Interference Alignment in Two-way Relay Channel with Compute-and-Forward

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Abstract

This paper analyzes interference alignment in the two-way relay network with compute-and-forward in both single relay and multiple relays networks. The advantages of compute-and-forward over other relaying strategies are that it can relay only linear combinations of the useful signals and remove the noise. The algorithm proposed in this paper adopts the criterion of maximum SINR to derive the pre-coding matrix. The experimental results show that the performance of interference alignment in two-way relay channel via compute-and-forward is better than that of amplify-and-forward, and the total sum rate in the two-way multiple relay networks is larger than that in the two-way single relay networks.

Keywords: Wireless communications, Interference alignment, two-way relay channel, compute-and-forward

1. Introduction

In past decades, interference is a primary problem to restrict the development of wireless network. Various methods have been proposed to deal with such issue in the multi-user communication networks. Recently, the interference alignment [1]-[4] algorithm has been presented as an efficient technique. In the target to improve the system capacity, interference alignment is employed in the procedures of the pre-coding at the transmitters and the interference suppression at the receivers, which achieves a better multiplexing gain in interference networks. It is now widely used in two-way relay channels via amplify-and-forward (AF) [5]-[7]. The feasibility conditions of interference alignment are analysized in multi-user two-way relay networks [5]. Later, a more relaxed alignment conditions are introduced to optimize the sum rate of network in [6]. Based on this idea, the sum degrees of freedom for the two-way relay network is further discussed with different antenna number of users and relay node [7]. Reports also show the amplify-and-forward strategy leads to relays of both transmitted message**s** and noise. Meanwhile, a compute-and-forward(CF) [8]-[10] strategy is developed to enable relays to decode linear equations of transmitted message using the noise linear combinations provided by the channel, and then only forwarding the desired message, which is based on nested lattice. More specifically, the transmitting procedure of compute-and-forward can be described as follows. Firstly, each transmitter maps its message from the finite field into an element of nested lattice code. Then the lattice codes are transmitted over the channel, where each relay decodes a linear equation of the lattice codes. Finally, these lattice equations are re-mapped in the finite field to obtain the desired linear combination of message. Following the aforementioned process, a new interference alignment algorithm is proposed via a channel diagonalization [11] approach based on the measure of the MAX-SINR, Moreover, this paper also analyze**s** the performance of the interference alignment in both two-way single relay networks and two-way multiple relays networks via compute-and-forward.

2. SYSTEM MODEL

In the two-way relay network, each of the nodes wants to transmit data streams to its corresponding partner, which means that a bidirectional communication is assumed. There is no direct link between the nodes but there are two communication phases between them. In the first phase, all the nodes transmit their signals to the relay, which is called multiple access(MAC) phase, as shown in **Fig.** 1 (a). In the second phase, the relay broadcasts the signals to the destination nodes, which is called broadcast(BC) phase, as shown in **Fig.** 1 (b).
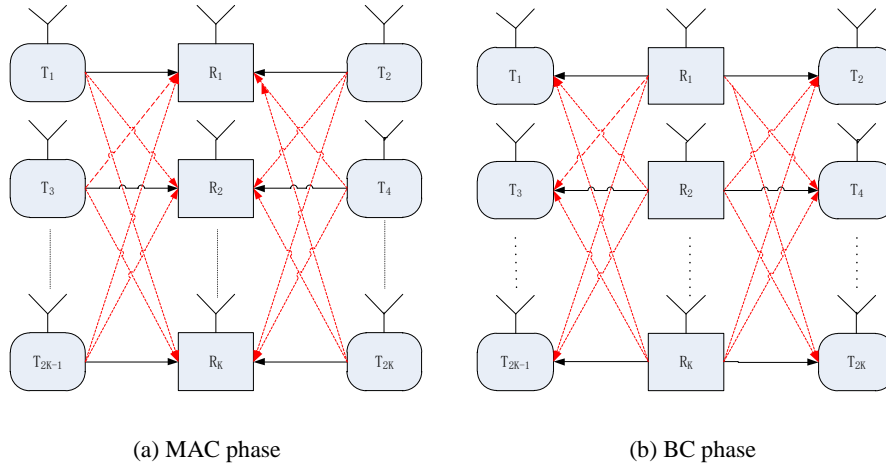
(a) MAC phase   (b) BC phase

Fig. 1. System model of the two-way relay interference channels.

It assumed that 2K nodes are to transmit data streams x_k to their corresponding partner. In the MAC phase, each transmitter maps its message into lattice codeword through codebook L.

$$t_k = L(x_k) \tag{1}$$

Then add dither vector d_k to lattice codeword, and transmit as w_k :

$$w_k = [t_k + d_k] \bmod \Lambda \tag{2}$$

where Λ is the coarse lattice.

All the transmitters transmit the codeword w_k to the relay node. The j-th relay node receives the linear combination of codeword and noise r_j, scales it by β_j and then removes the dithers:

$$[\beta_j r_j - \sum_{i=1}^{2K} \alpha_{ij} d_i] \bmod \Lambda = [\beta_j \sum_{i=1}^{2K} (H_{ji} w_i + z_i) - \sum_{i=1}^{2K} \alpha_{ij} d_i] \bmod \Lambda \tag{3}$$

$$= [\sum_{i=1}^{2K} \alpha_{ij}(w_i - d_i) + \sum_{i=1}^{2K} (\beta_j H_{ji} - \alpha_{ij}) w_i + \sum_{i=1}^{2K} \beta_j z_i] \bmod \Lambda$$

$$= [[\sum_{i}^{2K} \alpha_{ij} t_i] \bmod \Lambda + \sum_{i=1}^{2K} (\beta_j H_{ji} - \alpha_{ij}) w_i + \sum_{i=1}^{2K} \beta_j z_i] \bmod \Lambda$$

$$= [[\sum_{i}^{2K} \alpha_{ij} t_i] \bmod \Lambda + Z_{eff}(h, \alpha, \beta)] \bmod \Lambda$$

where α_{ij} is the coefficient vector of linear combination of the transmitted messages. Define $z_{eff}(h, \alpha, \beta) = \sum_{i=1}^{2K} (\beta_j H_{ji} - \alpha_{ij}) w_i + \sum_{i=1}^{2K} \beta_j z_i$. The last line of equation (3) is in the form of the nested codeword which is the linear combination of codeword and noise. Then relays use channel coefficient to decode. Let $a_{ij} = g^{-1}([\alpha_{ij}] \bmod p)$, and p is a prime number. Recall that g^{-1} maps finite messages to the corresponding element. The signals

received at the j-th relay node are given by:

$$y_{rj} = \sum_{i=1}^{2K} a_{ij} x_i \tag{4}$$

In the BC phase, the relay node firstly decodes y_{rj}, then broadcasts the signals to the destination nodes. Each receiver firstly removes the self-interference signals, then suppresses interference and finally has the desired signals.

3. Proposed Interference Alignment algorithms via CF

In multi-user two-way relay networks [5] [6], interference alignment is achieved by signal alignment and channel alignment. In MAC phase, the transmitted signals are pair-wise aligned at the relays, the signal alignment is given by

$$span(H_{ri}V_i) = span(H_{ri'}V_{i'}) \tag{5}$$

where V_i denotes the pre-coding matrix at the i-th node and H_{ri} denotes the channel between the i-th transmitter and the r-th relay. The i-th node and the $i'-th$ node are communication partners.

In the BC phase, channel alignment followed by zero forcing is performed to achieve interference alignment at the destination nodes. With channel alignment, the effective channels of the communication partners are made to span the same subspace. The channel alignment is given by

$$span(U_i G_{ir}) = span(U_{i'} G_{i'r}) \tag{6}$$

where U_i denotes the zero forcing matrix at the i-th receiver, G_{ir} denotes the channel between the r-th relay to the i-th receiver.

Assume each source node is equipped with L antennas to send L different messages, and the relay is equipped with N antennas. It is assumed that the antenna number of users and relay nodes satisfied the constraints of [3]. In this paper, the relaying strategy is compute-and-forward. Signal alignment means there exists linear relationship between coefficients a_{ij}. The main research of this paper is under the condition of channel alignment through channel diagonalization [11] to derive optimal pre-coding matrix in two-way single relay networks and two-way multiple relays networks respectively.

3.1 Interference alignment in two-way single relay networks

In BC phase, the relay node broadcasts the signal to all users after pre-coding. The signal received at the i-th user（$i = 1, \cdots, 2K$）is given by

$$y_i = D_i G_i P_r \sum_{j=1}^{2K} a_j x_j + D_i n_i \tag{7}$$

where D_i is interference suppression matrix at the i-th receiver, G_i is the channel matrix between the relay node and the i-th receiver, P_r is the pre-coding matrix at the i-th transmitter and n_i is an additive Gaussian noise. a_j denotes the linear coefficient of the signal which is decoded from the j-th transmitter.

After self-interference signals are cancelled, the signal to interference noise ratio (SINR) of the i-th user can be expressed as:

$$SINR_i = \frac{tr(D_iG_iP_ra_ia_iP_r^TG_i^TD_i^{\mathrm{T}})}{tr(\sum_{j\neq i,i'}^{2K} D_iG_iP_ra_ja_jP_r^TG_i^TD_i^{\mathrm{T}} + D_iD_i^{\mathrm{T}}\sigma_i^2)} \tag{8}$$

where σ^2 is the variance of noise and i' is the partner user of i.

According to (7) and (8), assume that pre-coding matrix P_r is known, the interference suppression matrices D_i and $D_{i'}$ are used to maximize the total SINR of the i-th user. D_i and $D_{i'}$ can figured out by solving an optimization problem under the constraint of (6), which is described as

$$\max_{D_i, D_{i'}} \quad SINR_i, SINR_{i'}$$
$$s.t. \quad D_iG_i = D_{i'}G_{i'} \tag{9}$$

This equation is the multi-objective optimization problem. The KKT conditions are used to solve the optimization problem with weighting method, the Lagrange function of (9) is

$$f(D_i, D_{i'}, \lambda) = SINR_i + SINR_{i'} + \lambda(D_iG_i - D_{i'}G_{i'}) \tag{10}$$

The KKT conditions are:

$$\frac{\partial SINR_i}{\partial D_i} + \lambda G_i = 0 \tag{11a}$$

$$\frac{\partial SINR_{i'}}{\partial D_{i'}} - \lambda G_{i'} = 0 \tag{11b}$$

Let $A = \sum_{j\neq i,i'}^{2K} G_iP_ra_ja_jP_r^TG_i^T + \sigma_i^2$ and $D_{i'} = D_iK$, then

$$D_iX(D_iAD_i^T - D_i^TD_iA)\left(D_iKA'(D_iK)^{\mathrm{T}}\right)\left(D_iKA'(D_iK)^{\mathrm{T}}\right) +$$
$$\left(AD_i^{\mathrm{T}}\right)\left(D_iAD_i^{\mathrm{T}}\right)D_iKD_iKY(D_iKA'(D_iK)^{\mathrm{T}} - (D_iK)^{\mathrm{T}}(D_iK)A') = 0 \tag{12}$$

But the close-form solution of (12) cannot be obtained. To solve this problem, the multi-objective optimization problem can be transformed into a single-objective one.

Let $\dfrac{\partial SINR_i}{\partial D_i} = 0$, then

$$\mathrm{D}_i^{opt} = \left(\left(G_i P_r \right) \left(G_i P_r \right)^T \sum_{j \neq i, i'}^{2K} a_j^2 + \sigma_i^2 \mathrm{I}_L \right)^{-1} \quad i = 1, \cdots, 2K \tag{13}$$

The feasible solutions of the corresponding optimization problem (9) are

$$D_i = (\sum_{j \neq i, i'}^{2K} G_i P_r a_j a_j P_r^T G_i^T + \sigma_i^2)^{-1} \tag{14a}$$

$$D_{i'} = (\sum_{j \neq i, i'}^{2K} G_i P_r a_j a_j P_r^T G_i^T + \sigma_i^2)^{-1} \cdot G_i \cdot G_{i'}^{-1} \tag{14b}$$

and

$$D_i = (\sum_{j \neq i, i'}^{2K} G_{i'} P_r a_j a_j P_r^T G_{i'}^T + \sigma_{i'}^2)^{-1} \cdot G_{i'} \cdot G_i^{-1} \tag{15a}$$

$$D_{i'} = (\sum_{j \neq i, i'}^{2K} G_{i'} P_r a_j a_j P_r^T G_{i'}^T + \sigma_{i'}^2)^{-1} \tag{15b}$$

By substituting the equation (14) into (8)

$$\mathrm{SINR}_i = \frac{tr\left(G_i P_r (G_i P_r)^T a_{i'}^2 \right)}{tr\left(G_i P_r (G_i P_r)^T \sum_{j \neq i, i'}^{2K} a_j^2 + \sigma_i^2 \mathrm{I}_L \right)} \quad i = 1, \cdots, 2K \tag{16a}$$

$$\mathrm{SINR}_{i'} = \frac{tr(G_i P_r a_i a_i P_r^T G_i^T)}{tr\left(\sum_{j \neq i, i'}^{2K} G_i P_r a_j a_j P_r^T G_i^T + G_i \cdot G_{i'}^{-1} \left(G_i \cdot G_{i'}^{-1} \right)^T \sigma_{i'}^2 \right)} \quad i = 1, \cdots, 2K \tag{16b}$$

Next, under interference alignment conditions, the optimal pre-coding matrix is derived. The relay node to the receiver channel can be reconstructed as virtual MIMO channels, which can be expressed as

$$G = \begin{bmatrix} G_1 & G_3 & \cdots & G_{2K-1} \end{bmatrix} \tag{17a}$$

$$G' = \begin{bmatrix} G_2 & G_4 & \cdots & G_{2K} \end{bmatrix} \tag{17b}$$

Apply SVD [11] to G and G'

$$G_i = V_i T U^H \tag{18a}$$

$$G_{i'} = V_{i'} T' U^H \quad i = 1, 3, \cdots, 2K - 1, i' = K + 1 \tag{18b}$$

where V_i and $V_{i'}$ are two unitary matrices. $T = \begin{bmatrix} \Lambda^T, 0^T_{(KL-N) \times N} \end{bmatrix}^T$, $T' = \begin{bmatrix} 0^T_{(KL-N) \times N}, \Lambda'^T \end{bmatrix}^T$, Λ and Λ' are nonnegative diagonal matrix. $T^T T + T'^T T' = I$. U is unitary matrices.

Then, the relay pre-coding matrix can be designed as

$$P_r = (\mathbf{U}^H)^{-1} W_r \tag{19}$$

where W_r is a diagonal matrix for relay power allocation. The ε -th diagonal element of W_r , $(UU^H)^{-1}$ and TT^H is defined as w_ε, k_ε and t_ε . By substituting the equation (18) and (19) into (16),

$$SINR_i = \frac{\left(\sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon\right) a_i^2}{\left(\sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon\right) \sum_{j \neq i, i'}^{2K} a_j^2 + \sigma_i^2} \tag{20a}$$

and

$$SINR_{i'} = \frac{\left(\sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon\right) a_i^2}{\left(\sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon\right) \sum_{j \neq i, i'}^{2K} a_j^2 + G_i \cdot G_{i'}^{-1} \left(G_i \cdot G_{i'}^{-1}\right)^T \sigma_i^2} \tag{20b}$$

From (20), it can be seen that both $SINR_i$ and $SINR_{i'}$ are monotonic functions of $\sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon$. In order to maximize $SINR_i$ and $SINR_{i'}$, the optimal w_ε can be derived by simplifying the optimization problem as

$$\max f(w_\varepsilon) = \sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon$$
$$s.t. \quad w_\varepsilon \geq 0, \sum_{\varepsilon=1}^{L} k_\varepsilon w_\varepsilon^2 \leq \rho_r \tag{21}$$

And the Lagrange function of equation (21) can be expressed as:

$$f(w_\varepsilon, \lambda) = \sum_{\varepsilon=1}^{L} w_\varepsilon^2 t_\varepsilon + \lambda(\sum_{\varepsilon=1}^{L} k_\varepsilon w_\varepsilon^2 - \rho_r) \tag{22}$$

Then according to the KKT conditions, the optimal solution can be derived as follows:

$$\omega_\varepsilon = \begin{cases} 0 & \varepsilon = 1, \cdots, \ \theta\text{-}1, \ \theta\text{+}1, \cdots \ L \\ \dfrac{\rho_\theta}{k_\theta} & \varepsilon = \theta \end{cases} \tag{23}$$

where $\dfrac{\rho_\theta}{k_\theta}$ is the maximum element in the set $\left\{\dfrac{\rho_\varepsilon}{k_\varepsilon}, \quad \varepsilon = 1, \cdots, L\right\}$.

By substituting w_ε into (20) , the maximum SINR of the i-th user and the partner are:

$$SINR_{i\max} = \frac{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)a_i^2}{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)\sum\limits_{j\neq i,i'}^{2K} a_j^2 + \sigma_i^2} \tag{24a}$$

$$SINR_{i'\max} = \frac{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)a_i^2}{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)\sum\limits_{j\neq i,i'}^{2K} a_j^2 + G_i \cdot G_i^{-1}\left(G_i \cdot G_i^{-1}\right)^T \sigma_i^2} \quad \varepsilon = 1,\cdots,L \tag{24b}$$

Similarly, according to the pre-coding matrix (15)，the maximum SINR of the i-th user and the partner are :

$$SINR_{i\max} = \frac{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)a_i^2}{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)\sum\limits_{j\neq i,i'}^{2K} a_j^2 + G_i \cdot G_i^{-1}\left(G_i \cdot G_i^{-1}\right)^T \sigma_i^2} \tag{25a}$$

$$SINR_{i'\max} = \frac{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)a_i^2}{\max\left(\frac{t_1}{k_1}\rho_R,\cdots,\frac{t_\varepsilon}{k_\varepsilon}\rho_R\right)\sum\limits_{j\neq i,i'}^{2K} a_j^2 + \sigma_i^2} \quad \varepsilon = 1,\cdots,L \tag{25b}$$

3.2 Interference alignment in two-way multiple relays networks

There are 2K users in two-way multiple relays networks. The number of relay node is K. After MAC phase, the signal that the r-th relay node broadcasts is as follows:

$$y_r = P_r \sum_{j=1}^{2K} a_{rj}x_j \qquad r = 1,\cdots K \tag{26}$$

where P_r denotes the pre-coding matrix of the r-th relay node and a_{rj} denotes the linear coefficient of the signal which is the r-th relay node decoding from the j-th transmitter.

At the BC phase, after interference suppression the signal of the i-th user can be expressed as:

$$y_i = D_i \sum_{r=1}^{K} G_{ir}P_r \sum_{j=1}^{2K} a_{rj}x_j + D_i n_i \qquad i = 1,\cdots,2K \tag{27}$$

where D_i is interference suppression matrix at the i-th receiver, G_{ir} is the channel matrix from the r-th relay node to the i-th receiver, n_i is an additive Gaussian noise at the i-th receiver.

After self-interference signals are cancelled, the SINR of the i-th user can be expressed as:

$$\text{SINR}_i = \frac{tr\left(\sum_{r=1}^{K} D_i G_{ir} P_r a_{ri'} a_{ri'} P_r^T G_{ir}^T D_i^T\right)}{tr\left(\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} D_i G_{ir} P_r a_{rj} a_{rj} P_r^T G_{ir}^T D_i^T + D_i D_i^T \sigma_i^2\right)} \quad i=1,\cdots,2K \quad (28)$$

where the i-th node and the $i'-th$ node are communication partners.

According to (27) and (28), assume pre-coding matrix P_r is known, the interference suppression matrices D_i and $D_{i'}$ are used to maximize the total SINR of the i-th user. Then, under the constraint of (6) by solving an optimization problem, D_i and $D_{i'}$ can be figured out as follows:

$$D_i = (\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} G_{ir} P_r a_{rj} a_{rj} P_r^T G_{ir}^T + I\sigma_i^2)^{-1} \tag{29a}$$

$$D_{i'} = (\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} G_{ir} P_r a_{rj} a_{rj} P_r^T G_{ir}^T + I\sigma_i^2)^{-1} G_{ir} G_{i'r}^{-1} \tag{29b}$$

and

$$D_i = (\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} G_{i'r} P_r a_{rj} a_{rj} P_r^T G_{i'r}^T + I\sigma_{i'}^2)^{-1} G_{i'r} G_{ir}^{-1} \tag{30a}$$

$$D_{i'} = (\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} G_{i'r} P_r a_{rj} a_{rj} P_r^T G_{i'r}^T + I\sigma_{i'}^2)^{-1} \tag{30b}$$

By substituting (29) and (30) into (28), the SINR of the i-th user and the partner user are

$$\text{SINR}_i = \frac{tr\left(\sum_{r=1}^{K} G_{ir} P_r a_{ri'} a_{ri'} P_r^T G_{ir}^T\right)}{tr\left(\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} G_{ir} P_r a_{rj} a_{rj} P_r^T G_{ir}^T + \sigma_i^2 I\right)} \tag{31a}$$

$$\text{SINR}_{i'} = \frac{tr\left(\sum_{r=1}^{K} G_{ir} P_r a_{ri} a_{ri} P_r^T G_{ir}^T\right)}{tr\left(\sum_{r=1}^{K}\sum_{j\neq i,i'}^{2K} G_{ir} P_r a_{rj} a_{rj} P_r^T G_{ir}^T + G_{ir} G_{i'r}^{-1} \left(G_{ir} G_{i'r}^{-1}\right)^T \sigma_{i'}^2\right)}$$
$$i=1,\cdots,2K \tag{31b}$$

The r-th relay node to the receiver channel can be reconstructed as virtual MIMO channels, which can be expressed as

$$G_{ir} = \begin{bmatrix} G_{1r} & G_{3r} & \cdots & G_{(2K-1)r} \end{bmatrix} \tag{32a}$$

$$G_{i'r} = \begin{bmatrix} G_{2r} & G_{4r} & \cdots & G_{(2K)r} \end{bmatrix} \tag{32b}$$

Apply SVD [11] to G_{ir} and $G_{i'r}$

$$G_{ir} = V_{ir} T_r U_r^H \tag{33a}$$

$$G_{i'r} = V_{i'r} T_r' U_r^H \quad i=1,3,\cdots,2K-1 \quad i'=K+1 \tag{33b}$$

where V_{ir} and V_{ir} are the unitary matrices, $T_r = \left[\Lambda_r^T, 0_{K(L-N)\times N}^T\right]^T$, $T_r' = \left[0_{(KL-N)\times N}^T, \Lambda_r'^T\right]^T$, Λ_r and Λ_r' are nonnegative diagonal matrix. Then $T_r^T T_r + T_r'^T T_r' = I$. U_r is unitary matrices.

The r-th relay node pre-coding matrix can be designed as $P_r = (U^H)^{-1} W_r$. Let $w_{\varepsilon r}$, $k_{\varepsilon r}$ and $t_{\varepsilon r}$ are the ε-th diagonal element of $W_r, (U_r U_r^H)^{-1}$ and $T_r T_r^H$. Then (31) can be expressed as:

$$SINR_i = \frac{\sum\limits_{r=1}^{K}\left(\sum\limits_{\varepsilon=1}^{L} w_{\varepsilon r}^2 t_{\varepsilon r}\right) a_{ri}^2}{\sum\limits_{r=1}^{K}\left(\sum\limits_{\varepsilon=1}^{L} w_{\varepsilon r}^2 t_{\varepsilon r}\right)\sum\limits_{j\neq i,i}^{2K} a_{rj}^2 + \sigma_i^2}$$

(34a)

$$SINR_i = \frac{\sum\limits_{r=1}^{K}\left(\sum\limits_{\varepsilon=1}^{L} w_{\varepsilon r}^2 t_{\varepsilon r}\right) a_{ri}^2}{\sum\limits_{r=1}^{K}\left(\sum\limits_{\varepsilon=1}^{L} w_{\varepsilon r}^2 t_{\varepsilon r}\right)\sum\limits_{j\neq i,i}^{2K} a_{rj}^2 + tr\left(G_{ir}G_{ir}^{-1}\left(G_{ir}G_{ir}^{-1}\right)^T\right)\sigma_i^2} \qquad i=1,\cdots,2K$$

(34b)

From (34), we can see that, the difference between $SINR_i$ and $SINR_i$ is only the noise term in the denominator. Therefore, $SINR_i$ and $SINR_i$ can achieve the maximum values under the same optimal $w_{\varepsilon r}$. Furthermore, when ρ_r is defined as the maximum transmit power of the r-th relay node, the Lagrange multipliers can be used to form the single-objective optimization problem as

$$f(w_{\varepsilon r} \quad \lambda_r) = \frac{\sum\limits_{r=1}^{K}\left(\sum\limits_{\varepsilon=1}^{L} w_{\varepsilon r}^2 t_{\varepsilon r}\right) a_{ri}^2}{\sum\limits_{r=1}^{K}\left(\sum\limits_{\varepsilon=1}^{L} w_{\varepsilon r}^2 t_{\varepsilon r}\right)\sum\limits_{j\neq i,i}^{2K} a_{rj}^2 + \sigma_i^2} + \sum\limits_{r=1}^{K} \lambda_r\left(\sum\limits_{\varepsilon=1}^{L} k_{\varepsilon r} w_{\varepsilon r}^2 - \rho_r\right)$$

(35)

The KKT conditions can be established as

$$\frac{\partial f}{\partial \lambda_r} = 0$$

(36)

then

$$\sum\limits_{\varepsilon=1}^{L} k_{\varepsilon r} w_{\varepsilon r}^2 - \rho_r = 0 \quad r=1,\cdots,K$$

(37)

If L=1, then $k_{1r} w_{1r}^2 = \rho_r$, and

$$w_{1r}^2 = \frac{\rho_r}{k_{1r}}$$

(38)

$$SINR_{i\,\max} = \frac{\sum_{r=1}^{K}\left(\frac{\rho_r}{k_{1r}}t_{1r}\right)a_{ri'}^2}{\sum_{r=1}^{K}\left(\frac{\rho_r}{k_{1r}}t_{1r}\right)\sum_{j\neq i,i'}^{2K}a_{rj}^2 + \sigma_i^2} \qquad (39)$$

If L=2, then $k_{1r}w_{1r}^2 + k_{2r}w_{2r}^2 = \rho_r$,

Let $\dfrac{\partial f}{\partial w_{1r}} = 0$ and $\dfrac{\partial f}{\partial w_{2r}} = 0$, then $w_{1r}t_{1r} = w_{2r}t_{2r}$,   and

$$w_{1r}^2 = \frac{\rho_r t_{2r}^2}{t_{2r}^2 k_{1r} + t_{1r}^2 k_{2r}} \qquad (40a)$$

$$w_{2r}^2 = \frac{\rho_r t_{1r}^2}{t_{2r}^2 k_{1r} + t_{1r}^2 k_{2r}} \qquad (40b)$$

$$SINR_{i\,\max} = \frac{\sum_{r=1}^{K}(\rho_r)a_{ri'}^2}{\sum_{r=1}^{K}(\rho_r)\sum_{j\neq i,i'}^{2K}a_{rj}^2 + \sigma_i^2} \qquad (41)$$

If $L = n$,then $k_{1r}w_{1r}^2 + \cdots + k_{nr}w_{nr}^2 = \rho_r$

let $\dfrac{\partial f}{\partial w_{1r}} = 0, \cdots, \dfrac{\partial f}{\partial w_{nr}} = 0$, then $w_{1r}t_{1r} = w_{2r}t_{2r} = \cdots = w_{nr}t_{nr}$,   and

$$w_{\varepsilon r}^2 = \frac{\rho_r}{t_{\varepsilon r}^2(\frac{k_{1r}}{t_{1r}^2} + \cdots + \frac{k_{nr}}{t_{nr}^2})} \qquad \varepsilon = 1, \cdots, n \ , \qquad (42)$$

$$SINR_{i\,\max} = \frac{\sum_{r=1}^{K}(\rho_r)a_{ri'}^2}{\sum_{r=1}^{K}(\rho_r)\sum_{j\neq i,i'}^{2K}a_{rj}^2 + \sigma_i^2} \qquad (43)$$

Then the SINR of the i'-th transmitter is:

$$SINR_{i'\,\max} = \frac{\sum_{r=1}^{K}(\rho_r)a_{ri}^2}{\sum_{r=1}^{K}(\rho_r)\sum_{j\neq i,i'}^{2K}a_{rj}^2 + tr\left(G_{ir}G_{i'r}^{-1}\left(G_{ir}G_{i'r}^{-1}\right)^T\right)\sigma_i^2} \qquad (44)$$

4. PERFORMANCE ANALYSIS

To evaluate the performance of the proposed interference alignment algorithm, the rate achieved by the i-th user and the sum rate [12] of the network are defined as
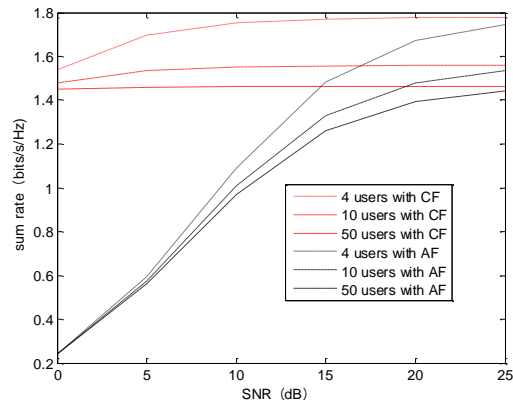
$$R_i = \log_2(1 + SINR_i) \tag{45}$$

$$SR = \sum_{i=1}^{2K} \log_2(1 + SINR_i) \tag{46}$$

In this section, the paper compares the performance of the proposed interference alignment in two-way relay channel via compute-and-forward (CF) with that via amplify-and-forward (AF) .
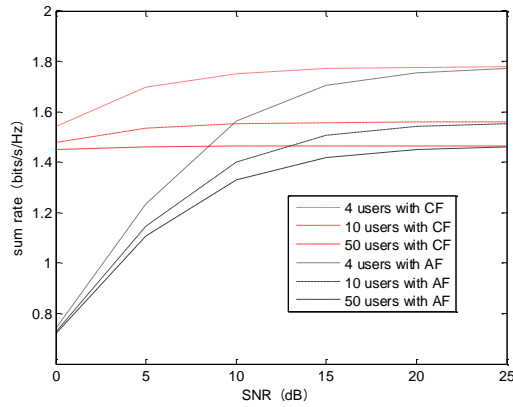
(a) $\rho_r \sigma_r^2 = 2K\sigma^2$

(b) $\rho_r \sigma_r^2 = 10K\sigma^2$

Fig. 2. Comparison between sum rate of AF and that of CF in two-way single relay interference networks

Fig. 2 (a) and **Fig. 2** (b) illustrate the relation between the sum rate and the SNR for AF and CF respectively in two-way single relay interference networks. In **Fig. 2** (a), we set $\rho_r \sigma_r^2 = 2K\sigma^2$ and in **Fig. 2** (b), set $\rho_r \sigma_r^2 = 10K\sigma^2$, where σ_r^2 and σ^2 denote the variance of the noise at relays and receivers respectively. As shown in **Fig. 2**, the system performance for CF increases with the increment of SNR, and is always better than the one for AF, especially in the low SNR region.
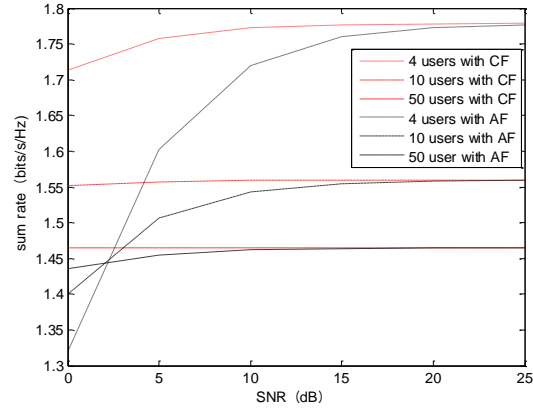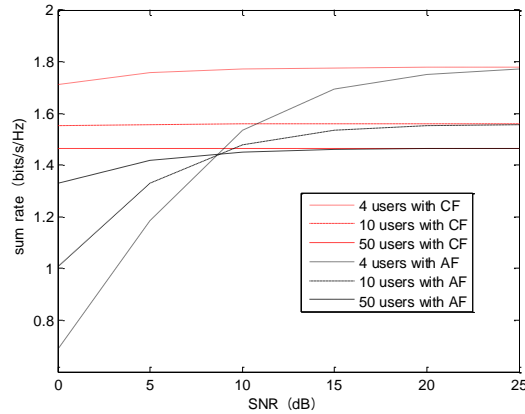
(a) $\rho_r\sigma_r^2 = 2K\sigma^2$

(b) $\rho_r\sigma_r^2 = 10K\sigma^2$

Fig. 3. Comparison between sum rate of AF and that of CF in two-way multiple relays interference networks under the condition of

Fig. 3 (a) and **Fig. 3** (b) illustrate the relation between the sum rate and the SNR for AF and CF respectively in the two-way multiple relays interference networks. In **Fig. 3** (a), we set $\rho_r\sigma_r^2 = 2K\sigma^2$ and in **Fig. 3** (b), set $\rho_r\sigma_r^2 = 10K\sigma^2$, where σ_r^2 and σ^2 denote the variance of the noise at relays and receivers respectively. As shown in **Fig. 3**, the system performance via CF increases with the increment of SNR, and is always better than that via AF, especially in the low SNR region.

It is observed from experiment results that the sum rate performance of CF strategy always better than AF, no matter in the two-way single relay networks or in the multiple relays networks, and no matter how many the number of the users is, especially in the low SNR region. It is mainly due to CF that enables relays to decode linear functions of transmitted messages according to their observed channel, only forwarding the desired messages. The relay of AF simply acts as a repeater, forwarding both the desired messages and noise. When the SNR is in the low region, the noise is similar to signals. CF has better sum rate performance in removing the noise. When the SNR is in the high region, the noise can be ignored, so there is little difference in the performance between CF and AF.
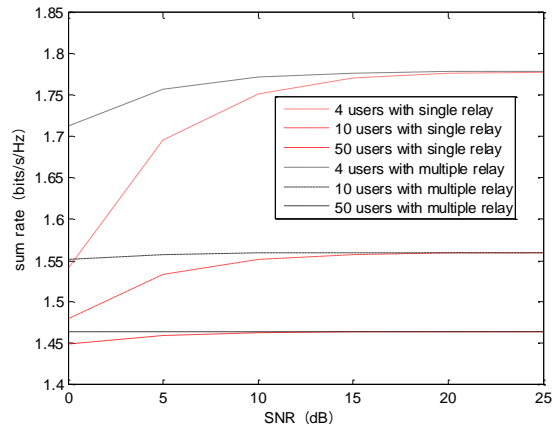
Fig. 4. Comparison between the sum rate for two-way single relay interference networks and that for two-way multiple relays interference networks.

Fig. 4 illustrates the relation between the sum rate and the SNR for two-way single relay networks and two-way multiple relays networks use of CF respectively. In **Fig. 4**, the number of users is set 4, 10 and 50 respectively. As shown in **Fig. 4**, the sum rate performance in the two-way multiple relays interference networks is better than single relay interference networks, especially when there are few users. The main reason is that every user receives both signals and interferences from multiple relays in the multiple relays networks . Therefor, compared to the single relay networks, the rate of single user increases when there are few users. But if the number of users is large, the number of relay nodes is also large in multiple relays networks. From equations (8) and (28), it can be seen that the user rate in multiple relays networks is almost the same as in single relay networks.

5. CONCLUSION

This paper analyzes interference alignment in the two-way single relay networks and two-way multiple relays networks with compute-and-forward. The use of compute-and-forward allows the relays to remove receiver noise, only forwarding useful signals. The simulation results are that the CF can enhance the sum rate performance of networks, and the sum rate performance of two-way multiple relays networks is better than two-way single relay networks when there are few users.

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