capacity levels where the firm has flexibility to allow both inventory-driven substitution and price-driven substitution for the two substitutable perishable products. We add both inventory-driven and price-driven substitution into joint pricing and capacity decisions based on the traditional newsvendor model. We develop the solution procedures to gain an understanding of the relationship between substitution and optimal solutions. The main contribution of this research is that we consider both price-driven and inventory-driven substitution at the same time, and we focus on demand substitution as a function of price differences based on an asymmetrical demand function. We consider stochastic optimization models rather than deterministic ones. The impact of substitution on the optimal price, production level and the expected revenue will be illustrated by numerical examination how optimal pricing and production decisions are affected.

2. The Mathematical Model

There are two segmented markets, and demand substitution occurs as a result of not only price changes but inventory stock-outs. The unit price in market A is higher than the unit price in market B. We consider the two segmented markets where the demands are classified by different market segment and price. Initially, the demands are assumed to be independent, identically distributed with known parameters and distributions. Later, demand dependencies are created by substitution. We assume that a price-driven substitution is realized from a market with a high-priced product (A) to a market with a low-priced product (B). We also assume that the demand in each market segment is dependent on the prices of both markets. Under the above assumptions, the revenue manager or supply chain manager has to confront a situation for determining the production quantities and prices. We discuss on how the decisions regarding prices and production levels are made. The following notation will be used throughout the paper :

r_{pd} : ratio of price-driven substitution
ρ : price-driven substitution rate
r_{id} : ratio of inventory-driven substitution
r_a : unit price of product A
r_b : unit price of product B
pc_a : per-unit variable cost of product A
pc_b : per-unit variable cost of product B
d_a : demand for product A
d_b : demand for product B
l_a : capacity level of product A
l_b : capacity level of product B
$f(\cdot)$: p.d.f.
$F(\cdot)$: c.d.f.
$\pi[\cdot]$: total expected profit

First, we consider that consumers substitute for a product that is out of stock by buying a similar substitutable product. If there are customers who cannot buy product A (at the price of r_a) then a fraction of the unsatisfied customers will shift to market B to buy product B at the price of r_b. In market B, the actual demand is the original demand for product B plus the diverted demand from market A.

Therefore, customers of both classes buy product B. The demands are assumed to be realized sequentially over time. The customers who prefer to buy high-priced products arrive before those who prefer low-priced products. This kind of one-way substitution problem is frequently found in the retail industries. For example, in the retail stores that sell perishable items, some customers who prefer to buy the product on the first day when the item is freshest and more expensive while other customers may buy less expensive products on the second day or later. Then, the actual sales at the two markets are given by:

Market A: $\min\{d_a,\ l_a\}$
Market B: $\min\{d_b + \max\{r_{id}(d_a - l_a),\ 0\},\ l_b\}$

We begin with a capacity level decision alone. The firm has to decide the optimal quantity of each product at given prices. We assume that a fixed fraction of customers who cannot buy a high-priced product may buy a low-priced one. But we do not consider price-driven substitution simultaneously. Under these assumptions, the optimization problem to consider is:

$$\begin{aligned}\underset{l_a l_b}{\text{Maximize}}\ \pi[l_a,\ l_b] &= r_a\left\{\int_0^{l_a} d_a f_a(d_a)\mathrm{d}d_a + \int_{l_a}^{\infty} l_a f_a(d_a)\mathrm{d}d_a\right\} \quad \text{(A)}\\ &+ r_b\left\{\begin{aligned}&\int_0^{l_a}\int_0^{l_b} d_b f_b(d_b)\mathrm{d}d_b f_a(d_a)\mathrm{d}d_a\\ &+\int_0^{l_a}\int_{l_b}^{\infty} l_b f_b(d_b)\mathrm{d}d_b f_a(d_a)\mathrm{d}d_a\\ &+\int_0^{l_b}\int_{la}^{\frac{l_b-d_b}{r_{id}}+l_a}(d_b + r_{id}(d_a - l_a))\\ &\quad f_a(d_a)\mathrm{d}d_a f_b(d_b)\mathrm{d}d_b\\ &+\int_0^{l_b}\int_{\frac{l_b-d_b}{r_{id}}+l_a}^{\infty} l_b f_a(d_a)\mathrm{d}d_a f_b(d_b)\mathrm{d}d_b\\ &+\int_{l_b}^{\infty}\int_{l_a}^{\infty} l_b f_a(d_a)\mathrm{d}d_a f_b(d_b)\mathrm{d}d_b\end{aligned}\right\}\\ &- pc_a l_a - pc_b l_b\end{aligned}$$

The expected profit function (A) is jointly concave so that there is a unique optimal solution: a unique maximum of (A) over l_a and l_b for $r_a > r_b$. The expected profit function is concave in l_a for a given l_b, and is concave in l_b for a given l_a. The profit function is also jointly concave in l_a and l_b. The first derivatives with respect to l_a and l_b are as follows:

$$\begin{aligned}\frac{\partial\pi[l_a,\ l_b]}{\partial l_a} &= r_a\{1 - F_a(l_a)\} - r_{id} r_b \int_0^{l_b}\left\{F_a\left(\frac{l_b - d_b}{r_{id}} + l_a\right) - F_a(l_a)\right\}\\ &\quad f_b(d_b)\mathrm{d}d_b - pc_a \qquad (1)\end{aligned}$$

$$\begin{aligned}\frac{\partial\pi[l_a,\ l_b]}{\partial l_b} &= r_b\left\{1 - F_b(l_b) + \int_0^{l_b}\left\{1 - F_a\left(\frac{l_b - d_b}{r_{id}} + l_a\right)\right\}\right.\\ &\quad \left. f_b(d_b)\mathrm{d}d_b\right\} - pc_b \qquad (2)\end{aligned}$$

The sufficient conditions for the existence of a unique maximum are satisfied as follows, where *He* denotes Hessian:

$$\begin{aligned}He_{11} &= -r_a f_a(l_a) - r_{id} r_b \int_0^{l_b}\left\{f_a\left(\frac{l_b - d_b}{r_{id}} + l_a\right) - f_a(l_a)\right\}\\ &\quad f_b(d_b)\mathrm{d}d_b < 0\\ He_{22} &= -r_b\left\{f_b(l_b)F_a(l_a) + \int_0^{l_b}\frac{1}{r_{id}} f_a\left(\frac{l_b - d_b}{r_{id}} + l_a\right) f_b(d_b)\mathrm{d}d_b\right\} < 0\\ He_{11}He_{22} - H_{12}^2 &= \left\{-r_a f_a(l_a) - r_{id} r_b \int_0^{lb}\right.\\ &\quad \left.\left\{f_a\left[\frac{l_b - d_b}{r_{id}} + l_a\right] - f_a(l_a)\right\} f_b(d_b)\mathrm{d}d_b\right\}\\ &\quad \times\left\{-r_b\left\{f_b(l_b)F_a(l_a) + \int_0^{l_b}\frac{1}{r_{id}}\right.\right.\\ &\quad \left.\left. f_a\left[\frac{l_b - d_b}{r_{id}} + l_b\right] f_b(d_b)\mathrm{d}d_b\right\}\right\}\\ &\quad - \left(r_b \int_0^{l_b}\left\{f_a\left[\frac{l_b - d_b}{r_{id}} + l_b\right]\right\} f_b(d_b)\mathrm{d}d_b\right)^2\\ &\quad > \left\{-r_a f_a(l_a) + r_{id} r_b \int_0^{l_b} f_a(l_a) f_b(d_b)\mathrm{d}d_b\right\}\\ &\quad \left\{-r_b \int_0^{l_b}\frac{1}{r_{id}} f_a\left[\frac{l_b - d_b}{r_{id}} + l_b\right] f_b(d_b)\mathrm{d}d_b\right\} > 0.\end{aligned}$$

After simplification from (1) and (2) we have the optimal capacity levels, l_a^*, and l_b^* :

$$F_a(l_a^*) = \frac{r_a - pc_a - r_{id}(r_b - pc_b)}{r_a - r_{id} r_b F_b(l_b^*)}$$

$$F_b(l_b^*) = \frac{r_b - pc_b}{r_b F_a(l_a^*)} - \frac{r_a(1 - F_a(l_a^*)) - pc_a}{r_{id} r_b F_a(l_a^*)}$$

From above, if $r_a - pc_a - r_{id}(r_b - pc_b) < 0$, then (1) is always negative and $l_a^* = 0$. If $r_{id} = 1.0$, then it is not beneficial to manufacture product A when the unit profit of product A is less than the unit profit of product B. If the unit profit of product A is sufficiently smaller than that of product B where $0.0 < r_{pd} < 1.0$ and $r_a - pc_a < r_{id}(r_b - pc_b)$; it is not beneficial to manufacture product A. Similarly, when $r_{id}(r_b - pc_b) > (r_a - pc_a) + \alpha$, that is, if the unit profit of product A is sufficiently smaller than that of product B, then (2) is always positive and the optimal quantity of product B is arbitrarily large. Considering (1) and (2), the first-order condition has an intuitive interpretation. Without the second term of (1) the optimal solution is equivalent to the single-period newsvendor problem with zero holding and shortage costs. The optimal quantity of product A is always less than the optimal solution of the equivalent newsvendor problem. If substitution ratio is 0, then the problem (A) becomes the newsvendor problem with zero holding and shortage costs. The optimal quantity to manufacture for market B is always greater than that of the equivalent newsvendor problem because the substituted demand flows from market A to market B. If there is any substituted demand between the markets, then the optimal solutions have to be adjusted from those of the newsvendor problems. As the substitution rate increases, the optimal quantity of product A is decreased while the optimal quantity of product B is increased. The price of product A affects the decision of the optimal level of product B and the price of product B also affects the decision of the optimal quantity of product A. For the reverse case of $r_a < r_b$, where the customers who prefer low-priced products arrive before those who prefer high-priced products, we show that this case has the same form of unique optimal solutions as that for the case when the customers who prefer high-priced products arrive before those who prefer low-priced products as long as $r_a > r_{id} r_b$. For $r_a < r_b$, there exists a unique maximum of (A) over l_a and l_b. The expected profit function is concave in l_a for a given l_b, and is concave in l_b for a given l_a. The expected profit function is jointly concave in l_a and l_b if $r_a > r_{id} r_b$ (See the proof of proposition 1; if $r_a > r_{id} r_b$, then $He_{11} He_{22} - He_{12}^2 > 0$).

We now consider price-driven substitution as well as inventory-driven substitution. We consider the optimization model by examining the optimal pricing decision under stochastic demand below. We present the linear stochastic demand model used in this research as:

$$d_a(r_a, r_b, \epsilon_a) = v_a - \omega_a r_a - r_{pd}(r_a - r_b) + \epsilon_a$$
$$d_b(r_a, r_b, \epsilon_b) = v_b - \omega_b r_b + \rho r_{pd}(r_a - r_b) + \epsilon_b$$

Consider a firm that manufactures and sells two similar products which may substitute each other there are two segmented markets and demand substitution occurs as a result of not only inventory stock-outs but price differences as well. The error terms ϵ_a, ϵ_b are assumed to be independent each other and price-independent: $E[\epsilon] = 0$, $\mathrm{var}[\epsilon] = \sigma^2$ r_{pd} is a factor of price-driven substitution and $r_{pd} \geq 0$. We assume that the parameters of the demands are carefully chosen so that the probability of negative demand is entirely zero and can be ignored. Note that ρ is the price-driven substitution rate and $0.0 \leq \rho \leq 1.0$. We substitute $d_a(r_a, r_b, \epsilon_a) = \mu_a(r_a, r_b) + \epsilon_a$, $d_b(r_a, r_b, \epsilon_b) = \mu_b(r_a, r_b) + \epsilon_b$ and define new variables: $sf_a = l_a - \mu_a(r_a, r_b)$ and $sf_b = l_b - \mu_b(r_a, r_b)$. Let error terms ϵ_i have p.d.f. of $f_{\epsilon_i}(\cdot)$ and c.d.f. of $F_{\epsilon_i}(\cdot)$ for $i = a, b$. The contribution maximization problem (A) can be transformed as follows:

$$\begin{aligned}
\underset{r_a, r_b}{\text{Miximize}}\ \pi[r_a, r_b] = r_a\Big\{ & \int_0^{sf_a} (\mu_0(r_a, r_b) + \epsilon_a) f_{\epsilon_a}(\epsilon_a) d\epsilon_a \\
& + \int_{sf_a}^{\infty} (\mu_a(r_a, r_b) + sf_a) f_{\epsilon_a}(\epsilon_a) d\epsilon_a \Big\}
\end{aligned}$$

$$+ r_b \left\{ \begin{array}{l}
F_{\epsilon_a}(sf_a) \left[\begin{array}{l} \int_0^{sf_b} (\mu_b(r_a, r_b) + \epsilon_b) f_{\epsilon_b}(\epsilon_b) d\epsilon_b \\ + \int_{sf_b}^{\infty} (\mu_b(r_a, r_b) + sf_b) f_{\epsilon_b}(\epsilon_b) d\epsilon_b \end{array} \right] \\
+ \int_0^{sf_b} \int_{sf_a}^{\frac{sf_b - \epsilon_b}{sf_a} + sf_a} (\mu_b(r_a, r_b) + \epsilon_b + r_{id}(\epsilon_a - sf_a)) \\
f_{\epsilon_a}(\epsilon_a) d\epsilon_a f_{\epsilon_b}(\epsilon_b) d\epsilon_b \\
+ \int_0^{sf_b} \int_{\frac{sf_b - \epsilon_b}{r_{id}} + sf_a}^{\infty} (\mu_b(r_a, r_b) + sf_b) \\
f_{\epsilon_a}(\epsilon_a) d\epsilon_a f_{\epsilon_b}(\epsilon_b) d\epsilon_b \\
+ \int_{sf_b}^{\infty} \int_{s_a}^{\infty} (\mu_b(r_a, r_b) + sf_b) f_{\epsilon_a}(\epsilon_a) d\epsilon_a f_{\epsilon_b}(\epsilon_b) d\epsilon_b
\end{array} \right\} \text{(B)}$$

$$- pc_a(\mu_a(r_a, r_b) + sf_a) - pc_b(\mu_b(r_a, r_b) + sf_b)$$

where:

$$\begin{aligned}
\Omega_a(sf_a) &= \int_{-\infty}^{sf_a} (sf_a - x) f_{\epsilon_a}(x) dx,\ \Upsilon(sf_a) \\
&= \int_{sf_a}^{\infty} (x - sf_a) f_{\epsilon_a}(x) dx \\
\Omega_a(sf_a) &= \int_{-\infty}^{z_b} (sf_b - x) f_{\epsilon_a}(x) dx,\ \Upsilon(sf_a) \\
&= \int_{sf_a}^{\infty} (x - sf_a) f_{\epsilon_a}(x) dx
\end{aligned}$$

and $\mathrm{T}(r_b \mid sf_a, sf_b) =$

$$r_b \left\{ \begin{array}{l}
\int_{-\infty}^{sf_b} \int_{z_a}^{\frac{sf_b - \epsilon_b}{r_{id}} + sf_a} (\epsilon_b + r_{id}(\epsilon_a - sf_a)) f_{\epsilon_a}(x) dx f_{\epsilon_b}(x) dy \\
+ \int_{-\infty}^{sf_b} \int_{\frac{sf_b - \epsilon_b}{r_{id}} sf_b}^{\infty} sf_b f_{\epsilon_a}(x) dx f_{\epsilon_b}(x) dy \\
+ \int_{sf_b}^{\infty} \int_{sf_a}^{\infty} sf_b f_{\epsilon_a}(x) dx f_{\epsilon_b}(x) dy
\end{array} \right\}$$

and

$$\begin{aligned}
&\Xi_a(r_a, r_b) = (r_a - pc_a)\mu_a(r_a, r_b), \\
&\Gamma_a(r_a \mid sf_a) = pc_a \Omega_a(sf_a) + (r_a - pc_a)\Upsilon_a(sf_a) \\
&\Xi_b(r_a, r_b) = (r_b - pc_b)\mu_b(r_a, r_b), \\
&\Gamma_b(r_b \mid sf_a, sf_b) = pc_b \Omega_b(sf_b) + (r_b F_{\epsilon_a}(sf_a) - pc_b)\Upsilon_b(sf_b)
\end{aligned}$$

We finally have a transformed contribution maximization problem for the model (B):

$$\begin{aligned}
\underset{r_a, r_b}{\text{Maximize}}\ \pi[r_a, r_b \mid sf_a, sf_b] = & \ \Xi_a(r_a, r_b) \\
& - \Gamma_a(r_a \mid sf_a) + \Xi_b(r_a, r_b) - \Gamma_b(r_b \mid sf_a, sf_b) \\
& + T(r_b \mid sf_a, sf_b)
\end{aligned} \qquad \text{(C)}$$

We also consider the decision of optimal sf_a, sf_b at a given price based on the above approach:

$$\underset{sf_a, sf_b}{\text{Maximize}}\ \pi[sf_a, sf_b \mid r_a, r_b] \qquad \text{(D)}$$

There exists a unique maximum of the transformed profit function in (C) over $r_a > r_b > 0$ at given sf_a and sf_b, and this is also the case for model (D). Consider the first derivatives with respect to price:

$$\begin{aligned}
\frac{\partial \pi[r_a, r_b]}{\partial r_a} &= 2(\omega_a + r_{pd})(\tilde{r}_a - r_a) - \Upsilon_a(sf_a) \\
\frac{\partial \pi[r_a, r_b]}{\partial r_b} &= 2(\omega_b + \rho r_{pd})(\tilde{r}_b - r_b) - F_{s_a}(sf_a)\Upsilon_b(sf_b) \\
&\quad + \delta(sf_a, sf_b)
\end{aligned}$$

where

$$\tilde{r}_a = \frac{v_a + r_{pd} r_b + (\omega_a + r_{pd}) pc_a + \rho r_{pd}(r_b - pc_b)}{2(\omega_a + r_{pd})},$$

$$\tilde{r}_b = \frac{v_b + \rho r_{pd} r_a + (\omega_b + \rho r_{pd}) pc_b + r_{pd}(r_a - pc_a)}{2(\omega_b + \rho r_{pd})}$$

and

$$\delta(sf_a,\ sf_b) = \left\{ \begin{array}{l} \displaystyle\int_{-\infty}^{sf_b} \int_{z_a}^{\frac{sf_b - \epsilon_b}{r_{id}}} (\epsilon_b + r_{id}(\epsilon_a - sf_a)) f_{\epsilon_a}(x) \mathrm{d}x f_{\epsilon_b}(x) \mathrm{d}y \\ \displaystyle + \int_{-\infty}^{sf_b} \int_{\frac{sf_b - \epsilon_b}{r_{id}} + sf_a}^{\infty} sf_b f_{\epsilon_a}(x) \mathrm{d}x f_{\epsilon_b}(x) \mathrm{d}y \\ \displaystyle + \int_{sf_b}^{\infty} \int_{sf_a}^{\infty} sf_b f_{\epsilon_a}(x) \mathrm{d}x f_{\epsilon_b}(x) \mathrm{d}y \end{array} \right\}$$

To ensure that a unique maximum exists, we show that the sufficient conditions are satisfied. The second derivatives are :

$$\frac{\partial^2 \pi[r_a,\ r_b]}{\partial r_a^2} = -2(\omega_a + r_{pd}) < 0,$$

$$\frac{\partial^2 \pi[r_a,\ r_b]}{\partial r_b^2} = -2(\omega_b + \rho r_{pd}) < 0$$

If $4(\omega_a + r_{pd})(\omega_b + \rho r_{pd}) - (1+\rho)^2 r_{pd}^2 > 0$, then the sufficient conditions for the existence of a unique maximum are satisfied. From the first derivatives, the optimal prices are given by :

$$r_a^* = \tilde{r}_a - \frac{\Upsilon_a(sf_a)}{2(\omega_a + r_{pd})}$$

$$r_b^* = \tilde{r}_b - \frac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b + \rho r_{pd})} + \frac{\delta(sf_a,\ sf_b)}{2(\omega_b + \rho r_{pd})}$$

Eliminating decision variables leads us to :

$$r_a^* = \frac{2(\omega_b + \rho r_{pd})[v_a + (\omega_a + r_{pd})pc_a - \rho r_{pd} pc_b - \Upsilon_a(sf_a)]}{4(\omega_a + r_{pd})(\omega_b + \rho r_{pd}) - r_{pd}^2(1+\rho)^2}$$

$$+ \frac{\begin{array}{c}(1+\rho) r_{pd}[v_b + (\omega_b + \rho r_{pd})pc_b - r_{pd} pc_a \\ - F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b) + \delta(sf_a,\ sf_b)]\end{array}}{4(\omega_a + r_{pd})(\omega_b + \rho r_{pd}) - r_{pd}^2(1+\rho)^2}$$

$$= \bar{r}_a^* + \frac{(1+\rho) r_{pd}[(1 - F_{\epsilon_a}(sf_a))\Upsilon_b(sf_b) + \delta(sf_a,\ sf_b)]}{4(\omega_a + r_{pd})(\omega_b + \rho r_{pd}) - r_{pd}^2(1+\rho)^2}$$

$$r_b^* = \frac{\begin{array}{c}2(\omega_a + r_{pd})[v_b + (\omega_b + \rho r_{pd})pc_b - r_{pd} pc_a \\ - F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b) + \delta(sf_a,\ sf_b)]\end{array}}{4(\omega_b + \rho r_{pd})(\omega_a + r_{pd}) - r_{pd}^2(1+\rho)^2}$$

$$+ \frac{(1+\rho) r_{pd}[v_a + (\omega_a + r_{pd})pc_a - \rho r_{pd} pc_b - \Upsilon_a(sf_a)]}{4(\omega_b + \rho r_{pd})(\omega_a + r_{pd}) - r_{pd}^2(1+\rho)^2}$$

$$= \tilde{r}_b^* + \frac{2(\omega_a + r_{pd})[(1 - F_{\epsilon_a}(sf_a))\Upsilon_b(sf_b) + \delta(sf_a,\ sf_b)]}{4(\omega_b + \rho r_{pd})(\omega_a + r_{pd}) - r_{pd}^2(1+\rho)^2},$$

where $\bar{r}_a^*$ and $\bar{r}_b^*$ are equal to the optimal prices of the stochastically equivalent model which considers only price-driven substitution. The optimal prices are unique solutions for model (C).

The optimal prices are affected by the capacity levels of both products. If we set the capacity level sufficiently high in order that there is no substitution demand as a result of stock-outs, then the contribution maximization problem (B) is identical to that of the stochastic price-driven substitution model. Since $(sf_a,\ sf_b)$ are fixed values in this model, $\Upsilon_a(sf_a)$, $\Upsilon_b(sf_b)$ and $\delta(sf_a,\ sf_b)$ are also fixed values, and $\delta(sf_a,\ sf_b)$ can be either positive or negative. Related to capacity level decisions, sf_a, sf_b provides another interpretation of the production decisions. If sf_a is bigger than the value of ϵ_a, then leftover occurs; if sf_a is smaller than the value of ϵ_a, then shortage occurs. We consider the decision of a set of the so called optimal stocking factors sf_a, sf_b at given fixed prices. The first derivatives with respect to sf_a, sf_b are :

$$\frac{\partial \pi[sf_a,\ sf_b]}{\partial sf_a} = r_a\{1 - F_{\epsilon_a}(sf_a)\}$$

$$- r_{id} r_b \int_{-\infty}^{sf_b} \left\{ F_{\epsilon_a}\left(\frac{sf_b - \epsilon_b}{r_{id}} + sf_a \right) - F_{\epsilon_a}(sf_a) \right\}$$

$$f_{\epsilon_b}(\epsilon_b) \mathrm{d}\epsilon_b - pc_a$$

$$\frac{\partial \pi[sf_a,\ sf_b]}{\partial sf_b} = r_b \left\{ 1 - \int_{-\infty}^{sf_b} F_{\epsilon_a}\left(\frac{sf_b - \epsilon_b}{r_{id}} + sf_a \right) f_{\epsilon_b}(\epsilon_b) \mathrm{d}\epsilon_b \right\} - pc_b$$

The sufficient conditions for the existence of a unique maximum are satisfied :

$$H_{11}=-r_a f_{\epsilon_a}(sf_a)-r_{id}r_b\int_{-\infty}^{sf_b}\left\{f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)-f_{\epsilon_a}(sf_a)\right\}$$

$$f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b<0$$

$$H_{22}=-r_b\left\{F_{\epsilon_a}(sf_a)f_{\epsilon_b}(sf_b)+\frac{1}{r_{id}}\int_{-\infty}^{sf_b}f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)\right.$$

$$\left.f_{\epsilon_a}(\epsilon_b)\mathrm{d}\epsilon_b\right\}<0$$

$$He_{11}He_{22}-He_{12}^2=$$

$$\left\{-r_a f_{\epsilon_a}(sf_a)+r_{id}r_b f_{\epsilon_a}(sf_a)\int_{-\infty}^{sf_b}f_{\epsilon_a}(\epsilon_a)\mathrm{d}\epsilon_a\right\}$$

$$\times\left\{-\frac{1}{r_{id}}r_b\int_{-\infty}^{sf_b}f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b\right\}>0$$

Therefore, a unique optimal solution exists. We finally consider the joint optimal pricing and capacity level decisions (where there are four decision variables). The transformed variables, sf_a, sf_b provides another interpretation of the production decisions. If sf_a is bigger than the value of ϵ_a, then leftover occurs. If sf_a is smaller than the value of ϵ_a, then shortage occurs. The first derivatives with respect to sf_a, sf_b are :

$$\frac{\partial\pi[\cdot]}{\partial sf_a}=r_a\{1-F_{\epsilon_a}(sf_a)\}\qquad(3)$$

$$-r_{id}r_b\int_{-\infty}^{sf_b}\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)-F_{\epsilon_a}(sf_a)\right\}$$

$$f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b-pc_a$$

$$\frac{\partial\pi[\cdot]}{\partial sf_b}=r_b\left\{1-\int_{-\infty}^{sf_b}F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b\right\}-pc_b\qquad(4)$$

The first derivatives with respect to price are :

$$\frac{\partial\pi[\cdot]}{\partial r_a}=2(\omega_a+r_{pd})(\tilde{r}_a-r_a)-\Upsilon_a(sf_a)\qquad(5)$$

$$\frac{\partial\pi[\cdot]}{\partial r_b}=2(\omega_b+\rho r_{pd})(\tilde{r}_b-r_b)-F_{\epsilon_a}(sf_a)\Upsilon_a(sf_b)\qquad(6)$$

$$+\delta(sf_a,sf_b)$$

The expected profit $\pi[\cdot]$ has a maximum with respect to sf_a, sf_b at a given r_a, r_b. Similarly, $\pi[\cdot]$ is concave for r_a, r_b at a given sf_a, sf_b. Thus, the simultaneous solution of the equations determines price and capacity level that maximizes the expected revenues. We obtain the optimal solutions through the four simultaneous equations. For (5) and (6) the optimal prices can be written as functions of sf_a, sf_b. These can be substituted into (3) and (4). Then, the contribution maximization problem can then be reduced to a two-variable search problem. Consider the first and second derivatives of $\pi[\cdot]$ with respect to sf and r. The first derivatives with respect to sf_a, sf_b can be rewritten :

$$\frac{\partial\pi[\cdot]}{\partial sf_a}=r_a\{1-F_{\epsilon_a}(sf_a)\}-r_{id}r_b\int_{-\infty}^{sf_b}$$

$$\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)-f_{\epsilon_a}(sf_a)\right\}f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b-pc_a$$

$$=\left\{\tilde{r}_a-\frac{\Upsilon_a(sf_a)}{2(\omega_a+r_{pd})}\right\}[1-F_{\epsilon_a}(sf_a)]$$

$$-r_{id}\left\{\tilde{r}_b-\frac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})}+\frac{\delta(sf_a,sf_b)}{2(\omega_b+\rho r_{pd})}\right\}$$

$$\int_{-\infty}^{sf_b}\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)-F_{\epsilon_a}(sf_a)\right\}$$

$$f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b-pc_a$$

$$\frac{\partial\pi[\cdot]}{\partial sf_b}=r_b\left\{1-\int_{-\infty}^{sf_b}F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b\right\}-pc_b$$

$$=\left\{\tilde{r}_b-\frac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})}+\frac{\delta(sf_a,sf_b)}{2(\omega_b+\rho r_{pd})}\right\}$$

$$\left\{1-\int_{-\infty}^{sf_b}F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b\right\}-pc_b$$

The second derivatives with respect to sf_a and sf_b are :

$$He_{11}=\frac{\partial^2\pi[\cdot]}{\partial sf_a^2}=-\frac{\Upsilon_a'(sf_a)}{2(\omega_a+r_{pd})}[1-F_{\epsilon_a}(sf_a)]$$

$$-\left\{\tilde{r}_a-\frac{\Upsilon_a(sf_a)}{2(\omega_a+r_{pd})}\right\}f_{\epsilon_a}(sf_a)$$

$$-r_{id}\left\{-\frac{f_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})}+\frac{\delta'(sf_a,sf_b)}{2(\omega_b+\rho r_{pd})}\right\}$$

$$\int_{-\infty}^{sf_b}\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right)-F_{\epsilon_a}(sf_a)\right\}f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b$$

$$- r_{id}\left\{\tilde{r}_b - \frac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \frac{\delta(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\}$$
$$\int_{-\infty}^{sf_b}\left\{f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) - f_{\epsilon a}(sf_a)\right\} f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b < 0$$

$$He_{22} = \frac{\partial^2\pi[\cdot]}{\partial sf_b^2} = -\left\{\frac{f_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \frac{\delta'(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\}$$
$$\left\{1 - \int_{-\infty}^{sf_b} F_{\xi_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b\right\}$$
$$-\left\{\tilde{r}_b - \frac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \frac{\delta(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\}$$
$$\int_{-\infty}^{sf_b} f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b < 0$$

Therefore, $\pi[\cdot]$ is concave for sf_a at a given sf_b, and $E[\pi]$ is also concave for sf_b at a given sf_a. The sufficient condition for the existence of a unique maximum is satisfied if:

$$\frac{\partial^2\pi[\cdot]}{\partial sf_a^2}\frac{\partial^2\pi[\cdot]}{\partial sf_b^2} - \frac{\partial^2\pi[\cdot]}{\partial sf_a\,\partial sf_b} < 0,$$

which is:

$$\left|\begin{array}{l}
-\dfrac{\Upsilon_a'(sf_a)}{2(\omega_a+r_{pd})}[1-F_{\epsilon_a}(sf_a)] - \left\{\tilde{r}_a - \dfrac{\Upsilon_a(sf_a)}{2(\omega_a+r_{pd})}\right\} f_{\epsilon_a}(sf_a) \\
- r_{id}\left\{-\dfrac{f_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \dfrac{\delta'(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\} \\
\displaystyle\int_{-\infty}^{sf_b}\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_b\right) - F_{\epsilon_a}(sf_a)\right\} f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b \\
- r_{id}\left\{\tilde{r}_b - \dfrac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b-\rho r_{pd})} + \dfrac{\delta'(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\} \\
\displaystyle\int_{-\infty}^{sf_b}\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) - f_{\epsilon_a}(sf_a)\right\} f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b
\end{array}\right|$$

$$\times\left|\begin{array}{l}
-\left\{\dfrac{f_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \dfrac{\delta'(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\} \\
\displaystyle\left\{1 - \int_{-\infty}^{sf_b} F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b\right\} \\
-\left\{\tilde{r}_b - \dfrac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \dfrac{\delta(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\} \\
\displaystyle\int_{-\infty}^{sf_b} f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b
\end{array}\right|$$

$$-\left|\begin{array}{l}
- r_{id}\left\{-\dfrac{F_{\epsilon_a}(sf_a)\Upsilon_b'(sf_b)}{2(\omega_b+\rho r_{pd})} + \dfrac{\delta'(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\} \\
\displaystyle\int_{-\infty}^{sf_b}\left\{F_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) - F_{\epsilon_a}(sf_a)\right\} f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b \\
- r_{id}\left\{\tilde{r}_b - \dfrac{F_{\epsilon_a}(sf_a)\Upsilon_b(sf_b)}{2(\omega_b+\rho r_{pd})} + \dfrac{\delta(sf_a,\ sf_b)}{2(\omega_b+\rho r_{pd})}\right\}\dfrac{1}{r_{id}} \\
\displaystyle\int_{-\infty}^{sf_b}\left\{f_{\epsilon_a}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) - f_{\xi_a}(sf_a)\right\} f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b
\end{array}\right| < 0.$$

The optimal prices can be obtained from the following two equations :

$$r_a^* = \tilde{r}_b - \frac{\Upsilon_a(sf_a)}{2(\omega_a+r_{pd})}$$
$$r_b^* = \tilde{r}_b + \frac{r_{id} r_b \Upsilon_b(sf_b)}{r_a(\omega_b+\rho r_{pd})}$$
$$\int_{-\infty}^{sf_b}\left\{F_{\epsilon_\epsilon}\left(\frac{sf_b-\epsilon_b}{r_{id}}+sf_a\right) - F_{\epsilon_a}(sf_a)\right\} f_{\epsilon_b}(\epsilon_b)\mathrm{d}\epsilon_b$$
$$+\frac{\delta(sf_a,\ sf_b)}{(\omega_b+\rho r_{pd})} - \frac{r_a - pc_a}{r_a(\omega_b+\rho r_{pd})} r_a$$

The transformed variable, sf_a represent the stocking factor defined as $sf_a = \mu_a + \mathrm{SF}\sigma_a$ where μ_a is the mean of ϵ_a and σ_a is the standard deviation of ϵ_a and SF denotes the safety factor. $\mathrm{SF} = l_a - Expexted\_Value[d_a(r_a, r_b, \epsilon_a)] / \sqrt{\mathrm{variance}[d_a(r_a, r_b, \epsilon_a)]}$. The optimal pricing and production decision is to manufacture $l_a^*(= sf_a^* + \mu_a(r_a^*, r_b^*))$ units at the price of r_a^*, r_b^* where sf_a^* and sf_b^* maximize the total expected revenues.

3. Numerical Examples

Consider an example of the optimization model B where ϵ_a and ϵ_b follow a Uniform demand distribution ranges : $t = 15,\ u = 10$. Let $v_a = 4{,}250$, $\omega_a = 10$, $v_b = 1{,}440$, $\omega_b = 5$, $pc_a = 200$, $pc_b = 200$ and 180, $t = 15$, $u = 10$, $sf_a = 1$, $sf_b = 1$ and $r_{pd} = 1$ and 5. The optimal prices and the expected revenues as a function of ρ(price-driven substitution rate) are displayed in <Table 1>. From the results, we see that the optimal price does not change significantly when r_{pd} is small (as ρ increases) while the expected revenues increase significantly. Comparing the results of the stochastic model with only considering price-driven substitution with those of our model in this research, the optimal prices do not change significantly. As ρ increases, the optimal price of products A increases and that of product B decreases, and the total expected profit increases.

〈Table 1〉 Numerical Results for the Optimal Pricing Decisions at Given sf_a and sf_b

$(pc_a = 200,\ pc_b = 200,\ sf_a = 1.0,\ sf_b = 1.0)$

($r_{pd} = 1$, inventory-driven substitution rate, $r_{id} = 0.1$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	304.58	304.83	305.31	305.81	306.30	306.80	307.05
r_b^*	254.03	254.02	254.01	254.04	254.07	254.12	254.15
π^*[1]	1.2733	1.2761	1.2816	1.2872	1.2928	1.2985	1.3013

[1] (in 100,000s).

($r_{pd} = 5$, inventory-driven substitution rate $= 0.1$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	289.62	290.44	292.13	293.89	295.71	297.61	298.58
r_b^*	288.37	284.83	279.58	275.99	273.48	271.73	271.08
π^*	1.1801	1.1817	1.1892	1.2011	1.2162	1.2336	1.2431

($r_{pd} = 1$, inventory-driven substitution rate $= 0.9$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	304.60	304.84	305.33	305.83	306.32	306.82	307.07
r_b^*	254.36	254.34	254.33	254.34	254.36	254.40	254.43
π^*	1.2817	1.2844	1.2900	1.2955	1.3012	1.3068	1.3097

($r_{pd} = 5$, inventory-driven substitution rate $= 0.9$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	289.68	290.50	292.16	293.95	295.78	297.67	298.65
r_b^*	288.74	285.16	279.86	276.24	273.71	271.94	271.27
π^*	1.1896	1.1910	1.1984	1.2101	1.2251	1.2425	1.2510

$(pc_a = 200,\ pc_b = 180,\ sf_a = 1.0,\ sf_b = 1.0)$

($r_{pd} = 1$, inventory-driven substitution rate $r_{id} = 0.1$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	304.12	304.41	305.00	305.58	306.16	306.75	307.05
r_b^*	243.98	243.97	243.98	244.01	244.05	244.11	244.15
π^*[1]	1.3126	1.3164	1.3242	1.3320	1.3399	1.3479	1.3520

[1] (in 100,000s).

($r_{pd} = 5$, $r_{id} = 0.1$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	287.80	288.79	290.83	292.94	295.13	297.41	298.58
r_b^*	277.47	274.01	268.93	265.51	263.19	261.64	261.08
π^*	1.1854	1.1915	1.2083	1.2299	1.2549	1.2829	1.2978

($r_{pd} = 1$, inventory-driven substitution rate $r_{id} = 0.1 = 0$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	304.14	304.43	305.01	305.60	306.19	306.78	307.07
r_b^*	244.31	244.30	244.29	244.31	244.34	244.39	244.43
π^*	1.3206	1.3244	1.3322	1.3400	1.3480	1.3560	1.3600

($r_{pd} = 5$, $r_{id} = 0.9$)

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	287.86	288.52	290.89	293.00	295.19	297.47	298.65
r_b^*	277.83	274.32	269.21	265.76	263.42	261.84	261.27
π^*	1.1945	1.2004	1.2172	1.2386	1.2636	1.2915	1.3063

We now demonstrate numerical examples for the four-variable decision problem) where ϵ_a and ϵ_b follow a Uniform demand distribution. Let $pc_a = 200$, $pc_b = 200$ & 180, $v_a = 4{,}250$, $\omega_a = 10$, $v_b = 1{,}440$, $\omega_b = 5$, $t = 15$, $u = 10$ and $r_{pd} = 1$ and 5. The numerical solutions for the joint pricing and capacity level decisions were done by standard numerical search procedures using <Table 2> displays the numerical results for the joint optimal prices and capacity levels as a function of ρ (price-driven substitution rate). As ρ increases at a given r_{id} (inventory-driven substitution rate), the optimal capacity level of product B increase while the optimal capacity levels of product A decrease. We consider the two cases for r_{id} = 0.1 and 0.9. Compared with the results of optimal pricing decision problem (B) in Table 1, the total expected profit increases but the optimal prices are very similar for any r_{pd} and ρ. If we choose different values of sf_a and sf_b the optimal prices also changes significantly. The optimal capacity level change significantly as r_{pd} and ρ changes.

<Table 2> Numerical Results for the Joint Optimal Prices and Capacity Levels

$(pc_a = 200,\ pc_b = 180)$

$(r_{pd} = 1$, inventory-driven substitution rate $r_{id} = 0.1)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	303.955	304.246	304.828	305.411	305.997	306.584	306.879
r_b^*	243.795	243.814	243.822	243.850	243.897	243.961	243.999
l_a^*	1144.801	1141.944	1135.588	1129.242	1122.882	1116.530	1113.340
l_b^*	216.220	221.679	233.895	246.232	258.684	271.254	277.583
π^{*1}	1.3224	1.3258	1.3336	1.3414	1.3493	1.3573	1.3613

[1] (in 100,000s).

$(r_{pd} = 5,\ r_{id} = 0.1)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	287.634	288.614	290.647	292.759	294.952	297.232	298.407
r_b^*	277.270	273.844	268.771	265.352	263.046	261.492	260.935
l_a^*	1315.195	1283.471	1227.810	1179.272	1134.978	1093.184	1072.863
l_b^*	49.408	73.787	124.448	177.073	231.759	288.634	317.927
π^*	1.1952	1.2012	1.2180	1.2395	1.2645	1.2923	1.3072

$(r_{pd} = 1$, inventory-driven substitution rate $r_{id} = 0.9$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	303.913	304.206	304.792	305.380	305.969	306.560	306.857
r_b^*	244.396	244.378	244.361	244.368	244.394	244.440	244.469
l_a^*	1144.036	1140.836	1134.420	1128.016	1121.618	1115.220	1112.005
l_b^*	219.337	225.391	237.588	249.900	262.339	274.888	281.211
π^*	1.3240	1.3278	1.3355	1.3434	1.3513	1.3592	1.3632

$(r_{pd} = 5,\ r_{id} = 0.9)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	287.673	288.600	290.661	292.784	294.986	297.272	298.448
r_b^*	278.166	274.828	269.500	265.949	263.539	261.918	261.334
l_a^*	1310.735	1281.614	1225.824	1177.194	1132.790	1090.920	1070.586
l_b^*	59.645	80.760	130.487	182.618	236.991	293.647	322.850
π^*	1.2008	1.2059	1.2221	1.2432	1.2680	1.2956	1.3104

$(pc_a = 200,\ pc_b = 200)$

$(r_{pd} = 1$, inventory-driven substitution rate, $r_{id} = 0.1)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	304.414	304.659	305.149	305.641	306.134	306.630	306.878
r_b^*	253.822	253.817	253.821	253.842	253.880	253.931	253.963
l_a^*	1150.199	1147.516	1142.162	1136.804	1131.451	1126.079	1123.399
l_b^*	165.038	170.146	180.437	190.832	201.319	211.914	217.240
π^{*1}	1.2839	1.2867	1.2922	1.2977	1.3033	1.3090	1.3118

[1] (in 100,000s)

$(r_{pd} = 5)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	289.451	290.264	291.955	293.709	295.531	297.426	298.402
r_b^*	288.148	284.641	279.399	275.814	273.315	271.571	270.915
l_a^*	1342.557	1312.919	1261.516	1217.440	1177.768	1140.773	1122.927
l_b^*	0	14.775	56.861	100.581	146.012	193.262	217.604
π^*	1.1901	1.1923	1.1998	1.2117	1.2267	1.2441	1.2536

$(r_{pd} = 1$, inventory-driven substitution rate, $r_{id} = 0.9)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	304.389	304.635	305.129	305.624	306.120	306.618	306.868
r_b^*	254.289	254.274	254.258	254.262	254.284	254.321	254.346
l_a^*	1149.462	1146.763	1141.358	1135.960	1130.567	1125.167	1122.462
l_b^*	167.932	173.035	183.312	193.691	204.267	214.747	220.070
π^*	1.2852	1.2880	1.2935	1.2991	1.3046	1.3103	1.3131

$(r_{pd} = 5)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
r_a^*	289.452	290.284	291.986	293.747	295.574	297.472	298.500
r_b^*	289.103	285.418	279.999	276.304	273.730	271.931	271.253
l_a^*	1341.940	1311.816	1260.196	1215.965	1176.179	1139.104	1120.465
l_b^*	0.072	20.087	61.624	105.006	150.208	197.285	221.830
π^*	1.1947	1.1959	1.2029	1.2145	1.2292	1.2465	1.2560

We now compare the expected profits of different optimization models. <Table 3> presents the expected profits for the eight different cases including the numerical results of the model with price-driven substitution alone (which do not consider inventory-driven substitution). In <Table 3>, π^{*1} represents the total expected profit for the traditional newsvendor model where r_a = 290, r_b = 255 or r_a = 305, r_b = 244. In this case, we consider a price-driven substitution for the demands of both products. The expected profits for the traditional newsvendor model are also shown at the first row in the tables. π^{*4} represents the total expected profit for the joint pricing and production decision model with price-driven substitution alone Simultaneous control of prices and capacity levels improves the expected profit. Considering both types of substitution also affects the total expected profit improving. In the joint pricing and capacity level decision model, inventory-driven substitution rate is also important factor which affects the total expected profit.

〈Table 3〉 Comparison of the (optimal) Expected Profit of Various Models

$(pc_a = 200,\ pc_b = 200)$

$(r_{pd} = 1)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
π^{*1}	1.2606	-	-	-	-	-	-
π^{*2}	1.2609	1.2628	1.2667	1.2705	1.2743	1.2782	1.2801
π^{*3}	1.2624	1.2643	1.2682	1.2720	1.2759	1.2797	1.2817
π^{*4}	1.2830	1.2860	1.2910	1.2970	1.3030	1.3080	1.3110
π^{*5}	1.2733	1.2761	1.2816	1.2872	1.2928	1.2985	1.3013
π^{*6}	1.2817	1.2844	1.2900	1.2955	1.3012	1.3068	1.3097
π^{*7}	1.2839	1.2867	1.2922	1.2977	1.3033	1.3090	1.3118
π^{*8}	1.2852	1.2880	1.2935	1.2991	1.3046	1.3103	1.3131

[1] The expected profit of the newsvendor model when $r_a = 290,\ r_b = 255$.
[2] The expected profit of the capacity level decision model when $r_a = 290,\ r_b = 255,\ r_{id} = 0.1$.
[3] The expected profit of the capacity level decision model when $r_a = 290,\ r_b = 255,\ r_{id} = 0.9$.
[4] The expected profit of joint pricing and capacity level decision without inventory-driven substitution; price-driven substitution alone
[5] The expected profit of the pricing decision model when $r_{id} = 0.1,\ sf_a = 1.0,\ sf_b = 1.0$.
[6] The expected profit of the pricing decision model when $r_{id} = 0.9,\ sf_a = 1.0,\ sf_b = 1.0$.
[7] The expected profit of the joint pricing and capacity level decision model when $r_{id} = 0.1$.
[8] The expected profit of the joint pricing and capacity level decision model when $r_{id} = 0.9$.
[1~8] (in 100,000s).

$(r_{pd} = 5)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
π^{*1}	1.0226	-	-	-	-	-	-
π^{*2}	1.0229	1.0362	1.0631	1.0899	1.1168	1.1436	1.1570
π^{*3}	1.0237	1.0371	1.0640	1.0908	1.1176	1.1445	1.1579
π^{*4}	1.1880	1.1920	1.1990	1.2110	1.2260	1.2430	1.2530
π^{*5}	1.1801	1.1817	1.1892	1.2011	1.2162	1.2336	1.2431
π^{*6}	1.1896	1.1910	1.1984	1.2102	1.2251	1.2425	1.2510
π^{*7}	1.1901	1.1923	1.1988	1.2117	1.2267	1.2441	1.2536
π^{*8}	1.1947	1.1959	1.2029	1.2145	1.2292	1.2465	1.2560

[1] The expected profit of the newsvendor model when $r_a = 305,\ r_b = 244$.
[2] The expected profit of the capacity level decision model when $r_a = 305,\ r_b = 244,\ r_{id} = 0.1$.
[3] The expected profit of the capacity level decision model when $r_a = 305,\ r_b = 244,\ r_{id} = 0.9$.
[4] The expected profit of joint pricing and capacity level decision without inventory-driven substitution
[5] The expected profit of the pricing decision model when $r_{id} = 0.1,\ sf_a = 1.0,\ sf_b = 1.0$.
[6] The expected profit of the pricing decision model when $r_{id} = 0.9,\ sf_a = 1.0,\ sf_b = 1.0$.
[7] The expected profit of the joint pricing and capacity level decision model when $r_{id} = 0.1$.
[8] The expected profit of the joint pricing and capacity level decision model when $r_{id} = 0.9$.
[1~8] (in 100,000s).

$(pc_a = 200,\ pc_b = 180)$

$(r_{pd} = 1)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
π^{*1}	1.2926	-	-	-	-	-	-
π^{*2}	1.2928	1.2955	1.3007	1.3060	1.3112	1.3164	1.3191
π^{*3}	1.2958	1.2984	1.3037	1.3089	1.3142	1.3194	1.3222
π^{*4}	1.3210	1.3250	1.3330	1.3410	1.3490	1.3570	1.3610
π^{*5}	1.3126	1.3164	1.3241	1.3320	1.3399	1.3479	1.3520
π^{*6}	1.3206	1.3244	1.3322	1.3400	1.3480	1.3560	1.3600
π^{*7}	1.3224	1.3258	1.3336	1.3414	1.3493	1.3573	1.3613
π^{*8}	1.3240	1.3278	1.3355	1.3434	1.3513	1.3592	1.3623

[1] The expected profit of the newsvendor model when $r_a = 290,\ r_b = 255$.
[2] The expected profit of the capacity level decision model when $r_a = 290,\ r_b = 255,\ r_{id} = 0.1$.
[3] The expected profit of the capacity level decision model when $r_a = 290,\ r_b = 255,\ r_{id} = 0.9$.
[4] The expected profit of joint pricing and capacity level decision without inventory-driven substitution
[5] The expected profit of the pricing decision model when $r_{id} = 0.1,\ sf_a = 1.0,\ sf_b = 1.0$.

[6] The expected profit of the pricing decision model when $r_{id}=0.9$, $sf_a=1.0$, $sf_b=1.0$.
[7] The expected profit of the joint pricing and capacity level decision model when $r_{id}=0.1$.
[8] The expected profit of the joint pricing and capacity level decision model when $r_{id}=0.9$.
[1~8] (in 100,000s).

$(r_{pd}=5)$

ρ	0.0	0.1	0.3	0.5	0.7	0.9	1.0
π^{*1}	1.0650	-	-	-	-	-	-
π^{*2}	1.0657	1.0852	1.1242	1.1633	1.2023	1.2413	1.2609
π^{*3}	1.0676	1.0872	1.1262	1.1652	1.2043	1.2433	1.2628
π^{*4}	1.1950	1.2010	1.2180	1.2390	1.2640	1.2920	1.3070
π^{*5}	1.1854	1.1915	1.2083	1.2299	1.2549	1.2829	1.2978
π^{*6}	1.1945	1.2004	1.2172	1.2386	1.2636	1.2915	1.3063
π^{*7}	1.1952	1.2012	1.2180	1.2395	1.2645	1.2923	1.3072
π^{*8}	1.2008	1.2059	1.2221	1.2432	1.2680	1.2956	1.3104

[1] The expected profit of the newsvendor model when $r_a=305$, $r_b=244$.
[2] The expected profit of the capacity level decision model when $r_a=305$, $r_b=244$, $r_{id}=0.1$.
[3] The expected profit of the capacity level decision model when $r_a=305$, $r_b=244$, $r_{id}=0.9$.
[4] The expected profit of joint pricing and capacity level decision without inventory-driven substitution
[5] The expected profit of the pricing decision model when $r_{id}=0.1$, $sf_a=1.0$, $sf_b=1.0$.
[6] The expected profit of the pricing decision model when $r_{id}=0.9$, $sf_a=1.0$, $sf_b=1.0$.
[7] The expected profit of the joint pricing and capacity level decision model when $r_{id}=0.1$.
[8] Theexpected profit of the joint pricing and capacity level decision model when $r_{id}=0.9$.
[1~8] (in 100,000s).

4. Conclusions

In this research, we study demand-management decisions in Yield Management. We develop a single-period model for deciding optimal prices and capacity levels. Substitution is an important factor in demand-management aspect in YM, and it has been actively researched in the area of Management Science. Most of the research papers deals with inventory-driven substitution. In this research, we investigated the impact of demand substitution as a result of both price differences (price-driven substitution) and based on inventory stock-outs (inventory-driven substitution) on the optimal pricing and production levels and the expected profits. We discussed structural properties of the substitution model and managerial implications. If we effectively control multiple decision variables, then we can improve the expected profit. The most significant contribution of this research is to develop analytical procedures for finding optimal solutions and we consider both types of substitution. We provide detailed theoretical analysis and numerical examples. Understanding the potential impact of demand substitution as a result of price differences as well as inventory stock-outs on the optimal solutions between multiple market segments, can make managers setup more appropriate policies.

References

[1] Baek, J.-G., C.O. Kim, and J. Jin, "Adaptive inventory control models in a supply chain with nonstationary customer demand," *Journal of the Korean Institute of Industrial Engineers*, Vol.31, No.2(2005), pp.106-1119.

[2] Cho, H.-R., J.-S. Yu, C.-N. Cha, and S.-K. Lim, "Analysis of pricing and efficiency control strategy between online and offline marketing channels," *Journal of the Korean Institute of Industrial Engineers*, Vol.27, No.2(2001), pp.181-189.

[3] Dong, L., P. Kouvelis, and Z. Tian, "Dynamic pricing and inventory control of substitutable products," *Manufacturing and*

Service Operations Management, Vol.11, No.3(2009), pp.317-339.

[4] Dutta, P. and D. Chakraborty, "Incorporating one-way substitution policy into the newsboy problem with imprecise customer demand," *European Journal of Operational Research*, Vol.200(2010), pp.99-110.

[5] Ingold, A., Yeoman, I., and McMahon, U., "Yield Management: Strategies for the Service Industries," Int. Thomson Business Press, 2001 U.K.

[6] Lee, K.B., C.M. Joo, and W.-S. Lee, "Pricing quality and service mix strategies for portal sites providing various services," *Journal of the Korean Institute of Industrial Engineers*, Vol.28, No.3(2002), pp.291-301.

[7] Lee, K.-K. and Y.-S. Kim, "Dynamic clearance pricing policy for durable goods," *Journal of the Korean Institute of Industrial Engineers*, Vol.26, No.1(2000), pp.66-72.

[8] Lim, S.Y., S. Hur, and Y.J. Park, "Determination of economic inventory quantity under probabilistic demands and cancellation of orders in production system with two different production speeds," *Journal of the Korean Institute of Industrial Engineers*, Vol.40, No.3(2014), pp.313-320.

[9] Karakul, M. and L. Chan, "Analytical and managerial implications of integrating product substitutability in the joint pricing and procurement problem," *European Journal of Operational Research*, Vol.190(2008), pp.179-204.

[10] Karaesmen, I. and G. van Ryzin, "Overbooking with substitutable inventory classes," *Operations Research*, Vol.52, No.1(2004), pp.83-104.

[11] Kim, S.-W., "A deterministic model for optimal pricing decisions with price-driven substitution," *Journal of the Korean Operations Research and Management Science Society*, Vol.33, No.1(2008), pp.1-17.

[12] Kim, E., "Optimal production controls in a two-stage production system with a component selling option," *Journal of the Korean Institute of Industrial Engineers*, Vol.41, No.5(2015), pp.447-452.

[13] Kocabiykoglu, A. and I. Popescu, "Revenue management with general stochastic price-sensitive demand," *INSEAD Working Paper*, France, 2007.

[14] Kuyumcu, H. and I. Popescu, "Deterministic price-inventory management for substitutable products," *Journal of Revenue and Pricing Management*, Vol.4(2006), pp.354-367.

[15] Lus, B. and A. Muriel, "Measuring the impact of increased product substitution on pricing and capacity decisions under linear demand models," *Production and Operations Management*, Vol.18(2009), pp.95-113.

[16] Park, K., "A note on the stochastic comparison in production yield management," *Journal of the Korean Institute of Industrial Engineers*, Vol.40, No.5(2014), pp.477-480.

[17] Rao, U., J. Swaminathan, and J. Zhang, "Multi-product inventory planning with downward substitution stochastic demand and setup costs," *IIE Transactions*, Vol.36 (2004), pp.36-59.

[18] Shinn, S.-W. and C.-Y. Song, "An optimal ordering policy under the condition of a free addition," *Journal of the Korean Insti-*

tute of Industrial Engineers, Vol.26, No.1 (2000), pp.48-53.

[19] Shumsky, R. and F. Zhang, "Dynamic Capacity management with substitution," *Operations Research*, Vol.57(2009), pp.671-684.

[20] Son, J.-D., "Optimal switching strategy between admission control and pricing control in an M/M/1/K queuing system," *Journal of the Korean Institute of Industrial Engineers*, Vol.35, No.2(2009), pp.171-177.

[21] Sung, C.S. and S.H. Park, "A combined approach of pricing and (S-1, S) inventory policy in a two-echelon supply chain with lost sales allowed," *Journal of the Korean Institute of Industrial Engineers*, Vol.30, No.2(2004), pp.146-158.

[22] Tang, C. and R. Yin, "Joint ordering and pricing strategies for managing substitutable products," *Production and Operations Management*, Vol.16, No.1(2007), pp.138-153.

[23] Wang, L. and R. Kapuscinski, "Coordination and price flexibility for multiple products with price-based substitution," Business School, Working Paper, University of Miami, Florida, 2009.

[24] Won, Y., "Note : proof of the conjecture on the consistent advantage of the newsvendor model under progressive multiple discounts," *Journal of the Korean Operations Research and Management Science Society*, Vol.37, No.3(2012), pp.13-21.

[25] Won, Y., "Note: proof of the conjecture on the consistent advantage of the newsvendor model under progressive multiple discounts," *Journal of the Korean Operations Research and Management Science Society*, Vol.37, No.3(2012), pp.13-21.

[26] Yu, M., "Joint pricing and inventory decisions under resource constraints and price-based substitution," *Journal of Revenue and Pricing Management*, Vol.14, No.1(2015), pp.19-27.

[27] Zhao, L., P. Tian, and L. Xiangyong, "Dynamic pricing in the presence of consumer inertia," *The international Journal of Management Science-Omega*, Vol.40, No.2(2012), pp.137-148.