

## CHROMATIC NUMBER OF BIPOLAR FUZZY GRAPHS

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**ABSTRACT.** In this paper, two different approaches to chromatic number of a bipolar fuzzy graph are introduced. The first approach is based on the  $\alpha$ -cuts of a bipolar fuzzy graph and the second approach is based on the definition of Eslahchi and Onagh for chromatic number of a fuzzy graph. Finally, the bipolar fuzzy vertex chromatic number and the edge chromatic number of a complete bipolar fuzzy graph, characterized.

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### 1. Introduction

In 1965, Zadeh [20] first proposed the theory of fuzzy sets. In 1994, Zhang [21, 22] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are extension of fuzzy sets whose range of membership degree is  $[-1, 1]$ . In a bipolar fuzzy sets, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree  $(0, 1]$  of a element indicates that the element somewhat satisfies the property, and the membership degree  $[-1, 0)$  of an element indicates that the element somewhat satisfies the implicit counter property. For example, sweetness of foods is a bipolar fuzzy set. If sweetness of foods has been given as positive membership values then bitterness foods is for negative membership values. Other tastes like salty, sour, pungent are irrelevant to the corresponding property. So these foods are taken as zero membership values. Graph theory is rapidly moving into the mainstream of mathematics because of its applications in diverse fields which include biochemistry (DNA double helix and SNP assembly problem), Chemistry (model chemical compounds) electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). Graph coloring is one

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of the most important concepts in graph theory and is used in many real time applications like Job scheduling, Aircraft scheduling, computer network security, Map coloring and GSM mobile phone networks, Automatic channel allocation for small wireless local area networks. In 1975 Rosenfeld [16] introduced the concept of fuzzy graphs. Akram [1] defined bipolar fuzzy graphs. Talebi and Rashmanlou investigated isomorphism on interval-valued fuzzy graphs [19]. Dey and Pal [4] introduced a coloring function based on  $\alpha$  cut of fuzzy graph  $G$ . Swaminathan [17] introduced fuzzy chromatic sum, fuzzy chromatic join, find a natural application in scheduling theory. Poornima and Ramaswamy [11] considered total coloring of a fuzzy graph. Ananthanarayanan and Lavanya [2] defined fuzzy chromatic number, fuzzy total chromatic number using  $\alpha$  cuts of the fuzzy graph which are crisp graphs. Firouzian and Nouri Jouybari [7] analyzed traffic lights problem by fuzzy chromatic number. The fuzzy vertex coloring of a fuzzy graph was defined by authors Eslahchi and Onagh [6] and another approach of vertex coloring of fuzzy graph was used in S. Munoz, T. Ortuno. Pal and Rashmanlou [10] studied irregular interval valued fuzzy graphs. Also, they defined antipodal interval valued fuzzy graphs [13], balanced interval valued fuzzy graphs [14], some properties of highly irregular interval valued fuzzy graphs [15]. In our paper, we determine bipolar fuzzy chromatic number of a bipolar fuzzy graph  $G$  whose edge and vertices both are bipolar fuzzy set.

## 2. Preliminaries

Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph. A third natural coloring called the total coloring. This three types of coloring are defined as follows:

(i) Vertex coloring of graph refers to assigning colors to vertices so that adjacent vertices are differently colored. Let  $N$  denote the set of all natural numbers and assume that a distinct and unique natural number is associated with each color. Given a graph  $G = (V, E)$ , a vertex coloring function is a function  $C_V : V \rightarrow N$  such that  $C_V(u) \neq C_V(v)$  for any two adjacent vertices  $u$  and  $v$ . A vertex  $K$ -coloring  $C_V^K$  is a vertex coloring function in which no more than  $K$  different colors are used. In other words  $C_V^K : V \rightarrow \{1, 2, \dots, K\}$ . A graph is said to be vertex  $K$ -colored if it admits a vertex  $K$ -coloring. The minimum value of  $K$  for which  $G$  is vertex  $K$ -colored is called vertex chromatic number of  $G$  and denoted by  $\chi_V(G)$ .

(ii) Edge coloring of a graph refers to assigning colors to edges so that adjacent edges are differently colored. It is known that every graph  $G$  can be edge colored with at most  $\delta(G) + 1$  colors and at least  $\delta(G)$  colors are always necessary. Given a graph  $G = (V, E)$ , an edge coloring function is a function  $C_E : E \rightarrow N$  such that  $C_E(i, j) \neq C_E(i, k)$  and  $C_E(i, j) \neq C_E(l, j)$  for all edges  $(i, j), (i, k)$  and  $(l, j) \in E$ . An edge  $K$ -coloring  $C_E^K$  is an edge coloring function in which no more than  $K$  different colors are used. In other words,  $C_E^K : E \rightarrow \{1, 2, \dots, K\}$ . A graph is said to be edge  $k$ -colored if it admits an edge  $K$ -coloring. The

minimum value of  $K$  for which  $G$  is edge  $K$ -colored is called edge chromatic number of  $G$  and is denoted by  $\chi_E(G)$ .

(iii) A total coloring of a graph  $G$  is a coloring of the vertices and edges of  $G$  in such a way that no two adjacent elements have the same color. Adjacent elements here refer either to two vertices connected by an edge or to two edges incident on a common vertex or to an edge and its end vertices. Given a graph  $G = (V, E)$ , a total coloring function is a function  $C_T : V \cup E \rightarrow N$  which satisfies the following conditions.

(i)  $C_T(u) \neq C_T(v)$ , for any two adjacent vertices  $u, v \in V$ .

(ii)  $C_T(i, j) \neq C_T(i, k)$  and

$C_T(i, j) \neq C_T(l, j)$ , for all edges  $(i, j), (i, k)$  and  $(l, j) \in E$ .

(iii)  $C_T(u) \neq C_T(u, v)$

$C_T(v) \neq C_T(u, v)$ , for any two adjacent vertices  $u, v \in V$ .

A total  $K$  coloring  $C_T^K$  is a total coloring function in which no more than  $K$  different colors are used. In other words  $C_T^K : V \cup E \rightarrow \{1, 2, \dots, K\}$ . A graph is said to be total  $K$ -colored if it admits a total  $K$ -coloring. The minimum value of  $K$  for which  $G$  is total  $K$ -colored is referred to as the total chromatic number of  $G$  and is denoted by  $\chi_T(G)$  (See [7]).

The triple  $G = (V, \sigma, m)$  is called a fuzzy graph on  $V$  where  $\sigma$  and  $m$  are fuzzy sets on  $V$  and  $V \times V$ , respectively, such that  $m(\{u, v\}) \leq \min\{\sigma(u), \sigma(v)\}$ , for all  $u, v \in V$ . Note that a fuzzy graph is a generalization of crisp graph in which  $m(v) = 1$ , for all  $v \in V$  and  $m(i, j) = 1$  if  $(i, j) \in E$ , otherwise  $m(i, j) = 0$  (See [2]).

**Definition 2.1** ([1]). Let  $X$  be a nonempty set. A bipolar fuzzy set  $B$  on  $X$  is an object having the form  $B = \{(x, m^+(x), m^-(x)); x \in X\}$ , where  $m^+ : X \rightarrow [0, 1]$  and  $m^- : X \rightarrow [-1, 0]$  are mapping. A bipolar fuzzy graph with an underlying set  $V$  is defined to be the pair  $G = (A, B)$  where  $A = (m_A^+, m_A^-)$  is a bipolar fuzzy set on  $V$  and  $B = (m_B^+, m_B^-)$  is a bipolar fuzzy set on  $E \subseteq V \times V$  such that

$$m_B^+(x, y) \leq \min\{m_A^+(x), m_A^+(y)\}, m_B^-(x, y) \geq \max\{m_A^-(x), m_A^-(y)\}$$

Let  $G = (A, B)$  and  $G' = (A', B')$  be two bipolar fuzzy graphs. A homomorphism  $h : G \rightarrow G'$  is a mapping  $h : V \rightarrow V'$  which satisfies the following conditions:

(i)  $m_A^+(x) \leq m_{A'}^+(h(x))$  and  $m_A^-(x) \geq m_{A'}^-(h(x))$ ,

(ii)  $m_B^+(xy) \leq m_{B'}^+(h(x)h(y))$  and  $m_B^-(xy) \geq m_{B'}^-(h(x)h(y))$ ,

**Theorem 2.2** ([3]). Let  $G$  be a crisp graph and  $H$  be a subgraph of  $G$ . Then  $\chi(H) \leq \chi(G)$ .

### 3. Main results

Let  $G = (A, B)$  be a bipolar fuzzy graph, where  $A = (m_A^+, m_A^-)$  and  $B = (m_B^+, m_B^-)$  be two bipolar fuzzy sets on a non-empty finite set  $V$  and  $E \subseteq V \times V$ , respectively. The positive and negative level set of fuzzy bipolar set  $A$  is defined

as  $L_A^+ = \{\alpha^+; m_A^+(u) = \alpha^+, \text{ for some } u \in V\}$ ,  $L_A^- = \{\alpha^-; m_A^-(u) = \alpha^-, \text{ for some } u \in V\}$  and the positive and negative level set of  $B$  is defined as  $L_B^+ = \{\alpha^+; m_B^+(u, v) = \alpha^+ \text{ for some } (u, v) \in V \times V\}$ ,  $L_B^- = \{\alpha^-; m_B^-(u, v) = \alpha^- \text{ for some } (u, v) \in V \times V\}$ . The fundamental set of the fuzzy bipolar graph  $G = (A, B)$  is defined as  $L^+ = L_A^+ \cup L_B^+$  and  $L^- = L_A^- \cup L_B^-$ . We define for  $\alpha^+ \in L^+$ ,  $\alpha^- \in L^-$ ,  $G_{\alpha^+} = (V_{\alpha^+}, E_{\alpha^+})$ , where  $V_{\alpha^+} = \{v \in V; m_A^+ \geq \alpha^+\}$  and  $E_{\alpha^+} = \{e \in E; m_B^+ \geq \alpha^+\}$  and  $G_{\alpha^-} = (V_{\alpha^-}, E_{\alpha^-})$ , where  $V_{\alpha^-} = \{v \in V; m_A^- \leq \alpha^-\}$  and  $E_{\alpha^-} = \{e \in E; m_B^- \leq \alpha^-\}$ .

**Definition 3.1.** The chromatic number of  $G$  is defined as  $(\chi_G^+, \chi_G^-) = (\max \chi(G_{\alpha^+}), \max \chi(G_{\alpha^-}))$ , where  $\chi_{\alpha^+}$  and  $\chi_{\alpha^-}$  are the chromatic number of  $G_{\alpha^+}$  and  $G_{\alpha^-}$ , respectively and  $\alpha^+$ ,  $\alpha^-$  are the positive and negative different membership value of vertices and edges of graph  $G$ . Moreover, the chromatic number of bipolar fuzzy graph  $G = (A, B)$ , is fuzzy number  $\chi(G) = \{(\chi_{\alpha^+}, \alpha^+), (\chi_{\alpha^-}, \alpha^-)\}$ , where  $\chi_{\alpha^+}$  and  $\chi_{\alpha^-}$  are the chromatic number of  $G_{\alpha^+}$  and  $G_{\alpha^-}$  which  $\alpha^+ \in L^+ \cup \{0\}$  and  $\alpha^- \in L^-$ . Two vertices  $u$  and  $v$ , for any strong edge in  $G$ , are called adjacent if  $m_B^+(u, v) = \min\{m_A^+(u), m_A^+(v)\}$  and  $m_B^-(u, v) = \max\{m_A^-(u), m_A^-(v)\}$ . A family  $\Gamma = (\Gamma^+, \Gamma^-) = (\{\gamma_1^+, \gamma_2^+, \dots, \gamma_{k_1}^+\}, \{\gamma_1^-, \gamma_2^-, \dots, \gamma_{k_2}^-\})$  of bipolar fuzzy sets on set  $V$ , is called a  $(k_1, k_2)$ -bipolar fuzzy coloring of  $G = (A, B)$ , if

- (i)  $(\vee \Gamma^+, \wedge \Gamma^-) = (m_1^+, m_1^-) = A$ ,
- (ii)  $\gamma_i^+ \wedge \gamma_j^+ = 0, \gamma_i^- \vee \gamma_j^- = 0$ ,
- (iii) for every strong edge  $(x, y)$  of  $G$ ,  $(\min\{\gamma_i^+(x), \gamma_j^+(y)\}, \max\{\gamma_j^-(x), \gamma_j^-(y)\}) = (0, 0)$ .

The minimum numbers  $k_1$  and  $k_2$  which there exists a  $(k_1, k_2)$ -fuzzy bipolar coloring is called the bipolar fuzzy chromatic number of  $G$  and denoted by  $\chi^f(G) = (\chi^{f^+}(G), \chi^{f^-}(G))$ .

**Theorem 3.2.** Let  $G = (A, B)$  and  $G' = (A', B')$  be two bipolar fuzzy graphs and  $h : G \rightarrow G'$  be a homomorphism. Then for all  $\alpha^+ \in L^+$  and  $\alpha^- \in L^-$ ,  $\chi(G_{\alpha^+}) \leq \chi(G'_{\alpha^+})$  and  $\chi(G_{\alpha^-}) \leq \chi(G'_{\alpha^-})$  and so  $\chi(G) \leq \chi(G')$ .

*Proof.* Let  $h(G_{\alpha^+}) = (V''_{\alpha^+}, E''_{\alpha^+})$ . We prove that  $h(G_{\alpha^+})$  is a subgraph of  $G'_{\alpha^+}$ . Let  $x'', y'' \in V''_{\alpha^+}$ . Then there exist  $x, y \in G_{\alpha^+}$  such that  $x'' = h(x)$  and  $y'' = h(y)$ . Since  $m_A^+(x) \geq \alpha^+$  and  $m_A^+(y) \geq \alpha^+$ ,  $m_{A'}^+(h(x)) \geq \alpha^+$  and  $m_{A'}^+(h(y)) \geq \alpha^+$ . Then  $m_{A'}^+(x'') \geq \alpha^+$  and  $m_{A'}^+(y'') \geq \alpha^+$  and so  $x'', y'' \in V'_{\alpha^+}$ . Now, let  $x''y'' \in E''_{\alpha^+}$ . Then there exist  $x, y \in E_{\alpha^+}$  such that  $x'' = h(x)$  and  $y'' = h(y)$ . Since  $m_B^+(xy) \geq \alpha^+$ , we have  $m_{B'}^+(h(x)h(y)) \geq \alpha^+$ . Then  $m_{B'}^+(x''y'') \geq \alpha^+$  and so  $x''y'' \in E'_{\alpha^+}$ . By Theorem 2.2,  $\chi(h(G_{\alpha^+})) \leq \chi(G'_{\alpha^+})$ . Since  $\chi(h(G_{\alpha^+})) = \chi(G_{\alpha^+})$ ,  $\chi(G_{\alpha^+}) \leq \chi(G'_{\alpha^+})$ . Similarly,  $\chi(G_{\alpha^-}) \leq \chi(G'_{\alpha^-})$ . So by definition of chromatic number of bipolar fuzzy graph,  $\chi(G) \leq \chi(G')$ .  $\square$

**Example 3.3.** Let  $G = (A, B)$  be a fuzzy bipolar graph, where  $A = (m_A^+, m_A^-)$  and  $B = (m_B^+, m_B^-)$  are two fuzzy bipolar sets on non-empty finite set  $V$  and  $E \subseteq V \times V$ . Moreover, let  $G$  has five vertices and membership value of those vertices that are  $A = \{(0.9, -0.5), (0.7, -0.4), (0.8, -0.5), (0.7, -0.5)\}$ ,

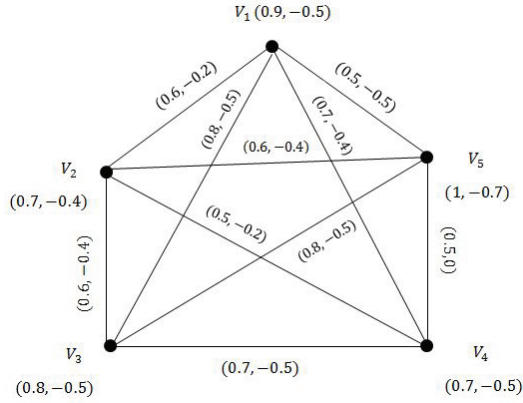


FIGURE 1

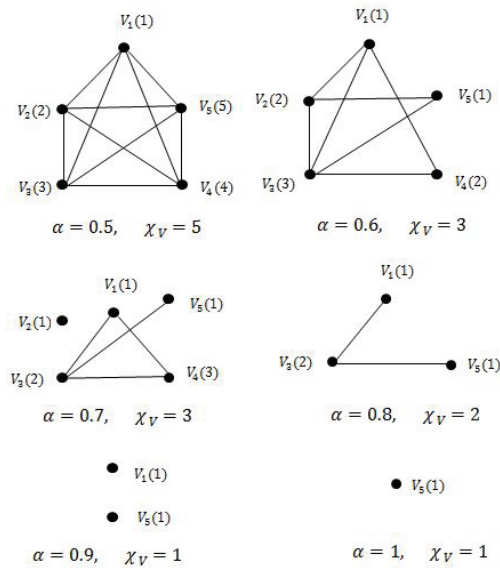


FIGURE 2

$(1, -0.7)$  and graph  $G$  has 10 edges. Consider the crisp graphs  $G_{\alpha^+}$  and  $G_{\alpha^-}$  corresponding to the values in the positive, negative level set of the bipolar fuzzy graph  $G$  given in Figure 1. For every values of  $\alpha^+ \in L^+ \cup \{0\}$  and  $\alpha^- \in L^-$ , we find graphs  $G_{\alpha^+}$  and  $G_{\alpha^-}$  are showed in Figure 2 and Figure 3 and find its fuzzy bipolar chromatic number.

Graphs  $G_{\alpha^-}$  as follows:

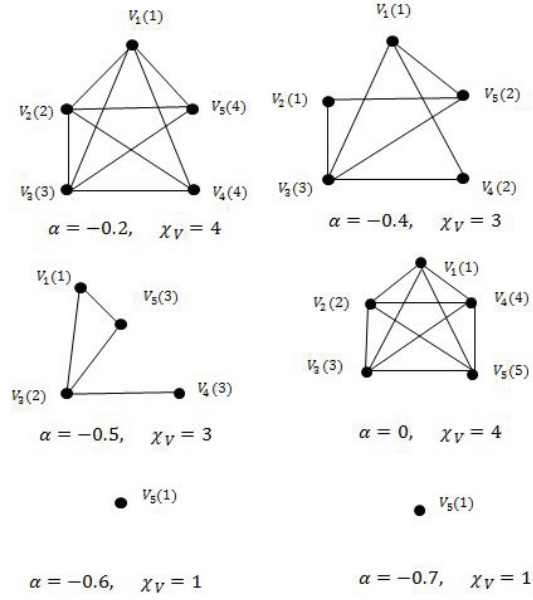


FIGURE 3

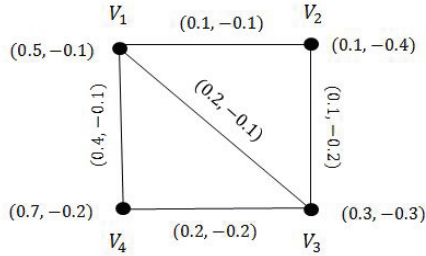


FIGURE 4

$\alpha$	$\chi_V$	$C_\alpha(1)$	$C_\alpha(2)$	$C_\alpha(3)$	$C_\alpha(4)$	$C_\alpha(5)$
0	5	1	2	3	4	5
0.5	5	1	2	3	4	5
0.6	3	1	2	3	2	1
0.7	3	1	1	2	3	1
0.8	2	1	0	2	0	1
0.9	1	1	0	0	0	1
1	1	0	0	0	0	1
-0.2	4	1	2	3	4	4
-0.4	3	1	1	3	2	2
-0.5	3	1	0	2	3	3
-0.6	1	0	0	0	0	1
-0.7	1	0	0	0	0	1

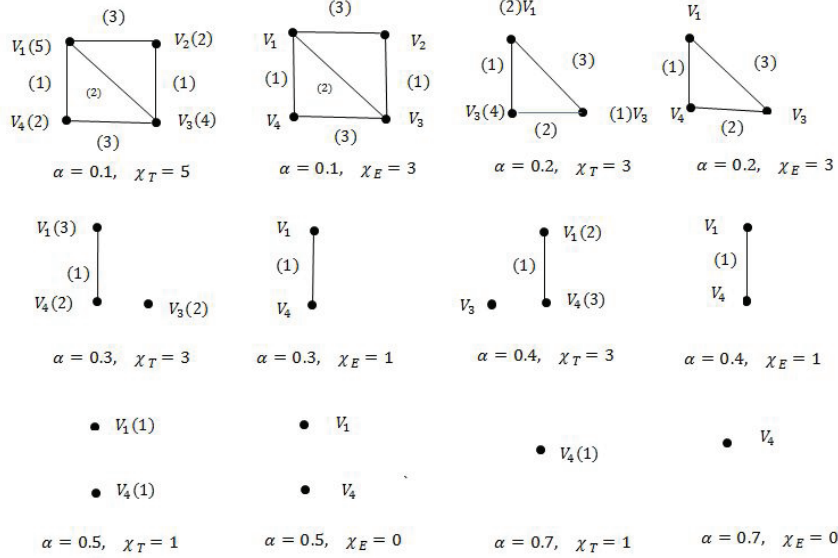


FIGURE 5

In the above example, six crisp graphs  $G_{\alpha^+} = (V_{\alpha^+}, E_{\alpha^+})$ , five crisp graphs  $G_{\alpha^-} = (V_{\alpha^-}, E_{\alpha^-})$  are obtained by considering the values for  $\alpha^+$  and  $\alpha^-$ . Now it can be shown that the chromatic number of bipolar fuzzy graph  $G$  is  $\chi_V(G) = \{(5, 0.5), (3, 0.6), (3, 0.7), (2, 0.8), (1, 0.9), (1, 1), (1, -0.7), (1, -0.6), (3, -0.5), (3, -0.4), (4, -0.2), (5, 0)\}$ .

**Example 3.4.** Consider the bipolar fuzzy graph  $G = (A, B)$ , where  $V = \{v_1, v_2, v_3, v_4\}$  and  $E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_3v_4\}$ .

Consider the crisp graphs  $G_{\alpha^+}$  and  $G_{\alpha^-}$  corresponding to the values in the positive, negative level set of the bipolar fuzzy graph  $G$  given in Figure 4,  $L^+ = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7\}$  and  $L^- = \{-0.4, -0.3, -0.2, -0.1\}$ . For every values of  $\alpha^+ \in L^+, \alpha^- \in L^-$ , we find graphs  $G_{\alpha^+}, G_{\alpha^-}$  and its edge coloring number and total coloring number. Graphs  $G_{0.1}, G_{0.2}, G_{0.3}, G_{0.4}, G_{0.5}, G_{0.7}$  are showed in Figure 5, Graphs  $G_0, G_{-0.4}, G_{-0.3}, G_{-0.2}, G_{-0.1}$  are showed in Figure 6.

In the above example, seven crisp graphs  $G_{\alpha^+} = (V_{\alpha^+}, E_{\alpha^+})$ , four crisp graphs  $G_{\alpha^-} = (V_{\alpha^-}, E_{\alpha^-})$  are obtained by considering the values for  $\alpha^+, \alpha^-$ . Now it can be shown that the edge, total chromatic number of bipolar fuzzy graph  $G$  is  $\chi_E(G) = \{(3, 0.1), (3, 0.2), (1, 0.3), (1, 0.4), (0, 0.5), (0, 0.7), (3, 0), (3, -0.1), (2, -0.2), (0, -0.3), (0, -0.4)\}$ ,  $\chi_T(G) = \{(5, 0.1), (3, 0.2), (3, 0.3), (3, 0.4), (1, 0.5), (1, 0.7), (5, -0.1), (4, -0.2), (1, -0.3), (1, -0.4), (5, 0)\}$ .

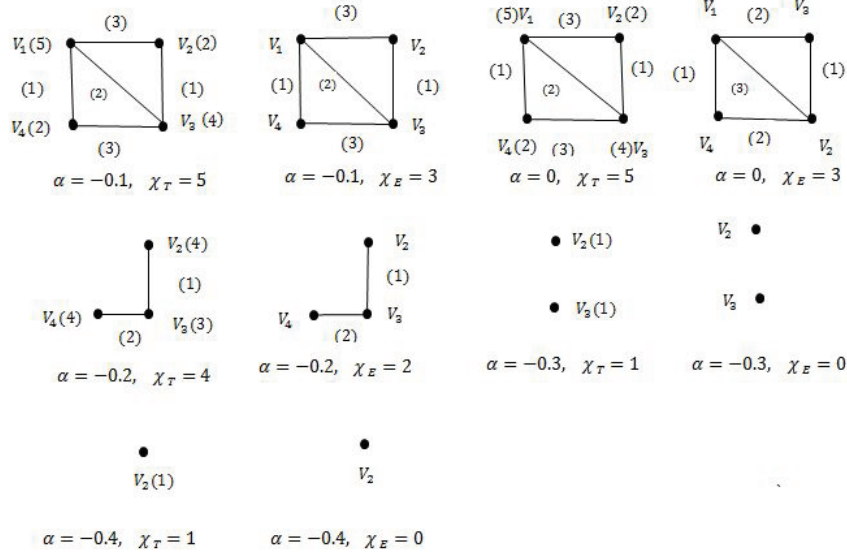


FIGURE 6

$\alpha$	0	0.1	0.2	0.3	0.4	0.5	0.7	-0.1	-0.2	-0.3	-0.4
$\chi_E$	3	3	3	1	1	0	0	3	2	0	0
$\chi_T$	5	5	3	3	3	1	1	5	4	1	1
$C_\alpha(1)$	5	5	2	3	2	1	0	5	0	0	0
$C_\alpha(2)$	2	2	0	0	0	0	0	2	4	1	1
$C_\alpha(3)$	4	4	1	2	0	0	0	4	3	1	0
$C_\alpha(4)$	2	2	3	2	3	1	1	2	4	0	0
$C_\alpha(12)$	3	3	0	0	0	0	0	3	0	0	0
$C_\alpha(13)$	2	2	3	0	0	0	0	2	0	0	0
$C_\alpha(14)$	1	1	1	1	1	0	0	1	0	0	0
$C_\alpha(23)$	1	1	0	0	0	0	0	1	1	0	0
$C_\alpha(34)$	3	3	2	0	0	0	0	3	2	0	0

**Remark 3.1.** For a bipolar fuzzy graph  $G = (A, B)$ , if  $\alpha_i^+ < \alpha_j^+$  and  $\alpha_i^- < \alpha_j^-$ , then  $\chi_{\alpha_i^+} \geq \chi_{\alpha_j^+}, \chi_{\alpha_i^-} \leq \chi_{\alpha_j^-}$ .

**Theorem 3.5.** For a bipolar fuzzy graph  $G = (A, B)$ ,  $\chi(G) = \chi^f(G)$ .

*Proof.* Let  $G = (A, B)$  be a bipolar fuzzy graph on  $n$  vertices  $\{v_1, v_2, \dots, v_n\}$  and  $\chi^f(G) = (k, k')$ . Then  $\Gamma = \{(\gamma_1^+, \gamma_2^+, \dots, \gamma_k^+), (\gamma_1^-, \gamma_2^-, \dots, \gamma_{k'}^-)\}$  is a  $(k, k')$ -bipolar fuzzy coloring. If  $C_{j^+}$  and  $C_{j^-}$  is the colors assigned to vertices in



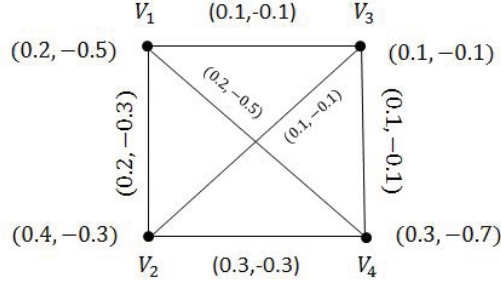


FIGURE 7

$\gamma_{j^+}^*$  and  $\gamma_{j^-}^*$ , where  $j^+ = 1, 2, \dots, k, j^- = 1, 2, \dots, k'$  and  $\gamma_{j^+}^* = \{v \in V; \gamma_j(v) > 0\}$ ,  $\gamma_{j^-}^* = \{v \in V; \gamma_j(v) < 0\}$ , then  $\{(\gamma_1^+, \gamma_2^+, \dots, \gamma_k^+), (\gamma_1^-, \gamma_2^-, \dots, \gamma_{k'}^-)\}$  is a family of bipolar fuzzy sets, where  $\gamma_{j^+}(v_i) = \{(v_j, m_{A^+}(v_j))\} \cup \{(v_i, m_{A^+}(v_i))\}$ ;  $m_{B^+}(v_i, v_j) = 0, i \neq j\}$  and  $\gamma_{j^-}(v_i) = \{(v_j, m_{A^-}(v_j))\} \cup \{(v_i, m_{A^-}(v_i))\}$ ;  $m_{B^-}(v_i, v_j) = 0, i \neq j\}$ . Also by Definition 3.1,  $\bigcup_{j=1}^k \gamma_{j^+}^* = \bigcup_{j=1}^{k'} \gamma_{j^-}^* = V$  and  $\gamma_{i^+}^* \cap \gamma_{j^+}^* = \gamma_{i^-}^* \cap \gamma_{j^-}^* = \emptyset, i^+ \neq j^+, i^- \neq j^-$ . Hence  $\gamma_{j^+}^*, \gamma_{j^-}^*$  are independent sets of vertices (i.e no two vertices in  $\gamma_{j^+}^*, \gamma_{j^-}^*$  are adjacent) for each  $j^+ = 1, 2, \dots, k, j^- = 1, 2, \dots, k'$ . Then by remark 3.5,  $\chi(G) = (\chi(G_t), \chi(G_{t'})) = (k, k')$ , where  $t = \min\{\alpha^+, \alpha^+ \in L^+\} = \max\{\chi_{\alpha^+}; \alpha^+ \in L^+\}$ ,  $t' = \max\{\alpha^-, \alpha^- \in L^-\} = \max\{\chi_{\alpha^-}; \alpha^- \in L^-\}$ . Therefore,  $\chi(G) = \chi^f(G)$ .  $\square$

**Example 3.6.** Consider the following bipolar fuzzy graph  $G = (A, B)$  and the crisp graphs  $G_{0.4}, G_{0.3}, G_{0.2}, G_{0.1}, G_0, G_{-0.7}, G_{-0.5}, G_{-0.3}, G_{-0.1}$  given in Figures 7,8 corresponding to the values in the positive and negative level set of the bipolar fuzzy graph,  $L^+ = \{0.1, 0.2, 0.3, 0.4\} \cup \{0\}$  and  $L^- = \{-0.7, -0.5, -0.3, -0.1\}$ . The fuzzy coloring is  $\Gamma = (\{\gamma_1^+, \gamma_2^+, \gamma_3^+, \gamma_4^+\}, \{\gamma_1^-, \gamma_2^-, \gamma_3^-, \gamma_4^-\})$ .

$$\begin{aligned} \gamma_1^+(x_i) &= \begin{cases} 0.2 & \text{if } i = 1 \\ 0 & \text{o.w} \end{cases} & \gamma_1^-(x_i) &= \begin{cases} -0.5 & \text{if } i = 1 \\ 0 & \text{o.w} \end{cases} \\ \gamma_2^+(x_i) &= \begin{cases} 0.4 & \text{if } i = 2 \\ 0 & \text{o.w} \end{cases} & \gamma_2^-(x_i) &= \begin{cases} -0.3 & \text{if } i = 2 \\ 0 & \text{o.w} \end{cases} \\ \gamma_3^+(x_i) &= \begin{cases} 0.1 & \text{if } i = 3 \\ 0 & \text{o.w} \end{cases} & \gamma_3^-(x_i) &= \begin{cases} -0.1 & \text{if } i = 3 \\ 0 & \text{o.w} \end{cases} \\ \gamma_4^+(x_i) &= \begin{cases} 0.3 & \text{if } i = 4 \\ 0 & \text{o.w} \end{cases} & \gamma_4^-(x_i) &= \begin{cases} -0.7 & \text{if } i = 4 \\ 0 & \text{o.w} \end{cases} \end{aligned}$$

Hence  $\chi^f(G) = (\chi^{f^+}(G), \chi^{f^-}(G)) = (4, 4)$  and the crisp graphs coloring yields  $\chi(G) = (\max\chi(G_{\alpha^+}), \max\chi(G_{\alpha^-})) = (4, 4)$ .

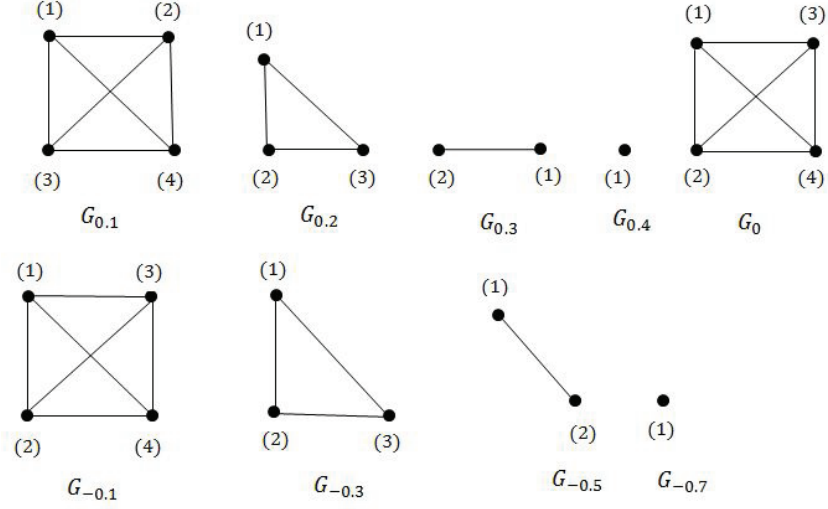


FIGURE 8

vertices	$\gamma_1^+$	$\gamma_2^+$	$\gamma_3^+$	$\gamma_4^+$	max
$v_1$	0.2	0	0	0	0.2
$v_2$	0	0.4	0	0	0.4
$v_3$	0	0	0.1	0	0.1
$v_4$	0	0	0	0.3	0.3

vertices	$\gamma_1^-$	$\gamma_2^-$	$\gamma_3^-$	$\gamma_4^-$	min
$v_1$	-0.5	0	0	0	-0.5
$v_2$	0	-0.3	0	0	-0.3
$v_3$	0	0	-0.1	0	-0.1
$v_4$	0	0	0	-0.7	-0.7

**Definition 3.7.** A bipolar fuzzy graph  $G = (A, B)$  is called a complete bipolar fuzzy graph on  $n$  vertices, denoted by  $K_n$ , if  $|V(G)| = n, xy \in E(G), m_{B^+}(x, y) = \min(m_{A^+}(x), m_{A^+}(y))$  and  $m_{B^-}(x, y) = \max(m_{A^-}(x), m_{A^-}(y))$ , for any distinct element  $x, y \in V(G)$ .

**Corollary 3.8.** The bipolar fuzzy vertex chromatic number of complete bipolar fuzzy graph  $G = (A, B)$  is  $(n, n)$ , where  $n$  is the number of vertices of  $G$ , i.e.  $\chi_V(G) = (n, n)$ .

*Proof.* Consider the crisp graphs  $G_{\alpha^+}$  and  $G_{\alpha^-}$ , where  $\alpha^- = \max L^-$  and  $\alpha^+ = \min L^+$ . Since  $G$  is a complete bipolar fuzzy graph, hence  $G_{\alpha^+}, G_{\alpha^-}$  are complete crisp graphs. Therefore,  $\chi_V(G) = (\chi_V(G_{\alpha^+}), \chi_V(G_{\alpha^-})) = (n, n)$ .  $\square$

**Corollary 3.9.** *The edge chromatic number of complete bipolar fuzzy graph  $G = (A, B)$  on  $n$  vertices is  $(n, n)$ , if  $n$  is odd and is  $(n - 1, n - 1)$ , if  $n$  is even.*

*Proof.* Consider the crisp graphs  $G_{\alpha^+}$  and  $G_{\alpha^-}$ , where  $\alpha^- = \max L^-$  and  $\alpha^+ = \min L^+$ . Since  $G$  is a complete bipolar fuzzy graph, hence  $G_{\alpha^+}$  and  $G_{\alpha^-}$  are complete crisp graphs. Therefore,  $\chi_E(G) = (\chi_E(G_{\alpha^+}), \chi_E(G_{\alpha^-})) = (n, n)$ , if  $n$  is odd and is equal  $(n - 1, n - 1)$ , if  $n$  is even.  $\square$

#### 4. Conclusion

In this paper, we compute bipolar fuzzy vertex, edge, total chromatic number based on  $\alpha^+$ -cut,  $\alpha^-$ -cut of a bipolar fuzzy graph whose edge and vertices both are bipolar fuzzy set. Also, we propose definition Eslahchi and Onagh [6] of bipolar fuzzy chromatic number and prove this two definition are equivalence.

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