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FILTERS OF BE-ALGEBRAS WITH RESPECT TO A CONGRUENCE

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Abstract. Some properties of filters are studied with respect to a congruence of BE-algebras. The notion of θ-filters is introduced and these classes of filters are then characterized in terms of congruence classes. A bijection is obtained between the set of all θ-filters of a BE-algebra and the set of all filters of the respective BE-algebra of congruences classes.

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1. Introduction

The notion of BE-algebras was introduced and extensively studied by H.S. Kim and Y.H. Kim in [5]. These classes of BE-algebras were introduced as a generalization of the class of BCK-algebras of K. Iseki and S. Tanaka [4]. Some properties of filters of BE-algebras were studied by S.S. Ahn and Y.H. Kim in [1]. In [8], the notion of normal filters is introduced in BE-algebras. In [2, 3], S.S. Ahn and Y.H. So and K.S. So introduced the notion of ideals in BE-algebras and proved several characterizations of such ideals. Also they generalized the notion of upper sets in BE-algebras, and discussed some properties of the characterizations of generalized upper sets related to the structure of ideals in transitive and self-distributive BE-algebras. In 2012, S.S. Ahn, Y.H. Kim and J.M. Ko [1] introduced the notion of a terminal section of BE-algebras and derived some characterizations of commutative BE-algebras in terms of lattice ordered relations and terminal sections. Recently in 2015, J.H. Park and Y.H. Kim [7] studied the properties of intersectional soft implicative filters of BE-algebras. In [10], A. Walendziak discussed some relationships between congruence relations and normal filters of a BE-algebra.

In this paper, two operations are introduced one from the set of all filters of a BE-algebra into the algebra of filters of its congruence classes and the other

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from the algebra of filters of the congruence classes into the set of all filters of the given BE-algebra. Later, it is shown that their composition is a closure operator on the set of all filters of a BE-algebra. The concept of θ-filters is introduced in BE-algebras with respect to a congruence. The θ-filters are also characterized in terms of congruence classes. A set of equivalent conditions is derived for every filter of a BE-algebra to become a θ-filter. Finally, a bijection is obtained between the set of all θ-filters of a BE-algebra and the set of all filters of it's algebra of congruence classes.

2. Preliminaries

In this section, we present certain definitions and results which are taken mostly from the papers [1], [5] and [8] for the ready reference of the reader.

Definition 2.1 ([5])**.** An algebra $(X, *, 1)$ of type $(2, 0)$ is called a BE-algebra if it satisfies the following properties:
(1) $x * x = 1$,
(2) $x * 1 = 1$,
(3) $1 * x = x$,
(4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$.

Theorem 2.2 ([5])**.** *Let $(X, *, 1)$ be a BE-algebra. Then we have the following:*
(1) $x * (y * x) = 1$
(2) $x * ((x * y) * y)) = 1$

We introduce a relation $\leq$ on a BE-algebra X by $x \leq y$ if and only if $x*y = 1$ for all $x, y \in X$. A BE-algebra X is called self-distributive if $x * (y * z) = (x * y) * (x * z)$ for all $x, y, z \in X$. In any self-distributive BE-algebra, the set $\langle a \rangle = \{x \in X \mid a \leq x\} = \{x \in X \mid a * x = 1\}$ is the smallest filter containing the element $a \in X$ which is known as the principal filter of X generated by a.

Definition 2.3 ([1])**.** Let $(X, *, 1)$ be a BE-algebra. A non-empty subset F of X is called a filter of X if, for all $x, y \in X$, it satisfies the following properties:
(a) $1 \in F$,
(b) $x \in F$ and $x * y \in F$ imply that $y \in F$.

Definition 2.4 ([8])**.** Let $(X, *, 1)$ be a BE-algebra. A non-empty subset F of X is called a normal filter of X if it satisfies the following properties:
(a) $1 \in F$,
(b) $x \in X$ and $a \in F$ imply that $x * a \in F$.

Definition 2.5 ([10])**.** Let $(X, *, 1)$ be a BE-algebra. A binary relation θ on X is called a congruence on X if $(x, y) \in \theta$ and $(z, w) \in \theta$ imply that $(x*z, y*w) \in \theta$ for all $x, y, z, w \in X$.

For any congruence θ on a BE-algebra X, the quotient algebra $X_{/\theta} = \{[x]_\theta \mid x \in X\}$, where $[x]_\theta$ is a congruence class of x modulo θ, is a BE-algebra with respect to the operation $[x]_\theta * [y]_\theta = [x * y]_\theta$ for all $x, y \in X$.

3. θ-filter in BE-algebras

In this section, the notion of θ-filters is introduced in BE-algebras. A bijection is obtained between the set of all θ-filters of a BE-algebra and the set of all filters of the BE-algebra of all congruence classes. We first prove the following crucial result which play a vital role in the forth coming results.

Theorem 3.1. *A non-empty subset F of a BE-algebra X is a filter of X if and only if it satisfies the following conditions for all $x, y \in X$.*
(1) $x \in F$ *implies* $y * x \in F$.
(2) $a, b \in F$ *implies* $(a * (b * x)) * x \in F$.

Proof. Assume that F is a filter of X. Let $x \in F$. We have $x * (y * x) = y * (x * x) = y * 1 = 1 \in F$. Since $x \in F$ and F is a filter, we get that $y * x \in F$. Let $a, b \in F$. Since $a * ((a * (b * x)) * (b * x)) = (a * (b * x)) * (a * (b * x)) = 1$, we get that $a \leq ((a * (b * x)) * (b * x))$. Since $a \in F$, it yields that $(a * (b * x)) * (b * x) \in F$. Hence $b * ((a * (b * x)) * x) \in F$. Since $b \in F$, it implies $(a * (b * x)) * x \in F$.

Conversely, assume that F satisfies the given conditions (1) and (2). By taking $x = y$ in the condition (1), it can be seen that $1 \in F$. Let $x, y \in X$ be such that $x, x * y \in F$. Then by the condition (2), we get $y = 1 * y = ((x * y) * (x * y)) * y = (x * ((x * y) * y)) * y \in F$. Therefore F is a filter in X. □

For any congruence θ on a BE-algebra $(X, *, 1)$, let us recall that the set $X_{/\theta}$ of all congruence classes forms a BE-algebra with respect to the operation $[x]_\theta * [y]_\theta = [x * y]_\theta$ for all $x, y \in X$. It also forms a partially ordered set ordered by set inclusion. The smallest congruence on X is given by $\theta_0 = \{(a, a) \mid a \in X\}$.

In the following, we first introduce two operations.

Definition 3.2. Let θ be a congruence on a BE-algebra X. Define operations α and β as follows:
(1) For any filter F of X, define $\alpha(F) = \{ [x]_\theta \mid (x, y) \in \theta \text{ for some } y \in F \}$
(2) For any filter $\widehat{F}$ of $X_{/\theta}$, define $\beta(\widehat{F}) = \{x \in X \mid (x, y) \in \theta \text{ for some } [y]_\theta \in \widehat{F}\}$.

In the following lemma, some basic properties of the above two operations α and β are observed.

Lemma 3.3. *Let θ be a congruence on a BE-algebra X. Then we have*
(1) *For any filter F of X, $\alpha(F)$ is a filter of $X_{/\theta}$,*
(2) *For any filter $\widehat{F}$ of $L_{/\theta}$, $\beta(\widehat{F})$ is a filter of X,*
(3) *α and β are isotone,*
(4) *For any filter F of X, $x \in F$ implies $[x]_\theta \in \alpha(F)$,*
(5) *For any filter $\widehat{F}$ of $X_{/\theta}$, $[x]_\theta \in \widehat{F}$ implies $x \in \beta(\widehat{F})$.*

Proof. (1). Let $[x]_\theta \in X_{/\theta}$ and $[a]_\theta \in \alpha(F)$. Then $(a, y) \in \theta$ for some $y \in F$. Hence $(x * a, x * y) \in \theta$ and $x * y \in F$ because of F is a filer. Thus $[x]_\theta * [a]_\theta = [x * a]_\theta \in \alpha(F)$. Let $[a]_\theta, [b]_\theta \in \alpha(F)$. Then $(a, x) \in \theta$ and $(b, y) \in \theta$ for some $x, y \in F$. Now, for any $t \in X$, we get

$$\begin{aligned}(b,y)\in\theta &\Rightarrow (b*t, y*t)\in\theta\\ &\Rightarrow (a*(b*t), x*(y*t))\in\theta \quad \text{since } (a,x)\in\theta\\ &\Rightarrow ((a*(b*t))*t, (x*(y*t))*t)\in\theta\end{aligned}$$

Since $x, y \in F$ and F is a filter, we get by Theorem 3.1 that $(x*(y*t))*t \in F$. Hence

$$\begin{aligned}([a]_\theta * ([b]_\theta * [t]_\theta)) * [t]_\theta &= [(a*(b*t))*t]_\theta\\ &= [(x*(y*t))*t]_\theta \in \alpha(F).\end{aligned}$$

Therefore by Theorem 3.1, it concludes that $\alpha(F)$ is a filter of $X_{/\theta}$.

(2). Let $x \in X$ and $a \in \beta(\widehat{F})$. Then $(a,y)\in\theta$ for some $[y]_\theta \in \widehat{F}$. Hence $(x*a, x*y)\in\theta$. Since $\widehat{F}$ is a filter, we get $[x*a]_\theta = [x*y]_\theta = [x]_\theta * [y]_\theta \in \widehat{F}$. Thus it yields $x*a \in \beta(\widehat{F})$. Again, let $a, b \in \beta(\widehat{F})$ and $t \in X$. Then we get $(a,x)\in\theta$ and $(b,y)\in\theta$ for some $[x]_\theta \in \widehat{F}$ and $[y]_\theta \in \widehat{F}$. Since $\widehat{F}$ is a filter, we get $[(x*(y*t))*t]_\theta = ([x]_\theta*([y]_\theta*[t]_\theta))*[t]_\theta \in \widehat{F}$. Since $(a,x)\in\theta$ and $(b,y)\in\theta$, it is clear that $((a*(b*t))*t, (x*(y*t))*t)\in\theta$. Since $[(x*(y*t))*t]_\theta \in \widehat{F}$, we get $(a*(b*t))*t \in \beta(\widehat{F})$. Therefore by Theorem 3.1, $\beta(\widehat{F})$ is a filter of X.

(3). Let F_1, F_2 be two filters in X such that $F_1 \subseteq F_2$. Let $[x]_\theta \in \alpha(F_1)$. Then, we get $(x,y)\in\theta$ for some $y \in F_1 \subseteq F_2$. Consequently, we get that $[x]_\theta \in \alpha(F_2)$. Therefore $\alpha(F_1)\subseteq\alpha(F_2)$. Again, let $\widehat{F_1}, \widehat{F_2}$ be two filters of $X_{/\theta}$ such that $\widehat{F_1}\subseteq\widehat{F_2}$. Suppose $x \in \beta(\widehat{F_1})$. Then, it infers that $(x,y)\in\theta$ for some $[y]_\theta \in \widehat{F_1}\subseteq\widehat{F_2}$. Hence $y \in \beta(\widehat{F_2})$. Therefore $\beta(\widehat{F_1})\subseteq\beta(\widehat{F_2})$.

(4). For any $x \in F$, we have $(x,x)\in\theta$. Hence it concludes $[x]_\theta \in \alpha(F)$.

(5). For any $[x]_\theta \in \widehat{F}$, we have $(x,x)\in\theta$. Hence we get $x \in \beta(\widehat{F})$. □

The following corollary is a direct consequence of the above lemma.

Corollary 3.4. *Let θ be a congruence on a BE-algebra X. Then we have*

(1) *For any normal filter F of X, $\alpha(F)$ is a normal filter of $X_{/\theta}$.*

(2) *For any normal filter $\widehat{F}$ of $L_{/\theta}$, $\beta(\widehat{F})$ is a normal filter of X.*

Lemma 3.5. *Let θ be a congruence on a BE-algebra X. For any filter F of X, $\alpha\beta\alpha(F) = \alpha(F)$.*

Proof. Let $[x]_\theta \in \alpha(F)$. Then $(x,y)\in\theta$ for some $y \in F$. Since $y \in F$, by Lemma 3.3(4), we get $[y]_\theta \in \alpha(F)$. Since $(x,y)\in\theta$ and $[y]_\theta \in \alpha(F)$, we get $x \in \beta\alpha(F)$. Hence $[x]_\theta \in \alpha\beta\alpha(F)$. Thus $\alpha(F)\subseteq\alpha\beta\alpha(F)$. Conversely, let $[x]_\theta \in \alpha\beta\alpha(F)$. Then $(x,y)\in\theta$ for some $y \in \beta\alpha(F)$. Since $y \in \beta\alpha(F)$, there exists $[a]_\theta \in \alpha(F)$ such that $(y,a)\in\theta$. Hence $[x]_\theta = [y]_\theta = [a]_\theta \in \alpha(F)$. Therefore $\alpha\beta\alpha(F)\subseteq\alpha(F)$. □

We now intend to show that the composition $\beta\alpha$ is a closure operator on the set $\mathcal{F}(X)$ of all filters of a BE-algebra X.

Proposition 3.6. *For any filter F of X, the map $F \longrightarrow \beta\alpha(F)$ is a closure operator on $\mathcal{F}(X)$. That is, for any two filters F, G of X, we have the following:*
(a) $F \subseteq \beta\alpha(F)$.
(b) $\beta\alpha\beta\alpha(F) = \beta\alpha(F)$.
(c) $F \subseteq G \;\Rightarrow\; \beta\alpha(F) \subseteq \beta\alpha(G)$.

Proof. (a). Let $x \in F$. Then by Lemma 3.3(4), we get $[x]_\theta \in \alpha(F)$. Since $(x, x) \in \theta$ and $\alpha(F)$ is a filter in $X_{/\theta}$, we get $x \in \beta\alpha(F)$. Therefore $F \subseteq \beta\alpha(F)$.
(b). Since $\beta\alpha(F)$ is a filter in X, by above condition (a), we get $\beta\alpha(F) \subseteq \beta\alpha[\beta\alpha(F)]$. Conversely, let $x \in \beta\alpha[\beta\alpha(F)]$. Then we obtain $(x, y) \in \theta$ for some $[y]_\theta \in \alpha\beta\alpha(F)$. Thus by above Lemma 3.5, we get that $[y]_\theta \in \alpha(F)$. Hence $x \in \beta\alpha(F)$. Therefore, it concludes $\beta\alpha[\beta\alpha(F)] \subseteq \beta\alpha(F)$.
(c). Suppose F, G are two filters of X such that $F \subseteq G$. Let $x \in \beta\alpha(F)$. Then we get $[x]_\theta \in \alpha(F)$. Hence $[x]_\theta = [y]_\theta$ for some $y \in F \subseteq G$. Since $y \in G$, we get $[x]_\theta = [y]_\theta \in \alpha(G)$. Therefore $x \in \beta\alpha(G)$. Hence $\beta\alpha(F) \subseteq \beta\alpha(G)$. □

Denoting by $\mathcal{F}(X_{/\theta})$ the set of all filters of $X_{/\theta}$, we can therefore define a mapping $\alpha : \mathcal{F}(X) \longrightarrow \mathcal{F}(X_{/\theta})$ by $F \mapsto \alpha(F)$ also another mapping $\beta : \mathcal{F}(X_{/\theta}) \longrightarrow \mathcal{F}(X)$ by $F \mapsto \beta(F)$. Then we have the following:

Proposition 3.7. *Let θ be congruence on a BE-algebra X. Then α is a residuated map with residual map β.*

Proof. For every $F \in \mathcal{F}(X)$, by Proposition 3.6(a), we have that $F \subseteq \beta\alpha(F)$. Let $F \in \mathcal{F}(X_{/\theta})$. Suppose $[x]_\theta \in F$. Then we get $x \in \beta(F)$. Since $\beta(F)$ is a filter of X, we get $[x]_\theta \in \alpha\beta(F)$. Hence, it yields $F \subseteq \alpha\beta(F)$. Conversely, let $[x]_\theta \in \alpha\beta(F)$. Then $[x]_\theta = [y]_\theta$ for some $y \in \beta(F)$. Since $y \in \beta(F)$, we get $[x]_\theta = [y]_\theta \in F$. Hence $\alpha\beta(F) \subseteq F$. Therefore for every $F \in \mathcal{F}(X_{/\theta})$, we obtain that $\alpha\beta(F) = F$. Since α and β are isotone, it follows that α is residuated and that the residual of α is nothing but β. □

We now introduce the notion of θ-filters in a BE-algebra.

Definition 3.8. Let θ be a congruence on a BE-algebra X. A filter F of X is called a θ-filter if $\beta\alpha(F) = F$.

For any congruence θ on a BE-algebra X, it can be easily observed that the filter $\{1\}$ is a θ-filter if and only if $[1]_\theta = \{1\}$. Moreover, we have the following:

Lemma 3.9. *Let θ be a congruence on a bounded BE-algebra X with smallest element* 0*. For any filter F of X, the following hold:*
(1) *If F is a θ-filter then $[1]_\theta \subseteq F$,* (2) *If F is a proper θ-filter then $F \cap [0]_\theta = \emptyset$.*

In the following theorem, a set of sufficient conditions is derived for a proper filter of a BE-algebra to become a θ-filter.

Theorem 3.10. *Let θ be a congruence on a BE-algebra X. A proper filter F of X is a θ-filter if it satisfies the following conditions:*
(1) *For $x, y \in X$ with $x \neq y$, either $x \in F$ or $y \in F$*
(2) *To each $x \in F$, there exists $x' \notin F$ such that $(x, x') \in \theta$*

Proof. Let F be a proper filter of X. Clearly $F \subseteq \beta\alpha(F)$. Conversely, let $x \in \beta\alpha(F)$. Then $(x,y) \in \theta$ for some $[y]_\theta \in \alpha(F)$. Hence $[y]_\theta = [a]_\theta$ for some $a \in F$. Since $a \in F$, there exists $a' \notin F$ such that $(a,a') \in \theta$. Since $(x,y) \in \theta, (y,a) \in \theta$ and $(a,a') \in \theta$, by the transitive property, we get $(x,a') \in \theta$. Since $a' \notin F$, by conditions (1) and (2), we get $x \in F$. Therefore $\beta\alpha(F) = F$. □

We now characterize θ-filters in the following:

Theorem 3.11. *Let θ be a congruence on a BE-algebra X. For any filter F of X, the following conditions are equivalent:*
(1) *F is a θ-filter;* (2) *For any $x,y \in X, [x]_\theta = [y]_\theta$ and $x \in F$ imply that $y \in F$;*
(3) *$F = \bigcup_{x\in F} [x]_\theta$;* (4) *$x \in F$ implies $[x]_\theta \subseteq F$.*

Proof. (1) ⇒ (2): Assume that F is a θ-filter of X. Let $x,y \in X$ be such that $[x]_\theta = [y]_\theta$. Then $(x,y) \in \theta$. Suppose $x \in F = \beta\alpha(F)$. Then $(x,a) \in \theta$ for some $[a]_\theta \in \alpha(F)$. Thus $(a,y) \in \theta$ and $[a]_\theta \in \alpha(F)$. Therefore $y \in \beta\alpha(F) = F$.
(2) ⇒ (3): Assume the condition (2). Let $x \in F$. Since $x \in [x]_\theta$, we get $F \subseteq \bigcup_{x\in F} [x]_\theta$. Conversely, let $a \in \bigcup_{x\in F} [x]_\theta$. Then $(a,x) \in \theta$ for some $x \in F$. Hence $[a]_\theta = [x]_\theta$. By the condition (2), we get $a \in F$. Therefore $F = \bigcup_{x\in F} [x]_\theta$.
(3) ⇒ (4): Assume the condition (3). Let $a \in F$. Then we get that $(x,a) \in \theta$ for some $x \in F$. Let $t \in [a]_\theta$. Then $(t,a) \in \theta$. Hence $(x,t) \in \theta$. Thus it yields $t \in [x]_\theta \subseteq F$. Therefore it can be concluded that $[a]_\theta \subseteq F$.
(4) ⇒ (1): Assume the condition (4). Clearly $F \subseteq \beta\alpha(F)$. Conversely, let $x \in \beta\alpha(F)$. Then $(x,y) \in \theta$ for some $[y]_\theta \in \alpha(F)$. Hence $[y]_\theta = [a]_\theta$ for some $a \in F$. Since $a \in F$, by condition (4), we get that $x \in [y]_\theta = [a]_\theta \subseteq F$. Thus $\beta\alpha(F) \subseteq F$. Therefore F is a θ-filter of X. □

In the following theorem, a set of equivalent conditions is obtained to characterize the smallest congruence in terms of θ-filters of BE-algebra.

Theorem 3.12. *Let θ be a congruence on a self-distributive BE-algebra X. Then the following conditions are equivalent:*
(1) *θ is the smallest congruence;*
(2) *Every filter is a θ-filter;*
(3) *Every principal filter is a θ-filter.*

Proof. (1) ⇒ (2): Assume that θ is the smallest congruence on X. Let F be a filter of X and $x \in F$. Let $t \in [x]_\theta$. Then $(t,x) \in \theta$. Hence $t = x \in F$. Thus we get $[x]_\theta \subseteq F$. Therefore by above Theorem 3.11, F is a θ-filter.
(2) ⇒ (3): It is obvious.
(3) ⇒ (1): Assume that every principal filter is a θ-filter. Let $x,y \in X$ be such that $(x,y) \in \theta$. Then $[x]_\theta = [y]_\theta$. Since $\langle y\rangle$ is a θ-filter, we get $x \in [x]_\theta = [y]_\theta \subseteq \langle y\rangle$. Since X is self-distributive, we get $y * x = 1$. Hence $y \leq x$. Similarly, we get $x \leq y$. Hence $x = y$. Therefore θ is the smallest congruence. □

Finally, this article is concluded by obtaining a bijection between the set of all θ-filters of a BE-algebra and the set of all filters of its quotient algebra.

Theorem 3.13. *Let θ be a congruence on a BE-algebra X. Then there exists a bijection between the set $\mathcal{F}_\theta(X)$ of all θ-filters of X and the set of all filters of the BE-algebra $X_{/\theta}$ of all congruence classes.*

Proof. Define a mapping $\psi : \mathcal{F}_\theta(X) \mapsto \mathcal{F}(X_{/\theta})$ by $\psi(F) = \alpha(F)$ for all $F \in \mathcal{F}_\theta(X)$. Let $F, G \in \mathcal{F}_\theta(X)$. Then $\psi(F) = \psi(G) \Rightarrow \alpha(F) = \alpha(G) \Rightarrow \beta\alpha(F) = \beta\alpha(G) \Rightarrow F = G$(since $F, G \in \mathcal{F}_\theta(X)$). Hence ψ is one-one. Again, let $\widehat{F}$ be a filter of $\mathcal{F}(X_{/\theta})$. Then $\beta(\widehat{F})$ is a filter in X. We now show that $\beta(\widehat{F})$ is a θ-filter in X. We have always $\beta(\widehat{F}) \subseteq \beta\alpha\beta(\widehat{F})$. Let $x \in \beta\alpha\beta(\widehat{F})$. Then we get $(x, y) \in \theta$ for some $[y]_\theta \in \alpha\beta(\widehat{F}) = \widehat{F}$. Hence $x \in \beta(\widehat{F})$. Therefore $\beta(\widehat{F}) = \beta\alpha\beta(\widehat{F})$. Now for this $\beta(\widehat{F}) \in X$, we get $\psi[\beta(\widehat{F})] = \alpha\beta(\widehat{F}) = \widehat{F}$. Therefore ψ is onto. Therefore ψ is a bijection between $\mathcal{F}_\theta(X)$ and $\mathcal{F}(X_{/\theta})$. ☐

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