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Identifying Top K Persuaders Using Singular Value Decomposition

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Abstract

Purpose - Finding top K persuaders in consumer network is an important problem in marketing. Recently, a new method of computing persuasion scores, interpreted as fixed point or stable distribution for given persuasion probabilities, was proposed. Top K persuaders are chosen according to the computed scores. This research proposed a new definition of persuasion scores relaxing some conditions on the matrix of probabilities, and a method to identify top K persuaders based on the defined scores.
Research design, data, and methodology - A new method of computing top K persuaders is computed by singular value decomposition (SVD) of the matrix which represents persuasion probabilities between entities.
Results - By testing a randomly generated instance, it turns out that the proposed method is essentially different from the previous study sharing a similar idea.
Conclusions - The proposed method is shown to be valid with respect to both theoretical analysis and empirical test. However, this method is limited to the category of persuasion scores relying on the matrix-form of persuasion probabilities. In addition, the strength of the method should be evaluated via additional experiments, e.g., using real instances, different benchmark methods, efficient numerical methods for SVD, and other decomposition methods such as NMF.

Keywords: Word-of-Mouth, Persuaders, Social Network Analysis, SVD.

JEL Classifications: C55, D85, M31, M37.

1. Introduction

One of the influential factors which affect a consumer's purchasing decision is an opinion of his or her acquaintances (Hill et al., 2006; Kim et al., 2014; Jung et al., 2014). In marketing, these opinions have been known as word of mouth (WOM) (Arndt, 1967; Oluwafemi & Dastane, 2016), and used as the following form of marketing campaign: give a message to a set of consumers (a.k.a., seeds or persuaders) to spread the word about a product to other consumers (Kozinets et al., 2010). Hence, to deploy WOM marketing campaign successfully, a company should know who are the seeds or persuaders (Hinz et al., 2011) and which types of messages are effective for chosen seeds (Phelps et al., 2004).

According to (Hinz et al., 2011), there are four keys to success in viral marketing (Van der Lans et al., 2010), a form of WOM marketing. These are (i) content (Porter & Golan, 2006), (ii) structure of the social network (Bampo et al., 2008), (iii) behavioral characteristics of the persuaders and their incentives for sharing the message (Arndt, 1967), and (iv) seeding strategy (Kalish et al., 1995; Libai et al., 2005; Bampo et al., 2008; Fang & Hu, 2016), i.e., the set of persuaders. Among them, we are interested in the fourth factor, i.e., the seeding strategy. More specifically, we want to solve the problem of predicting top K persuaders or seeds in which other factors are assumed to be given. The first three factors which are assumed to be given are typically described by the matrix whose entry represents the probability that a social entity persuades the other one, e.g., (Fang & Hu, 2016).

Recently, this problem is attacked by (Fang & Hu, 2016). They first define the matrix P of persuasion probabilities, i.e., P_{ij} is the probability that the entity i persuades the

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entity j. This matrix is constructed by considering first three factors mentioned in (Hinz et al., 2011). Since we are focusing on the fourth factor, i.e., seeding or predicting top K persuaders, we will not explain further details for the construction of the matrix. For more details, the interested reader is referred to (Fang & Hu, 2016). Secondly, for each entity i, the numerical value, called the persuasion score c_i is computed. The vector of persuasion scores is defined as a solution of $c = Pc$, of which precise meaning will be explained in later section. Finally, top K persuaders are chosen according to the order of persuasion scores.

The idea of our paper is motivated by (Fang & Hu, 2016). We observe that the defining equation of the persuasion score, i.e., $c = Pc$ seems to be restrictive. For example, if P is Markov matrix, it has the eigenvalue of 1 and corresponding eigenvector is the one-vector whose entries are all 1's. Indeed, this eigenvector is a solution of $c = Pc$. In addition, the underlying graph over social entities is assumed to be a complete graph, i.e., for any two entities there exists an edge between them. This assumption may fail in many real instances. To overcome this restriction on the class of persuasion matrices, we propose a new definition and an algorithm for persuasion scores based on persuasion matrix suggested by (Fang & Hu, 2016). In Section 2, we will review some previous studies on predicting K persuaders or seeds, and SVD (singular value decomposition) we will use extensively in the development of the proposed definition and algorithm. Section 3 is devoted to explain our new definition of persuasion scores and algorithm finding them. This definition is evaluated and interpreted for a randomly generated instance in Section 4. Finally, concluding remarks and possible future research directions are suggested in Section 5.

2. Literature Review

Finding the top K influential persuaders, i.e., opinion leaders have been extensively studied in the field of social network analysis (Wasserman & Faust, 1994). A social network is a network modeling social structure which consists of a set of social entities (e.g., individuals, organizations) represented by nodes and relationships represented by edges between social entities (Wasserman & Faust, 1994). Thus, if we define the relationships having marketing-friendly meaning, the techniques used in social network analysis can also be used for our problem. In the context of social network analysis, top K persuaders are typically explained with the concept of centrality (Borgatti et al., 2009; Cha et al., 2010). There are many realizations of the concept of centrality: for example, degree centrality, topological centrality, and eigenvector centrality (see <Table 1>).

<Table 1> Literature Survey on the Concepts of Centrality

Concept of Centrality	Literature
Degree Centrality	(Walter et al., 1996; Albert et al., 2000)
Topological Centrality	(Brass, 1984; Borgatti et al., 2009)
Eigenvector Centrality	(Bonacich, 1972; Ballester et al., 2006)

The degree centrality (Walter et al., 1996; Albert et al., 2000) is based on the assumption that an entity with many neighbors possesses more persuasion power. In the topological centrality (Brass, 1984; Borgatti et al., 2009), on the other hand, the persuasion power of a particular entity is determined by two factors, the sum of distances or closeness between other entities and itself, and total frequencies it appears in the shortest paths for all pairs of entities in the network. Finally, eigenvector centrality (Bonacich, 1972; Ballester et al., 2006) is defined as a solution of $\lambda x^T = x^T A$ where A is the adjacency matrix, i.e., left eigenvector of the adjacency matrix. It can be interpreted that edges or relationships to entities with high score contribute more to the score of the entity in question.

The singular value decomposition (Golub & Van Loan, 1996) of a matrix A is the factorization (or decomposition) of A into the product of three matrices $A = U\Sigma V^T$. U and V are orthonormal matrices and Σ is diagonal with positive real entries, called singular values. The columns of U are called the left singular vectors of A and the columns of V are called right singular vectors. Among the singular vectors, we are interested in the left singular vector corresponding to the maximum singular value which is the objective of the following optimization problem: $\sigma_1 = \max_{\| v \|_2 = 1} \| Av \|_2$.

Suppose that $\sigma_1 = \| Av_1 \|_2$ and $u_1 = \frac{1}{\sigma_1} Av_1$, i.e., v_1 and u_1 are right and left singular vectors corresponding to the largest singular value. v_1 is known to the best direction which approximates the rows of A in the least squares criterion. Then, Av_1 can be seen as the vector whose entry i represents the similarity between the ith row of A and v_1, and hence, u_1, which is normalized by σ_1, has the same information. In Section 3, we will give a similar view of SVD which is more relevant to our application of SVD.

3. Methodology

Let's define the persuasion score of a social entity, a target of marketing, as the extent to which that entity can persuade other entities to adopt some products. Then, predicting top K persuaders refer to choose the highest K persuasion scores. Let $c \in R^n$ be the vector of persuasion scores for n social entities. Recently, (Fang & Hu, 2016) proposed the algorithm which estimates the matrix $P \in R^{n \times n}$ of persuasion probabilities. Here, P_{ij} is the probability that the entity i persuades the entity j to adopt. Then, if the entity i has a persuasion power by which the entity j of persuasion score c_j can be persuaded, the amount of $P_{ij}c_j$ should be reflected to c_i, the persuasion score of the entity i. In this respect, (Fang & Hu, 2016) enforces c to satisfy $c = Pc$, i.e., for each entity i, $c_i = \sum_{j=1}^{n} P_{ij} c_j$. Thus, c is a fixed point of the operator P. But, the method of (Fang & Hu, 2014) cannot be applied to some useful subclass of matrices P.

Case (i)

If we define P satisfying $\sum_{j=1}^{n} P_{ij} = 1$, i.e., P satisfies the Markov assumption, one of the solution of $c = Pc$ becomes $(1,\dots,1)^T \in R^n$, which is the eigenvector corresponding to the eigenvalue of 1. Note that if we use power iteration method, used in (Fang & Hu, 2016), we may obtain the vector c which is not equal to $(1,\dots,1)^T$.

Case (ii)

To find c satisfying $c = Pc$ with the power iteration method, we presumed some conditions on P. One of the conditions which guarantees the convergence of the power iteration method is $P_{ij} > 0$ for all $i \neq j$. In other words, the underlying graph is assumed to be complete graph, which can be violated in many real instances (see <Figure 1>). For example, for a pair of social entities any relevant data which are used for estimating persuasion probabilities may not be available. In addition, estimating the persuasion probabilities between all pairs of social entities requires infeasible computational resources if the number of social entities is large enough.

To overcome the above drawbacks, we propose the algorithm based on SVD (Singular Value Decomposition). More specifically, for a given persuasion matrix P (which can violate the conditions in case (i) and (ii)),

Step 1. Compute the singular value decomposition (SVD) of P, $P = U\Sigma V^T$ where $U \in R^{n \times r}$, $\Sigma \in R^{r \times r}$, $V \in R^{m \times r}$, and r is the rank of P.

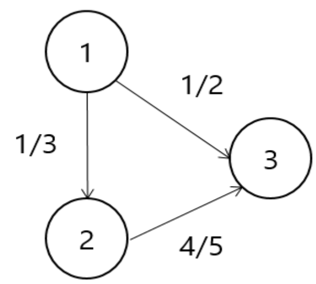
Step 2. Choose the column u of U, corresponding to the maximum singular value.

Step 3. Choose top K persuaders who are those with the highest K absolute persuasion scores in u.

Now, let's interpret the proposed method. According to the interpretation of SVD introduced in Section 2, the right singular vector v, corresponding to the largest singular value, can be interpreted as a virtual persuader whose persuasion probabilities are the best approximation of the whole persuaders' probabilities. Then, as mentioned in Section 2, the left singular vector u represents the similarities between each persuader and the representative entity, the right singular vector for the largest singular value. Note that the signs of the entries of u are not important, and hence we consider only the absolute values of them.

4. Results

In <Figure 1>, a simple example of the persuasion network and the result (see <Table 2>) obtained by our method are presented. Note that the method proposed by (Fang & Hu, 2016) cannot be applied to this example since there are missing edges (2, 1), (3, 1) and (3, 2). In this example, node 2 has the largest absolute value, and hence it is the most influential persuader.



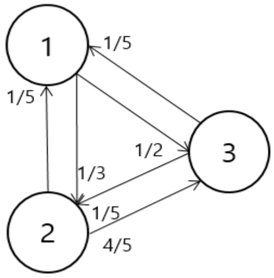
Max. Singular Value	0.96
Singular vector	[-0.58, -0.82, 0.0]

<Figure 1> An Example of Non-Complete Graph

To apply the method proposed by (Fang & Hu, 2016) to the example in <Figure 1>, we modify the example as shown in <Figure 2>. To check whether these two methods shows similar results or not, the added edges (2, 1), (3, 1) and (3, 2) have the relatively small probabilities of 1/5.

As you can see in <Figure 2> below, the orders of importance of persuaders are same for both methods.

One natural question is whether these two methods are essentially same or not. Let's consider a matrix of persuasion probabilities shown in <Table 1>.



Max. Singular Value	0.97818
Singular vector	[0.56, 0.82, 0.08]
(Fang & Hu, 2016)	[0.95, 1.00, 0.59]

<Figure 2> A Modification of Figure 1

<Table 2> An Example of Persuasion Probabilities

0.00000	0.05656	0.82864	0.15432	0.51395
0.93967	0.00000	0.96864	0.10943	0.05334
0.56217	0.64669	0.00000	0.69656	0.77500
0.68064	0.94487	0.74247	0.00000	0.39859
0.92610	0.42429	0.31150	0.20559	0.00000

For this example, the order of persuasion scores of 5 social entities obtained by the method of (Fang & Hu, 2016) is (4, 3, 2, 5, 1). On the other hand, the order of persuasion scores obtained by our method is (4, 2, 3, 5, 1). Thus, we can conclude that two methods are essentially different even if some instances share the same ordering.

Indeed, the persuasion probabilities of the representative persuader is (0.625234, 0.429996, 0.534419, 0.204329, 0.311179) in which the order of persuasion probabilities is (1, 3, 2, 5, 4). Since the orders of persuasion probabilities of the second and third entities are (3, 1, 4, 5) and (5, 4, 2, 1), respectively the order of the second entity is more compatible to that of the representative persuader than the third entity.

5. Conclusion

5.1. Summary

To launch a successful WOM marketing campaign, it is mandatory to predict some of the most influential persuaders. Previously, (Fang & Hu, 2016) proposed the method to identify top-K persuaders based on finding the fixed point for a given matrix of persuasion probabilities. Their method, however, cannot be generally extended to handle more general matrices, e.g. probability matrices corresponding to non-complete graph. To overcome this restriction, we propose the method based on singular value decomposition (SVD) which can be applied to any matrices. This method is shown to have similar results compared with those from (Fang & Hu, 2016), but not necessarily same.

5.2. Limitations

However, this method is limited to the category of persuasion scores computed from the matrix-form of persuasion relationships between entities. It is obvious the result of our method is highly dependent on the meaning and validity of the corresponding definition of persuasion probabilities. For example, it cannot provide a way to define persuasion probabilities from other variables, e.g. (Kim et al., 2014). The strength of the method is not fully evaluated, and hence additional experiments with real data (Kim et al., 2014; Oluwafemi & Dastane, 2016) or comparing the result with other methods are needed.

5.3. Implications

Before concluding the paper, we introduce some possible directions for future research. First, it is necessary to check validity of the proposed method via real data. To apply our method to a large-scale real probability matrix, an efficient method for calculating SVD or computing the left singular vector corresponding to the largest singular value should be used. Second, there may be other ways to use the result of SVD, e.g., by using more than one representative persuaders. In addition, there may be other factorization techniques, such as NMF(Non-negative Matrix Factorization), which can give us some new interpretations and derived algorithms. Finally, to answer the strength of the proposed method definitely, some additional experiments comparing the proposed method with the methods other than (Fang & Hu, 2016) are necessary.

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