An Energy Efficient Algorithm Based on Clustering Formulation and Scheduling for Proportional Fairness in Wireless Sensor Networks

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Received September 14, 2015; revised November 27, 2015; accepted January 10, 2016; published February 29, 2016

Abstract

In this paper, we investigate the problem of achieving proportional fairness in hierarchical wireless sensor networks. Combining clustering formulation and scheduling, we maximize total bandwidth utility for proportional fairness while controlling the power consumption to a minimum value. This problem is decomposed into two sub-problems and solved in two stages, which are Clustering Formulation Stage and Scheduling Stage, respectively. The above algorithm, called CSPF_PC, runs in a network formulation sequence. In the Clustering Formulation Stage, we let the sensor nodes join to the cluster head nodes by adjusting transmit power in a greedy strategy; in the Scheduling Stage, the proportional fairness is achieved by scheduling the time-slot resource. Simulation results verify the superior performance of our algorithm over the compared algorithms on fairness index.

Keywords: Proportional fairness, clustering formulation, scheduling, energy efficiency, power control.

We express our thanks to Dr. Hongying Tang who checked our manuscript. We would like to thank the editor and anonymous reviewers for their constructive and helpful comments.

http://dx.doi.org/10.3837/tiis.2016.02.007 **ISSN : 1976-7277**

1. Introduction

In wireless sensor networks, research has been carried out to study the fairness of resources allocation, which provide supports in many important scenarios, such as target detection sensor system and wireless multimedia sensor networks. In [1], the authors gave a detailed survey on many kinds of fairness. Among them, proportional fairness is one such criterion, which can bring an allocation balance in a resource scheduling process and prevent the situation where heavy resource users benefit more while the poor ones get little from the resource provider. Hence, it is ofen used for optimizing scheduing and allocating resources [2, 3].

The existing works that aims at improving network performance mostly focus on maximizing throughput [4], minimzing delay and their trade-off [5]. As one metric of network performance, proportional fairness is well studied in issues [2, 6]. Li Li [6] proposed the basic proportional fairness problem in multi-rate wireless networks and solved it using a distributed and heuristic algorithm [6]. Wei Li investegated the same problem in [6] and proposed a different solution which adopts relaxation to obtain a result that is better than half of the optimal [2]. These two solutions are both solved in muliti-rate WLAN via associaiton control.

Another issue encountered in wireless sensor networks is to achieve energy efficency by adopting power control strategy. With power control, energy consumption can be reduced and the period of lifttime will be extended. Obviously, bigger transmit power results in longer transmission distance, but meanwhile it also leads to shorter network lifetime. Howerver, in previous works [2, 3, 6-9], energy consumption is not considered in system model. Therefore, we hope to seek an energy efficient algorithm for scheduling resources under proportional fairness criteria.

In this paper, we research on a problem about achieving proportional fairness in hierarchical wireless sensor networks, which aims at finding a trade-off between network utilities and energy consumption. From the crose-layer perspective, we formulate the problem as a non-linear mathematical programming and prove that it is an NP-hard problem, which is hard to determine the optimal solution efficiently in a polynomial time. Therefore, we resort to an approximation solution by jointly determining the node association, power control and time allocation. More specifically, we divide the problem into two sub-problems. Consequently, a two-stage algorithm is proposed. In the first stage, node assoication and transmit power are determined in a greedy strategy. Based on these association coefficients, clusters will be formed by letting sensor nodes join to the designated cluster head nodes. In the second stage, we schedule time resources to achieve fairness. Through the proposed approximation algorithm, the original problem can be solved in a polynomial time. Simulation results indicate that our algorithm has a better performance over compared methods. We also simulate the dynamic adjustment procedure of transmit power allocation.

It is worth noting that, the authors in [10] investigated the similar problem and proposed an algorithm jointly considering power control and AP(Access Point) association. Meanwhile, it should also be pointed out that their transmit power is set equally between all Aps and is related to bit rate. In contrast, our power control is devised to have an impact on association while bit rate is set as fixed value. In addition, the procedure of transmit power adjusting is one by one in greedy strategy. Very recently, Sun discussed the problem of combining BS(base station) association and power control in [11]. With fixed BS association, a binary search strategy for power allocation was used to achieve downlink max-min SINR(Signal to Interfernce plus Noise Ratio). Indeed, Sun's paper studied a different system model from ours.

The rest of our paper is organized as follows: Related work is in section 2. System model and problem formulation are introduced in section 3. Our proposed algorithm is discussed in section 4. After simulation, the results and discussions are shown in section 5. We conclude this paper in section 6.

2. Related Work

Fairness in wireless networks has been studied in the past. Futhermore, to implement fairness is mainly under the domain of data-link layer and resource allocation. In [1], the authors concluded the work of fairness in wireless networks, and they presented a general view of fairness studies. In their opinion, fairness measures can be divided into two groups: Quantitative Fairness Measures and Qualitative Fairness Measures. Specifically, Jain's index and Entropy belong to the quantitative group; Max-min Fairness, Proportional Fairness and the Tian Lan's Model are in the qualitative group. Besides, fairness can be divided into time-based fairness and throughput-based fairness. In issue [12, 13], time-based fairness is introduced. Throughput-based fairness is compared with time-based fairness in issue [13]. As one category of fairness, proportional fairness is researched in [3, 7-9, 14]. The authors in [15] determined reach a network wide proportional fairness in an analytic way. In [7], the authors formulated and studied a generalized proportional fairness problem with user associations to base stations, which a generalized proportional fairness objective function is acheived. In [3], the authors considered how spatial reuse impact on proportional fairness. In [8], the authors studied the proportional fair scheduling (PFS) problem, jointly considering the user selection and utility maximization, in an HM-aided wireless network. For proportional fairness, a fairness criterion is proposed by Kelly [16] and a style of proportional fairness is defined in [2]. Consequently, measuring fairness needs a fairness metric. Specifically, Jain's index [17] is the most widely-used fairness index in issues. Jain's index is between 0 and 1, and a larger value of Jain's index means a better fairness.

Besides fairness, energy efficiency is imminently needed in wireless sensor networks, while the lifetime of nodes is constrainted by the energy supplement technology. Moreever, one way to achieve energy efficiency is controlling the transmit power. The problem about energy efficiency always can be an optimization problem, and the objective function can be throughput while under the energy consumption constraint. In [18], the authors formulated an energy-constrained optimization problem for link scheduling, power control and routing in ad hoc wireless networks. The authors in [19] presented a cross-layer design framework in contention-based wireless ad hoc networks for the multiple access problem , and the authors in [4] examined joint link scheduling and power control with throughput improvement. In issue [5], the problem for joint scheduling with either power control or rate control or both is formulated.

Joint AP association and fairness are proposed by Wei [2]. They proposed their algorithm in static scenario and exposed it to dynamic situation. Similarly, AP association in WLAN can be treated as clustering formulation in wireless sensor networks. Clustering is a classical network formulation method in wireless sensor networks, and the network will be hierarchical. When dealing with clustering, game theory is ofen investigated [20]. In addition to game theory, interference has an influence on the choice of cluster head nodes, and it further affects the effective bandwidth allocation. In [3], Douglas et al. demonstrated the problems of applying fairness concepts to wireless networks, which with interference caused by spatial reuse.

3. System Model and Problem Formulation

3.1 System Model

We consider a scenario in which wireless sensor networks are hierarchical. In this scenario, there exist N cluster head nodes named as $a_1, a_2, ..., a_i ..., a_N$ and M sensor nodes which can be associated to cluster head nodes and numbered as $u_1, u_2, ..., u_j, ...u_M$. Node controller c runs the proposed algorithm and exchanges messages with cluster head node a_i. The cluster head node a_i is a node which can achieve fairness while the allocated resource is time slot in TDMA (Time Division Multiple Access) wireless systems. Sensor node u_j can only associate with one cluster head node. They are all in the same general area and these nodes can move. We assume that the hierarchical wireless network is dynamic and the network topology varies in case of node mobility. The cluster head nodes have multi-rate ability. Multi-rate wireless sensor network is a kind of wireless sensor network whose links with different data rates coexist.

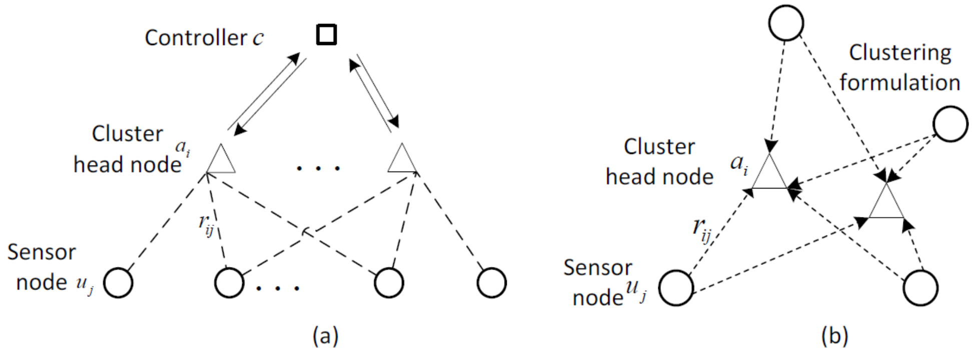


Fig. 1. System scenario. (a) Network model and sructure. (b)Nodes in real scenario

Fig. 1 shows the wireless scenario of system. Sensor node u_j joins to cluster node a_i and can only join to one cluster head node. u_j can choose which a_i to join in. This procedure of association can be treated as clustering formulation with designated cluster head nodes.

In this paper, we consider a time-slot system with limited bit rate wireless links. The total transmission time of a cluster head node is assumed to be 1. The wireless bit rate of links between a_i and u_j is denoted as r_{ij}, which represents the quality of wireless links. We assume that it is a known parameter. While under SINR interference model, I_{ij} represents the quality of wireless links and is related to the interference from other wireless links. We have

$$\mathrm{I}_{ij} = g\left(\frac{Rss_{ij}}{\sum_{n\in[1,N];m\in[1,M],m\neq j} Rss_{nm} + N_0}\right) \tag{1}$$

where I_{ij} is the total interference influence from cluster head i and other nodes to sensor node j. Specifically, n and i are the indexes of cluster head nodes, m and j are the indexes of sensor nodes. Rss_{nm} is the received signal strength from cluster head node n to sensor node m. $\sum_{n\in[1,N];m\in[1,M],m\neq j} Rss_{nm}$ represents the total interference from all wireless links except himself. g is an increasing function. I_{ij} increases with SINR increasing. We assume that the transmit power of cluster head node is higher than that of sensor node. We employ a regular model to represent the wireless channel condition, which is expressed as:

$$Rss = Pt - PL\left(\mathrm{d}_0\right) - 10\eta\log(dist / d_0) \tag{2}$$

where Rss represents received signal strength and Pt is the transmit power. Transmit power minus path loss is received signal strength in reception node. Equation (2) is a large scale path loss model which adopts logarithm distance named as logarithm-distance radio propagation model. d_0 is a reference distance and $PL(d_0)$ represents the received power in distance d_0. The $dist$ represents distance from transmission node to reception node. Symbol η is path loss exponent, which ranges from 2 indoor to 4 outdoor.

The energy consumption is related to the transmit power. In this paper, we consider the power control in cluster head nodes and use vector $\mathbf{Pt} = [Pt_1, Pt_2, ..., Pt_i, ...Pt_N]^T$ to represent the transmit power, where Pt_i represent the transmit power in cluster head node a_i. We assume that $Pt_{\min} \le Pt_i \le Pt_{\max}$, where $Pt_{\max}$ is the maximum transmit power and $Pt_{\min}$ is the minimum transmit power in a_i. We adopt the energy consumption equation in [21]. The total energy consumption can be divided into three parts. We simplify it and assume that transmission nodes have the same length of transmit packet and d^n varies directly with Pt_i. d^n represents the nth power of distance d, and d is the distance. We have

$$E_i^c = E^d + \varepsilon^c \cdot Pt_i \tag{3}$$

where E_i^c represents the energy consumption in a_i. E^d is a constant parameter related to the transceiver circuit. ε^c is assumed to be a constant and is related to the amplifier's electronic parameter.

3.2 Problem Formulation

A fairness criterion was proposed by [16] and proportional fairness was described in [2]. We adopt Jain's Fairness Index [17] as the measurement of fairness index, which is denoted as J. It is an index in the range of [0, 1]. The higher the value, the more equitable. The bandwidth utility for proportional fairness formulation is given by [6].

$$f\left(x, p\right) = \sum_{j\in U} \omega_j log b_j \tag{4}$$

where U represents the sensor nodes set. b_j is the effective bandwidth allocated to sensor nodes from cluster head nodes. Let $b_j = \sum_{i=1}^{N} x_{ij} p_{ij} r_{ij}$, x_{ij} is the association between u_j and a_i . One node can only associate with one cluster head node, it can be denoted as $x_{ij} \in \{0,1\}$. p_{ij} is the transmission time that a_i allocates to u_j . ω_j is the weight of u_j . We use and reference the optimization formulation in [2, 6] and propose our optimization equations, which are shown as follows:

$$\begin{aligned}
&\text{maximize} && \sum_{j\in U}\omega_j log(\sum_{i=1}^{N} x_{ij} p_{ij} r_{ij}) && (a)\\
&\text{minimize} && \sum_{i=1}^{N}(E^d + \varepsilon^c \cdot Pt_i) && (b)\\
&s.t && \sum_{i=1}^{N} x_{ij} = 1 && (c)\\
& && x_{ij} \in \{0,1\} && (d)\\
& && \sum_{j=1}^{M} x_{ij} p_{ij} = 1 && (e)\\
& && p_{ij} \in [0,1] && (f)\\
& && \mathrm{i} \in [1, \mathrm{N}], \mathrm{j} \in [1, \mathrm{M}] && (g)
\end{aligned} \tag{5}$$

This group of formulations is an NP-hard problem. We prove it in **Proposition 1**. We name it as AFME Problem (Achieving Fairness and Minimizing Energy Consumption Problem). The unknown parameters that need to figure out are x_{ij} , p_{ij} and Pt_i . We aim at achieving proportional fairness for bandwidth allocation while ensuring a less energy consumption. Objective function (a) is to maximize bandwidth utility for proportional fairness. Objective function (b) represents minimizing the sum of energy consumption in each cluster heads. Constraint (c) and (d) say that node u_j can only associate to one cluster head node a_i simultaneously. Constraint (e) says that the total transmission time of a_i is 1. Constraint (f) and (g) define the range of p_{ij} , i and j .

Proposition 1. Formulation (5) is an NP-hard problem.

Proof. We bring in a problem named GPF1(Generalized Proportional Fairness) introduced in paper [7]. This problem has been proved to be NP-hard. As same as GPF1, GPF2 (restricted version of the GPF1 problem) is also an NP-hard problem. What we need to do is to find a polynomial-time reduction function F which makes $GPF2 \le_p MFME$. Symbol $\le_p$ represents polynomial-time reduction. Since GPF2 is an NP-hard problem, we can judge that AFME problem is also an NP-hard problem if there have a F . GPF2 problem can be shown as follows [7]:

$$\begin{aligned}
&maximize && \sum_{u\in U}\sum_{a\in S_u} x_{ua} \log(r_{ua}\frac{G(y_a)}{y_a}) && \text{(a)}\\
&\text{s.t.} && \sum_{a\in S_u} x_{ua} = 1, \forall u \in U && \text{(b)}\\
& && y_a = \sum_{u:a\in S_u} x_{ua}, \forall a \in A && \text{(c)}\\
& && x_{ua} = \{0,1\} && \text{(d)}
\end{aligned} \tag{6}$$

where a user u 's average data rate associating with a BS(Base Station) a is denoted r_{ua} . x_{ua} is the association variable, $x_{ua}=1$ if u is associated with a . $S_u=\{a \mid r_{ua}>0, \forall a \in A\}$, A is the set of BSs. In addition, $y_a=\{u \mid x_{ua}=1, \forall u \in U\}$, U is the set of users. $G(y_a)=E\{\max_{u:x_{ua}=1} \mathrm{Y}_{ua}\}$, where Y_{ua} are independent and identicaly distributed copies. Besides, $\gamma_{ij}=G(y_a) r_{ua} / y_a$ where γ represents the actual bandwidth allocation to user u by the network.

First, we can find out that AFME and GPF2 have the same meaning of constraints. They have $\sum_{i=1}^{N} x_{ij}=1, x_{ij} \in\{0,1\}$. $\sum_{j=1}^{M} x_{ij} p_{ij}=1$ and $y_a=\sum_{u:a \in S_u} x_{ua}$ have the same meaning and the total allocation time in cluster head nodes is assumed to be 1 in problem AFME. Second, AFME has two object functions while GPF2 has one objective function. Here, we use the method of weighted sum of objectives as a single objective to deal with this optimization problem, which will formulate a new utility function. It can be expressed as:

$$\lambda_1\left(\sum_{j \in U} \omega_j log\left(\sum_{i=1}^{N} x_{ij} p_{ij} r_{ij}\right)\right)-\lambda_2 \sum_{i=1}^{N} E_i^{c} \tag{7}$$

where λ_1 and λ_2 are the weight of proportional fairness function and energy consumption function. Besides, both of them can also be normalized coefficients. Furthermore, parameter λ_1 and λ_2 can be set utilizing expert's experience. Consequently, we maximize the total utility function with respect to bandwidth allocation for proportional fairness while controlling the power consumption to a minimum value, which will bring an energy efficient bandwidth assignment for proportional fairness. Finally, since the two object functions of two problems are polynomial, we can find a polynomial time function F that makes $GPF2 \leq_p MFME$. Then the AFME is an NP-hard problem.

4. Proposed Algorithm

According to the optimal model, there will be a balance between fairness and energy consumption. The unknown parameters which need to be solved are the association relationship x_{ij} , scheduling time slot result p_{ij} and power allocation information Pt_i . To solve AFME problem, we divide the problem into two sub-problems and each sub-problem corresponds to one stage. The first stage formulates the network structure, and sensor nodes join to the cluster head nodes. The first sub-problem is in the stage of clustering formulation based on SINR interference model and power control, which can obtain x_{ij} and Pt_i ; in the second stage, after the networks structure has formulated, we use the information got from the first stage, and then scheduling. We can obtain p_{ij} in this stage. The resource in this stage that can be allocated is time-slot resource.

4.1 Clustering Formulation Stage

In this stage, we firstly focus on figuring out the x_{ij} , which means that sensor node u_j joins to cluster head node a_i . This process can be treated as clustering formulation while cluster head nodes are designated. It deletes the unimportant potential association depending on r_{ij} , Rss

and E_i^c. We set up a liner function to express the contributions of each factors to the final association result.

$$\Delta_{ij} = \zeta_1 \eta_{ij} + \zeta_2 \mathrm{I}_{ij} - \zeta_3 E_i^c \tag{8}$$

where Δ_{ij} represents the association strength with respect to bit rate, interference and transmit power. For each u_j, we choose the maximum Δ_{ij} and find the corresponding a_i. Let u_j associate to a_i and set $x_{ij} = 1$, otherwise set as 0. ζ_1, ζ_2 and ζ_3 are normalizing ratio, and it was set considering the impact of weight. In the first part $\eta_{ij} = r_{ij} / (\sum_{j=1}^{M} r_{ij})$, taking the bit rate's influence into account. The second part introduces the impact of interference, and the third part is the impact of energy consumption. It can be rewritten in the following format:

$$\Delta_{ij} = \zeta_1 r_{ij} / (\sum_{j=1}^{M} r_{ij}) + \zeta_2 g(SINR(\frac{Rss_{ij}}{\sum_{n\in[1,N];m\in[1,M],m\neq j} Rss_{nm} + N_0})) - \zeta_3 (E^d + \varepsilon^c \cdot Pt_i) \tag{9}$$

In this formulation, we first find the unknown parameters and can figure out the relationship between input parameters and output result. I_{ij} is related to $\boldsymbol{Pt} = [Pt_1, Pt_2, ..., Pt_i, ...Pt_N]^T$. We need to find a new way to figure out the maximum Δ_{ij}.

We adopt a greedy strategy to deal with the $\boldsymbol{Pt}$ in the process of computing Δ_{ij}. Since $Pt_i \in [Pt_{\min}, Pt_{\max}]$, we discrete Pt_i into K pieces, and k is the index of pieces. Thus the value matrix can be shown as follows:

$$\boldsymbol{Pt} = [Pt_1, Pt_2, ..., Pt_i, ...Pt_N]^T \longrightarrow \begin{bmatrix} Pt_{11} & Pt_{21} & ... & Pt_{N1} \\ Pt_{12} & Pt_{22} & ... & Pt_{N2} \\ ... & ... & Pt_{ik} & ... \\ Pt_{1K} & Pt_{2K} & ... & Pt_{NK} \end{bmatrix}^T \tag{10}$$

$$Pt_{ik} = Pt_{\min} + (k-1) * (Pt_{\max} - Pt_{\min}) / (K-1) \tag{11}$$

where Pt_{ik} represents the kth piece of Pt_i. To find the maximum Δ_{ij} for each j, different Pt_{ik} has different Δ_{ij}. For different a_i, we select a Pt_i, and we have $(Pt_{ik}, r_{ij}) \rightarrow \Delta_{ijk}$, for each a_i, we choose Pt_{ik} and the related k in the guidance equation which can be expressed as:

$$\max_{k} \sum_{j=1}^{M} \Delta_{ijk} \tag{12}$$

For different a_i, we adopt a greedy strategy to avoid uncertain situation. For a_i, we need to know about $Pt_{1k_1}, ..., Pt_{(i-1)k_{i-1}} Pt_{ik} Pt_{(i+1)k_{i+1}}, ..., Pt_{Nk_N}$ according to Δ_{ij} when selecting Pt_{ik}, where $k_1...k_{i-1}k_{i+1}...k_N$ is the chosen k for each a_i. We vary Pt_{ik} for each k and fix the value of $Pt_{1k_1}, ..., Pt_{(i-1)k_{i-1}} Pt_{ik} Pt_{(i+1)k_{i+1}}, ..., Pt_{Nk_N}$. We found k according to formulation (12). We update the value of Pt_{ik} at each round for a_i. Finally, we can find appropriate k for each column in the matrix, then Pt_{ik} can be figured out. According to formulation (12) with the corresponding i and k, nodes association x_{ij} can be determined.

In the procedure of adjustment, the transmit power varies from the minimum value and find k to satisfy the strategy. It can bring a less energy consumption and make the algorithm energy efficient.

4.2 Scheduling Stage

After the clustering formulation stage, we have figured out x_{ij} and Pt_{ik}, while the network structure formulated. In this stage, only p_{ij} is left to be determined. Different p_{ij} results in a different fairness performance. The scheduling is needed in this stage. We assume that the transmit power are fixed since the dynamic power allocation adjustment has finished. This optimal problem can be simplified as follows:

$$\begin{aligned} \text{maximize} \quad & \sum_{j\in U} \omega_j log(\sum_{i=1}^{N} x_{ij} p_{ij} r_{ij}) \\ s.t \quad & \begin{cases} p_{ij} = 0 & if \ x_{ij} = 0 \\ \sum_{j=1}^{M} x_{ij} p_{ij} = 1 & if \ x_{ij} = 1 \end{cases} \\ & p_{ij} \in [0,1] \end{aligned} \tag{13}$$

It is solve_opt (Algorithm 1) in the proposed algorithm. As is shown in the equation, p_{ij} is corresponding to x_{ij}. When $x_{ij}=0$, it means that u_j has no association with a_i, not to mention that a_i allocates time-slot to u_j. Otherwise, the sum of time-slot that a_i allocates to u_j is 1 and this has been assumed. This model can be solved in polynomial time and has an optimal solution.

Proposition 2. Formulation (13) can be solved in polynomial time. It satisfies the KKT necessary and sufficient condition.

Proof. We make matrix p_{ij} head-to-tail transforms to be a vector $\mathbf{P}_v$ by column, and $\sum_{i=1}^{N} x_{ij} p_{ij} r_{ij}$ can be expressed as $\mathbf{\Omega P}_v$, where $\mathbf{\Omega}$ represents a known constant matrix. Formulation (13) can be $\mathbf{\Gamma} = \mathbf{\Phi} log(\mathbf{\Omega P}_v)$, where $\mathbf{\Gamma}$ represents the target function and $\mathbf{\Phi}$ represent the known constant matrixes corresponding to $\boldsymbol{\omega}$. Equal constraint can be expressed as $\mathbf{YP}_v - 1 = 0$, where $\mathbf{Y}$ is a constant matrix corresponding to $\mathbf{x}$. Then KKT necessary and sufficient condition can be expressed as

$$\begin{aligned} & \mathbf{\Gamma}' - \nu(\mathbf{YP}_v - 1)' = 0 \\ & (\mathbf{\Phi} \log \mathbf{\Omega} + \mathbf{\Phi} \log \mathbf{P}_v)' - \nu \mathbf{Y} = 0 \\ & \mathbf{\Phi} / \mathbf{P}_v - \nu \mathbf{Y} = 0 \end{aligned} \tag{14}$$

where ν represents a non-negative vector. $\overline{\mathbf{P}_v} = \mathbf{\Phi} / \nu \mathbf{Y}$ and $\overline{\mathbf{P}_v}$ is the KKT point. Due to the fact that logarithm function is a convex function and equal constraint is linear, KKT point $\overline{\mathbf{P}_v}$ is the optimal result for this non-liner problem. Formulation (13) can be solved in polynomial time. It satisfies the KKT necessary and sufficient condition.

4.3 CSPF_PC Algorithm

Based on the stage mentioned above, we proposed an algorithm which is divided into two stages. In each stage, we figure out part of the optimal problem. The first stage figures out the clustering formulation problem with respect to power control and finds out the association information and proper transmit power in cluster head nodes. The second stage figures out the time-slot allocation or scheduling problem. The target function for this optimal problem is to achieve proportional fairness in cluster head nodes while considering minimizing the energy consumption. This algorithm can be shown in **Algorithm 1**.

Algorithm 1 indicates the key steps of proposed method to figure out this kind of problem. Row 1 says that system can obtain the channel state information by exchanging messages. Row 2~9 indicates the key steps of Clustering Formulation Stage, which has been introduced before. Row 10 is the step of Scheduling Stage, and the problem can be solved in classical algorithms through mathematical tools. Row 11 indicates the algorithm's result.

Algorithm 1	Clustering formulation and scheduling for proportional fairness based on power control algorithm(CSPF_PC Algorithm)
Input	1、 Bit rate r_{ij} between a_i and u_j 2、 Weight ω_j for each u_j . 3、 The range of transmit power $[Pt_{\min}, Pt_{\max}]$
1	Cluster head nodes send packets and nodes return response packets including channal stage information
2	For each $a_i, i \in [1,N]$ u_j , $\mathrm{j} \in [1,\mathrm{M}]$ do
3	$\eta_{ij} = r_{ij} / (\sum_{j=1}^{M} r_{ij})$
4	$\mathrm{I}_{ij} = g(SINR(Rss_{ij} \Big/ \sum_{n\in[1,N];m\in[1,M],m\neq j} Rss_{nm} + N_0))$
5	$E_i^c = E^d + \varepsilon^c \cdot Pt_i$
6	$\Delta_{ij} = \zeta_1 \eta_{ij} + \zeta_2 \mathrm{I}_{ij} - \zeta_3 E_i^c$
7	Choose Pt_{ik} in the described greedy strategy $\max_k \sum_{j=1}^{M} \Delta_{ijk}$ for each a_i
8	set $x_{ij'} = 1$ if $\Delta_{ij'}$ ==max($\Delta_{ij'}$) for each u_j ,otherwise $x_{ij'} = 0$
9	end
10	p_{ij}=solve_opt(x_{ij} , r_{ij} , ω_j)
11	Controller c sends results to each cluster head node a_i

The algorithm is finite and can have an optimal result. In the first stage, for a_i and u_j , the loop will end while $i \in [1,N]$ and $\mathrm{j} \in [1,\mathrm{M}]$. When choosing Pt_{ik} and finding the $\max(\Delta_{ij})$, the procedure is finite, for the discrete number is limited and $\mathrm{j} \in [1,\mathrm{M}]$. In the second stage, there will be a KKT point in the optimal problem and it can be solved using a classical algorithm.

In the first stage, the computation complexity of loops is $\Theta(MN)$, and choosing Pt_{ik} can cost $\Theta(NK)$ for adopting greedy strategy. It can be treated as a branch bounding method which selects a branch that has the suboptimal value of transmit power, cutting down the branches at each time for reducing the computation complexity evidently. The computation complexity of choosing maximum Δ_{ij} is included in the loop of j. In the second stage, since it can be solved in polynomial time, we treat it as $\Theta(MN)$. Finally, the computation complexity of this problem is $2*\Theta(MN)+\Theta(NK)$.

5. Simulation and Discussion

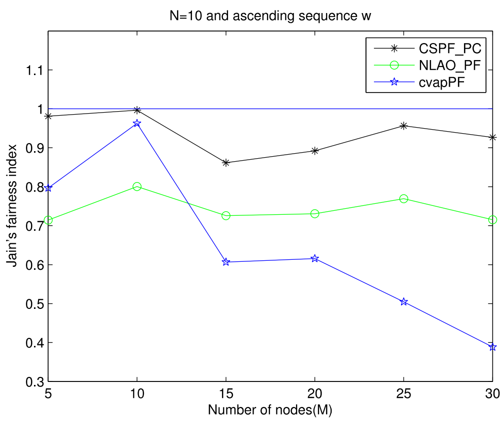
We simulate the performance of our new proposed algorithm compared to NLAO-PF algorithm [2] and cvapPF algorithm [6]. We choose these two algorithms for the reason that this paper comes from the two compared issues, and the flows of algorithms are similar. The AP association can be treated as clustering formulation in wireless sensor networks.

We adopt Jain's fairness index as the compared metric in this paper. Jain's fairness index is set between 0 and 1 and widely used in issues about fairness. The larger Jain's fairness index, the better fairness. The definition of Jain's fairness index in this paper is

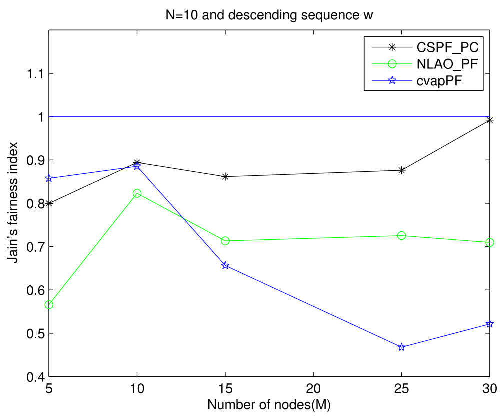
$$J = \frac{[\sum_{j=1}^{M} b_j]^2}{M \sum_{j=1}^{M} b_j^{\ 2}} \tag{15}$$

where J represents Jain's fairness index and b_j is bandwidth.

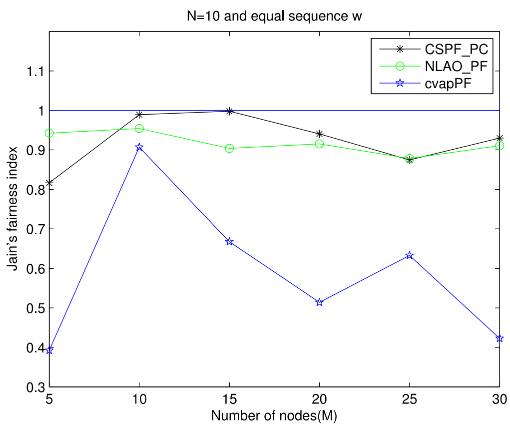
We set some of the parameters as follows: reference distance d_0=1$[m]$, path loss for reference distance d_0 as $PL(\mathrm{d}_0)=55[dB]$, noise $N_0=-80[dB]$, path loss exponent η=4, the width of simulation square L=200[m]. We set maximum value of transmit power $Pt_{\max}=\mathbf{40}[mW]$ and minimum value of transmit power $Pt_{\min}=10[mW]$, K=10. The area of simulation network is set to be $200m \times 200m$. In our simulation scenario, nodes are randomly deployed in a square region and L is the width of square.



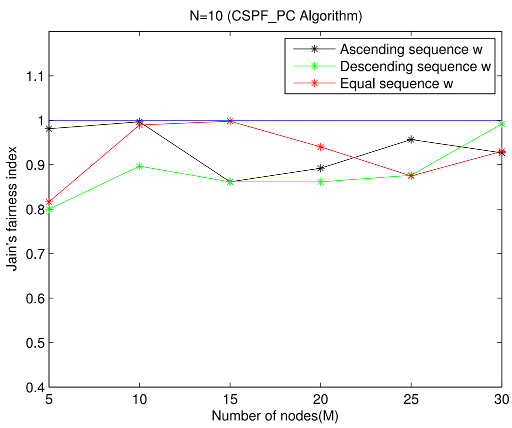
(a)



(b)



(c)



(d)

Fig. 2. Simulation results of different M and ω_i. (a)Ascending sequence ω_i; (b) descending sequence ω_i; (c) equal sequence ω_i; (d) different sequence ω_i for CSPF_PC algorithm.

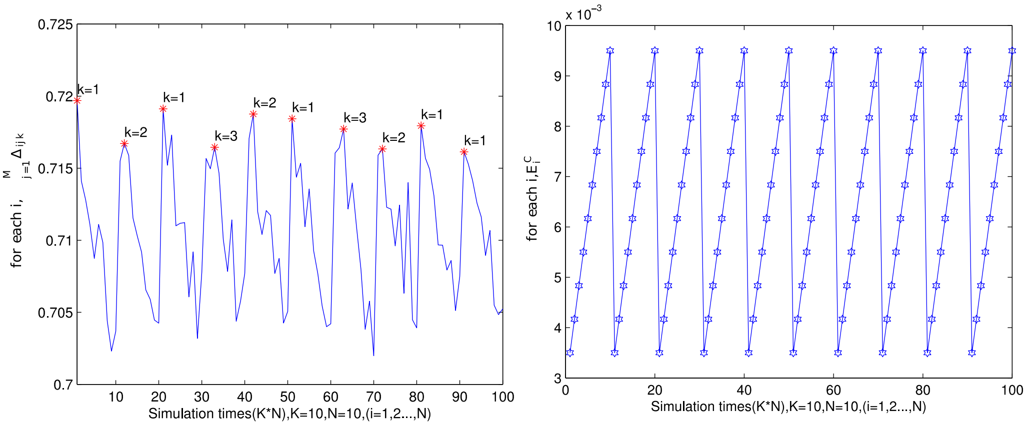
We compare the algorithms under different M values. We use $N=10$ and let M varies in the range of [5, 30].

Fig. 2 shows the impact of nodes size M and weight ω_i on fairness index. We firstly set weight ω_i in an ascending sequence, and we obtain the Jain's fairness index of three algorithms as is shown in

Fig. 2(a). As the number of nodes increases, the fairness index remains above 85% and CSPF_PC increases the fairness index by 18%~37% compared with NLAO_PF algorithm. Then we evaluate the descending sequence of ω_i and equal sequence of ω_i, and we can observe that when N=M, the fairness index of three algorithms will get its locally optimal point. This is due to the fact that ω_i is in the fairness formulation, which has a great impact on final result.

Fig. 2(b) and (c) show that as ω_i becomes equal, the fairness index becomes stable except cvapPF algorithm. From these three figures, we can observe that CSPF_PC has a better performance on fairness index than the other two algorithms as M and ω_i vary. We use different values of ω_i for comparison. First we get a sequence of ω_i which satisfies $\sum_{j=1}^{M} \omega_j = 1$ and order it in an ascending sequence. Then we get descending and equal sequence.

Fig. 2(d) indicates that the impact of different weight ω_i on fairness index using CSPF_PC algorithm. We average the results and it is 0.9358, 0.8814 and 0.9248. It shows that ascending sequence ω_i works best in this situation.



(a) (b)

Fig. 3. Simulation result of transmit power adjustment procedure using CSPF_PC algorithm. (a) The procedure of Δ varies using greedy strategy. (b) Transmit power adjustment procedure.

Fig. 3(a) shows the procedure of $\sum_{j=1}^{M}\Delta_{ijk}$ varies when adopting fixed i for each iteration. We choose the k using greedy strategy labeled in the figure and use it in Pt_{ik} . Then, we can find out the suitable value in this procedure. **Fig. 3**(b) shows the procedure of transmit power varies. From **Fig. 3**, we can see that $\sum_{j=1}^{M}\Delta_{ijk}$ for each i varies while transmit power varies, but the variation trend is not synchronous. The reason due to the fact that the weight ζ_1,ζ_2,ζ_3 have different impact on the result in this procedure.

6. Conclusion

In this paper, we developed a polynomial-time algorithm which is called CSPF_PC algorithm to solve the problem of achieving proportional fairness in wireless sensor networks. The goal is to maximize bandwidth utility with less energy consumption under the constraints that (i) sensor node can only associate to one cluster head node, and (ii) the total transmission time is assumed to be 1. Through the algorithm, optimization variables were determined in two stages. Simulation results demonstrate that CSPF_PC outperforms in terms of fairness metric in different input parameters.

In future research, we will consider to find an optimal solution of to the problem. In addition, we will also consider power allocation in both of the stages. Moreover, we will consider a contention based scenario and develop a distributed algorithm in future.

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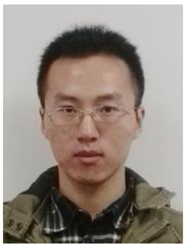

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