# Efficient Channel Assignment Scheme Based on Finite Projective Plane Theory

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## Abstract

This paper proposes a novel channel assignment scheme that is based on finite projective plane (FPP) theory. The proposed scheme involves using a Markov chain model to allocate $N$ channels to $N$ users through intermixed channel group arrangements, particularly when channel resources are idle because of inefficient use. The intermixed FPP-based channel group arrangements successfully related Markov chain modeling to punch through ratio formulations proposed in this study, ensuring fair resource use among users. The simulation results for the proposed FPP scheme clearly revealed that the defined throughput increased, particularly under light traffic load conditions. Nevertheless, if the proposed scheme is combined with successive interference cancellation techniques, considerably higher throughput is predicted, even under heavy traffic load conditions.

***Keywords:*** Channel assignment, collision avoidance, collision tolerance, finite projective plane (FPP), steady-state probability (SSP), punch through ratio (PTR), effective punch through ratio (EPTR).

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## 1. Introduction

Technological advancements in handheld wireless terminals have facilitated the rapid growth of the field of wireless communications and driven the tremendous growth in the wireless and mobile user population. This population growth coupled with the bandwidth requirements of multimedia applications requires the efficient reuse of the scarce radio spectrum allocated for wireless and mobile communications.

Conventionally, the usage of the radio spectrum and the regulation of radio emissions are coordinated by national regulatory bodies. Such bodies divide the radio spectrum into numerous frequency bands and allocate them to licensed users, often for exclusive use [1-3]. Depending on the type of radio service provided by the licensees, frequency bands are often idle in numerous areas because of inefficient use. Over the past decade, three main channel assignment approaches, namely fixed, dynamic, and hybrid channel assignment, have been used for cellular mobile communication systems [4, 5]. A previous study presented a comprehensive survey of different channel assignment schemes and algorithms for cellular mobile telecommunication systems [1]. In cellular networks, data traffic and signaling control traffic are typically carried in separate channels. Various channel assignment strategies in cellular domains have been designed and implemented according to this separate-channel concept [2, 3]. In surveys on state-of-the-art channel assignment schemes in IEEE 802.11-based wireless local area networks (WLANs), the authors concluded by listing several research problems [6-8]. However, the channel assignment techniques employed in cellular mobile systems cannot be applied directly in WLAN scenarios. In WLANs, both data and control traffic should share the same channel [9]. In addition, cognitive radio is a promising technology for wireless networks (cognitive radio networks). This technology exploits underutilized spectrum bands and overcomes the problem of overutilization of free bands; in particular, unlike existing wireless network technology, it enables users to access any unused portion of the spectrum without limiting their access to specific free frequencies [10-11].

Collision resolution is crucial in wireless networks. Collision resolution schemes for improving channel utilization can be classified into the following categories: collision avoidance, collision tolerance, and collision recovery [12]. Collision avoidance is the most prevalent collision resolution scheme, and numerous collision avoidance protocols have been proposed in recent years [13, 14]. According to the mechanism used for avoiding collisions, collision avoidance schemes can be divided into schedule-based (e.g., time division multiple access and frequency division multiple access) and contention-based (e.g., carrier sense multiple access) strategies [15]. Unlike collision avoidance, the concept of collision tolerance is to allow collisions [16, 17]. Collision recovery involves recovering collided signals by using advanced physical layer techniques. This scheme entails iteratively decoding a collision-free part in collided signals first and then removing it from the collided signals. A major approach is successive interference cancellation (SIC), which involves resolving different users sequentially; in other words, the interference associated with resolved users is subtracted before resolving other users [18, 19].

A common limitation of existing collision tolerance schemes is that they can be applied only in flooding or broadcasting scenarios, where all transmitted packets must carry the same data. This requirement considerably limits their application scope. The reason behind the limitation is that these schemes fail to discern the basic timing and concurrency requirements of collision tolerance. Collision recovery entails recovering collided signals by using advanced physical layer techniques. This scheme involves iteratively decoding a collision-free part in collided signals first and then removing it from the collided signals. The limitations of these schemes are that they require a specially modified physical layer and are not supported by commercial hardware.

Although various collision resolution schemes such as the aforementioned schemes have been devised, little attention has been paid to describing the relationship between multiple channel assignment and collision tolerance. In general, as shown in **Fig. 1**, $N$ users attempt to access the base station channel resources ($N$ channels) in a fair and monopolistic manner (i.e., fixed channel allocation). In reality, channel utilization typically involves "unfair" access because of different user requirements; specifically, some channels are often in an idle state and the others are in a busy state all the way. In this situation, the channel utilization ratio is low. The main objective of the current study was to enhance channel utilization by applying a novel finite projective plane (FPP)-based scheme involving Markov chain modeling of $N$ channels and $N$ users, particularly when channel resources are often idle because of inefficient use. The main contribution of this study is the successful relation of the Markov chain modeling to FPP-based punch through ratio (PTR) formulations, which we propose. Moreover, simulation results pertaining to the effective PTR (or throughput) with and without SIC are presented to demonstrate the enhancement of channel utilization.

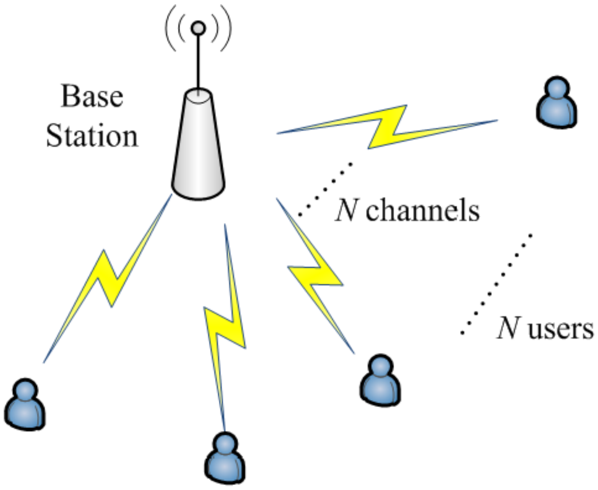


**Fig. 1.** System scenario with $N$ users accessing $N$ channels

**Fig. 2** illustrates the relationship between channel and time slots in occupied, collided, and idle states for a fixed channel (fixed channel assignment (FCA) scheme) and the proposed intermixed channel (FPP scheme). In addition to FCA, another traditional dynamic channel assignment (DCA) scheme is, in reality, a variant of FCA, and can be also available to improve the FCA disadvantages. Without loss of generality, we can assume that there exist some additional estimation and signaling channels for the DCA scheme when in comparison with the FCA scheme. In the FCA scheme (**Fig. 2(a)**), each channel is simply allocated to a specific user (i.e., User A, B, or C). Nevertheless, these allocated channels are not always used by users, generally resulting in numerous time idle slots. Many sophisticated dynamic channel assignment schemes can be applied to use these idle time slots more efficiently, but as mentioned, they have the drawback of high complexity. In the FPP scheme (**Fig. 2(b)**), two channels are allocated to each user simultaneously; for example, Channels 1 and 2 are allocated to User A, Channels 2 and 3 are allocated to User B, and Channels 1 and 3 are allocated to User C. In this intermixed scheme, idle time slots are reduced and the channel utilization clearly increases. This is an efficient FPP scheme (of order $m = 1$) and does not involve dynamic and complex assignment algorithms. For higher orders, the basic principles of intermixed grouping arrangements of channel resources are presented in the next section. Without loss of generality, the increase in the number of users and at least two self-evident crucial factors, namely the PTR and probability of steady state, can be considered for evaluating the collision tolerance performance of the conventional FCA scheme and the proposed FPP scheme, which are illustrated in **Figs. 2(a)** and **2(b)**, respectively.

For each user, it is a continuous-time stochastic process. In probability theory, a continuous-time Markov chain is a mathematical model that derives values in a specific finite set and for which the time spent in each state is a nonnegative real value; the time intervals spent in the different states demonstrate an exponential distribution. It is a continuous-time stochastic process with a Markov property, implying that the future behavior of the model (both remaining time in the current state and the time in the next state) depends on only the current state of the model and not on the model's previous behavior [20].

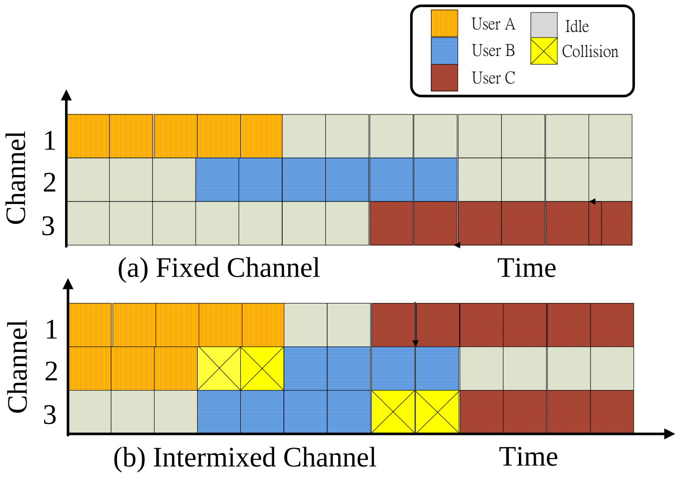


**Fig. 2. (a)** Fixed channel and **(b)** intermixed channel

In this paper, a continuous-time Markov chain modeling process of spectrum allocation of channel access is presented and investigated. Furthermore, an efficient channel assignment scheme that is based on FPP theory is proposed. In this scheme, timing slots and frequencies are equitably arranged in an intermixed grouping style without resorting to special and complex channel avoidance, tolerance, or physical layer modification techniques. Channel utilization can be improved by allowing users to transmit data through multiple channels corresponding to the point numbers of a set of FPP tables.

The rest of this paper is organized as follows. Section 2 presents the basics of FPP theory. Section 3 describes the conventional fixed, dynamic, and FPP-based channel assignment through Markov chain modeling. Section 4 presents the simulation results for both schemes. Finally, Section 5 presents the conclusion.

## 2. FPP Basics

Determining an efficient channel scheme with high channel utilization has attracted considerable interest. Before the proposed FPP-based channel assignment scheme is described, the basics of FPP theory are presented in this section. An FPP is basically a geometry that satisfies the condition that any two lines intersect at exactly one point. In short, an FPP of $N$ points has the following inherent properties [21, 22], which are of interest and worthy of investigation and expansion for use in numerous aspects of communications:

(1). An FPP of $N$ points comprises $N$ sets of points.
(2). Each set has exactly $m + 1$ points, where $m^2 + m + 1 = N$. The value of $m$ is also called the FPP order; for example, $N = 7$ if $m = 2$.
(3). Two distinct sets intersect at exactly one point. For example, an FPP of 3 points ($m = 1$) has sets $A_1 = (1, 2)$, $A_2 = (1, 3)$, and $A_3 = (2, 3)$. An FPP of 7 points ($m = 2$) is called a Fano plane (**Fig. 3**) and has the (line) sets $A_1 = (1, 2, 3)$, $A_2 = (1, 4, 5)$, $A_3 = (1, 6, 7)$, $A_4 = (2, 4, 6)$, $A_5 = (2, 5, 7)$, $A_6 = (3, 5, 6)$, and $A_7 = (3, 4, 7)$.

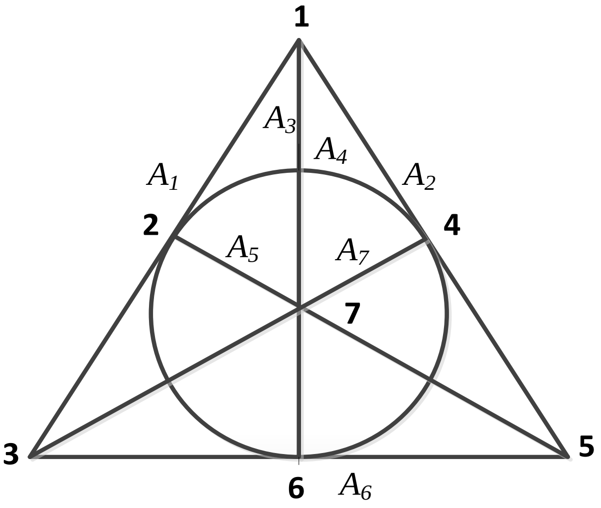


**Fig. 3.** Fano plane of order 2 and having three basic FPP properties

For example, as shown in **Table 1**, for $m = 2$, $A_1$ intersects $A_2$ to $A_7$ at exactly only one point. **Table 1** also shows the typical FPP intermixed resource allocation schemes for $m = 2$ and 3, which are combined with Latin squares of dimensions three and four, respectively. Therefore, FPPs have at least two critical properties. First, each pair of sets intersects at exactly one point. Second, the number of occurrences of a point number among the sets is constant (i.e., $m + 1$). This holds for high-order FPPs. Therefore, if a node competes with different groups of nodes, fairness is guaranteed.

Previous studies have investigated the characteristics of FPP and the application of FPP to decentralized consensus protocols [23-25]. In addition, numerous studies have investigated FPP theory and its applications. A previous study proposed an alternative to Walsh functions for variable spreading codes, which are essential for multirate services in the Third Generation Partnership Project [26]. Another study proposed a complex FPP-based orthogonal design that can be used in certain applications such as sensor networks and deep space exploration, in which a limit might be imposed on the peak transmit power [27].

A cyclic FPP plane was demonstrated to be equivalent to that of a difference set [24]. A set of $m + 1$ residues $D{:}\{d_1\ldots,d_{m+1}\} \oplus (m^2 + m + 1)$ is called an FPP $(m^2 + m + 1, m + 1, 1)$-difference set if for every $(q \neq 0) \oplus (m^2 + m + 1)$, there exists exactly one ordered pair $(d_u, d_v)$, where $d_u, d_v \in D$ such that $d_u - d_v \equiv q \oplus (m^2 + m + 1)$; for example, $D = \{0, 1, 3\}, \{0, 2, 6\}, \{2, 3, 7\}$ for $m = 2$ and $D = \{0, 1, 3, 9\}, \{0, 2, 5, 6\}, \{0, 3, 5, 12\}$ for $m = 3$. Hence, we can construct $m^2 + m + 1$ FPPs, with each FPP having a dual index. We can then generate $(m^2 + m + 1)$ points and $(m^2 + m + 1)$ lines such that the points are incident with the lines. Let $D$:$\{d_1\ldots,d_{m+1}\}$ be an $(m^2 + m + 1, m + 1, 1)$-difference set. Subsequently, we can use an equation to generate $(m^2 + m + 1)$ points and $(m^2 + m + 1)$ lines such that the points are incident with the lines.

Similarly, for an FPP of 13 points ($m = 3$), as shown in **Table 2** with $m = 3$, the sets are as follows: $A_1 = (1, 2, 3, 4)$, $A_2 = (1, 5, 6, 7)$, $A_3 = (1, 8, 9, 10)$, $A_4 = (1, 11, 12, 13)$, $A_5 = (2, 5, 8, 11)$, $A_6 = (2, 6, 9, 12)$, $A_7 = (2, 7, 10, 13)$, $A_8 = (3, 5, 10, 12)$, $A_9 = (3, 6, 8, 13)$, $A_{10} = (3, 7, 9, 11)$, $A_{11} = (4, 5, 9, 13)$, $A_{12} = (4, 6, 10, 11)$, and $A_{13} = (4, 7, 8, 12)$.

**Table 1.** Seven-point FPP with seven sets of lines ($m = 2$)

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | $B_7$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 1 | 2 | 3 | | | | |
| $A_2$ | 2 | | | 3 | 1 | | |
| $A_3$ | 3 | | | | | 2 | 1 |
| $A_4$ | | 3 | | 2 | | 1 | |
| $A_5$ | | 1 | | | 3 | | 2 |
| $A_6$ | | | 1 | | 2 | 3 | |
| $A_7$ | | | 2 | 1 | | | 3 |

**Table 2.** Thirteen-point FPP with 13 sets of lines ($m = 3$)

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | $B_7$ | $B_8$ | $B_9$ | $B_{10}$ | $B_{11}$ | $B_{12}$ | $B_{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_1$ | 1 | 2 | 3 | 4 | | | | | | | | | |
| $A_2$ | 2 | | | | 1 | 4 | 3 | | | | | | |
| $A_3$ | 3 | | | | | | | 4 | 1 | 2 | | | |
| $A_4$ | 4 | | | | | | | | | | 3 | 2 | 1 |
| $A_5$ | | 3 | | | 4 | | | 1 | | | 2 | | |
| $A_6$ | | 4 | | | | 1 | | | 2 | | | 3 | |
| $A_7$ | | 1 | | | | | 2 | | | 4 | | | 3 |
| $A_8$ | | | 4 | | 2 | | | | | 3 | | 1 | |
| $A_9$ | | | 1 | | | 3 | | 2 | | | | | 4 |
| $A_{10}$ | | | 2 | | | | 4 | | 3 | | 1 | | |
| $A_{11}$ | | | | 1 | 3 | | | | 4 | | | | 2 |
| $A_{12}$ | | | | 3 | | 2 | | | | 1 | 4 | | |
| $A_{13}$ | | | | 2 | | | 1 | 3 | | | | 4 | |

**Tables 1** and **2** also show the typical FPP intermixed resource allocation schemes of orders 2 and 3. The schemes are combined with Latin squares of dimensions three and four, respectively. Therefore, FPPs have at least two vital properties. First, each pair of sets intersects at exactly one point. Second, the number of occurrences of a point number among the sets is constant (i.e., $m + 1$). The link between **Fig. 3** and **Table 1** is clear, where each line set (e.g., $A_3$,= (1, 6, 7)) has $m + 1 = 3$ points since $m = 2$. Similarly, each point set (e.g., $B_3 = (1, 6, 7)$) has $m + 1 = 3$ lines because $m = 2$. The numbers shown in **Tables 1** and **2** are listed for labeling the $m + 1$ group members of each line set or point set.

**Table 3** shows a resource allocation scheme ($m = 2$) similar to that shown in **Table 1**, but each number shown is divided by six, which is the the sum of the point numbers in each row or column (i.e., the sum of probabilities in a row or column is one). The case of high-order FPPs is similar (e.g., **Table 2** for order $m = 3$). Therefore, if a node competes with different groups of nodes, fairness is guaranteed. The fraction numbers shown in **Table 3** are listed to demonstrate that the $m + 1$ group members of each line set or point set can be manipulated to be fair, with the sum of probabilities being 1 (e.g., (1/3, 1/3, 1/3) or (1/6, 2/6, 3/6)).

**Table 3.** FPP scheme ($m = 2$) with intermixed grouping and guaranteed fairness

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | $B_7$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 1/6 | 2/6 | 3/6 | | | | |
| $A_2$ | 2/6 | | | 3/6 | 1/6 | | |
| $A_3$ | 3/6 | | | | | 2/6 | 1/6 |
| $A_4$ | | 3/6 | | 2/6 | | 1/6 | |
| $A_5$ | | 1/6 | | | 3/6 | | 2/6 |
| $A_6$ | | | 1/6 | | 2/6 | 3/6 | |
| $A_7$ | | | 2/6 | 1/6 | | | 3/6 |

## 3. FPP-Based Channel Assignment Scheme

The main property of a Markov chain model is that the future behavior of the model depends on only the current state of the model and not on its historical behavior. This section presents the continuous-time Markov chain model used for spectrum allocation for channel access. This model was applied to both the FCA and FPP channel assignment schemes for comparisons.

### 3. 1 Markov Chain Modeling

In this subsection, we present formulas that can be used to determine the steady-state operating characteristics of channel assignment schemes through Markov chain modeling. The formulas are applicable if the arrivals of users follow a Poisson probability distribution and the service channels follow an exponential probability distribution. Because these assumptions apply to the channel assignment problem introduced in previous sections, we show how the formulas can be used to determine the operating characteristics of a channel assignment scheme. The mathematical methodology used to derive the formulas for the operating characteristics of channel assignment schemes is rather complex. Here, our purpose is not to detail the theoretical development of models, but to show how the developed formulas can provide information about operating characteristics of channel assignment schemes. First, we assume that both the number of channels and users in a scheme are identical (i.e., $N$). The available communication bandwidth is then divided into $N$ channels for allocation to $N$ independent users. The number $N$ is limited by the FPP order since $N = m^2 + m + 1$. **Fig. 4** illustrates the Markov chain model used for allocating $N$ channels to $N$ users, where the steady-state number is $N + 1$, $\lambda$ is the mean number of arrivals per unit time (mean arrival rate), and $\mu$ is the mean number of services per unit time (mean service rate). The ratio of $\lambda$ to $\mu$ (denoted by $\alpha$) is defined as the channel utilization ratio.

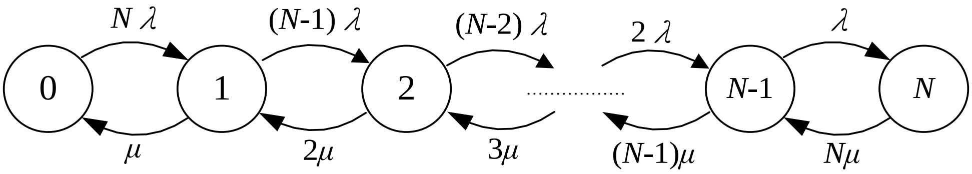


**Fig. 4.** Markov chain model allocating $N$ channels to $N$ users

As illustrated in **Fig. 4**, the associated states **S** are related to the probability of each steady state. Transitions between states in the Markov model are characterized by an infinitesimal generator matrix **B,** which is expressed as follows **[28]**:

$$\mathrm{B} = \left\{ \begin{array}{cccccccc} \Phi_0 & N\lambda & 0 & & & 0 & 0 & 0 \\ \mu & \Phi_1 & (N-1)\lambda & & & 0 & 0 & 0 \\ 0 & 2\mu & \Phi_2 & \cdots & \cdots & 0 & 0 & 0 \\ & \vdots & 3\mu & \ddots & & & & \\ & \vdots & & & \ddots & 3\lambda & & \\ 0 & 0 & 0 & & & \Phi_{N-2} & 2\lambda & 0 \\ 0 & 0 & 0 & \cdots & \cdots & (N-1)\mu & \Phi_{N-1} & \lambda \\ 0 & 0 & 0 & & & 0 & N\mu & \Phi_N \end{array} \right\}. \tag{1}$$

Moreover,

$$\mathbf{SB} = 0. \tag{2}$$

In the preceding equations, $\mathbf{S} = [S_0, S_1, S_2, \ldots, S_N]$ is the steady-state probability (SSP) vector, and $\Phi_0 = -N\lambda$, $\Phi_1 = -\mu - (N-1)\lambda$, $\Phi_2 = -2\mu - (N-2)\lambda$ . We can then derive $\mathbf{S} = [1, P_1, P_2, \ldots, P_{N\text{-}1}, P_N]P_0$ with $\lambda/\mu = \alpha$, where

$$P_1 = Na,\ P_2 = \frac{N!}{(N-2)!2!}a^2, \ldots,\ P_{N-1} = \frac{N!}{(N-(N-1))!(N-1)!}\alpha^{N-1},\ P_N = \alpha^N \tag{3}$$

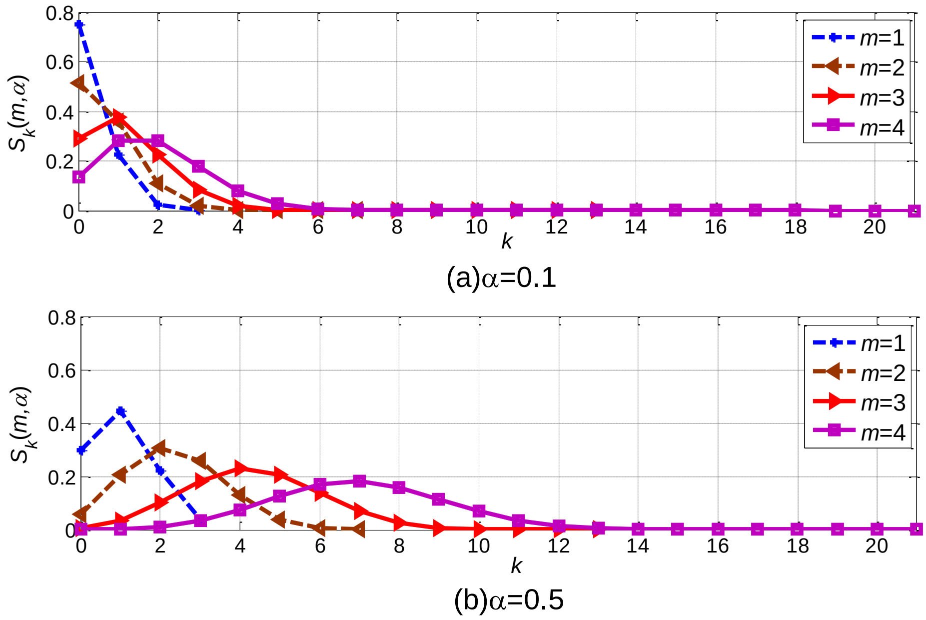
and

$$P_0(m,\alpha) = \left[\sum_{k=0}^{N} \frac{N!}{(N-k!)k!}\left(\frac{\lambda}{\mu}\right)^k\right]^{-1} = \left[\sum_{k=0}^{N} \frac{N!}{(N-k!)k!}\alpha^k\right]^{-1}, \tag{4}$$

which is the probability that no channel is used (i.e., idle channel) in the system. When $P_0$ is lower, the system is busier; by contrast, when $P_0$ is higher, the system is less busy. Therefore, the general formula of the SSP of a Markov chain model for channel assignment schemes (detailed subsequently) can be expressed as

$$S_k(m,a) = \frac{N!}{(N-k)!k!}a^k \times P_0(m,a),\ k=0, 1, 2, \ldots, N, \tag{5}$$

where $k$ is the number of users present (possibly from 0 to $N$ (= $m^2 + m + 1$)), $m$ is the FPP order, and $\alpha$ is the channel utilization ratio defined previously. Eq. (5) is applied to both the FCA scheme and FPP channel assignment scheme, in which $N$ channels are assigned to $N$ users. **Fig. 5** shows the SSP distribution of the Markov chain model for variable $m$ and channel utilization ratios of 0.1 and 0.5. For example, if $m$ increases from 1 to 4, the number of users and number of channels available also increase (since $N = m^2 + m + 1$). As shown in **Fig. 5(a)**, under light load conditions ($\alpha = 0.1$), the SSP distribution is concentrated in the area of low $k$ values when $m$ increases from 1 to 4. The system is essentially less busy. In addition, the idle probability ($k = 0$) decreases from 0.75 to 0.14 when $m$ is increased from 1 to 4, implying that the system is busier when $m$ is higher (i.e., when the number of users and channels is increased). Nevertheless, as shown in **Fig. 5(b)**, under high load conditions ($\alpha = 0.5$), the SSP distribution shifts to the right and is concentrated in the area of higher $k$ values when $m$ is varied from 1 to 4, which means the system is busier because more channels are being used by users.



(a)α=0.1

(b)α=0.5

**Fig. 5.** SSP $S_k(m, \alpha)$ for (a) $\alpha = 0.1$ and (b) $\alpha = 0.5$

## 3.2 FCA Scheme

According to (5), the SSP distribution of the Markov chain model can be directly applied to the FCA scheme. Basically, in the FCA scheme, the entire bandwidth is divided into $N$ channels and is allocated to $N$ independent users, with each user being assigned one channel. Each user has exclusive rights to the assigned channel. This scheme can effectively prevent collisions, but at the cost of a low PTR, which is defined as the ratio of the number of users punched through the available channels. The PTR for the FCA scheme is expressed as follows:

$$W_k^{FCA}(m) = \frac{k}{N} \qquad \text{for} \qquad k = 1, 2, 3 \ldots N. \tag{6}$$

**Fig. 6** illustrates the PTRs for the FCA scheme with the value of $m$ ranging from 1 to 4 (relative to channel number $N$). The PTR varies linearly with the number of user $k$, as shown in (6).

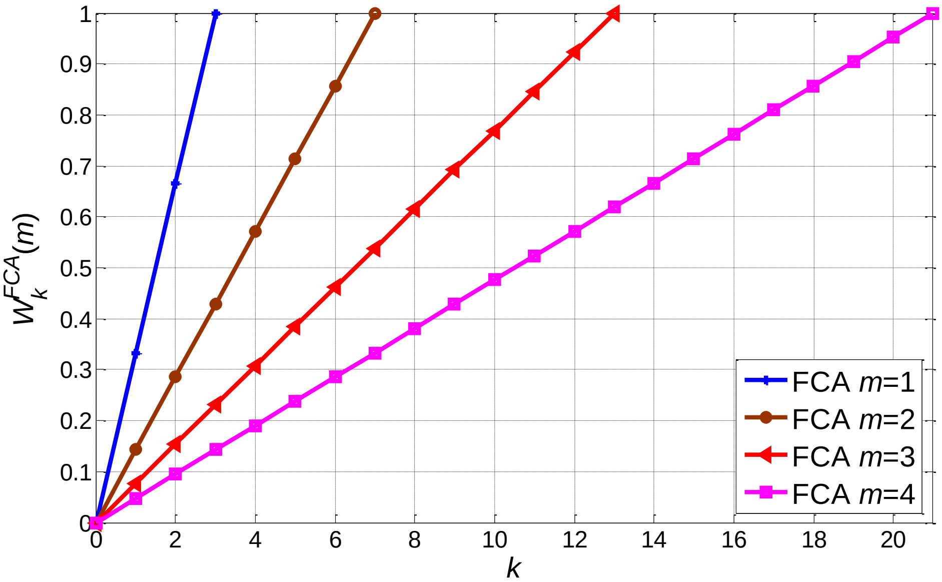


**Fig. 6.** PTR $W_k^{FCA}$ for the FCA scheme

To evaluate the channel assignment performance, the sum of the products of the SSPs and PTRs (i.e., (5) and (6)) is defined as the effective PTR (EPTR) of the FCA scheme and is expressed as

$$\rho^{FCA}(m,\alpha) = \sum_{k=1}^{N} S_k(m,\alpha) \cdot W_k^{FCA}(m) \,. \tag{7}$$

The EPTR of the FCA scheme is essentially a function of *m* (and therefore indirectly related to the number of channels or users) and $\alpha$.

### 3.3 FPP-Based Channel Assignment Scheme

For the FPP channel assignment scheme, the entire bandwidth is divided into *N* channels and are allocated to *N* independent users. Nevertheless, according to the FPP characteristics described in Section 2, at most $(m+1)N\lambda$ arrivals and $(m+1)N\mu$ services exist (with specific $\alpha$). Therefore, the distribution of the SSP of Markov chain modeling in (5) can be directly applied to the FPP channel assignment scheme. Moreover, the PTR for the FPP channel assignment scheme is expressed as follows:

$$W_k^{FPP}(m) = \begin{cases} (m+1)/N & ,k=1 \\ (m \cdot k)/N & ,k=2,\ldots,m+1 \\ \left(2m^2+(2-k)m-1\right)/N & ,k=m+2,\ldots,2m+1 \\ \left(m-1-\left\lceil \dfrac{k-(2m+1)}{m} \right\rceil\right)/N & ,k=2m+2,\ldots,N \end{cases} \tag{8}$$

where *k* is the number of users present and the number of stages *k* for $W_k^{FPP}$ ranges from 1 to $N = m^2 + m + 1$ (**Fig. 7**). Fig. 8 shows noteworthy characteristics of the PTR in the FPP channel assignment scheme.

**Fig. 7.** Stages of $k$ for $W_k^{FPP}$ in Eq. (8)

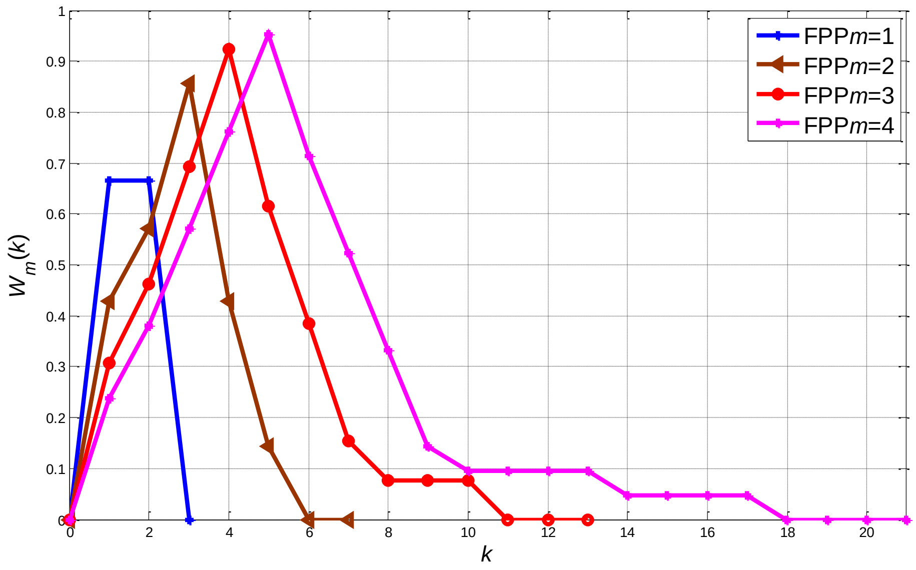


**Fig. 8.** PTR characteristics ($W_k^{FPP}$) for $m$ in the range of 1 to 4

For $k = 0$ (no user), $W_k^{FPP}$ is also zero, and therefore, the number of stages starts from $k = 1$; because no collision occurs for $k = 1$ (one user), the PTR is expressed as $(m + 1)/N$. For the stage from $k = 2$ to $m + 1$, collisions among users are limited. In this range, collisions increase with $k$ and reach the peak PTR value at $k = m + 1$. Subsequently, when $k$ ranges from $m + 2$ to $2m + 1$, collisions occur among users, causing the PTR to drop $m$-fold. When $k$ is increased from $2m + 2$ to $N = m^2 + m + 1$, PTR gradually decreases and finally returns to its minimum value. For example, **Table 4** shows $W_k^{FPP}$ values ($m = 2$) for seven users and seven channels, where **S** represents the SSP vector; $W_k^{FPP} = 0$ for $S_0$, and $W_k^{FPP}$ reaches its peak value in the $S_3$ state ($m + 1 = 3$). Next, $W_k^{FPP}$ begins to gradually decrease to ground. It can be derived using a similar approach to that of obtaining $W_k^{FPP}$ for high $m$. Furthermore, in comparison with (8), $W_k^{FPP}$ with SIC algorithm ($W_k^{FPP\text{-}S}$) can be expressed as follows:

$$W_k^{FPP-S}(m) = \begin{cases} (m+1)/N & ,k=1 \\ \left(\left(m \cdot k\right)+1\right)/N & ,k=2,\ldots,m+1 \\ 1 & ,k=m+2,\ldots,N \end{cases} \tag{9}$$

Because contemporary wireless systems are becoming increasingly susceptible to interference, using advanced interference mitigation techniques to improve network performance in addition to the conventional approach of treating interference as background noise have received increasing interest. As shown in **Fig. 8** and **Table 4**, overlapping channels mutually interfere with each other. Interference mitigation techniques should be developed to prevent this mutual interference. A major approach is SIC, which is based on the concept of resolving users sequentially. In this approach, interference associated with resolved users is removed before resolving other users [20, 21]. Although SIC is not always the optimal multiple access scheme in wireless networks, in many cases, it is easy to implement and attains boundaries of the capacity regions in multiuser systems. Therefore, if the proposed scheme is combined with an SIC technique (i.e., if $W_k^{FPP}$ is combined with SIC as shown in **Table 4**), $W_k^{FPP}$ reaches its maximum level (one) ideally.

**Table 4.** Variation of $W_k^{FPP}$ for $m$ = 2

| S | Description | $W_k^{FPP}$ | $W_k^{FPP}$ w/ SIC |
|---|---|---|---|
| $S_0$ | All channels idle. | 0 | 0 |
| $S_1$ | 1 user occupies 3 channels w/o overlap | 3/7 | 3/7 |
| $S_2$ | 2 users occupy 5 channels w/ 1 overlap | 4/7 | 5/7 |
| $S_3$ | 3 users occupy 7 channels w/ 1 overlap | 6/7 | 1 |
| $S_4$ | 4 users occupy 7 channels w/ 4 overlap | 3/7 | 1 |
| $S_5$ | 5 users occupy 7 channels w/ 6 overlap | 1/7 | 1 |
| $S_6$ | 6 users occupy 7 channels w/ 7 overlap | 0 | 1 |
| $S_7$ | 7 users occupy 7 channels w/ 7 overlap | 0 | 1 |

In the FPP scheme, if the number of channels assigned to each user is increased to $m$ + 1 and one of the collided channels is left unassigned, then the PTRs of the first $m$ + 1 users increase $m$-fold. After the first $m$ + 1 users use all the channels, subsequent users would face the problem of collisions. The available factors greatly decrease $m$-fold. As the number of users exceeds 2$m$ + 1, the PTRs demonstrate a stable decrease until they reach zero. Furthermore, as the number of users increases, the PTR shows a swift increase initially. The optimal value is obtained for $m$ + 1 users, and it is equal to $(N − 1)/N$.

**Fig. 9** shows that the peak PTR values in the FPP scheme are obtained for $k = m + 1$ (e.g., $k$ = 5 for $m$ = 4). The expressions related to the simulation curves in **Fig. 9** are presented in (6) and (8) for a constant number of users. In this figure, the original linear curve for the FCA scheme with $m$ = 4 is presented only for comparison with the curves of the FPP scheme with $m$ values ranging from 1 to 4. For a clear illustration, the linear curves of the FCA scheme with $m$ values ranging from 1 to 4 are also shown for comparison. The value of $W_k^{FCA}$ varies linearly with $k$. By contrast, the value of $W_k^{FPP}$ peaks for $k = m + 1$ and then gradually decreases to ground, reflecting favorable punch through characteristics, particularly for light traffic load.

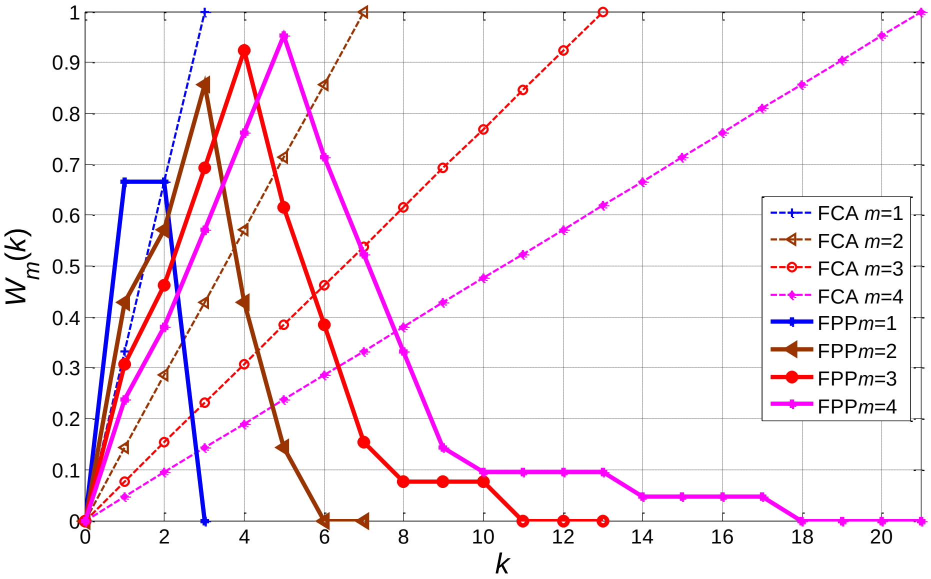


**Fig. 9.** Comparison of PTR characteristics of $W_k^{FCA}$ and $W_k^{FPP}$ for $m$ in the range from 1 to 4

Similar to (7), to evaluate the channel assignment performance, the sum of the products of the SSPs and PTRs (i.e., (5) and (8)) is defined as the EPTR of the FPP channel assignment, and it is expressed as follows:

$$\rho^{FPP}(m,\alpha)=\sum_{k=1}^{N} S_k(m,\alpha)\cdot W_k^{FPP}(m) \tag{10}$$

If the FPP scheme is with SIC, the EPTR performance evaluation can beavailable from (10) by replacing the PTR item with (9).

### 3.4 DCA Scheme

This paper proposes an FPP channel assignment scheme that is based on FPP theory with the basic assumptions that the available channels are all healthy. The proposed scheme involves using a Markov chain model to allocate $N$ channels to $N$ users through intermixed channel group arrangements, particularly when channel resources are idle because of inefficient use. In addition to FCA, another traditional DCA scheme can be also used to improve the FCA disadvantages, especially when the traffic loads are low. Basically, the DCA scheme is designed for more flexible channel access. Nevertheless, it needs additional estimation and signaling channels for accessing the available traffic channels. Moreover, its implementation complexity is increased because of real time channels estimation. Therefore, without loss of generality, we can assume that there exist $m$ estimation and signaling channels for the DCA scheme when in comparison with the FPP scheme with $m$ order and $N$ users and available channels, where $N = m^2 + m + 1$. In comparison with (6), (8) and (9), the PTR for the DCA scheme is expressed as follows:

$$W_k^{DCA}(m)=\frac{N-m}{N} \quad ,k=1,\ldots,N \tag{11}$$

The EPTR performance evaluation for the DCA scheme can be also available from (7) by replacing the PTR item with (11).

## 4. Simulation Results

In this section, to evaluate the performance of the FCA scheme and the proposed FPP channel assignment scheme according to (7)–(9), the EPTRs for both schemes are demonstrated for the variables $\alpha$ and $k$.

**Fig. 10** shows plots of $\rho^{FPP}$ against $\alpha$ for different $m$ values along with a plot of $\rho^{FCA}$ against $\alpha$ (for $m = 4$) for comparison. For example, when the FPP order is set as $m = 1, 2, 3$, and 4, the peak EPTRs occur at $\alpha = 1, 0.6, 0.4$, and 0.3, respectively, and the FPP curve intersects the FCA curve at $\alpha$ =1, 0.98, 0.8, and 0.62, respectively. The performance of the proposed FPP channel assignment scheme is enhanced, particularly when $\alpha$ is lower than approximately 0.6 for $m = 4$. The EPTR values reach 0.65 and 0.60 for $m = 4$ and $m = 3$, respectively. In short, the proposed scheme improves channel utilization, particularly under a light traffic load. When m approaches infinity, the EPTR value abruptly reaches 1 at $\alpha = 0.1$. It shows obviously that the EPTR characteristic is improved in the light traffic load condition because of the inherent intermixed characteristics of the FPP scheme, which allows for channel sharing and ensure fair resource use among users simultaneously. Nevertheless, the EPTR performance degrades even worse as $m$ increases in the heavy traffic load condition because of more users and collisions.

In addition to the traditional FCA scheme, another traditional DCA scheme can be designed for more flexible channel assignments. Without loss of generality, we can assume that there exist $m$ estimation and signaling channels for the DCA scheme when in comparison with the proposed FPP scheme with $m$ order and $N$ users and available channels, where $N = m^2 + m + 1$. In order to differentiate the FPP scheme from the DCA scheme, **Fig. 10** also shows plots of EPTR performance curves against $\alpha$ for the DCA scheme and the FPP schemes without SIC ($m = 1$ to 4). From this figure, the DCA scheme outperforms the FPP scheme without SIC under any traffic load conditions in addition to the case with $m = 1$. Nevertheless, it needs additional and complex estimation and signaling channels for accessing the available traffic channels because of requirements of real time channels estimation.

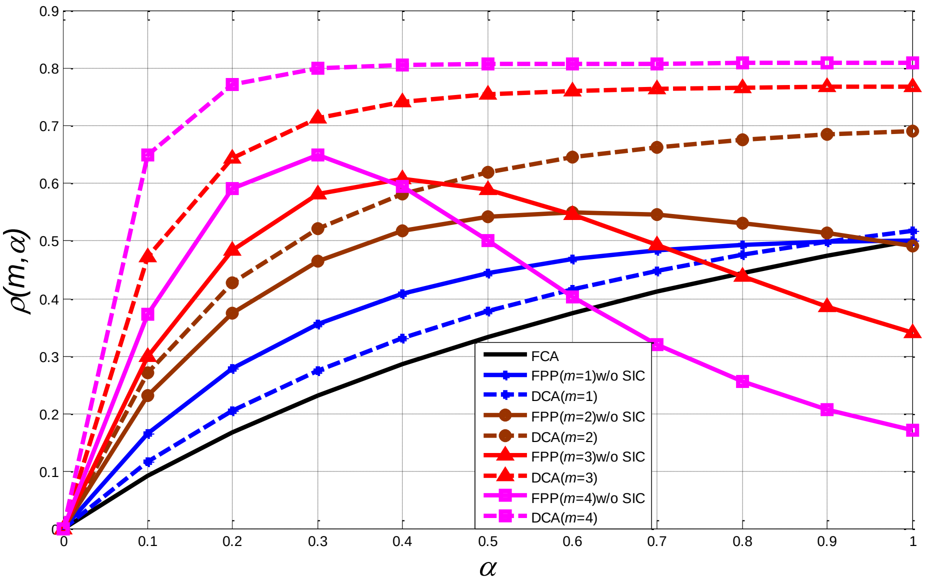


**Fig. 10.** Comparison of plots of $\rho^{FCA}$, $\rho^{DCA}$, and $\rho^{FPP}$ (with $m$ in the range from 1 to 4) against $\alpha$

**Fig. 11** shows a comparison of plots of $\rho^{FPP}$ against $\alpha$ for the proposed scheme with and without SIC ($m$ = 1 to 4). The proposed scheme with SIC processing improves channel utilization under heavy traffic load. Clearly, the EPTR curves for the scheme with SIC gradually approach 1 under heavy traffic load conditions, particularly for a higher $m$. The EPTR performance with SIC improves even better as $m$ increases in any traffic load conditions because of collision recovery. The EPTR performance with SIC improves further as $m$ increases while in comparison with the case without SIC processing. Moreover, under the same traffic load conditions, it also shows that the EPTR performance is enhanced for the FPP scheme with SIC because of the PTR characteristics from (8) and (9), where the peaks occur at $k = m + 1$ relative to the user number $N = m^2 + m + 1$.

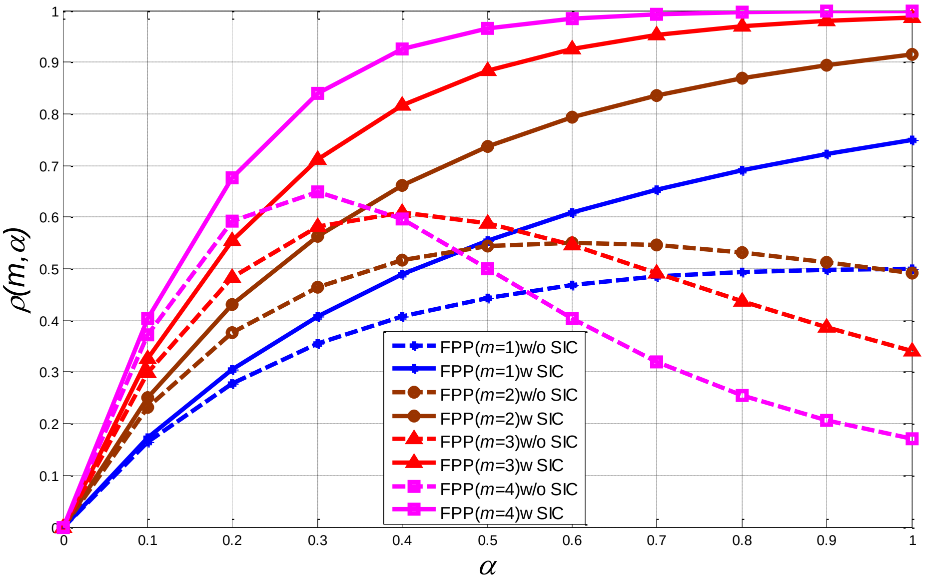


**Fig. 11.** Comparison of plots of $\rho^{\mathrm{FPP}}$ against $\alpha$ for the FPP scheme with and without SIC

**Fig. 12** illustrates plots of $\rho^{FPP}$ against $\alpha$ for both the FCA and FPP schemes with and without SIC processing with a constant $m$ value ($m$ = 4) and various values ($\alpha$ = 0.1 and 1.0). It indicates that the EPTR values peak at 0.40, 0.37 and 0.09 for the proposed FPP scheme with and without SIC and the FCA schemes, respectively, under light traffic load conditions ($\alpha$ = 0.1). The EPTR values peak at 1.00, 0.17 and 0.50 for the proposed FPP scheme with and without SIC and the FCA schemes, respectively, under heavy load conditions ($\alpha$ = 1.0). The EPTR values peak at specific levels for both schemes as $k$ is increased. When the FPP scheme is used with SIC, particularly under heavy load conditions ($\alpha$ = 1.0), the EPTR values peak at 1.0 for a $k$ value of approximately 16. Under light traffic load condition, the FPP schemes with and without SIC perform approximately the same; on the contrary, the proposed scheme with SIC outperforms the proposed scheme without SIC under heavy traffic load condition. Collision recovery through SIC processing can enhance EPTR performance evidently.

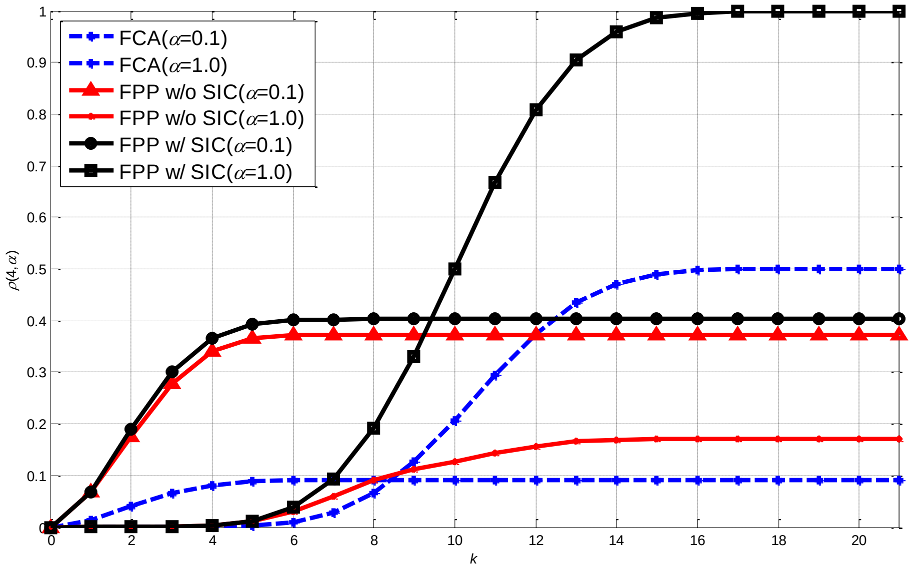


**Fig. 12.** Comparison of plots of $\rho^{FCA}$ and $\rho^{FPP}$ against $k$ for $\alpha = 0.1$ and 1.0 and $m = 4$

As aforementioned, the DCA scheme outperforms the FPP scheme without SIC under any traffic load conditions in addition to the case with $m = 1$. In order to differentiate the FPP scheme with SIC from the DCA scheme, **Fig. 13** show plots of EPTR performance curves against α for the DCA scheme and the FPP schemes with SIC ($m = 1$ to 4), respectively. The FPP scheme with SIC outperforms the DCA scheme under $\alpha > 0.3$ traffic load conditions while $m$ order is larger than 1.

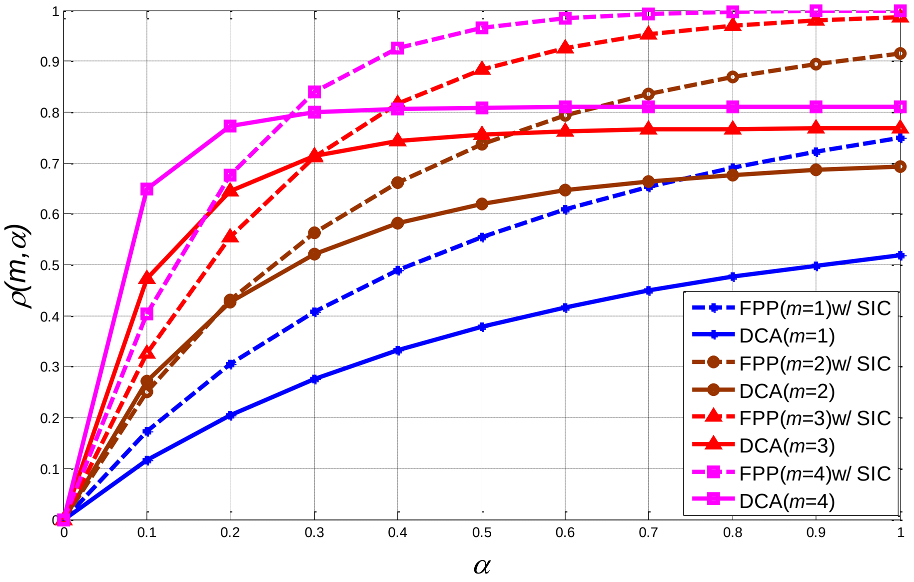


**Fig. 13.** Comparison of plots of $\rho^{FPP}$ with SIC and $\rho^{DCA}$ against $\alpha$

## 5. Conclusion

We propose an efficient FPP channel assignment scheme that allows each user in a system to be allocated multiple channels in an intermixed grouping style to improve the channel utilization performance, which is restricted in conventional assignment schemes such as the FCA scheme. A Markov chain model was used to analyze the performance of the FCA, the DCA schemes, and the proposed FPP channel assignment scheme. The EPTR increased and fairness was guaranteed, particularly when the offered traffic load was light because of the proposed intermixed assignment of channels according to FPP theory. Nevertheless, if the proposed channel assignment scheme is combined with SIC techniques, considerably higher EPTR performance is predicted, even under heavy traffic load conditions.

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