# Third Harmonic Generation of Shear Horizontal Guided Waves Propagation in Plate-like Structures

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Abstract The use of nonlinear ultrasonics wave has been accepted as a promising tool for monitoring material states related to microstructural changes, as it has improved sensitivity compared to conventional non-destructive testing approaches. In this paper, third harmonic generation of shear horizontal guided waves propagating in an isotropic plate is investigated using the perturbation method and modal analysis approach. An experimental procedure is proposed to detect the third harmonics of shear horizontal guided waves by electromagnetic transducers. The strongly nonlinear response of shear horizontal guided waves is measured. The accumulative growth of relative acoustic nonlinear response with an increase of propagation distance is detected in this investigation. The experimental results agree with the theoretical prediction, and thus providing another indication of the feasibility of using higher harmonic generation of electromagnetic shear horizontal guided waves for material characterization.

Keywords: Third Harmonic Generation, Shear Horizontal Guided Waves, Magnetostrictive Transducer, Perturbation Method

### 1. Background

Guided waves have demonstrated their ability as an attractive alternative for rapid inspection of large areas of structure. Possibility to inspect inaccessible or hidden area and great costeffectiveness are the key advantages of guided wave-based damage detection approaches [1]. Shear horizontal (SH) guided wave, in which the particle displacement is in parallel to the surface of specimen, is an attractive technique for surface defect characterization. Theoretical studies and exploration of SH guided wave applications have got increasingly concern. However, most SH guided wave technologies which based on linear acoustic features, are sensitive to gross defects, but much less sensitive to material micro-damages or degradation [2-5]. The use of nonlinear ultrasonic wave has been accepted as one of the most promising methods for evaluating material micro-structural changes in early stage [6-8]. One of the typical nonlinear phenomena of ultrasonic propagation is the generation of higher harmonics. Higher harmonic generation of SH guided waves, combines the great advantages of SH guided waves with the sensitivity of nonlinear ultrasonic techniques, is an attractive option for monitoring states related to microstructural changes.

However, although nonlinear ultrasonic measurement has been a subject of considerable interest, most of the researches were about the nonlinear bulk and surface waves [9-11]. There is rarely investigation of nonlinear SH guided waves in plate-like structure. In addition, generation and detection of SH guided waves are not as easy as other guided wave modes for the reason that the particle displacements of SH

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waves are parallel to the surface of waveguide. Recently, the interaction of SH0 guided wave mode with Rayleigh Lamb waves in weakly nonlinear plates were investigated in [12]. The cumulative second-harmonic generation accompanying nonlinear shear horizontal mode propagation in a solid plate was reported in [13].

In paper, theoretical studies this and experimental observations of higher harmonic generation of SH guided waves propagation in an isotropic plate are presented. Electromagnetic transducers are designed to generate and detect the nonlinear SH guided waves. Measurement of the strongly nonlinear response of shear horizontal guided waves is conducted to verify the accumulative growth of relative acoustic nonlinear response with propagation distance. The experimental results have a good agreement with the theoretical prediction, and thus provide another indication of the feasibility of using higher harmonic generation of electromagnetic shear horizontal guided waves for material characterization.

#### 2. Theory

The shear horizontal waves caused the particle vibrations (displacement and velocities) are in plane that is parallel to the surface of the layer. A Cartesian coordinate system is shown in Fig. 1, in which the  $x_3$  axis coincides with the center of the solid plate, and the  $x_1$  axis is normal to the boundaries of the plate. The acoustic fields of SH guided waves can be decomposed into two oblique shear wave components that with self-inactions and cross-inactions in the plate.

The distinguishing feature of nonlinearity in shear waves is that is primarily cubic rather than quadratic, so, the shear wave propagation generally does not permit second harmonic generation directly by the fundamental. Thus, energy amplitude of third harmonic is higher



Fig. 1 Acoustic fields of SH mode propagation in a solid plate

than the second harmonics in this investigation.

Referenced our earlier work [14], nonlinear parameters of SH guided wave in isotropic plate can be represented as,

$$\beta_{s} = \frac{24}{k^{3}x} \frac{A_{3}}{A_{1}^{3}} F \tag{1}$$

where  $\beta_s$  is nonlinear parameter;  $A_1$  and  $A_3$  are fundamental wave and higher harmonic wave amplitude, respectively; k is wave number of SH guided wave; x is wave propagation distance; F is nonlinear feature function of SH guided wave. In this work, since the identical wave mode is chosen to detect acoustic nonlinearity in the specimens with same thickness at a fixed input fundamental frequency, the influence of the feature function is neglected since the interest is focused on the acoustic nonlinearity of SH0 mode at 0.5 MHz only.

In the experimental work, the relative acoustic nonlinear response can be represented as the ratio of the third harmonic amplitude divided by the cubic of the fundamental wave amplitude,  $(A_3/A_1^3)$ , which corresponds to the slope of the line correlating the actual acoustic nonlinear parameter,  $\beta_s$  with wave propagation distance, x, for a fixed wave number k and the nonlinearity feature function F. Then, the relative acoustic nonlinear response is,

$$\overline{\beta} = \frac{A_3}{A_1^3} \propto x \beta_s \tag{2}$$

It is found that, the measured amplitude ratio of fundamental and higher harmonic waves is a function of wave propagation distance, thus if the material nonlinearity is fixed, the acoustic nonlinear response of amplitude ratio  $(A_3/A_1^3)$  will have accumulative effect with propagation distance (x).

The cumulative effect of the higher harmonic amplitude is of great advantage for detection in experimental work to measure the nonlinear effect with sufficient signal-to-noise ratio. The higher harmonic amplitude grows linearly with propagation direction, when nonzero power flux and phase matching conditions are satisfied.



Fig. 2 Phase velocity (a) and group velocity (b) dispersion curves for SH guided waves in an aluminum plate

To choose the guided wave mode which satisfies the phase matching condition, we use numerical dispersion curves to find the "phase matching" modes. As shown in Fig. 2, the SHO mode is selected in this work for several reasons: (1) the SHO guided wave modes have no mode conversion and are non-dispersive, so it is relatively easy to excite and detect by magnetostricitve transducers; (2) the SH0 guided modes have high resolution wave and combination of high velocities; (3) both the phase and group velocity of the fundamental and higher harmonic SH0 wave modes are identical.

## 3. Experiments

The specimens used in this study are aluminum plate with 1 mm thickness. The dimensions are 500 mm  $\times$  500 mm. From the dispersion curves, the SH0 guided mode at fd = 0.5 MHz mm with the phase velocity of 3.14 km/s is chosen. Since the thickness of the specimen is 1 mm, therefore the generated fundamental frequency is 0.5 MHz. And the wavelength can be calculated as,

$$\lambda = \frac{Cp}{f} = \frac{3.14}{0.5} mm \approx 6.3mm \tag{3}$$

The equipment setup to measure the nonlinear ultrasonic wave is shown in Fig. 3. A high voltage sinusoidal tone burst system with



Fig. 3 Block diagram of experimental measurement system (a) and (b) the differences of generator and receiver

MS sensors is employed to generate the ultrasonic signal of 10 cycles at 0.5 MHz. The attenuator and receiver unit are equipped with amplifier connected to the generator. an Consistent loading is applied on the transducers to ensure consistent bonding conditions between and specimens. transducers The frequency bandwidth can readily be limited by windowed tone burst signals that generated by the measurement system. A meander coil is placed over each foil and then a permanent magnet placed over the coil to form a MS transducer. The spacing of the coils dictates the wavelength to be excited or received. To detect the corresponding third harmonic frequency components primarily, the spacing of the coils in receivers is designed as one third of that of generator.

The use of MS patch transducer is an efficient method to generate and measure SH waves [15]. The cross-sectional diagram of MS transducer module is shown in Fig. 4. This novel small-sized SH guided wave transducer can be used to generate and detect SH guided wave signals efficiently. It is composed of ferromagnetic element made by Fe-Co alloy to detect magnetic field from ferromagnetic elements. The periodic planar solenoid is used in the transducer module. When the applied dynamic magnetic field is orthogonal to the static bias magnetic field in the MS patch, shear deformation will be developed. Therefore, small



Fig. 4 Magnetostrictive transducer bonded onto a plate-like structure

rectangular elements in the path will have shearing deformations. The transducers can be installed without any coupling media. The transducers can be applied to non-ferromagnetic structures as well as ferromagnetic ones. Another advantage is that the transducers can be applied to either plate or the structures with curved surface.

To generate expected wave mode, the frequency of wave signal is emitted by the function generator, while the wavelength is controlled by the spacing of adjacent coil. The adjacent coil distance is half of the wavelength of expected wave mode. This is the principle of magnetostrictive transducer design for generate and measure the expected guided wave mode.

## 4. Results and Discussions

The physical effect of nonlinear ultrasonic technique is the generation of higher harmonic waves. After the SH guided wave propagating in the specimen with material nonlinearity, the waveform will be distorted and thus the higher harmonic could be generated. As shown in Fig. 5, a typical distorted waveform of nonlinear SH guided wave propagation a certain distance is provided in both time domain and frequency domain. In the frequency spectrum of the signal, it can find the third harmonic waves at triple frequency at 1.5 MHz in the Fig. 5(b). Since it has been widely recognized that the cubic nonlinearity will be induced by shear ultrasonic wave propagation, the experimental results have a good agreement with the prediction. It is important to note that the receiving signal at triple frequency of fundamental wave mode is the primary higher harmonic generation of SH0 guided wave propagation. So, the differences between nonlinear SH guided wave and other types of guided waves, such Rayleigh surface wave or Lamb wave, is that: the centre frequency of detector is triple of that of



Fig. 5 Nonlinear SH guided wave signal in time domain (a) and (b) frequency domain

generator for nonlinear SH guided wave testing, while others are double frequency.

As discussed in the theoretical part, the SHO guided wave modes are not dispersive, its third harmonic wave mode is internally resonant. Third harmonic fields of SHO guided wave modes in waveguides are also SHO guided wave modes at triple frequency. In this work, the third harmonic mode which is in resonance with primary wave, plays a dominant role in nonlinear acoustic field, the amplitude of this higher harmonic mode has accumulative effect. The cumulative effect of the third harmonic amplitude verifies that, under certain conditions, acoustic nonlinearity of the SH ultrasonic guided wave can be applied for tracking micro-damages in waveguide. As shown in Fig. 6, the relative higher harmonic amplitude grows with the propagation distance via the accumulative effect to a certain point when material attenuation



Fig. 6 Relative acoustic nonlinear response  $(A_3/A_1^3)$  versus propagation distance

becomes dominant. To ensure that the measurements from the specimens are not due to the one arising from the measurement system uncertainty, but due to the damage-induced nonlinearity, the demonstration of this cumulative effect is essential.

### 5. Conclusions

In this study, nonlinear SH guided wave propagation in an isotropic plate with weakly nonlinearity is investigated by using MS sensors. It is found that third harmonic wave with triple frequency of that of fundamental wave is generated during the wave propagation in the specimen. Measurement of the strongly nonlinear response of shear horizontal guided waves is presented with the accumulative growth of relative acoustic nonlinear response as propagation distance. The experimental results have a good agreement with the theoretical prediction and thus show that the use of electromagnetic shear horizontal guided wave is favorable to detect the cubic material nonlinearity.

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