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역 필터 순서와 파워 스펙트럼 밀도에 기초한 이미지 복원

Image Restoration Based on Inverse Filtering Order and Power Spectrum Density

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요약 본 연구에서는, 웨이블릿 노이즈 감쇠에 고속 푸리에 역 변환을 포함하는 방법을 제안한다. 위너 필터링에 인자를 채용하여 역 필터링을 나타내고, 최적의 계수는 전체 평균 제곱 오차를 최소화하도록 선택된다. 위너 필터를 적용하기 위해, 손상된 그림에서 원 화상의 파워 스펙트럼을 계산한다. 위너 필터링은 역 필터링 처리를 포함하기 때문에 블러링 필터가 반전되지 않을 때 노이즈는 확장한다. 큰 노이즈를 제거하려면 최고의 웨이블릿 임계값을 사용하여 노이즈를 제거하는 것이다. 웨이블릿 노이즈 감쇠 단계는 역 필터링 및 웨이블릿 기능으로 노이즈 감소로 구성된다. 실험 결과는 전체 재생 성능 이상의 다른 방법을 능가하지는 않았다.

Abstract In this paper, we suggest a approach which comprises fast Fourier transform inversion by wavelet noise attenuation. It represents an inverse filtering by adopting a factor into the Wiener filtering, and the optimal factor is chosen to minimize the overall mean squared error. in order to apply the Wiener filter, we have to compute the power spectrum of original image from the corrupted figure. Since the Wiener filtering contains the inverse filtering process, it expands the noise when the blurring filter is not invertible. To remove the large noises, the best is to remove the noise using wavelet threshold. Wavelet noise attenuation steps are consisted of inverse filtering and noise reduction by Wavelet functions. experimental results have not outperformed the other methods over the overall restoration performance.

Key Words : Image restoration, Inverse filter, Power spectrum, Wiener filter, Wavelet transform.

1. Introduction

An image is usually corrupted by various noises during the image acquisition or transmission. These noises decrease the visual analysis and performance. The noise reduction process can be described as to increase the visual performance while retaining the quality of processed image^[1]. The traditional algorithms to remove the noise from an image use a low or band

pass filter with several thresholds. These techniques can remove a relevant of the noise. But, they are incapable if the noises are in the band of the signal to be analyzed. Thus, many noise removing techniques have been discussed to overcome this problem. The signal processing algorithms can be also used for images because an image can be considered as a two dimensional signal. Therefore, the digital signal

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processing techniques for a one dimensional signal can be adapted to process two dimensional images^{[2][3]}.

Wavelet transform has been successfully used in many scientific fields such as signal processing, image compression, and computer graphics^[4]. On contrary the traditional Fourier transform, the Wavelet transform is very suitable for the applications of non-stationary signals that may instantaneous vary in time^[5-7]. It is important to analyze the time-frequency characteristics of the signals that had transient signals to represent the exact features of such signals^[8-9]. For this reason, many researches has been focused on continuous Wavelet transform which shows more reliable time-scale analysis rather than Fourier transform giving a time-frequency analysis. The wavelet coefficient is a similarity measure in the frequency content between a signal and a chosen wavelet function. It is computed as a convolution of the signal and the scaled wavelet function, which is regarded as a dilated band-pass filter because of its band-pass like spectrum.

The purpose of image restoration is to remove defects that degrade an image. Degradation appears in many forms such as motion blur, camera out of focus, etc. In case of motion blur, it is possible to find very good estimates of the actual blurring function and reduce the blur to restore the original image. In cases where the image is corrupted by noise, the best is to compensate for the degradation it caused. The major concept of the wavelet noise reduction is to obtain the real components of the image from the noisy image. It requires the estimation of the noise level. The noise level estimation is used to threshold the small coefficient regarded as noise. Specifically, discrete Wavelet transform based noise removing is consist of three levels; decomposition of the image, threshold work and reconstruction of the image. Several methods use this idea proposed and implements it in different ways. When attempting to decrease the influence of noise wavelets coefficient, it is possible to do this in particular ways, also the need of information of the

underlying signal leads to different statistical treatments of the available information. However, it is clear that there is no single optimal wavelet based noise removal method. The methods and their parameters should be chosen according to the signals in hand.

Curvelet transform can represent edge singularity much more efficiently than the traditional wavelet transform. It combines multi-scale analysis and geometrical concept to achieve the optimal rate of convergence by simple threshold techniques. Multi-scale decomposition captures point discontinuities into linear structures^[10-11]. But a drawback of the linear structure is that they are not able to preserve edges in a good way. In this paper, we suggest a simple image restoration method which involves fast Fourier transform inversion by wavelet noise reduction. It does the inverse filtering by applying a parameter into the Wiener filtering, and the optimal parameter is chosen to minimize the overall MSE. The compromise of Wiener filter and reasonable inverse filter represents the power spectrum of original image. Since the Wavelet coefficients of image are better estimates of the power spectrum, we substitute the Wavelet transform into the order of inverse filtering. The advantage of this approach is that we can perform different appropriate inverse filtering.

II. Wiener Filtering and DFT

For an original image I , suppose that b is some kind of a low pass filter and I^B is a blurred image. Then, a blurred image can be modeled by

$$I^B(i, j) = I(i, j) * b(i, j) \quad (1)$$

To get back the original image, we would just have to convolve the blurred function with some kind of a high pass filter

$$I(i, j) = I^B(i, j) * h(i, j) \quad (2)$$

A problem is to find the high pass filter h . In the real case, we would just invert all the elements of b to get

a high pass filter. However, notice that a lot of the elements in b have values either at zero or very close to it. Inverting these elements would give us either infinities or some extremely high values. In order to avoid these values, we will need to set some sort of a threshold on the inverted element. So instead of making a full inverse out of b , we can obtain an almost full inverse by the following:

$$H(i, j) = \begin{cases} 1/B(i, j), & 1/B(i, j) < \delta \\ \delta, & \text{otherwise} \end{cases} \quad (3)$$

Thus, the higher d we set, the closer H is the full inverse filter.

The iterative method is to start a certain initial guess of I by using I^B and to update that guess after every iteration. The method has the following equations.

$$\begin{aligned} I_0(i, j) &= \lambda I^B(i, j) \\ I_{n+1}(i, j) &= I_n(i, j) + \lambda(I^B(i, j) - I_n(i, j) * b(i, j)) \end{aligned} \quad (4)$$

Here I_0 is an initial guess based on I^B . If I_n is a desirable image, I_n convolved with b will be close to I^B . The second term in the I_{n+1} equation will disappear and I_n and I_{n+1} will converge. λ denotes a convergence factor and it determines how fast I_n and I_{n+1} converge. If both of the above equations to the frequency domain are taken, we can get the following.

$$\begin{aligned} F_0 &= \lambda F^B \\ F_{n+1} &= F_n + \lambda(F^B - F_n B) \end{aligned} \quad (5)$$

Recursively, solving for F_n , it can be shown that

$$\begin{aligned} F_n &= \lambda F^B [1 + (1 - \lambda B) + \dots + (1 - \lambda B)^n] \\ &= F^B [1 - (1 - \lambda B)^{n+1}] / B \end{aligned} \quad (6)$$

Therefore, as n goes to infinity, we can get the result as obtained by the inverse filter. In general, this method will not give the exact same results as inverse filtering, but can be less sensitive to noise in some cases.

Image restoration with Wiener filter provides us with the optimal trade-off between noise elimination and inverse filtering. The inverse filtering is a restoration technique for de-convolution, i.e., when the image is blurred by a known low pass filter, it is

possible to recover the image by inverse filtering or generalized inverse filtering. However, inverse filter is very sensitive to additive noise. The approach of reducing degradation at a time allows us to develop a restoration algorithm for each type of degradation and simply combine them. Wiener filtering executes an optimal tradeoff between inverse filtering and noise smoothing. It removes the additive noise and inverts the blurring simultaneously. The Wiener filtering is optimal in terms of the mean square error. It minimizes the overall mean square error in the process of inverse filtering and noise smoothing. The Wiener filtering is a linear estimation of the original image. The approach is based on a stochastic framework. The orthogonal property implies that the Wiener filter in Fourier domain can be expressed as follows:

$$\begin{aligned} W(I_1, I_2) &= H(I_1, I_2) S_{xx}(I_1, I_2) / \\ &[|H(I_1, I_2)|^2 S_{xx}(I_1, I_2) + S_{nn}(I_1, I_2)] \end{aligned} \quad (7)$$

Here S_{xx} and S_{nn} are power spectra of the original image and the noise addition, respectively. H is a blurring filter. Wiener filter has two separate parts, an inverse filtering part and a noise smoothing part. It not only performs the de-convolution by inverse filtering but also removes the noise with a compression operation (say, low pass filtering).

To implement the Wiener filter, it must be estimated the power spectra of the original image and the noise addition image. For white additive noise the power spectrum is equal to the variance of the noise. To estimate the power spectrum of the original image many methods can be used. A direct estimate is the period-gram estimate of the power spectrum computed from the observation:

$$S_{yy}^P = [Y(k, l) Y^*(k, l)] / N^2 \quad (8)$$

$Y(k, l)$ is DFT of the observation, and the merit of this estimate is that it can be implemented very easily without the singularity of inverse filtering. Another estimate which leads to a cascade implementation of the inverse filtering and the noise smoothing is the following.

$$S_{xx} = (S_{yy} - S_{nn}) / |H|^2 \quad (9)$$

The power spectrum can be estimated directly from the observation using the periodogram estimate. It results in a cascade implementation of inverse filtering and noise smoothing:

$$W = (S_{yy}^p - S_{nn}) / (S_{yy}^p H) \quad (10)$$

The disadvantage of this implementation is that when the inverse filter is singular, we have to use the generalized inverse filtering. It also suggests the power spectrum of the original image can be estimated based on a model.

Instead one-dimensional signal that represents the changes of amplitude in time, we apply with two-dimensional signals which represent the intensity variations in space. These signals have the form of images. Because the signals are discrete, we use an analog of the one dimensional DFT for two dimensional signals. It is the following pair of transforms:

$$F = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I(x, y) \exp[-j2\pi(ux/M + vy/N)]$$

$$I = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \exp[-j2\pi(ux/M + vy/N)] \quad (11)$$

Here, the $M \times N$ image has the $M \times N$ set of Fourier coefficients. The two dimensional DFT is divided into two one dimensional DFT which can be done with FFT algorithm.

$$F = \frac{1}{M} \sum_{x=0}^{M-1} \exp(-j2\pi ux/M) \cdot \sum_{y=0}^{N-1} I(x, y) \exp(-j2\pi vy/M) \quad (12)$$

The DFT coefficients by the above equation are arranged in an intuitive way as Fig 1.

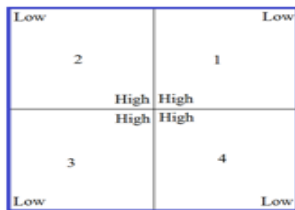


그림 1. DFT 계수의 직관적 표현
Fig. 1. Intuitive representation of DFT coefficients

It is very intuitive to have low frequency in the center of the image and high frequency on the outsides of the image. According to the frequency periodicity and the DFT over any period of the image, we can modify the frequency representation by interchanging the 1st and 3rd quadrants and 2nd and 4th quadrants(see Fig 2.).

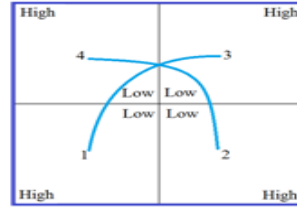


그림 2. DFT 계수의 보다 직관적 표현
Fig. 2. More intuitive representation of DFT coefficients

Because the usual image has slowly varying features as opposed to irregular intensity change, this positions the majority of the component around the center of frequency spectrum. It also means that some properties held, and gives some relations between the spatial and frequency domain.

As with the one dimensional DFT, there are many properties of the transformation that give insight into the components of the frequency domain representation of an image and allow us to manipulate the images in one domain or the other. First, there is a simple relation that can be derived for shifting an image in one domain or the other. Since both the space and frequency domains are regarded as periodic, shifting means rotating around the boundaries. Shifting the scaled pulse to the upper left-hand corner shifts the phase along the diagonal direction while leaving the magnitude untouched. Next, in one-dimensional domain, shrinking raises expansion in the other direction for the two-dimensional DFT. It notes that as a figure grows in an image, the corresponding features in the frequency domain are expanded. Finally, rotation is a property of the two-dimensional DFT. Because of the separation of transformed equations, the frequency

components are positioned based on the location of the figure in the spatial domain. It notes that rotating the spatial contents rotates the frequency contents.

Before the image filtering, it must be examined what the frequency components of image is. In a time-based signal, a low frequency is one that changes slowly, whereas a high frequency represents a more rapid change. To extend this idea to a spatial signal, it is easy to see that low-frequency data occurs where intensity values change slowly. According to these concepts, we can now anticipate the results of filtering an image. Trying to use a low-pass filter in the Y direction, two facts are appeared. First, an ideal filter cannot be used because it creates ringing figures, the same as in a one-dimensional transform. The second and more important realization is that a filter varying only in the Y frequency direction, and equal across all X , has its effects only in the Y direction of the image. It can be expected from the rotation property, and from this we can compute, properly it turns out, that a filter is just as separable as the transform, and therefore a filter direction is its effect one. Notice the way the shadows ripple up and down from horizontal lines in the source image, whereas vertical lines are unaffected.

III. Compromise between noising and inverse filtering

Wiener filtering is the optimal tradeoff of inverse filtering and noise smoothing. However, in the case when the blurring filter is singular, it really extends the noise. So, a noise removing step is required to reduce the diffusion noise. Wavelet-based denoising is a natural technique for this purpose. The image restoration contains two separate steps: Fourier-domain inverse filtering and wavelet-domain image without noising (see Fig 3.).

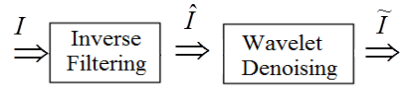


그림 3. 두 단계의 이미지 복원
 Fig. 3. Image restoration with two steps

Wavelet-based denoising for image restoration improves the performance. However, in the case when the blurring function is not invertible, the algorithm is not applicable. Furthermore, since the two steps are separate, there is no handling over the total performance of the restoration. A wavelet-based deconvolution method can be used for this problem. The idea is to apply both Fourier-domain Wiener-like and wavelet-domain regularization. The regularized inverse filter is to modify the Wiener filter with a new regular parameter as the following.

$$G_\alpha = (H * S_{xx}) / (|H|^2 S_{xx} + \alpha S_m) \quad (13)$$

The parameter α can be selected to minimize the total mean-square error (see Fig 4.).

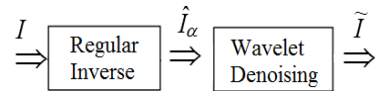


그림 4. 정규화 역을 이용한 이미지 복원
 Fig. 4. Image restoration using regularized inverse

The regular inverse filter involves the power spectrum of original image. Since wavelet transform has good independent property, the wavelet coefficients of image can have better probability model, and the power spectrum can be better estimated. It suggests a new approach changing the order of regular inverse filtering and the wavelet transform (see Fig 5.).

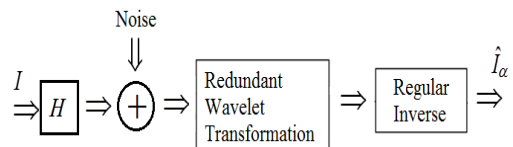


그림 5. 일반적인 역 필터와 웨이블릿 변환
 Fig. 5. Regular inverse filter and wavelet transform

Both inverse filtering and noise smoothing can be performed in wavelet domain. Specifically, the image power spectrum in a same sub-band can be computed under the condition that the wavelet coefficients are not dependent. So, the power spectrum is just the variance of the wavelet coefficients. Trade-off of the inverse filtering and the wavelet transform is only valid in case that continuous wavelet transform is used and the blurring function is separable. We can substitute the wavelet transform into the order of the blurring operation. It means that the inverse filtering suppresses the blurring in the wavelet domain. Thus, the result by wavelet threshold becomes a reasonable estimate (see Fig. 6.).

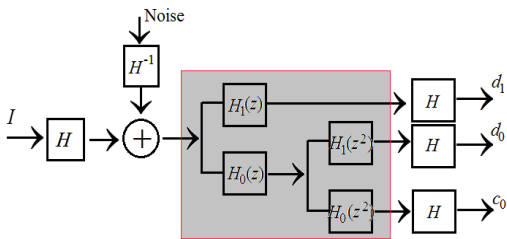


그림 6. 합리적인 변환
Fig. 6. A reasonable transform

After loading the input image, the calculations for the local adaptive image de-noising are done by a Wavelet de-noising function. This function calls several sub functions. The implementation can be summarized as the following.

The signal variance of a coefficient is computed using neighboring coefficients in a rectangular area with the window size. Also, set how many steps are used for the wavelet transform.

The noisy image is extended using symmetric extension to reduce the boundary problem with a symmetric extended function. Perform the forward wavelet transform.

Compute the noise variance. The noise variance is calculated using the robust median estimator.

The coefficient and the corresponding parent nodes are prepared for each sub-band, and the parent node is

expanded using the diffusion function to make the node size the same as the coefficient node.

Estimate the signal variance and threshold value: The signal variance for each coefficient is estimated using the window size, and the threshold value for each coefficient is calculated and stored in a node with the same size as the coefficient node.

The coefficients are estimated using the noisy coefficient, its parent, and the estimated threshold value with the bi-shrink function. Finally, calculate the inverse wavelet transform, and extract the image.

Finally, we suggest a simple technique for estimating h based on degraded image. To this point, we have discussed some restoration techniques, when we knew the blurring function h . Also, we have assumed that we knew the image spectrum S_{uu} and spectrum noise S_{nn} as well. Two restoration filters are the basis for the above algorithm. One is the Wiener Filter, which exhibits the better property in the MSE sense between inverse filtering and noise smoothing. Another filter is to restore the power spectrum of degraded image. It is similar to the power spectrum equalization. Our degradation model is assumed that the input image is blurred through convolution with a low pass filter h and then Gaussian noise is added. Moreover, because the power spectrum equalization works best assuming the low-pass filter is without phases, we generate a low pass filter to have zero phases. Let's begin by introducing the Wiener filter.

$$G = (H * S_{xx}) / (|H|^2 S_{xx} + S_{nn}) \quad (14)$$

Here, H is the Fourier Transform of low pass filter h , and S_{xx} and S_{nn} are defined as above. This degradation model is the same as a convolution plus noise. In the frequency domain, convolution becomes multiplication. If the noise addition is ignored, it can be taken the log of the multiplication and got addition. Thus, the log Fourier transform of degraded image I_d is equal to the log Fourier transform of the original image I_o plus the log of transfer function H . We can use a statistical estimation to obtain H and thus solve for I_o . The problem with this procedure is in fact that

the noise can be not ignored. Therefore, it is required ways to estimate the log multiplication of I_o and H plus the noise spectrum. Let U_k and V_k be obtained by breaking the input image and degraded image into M smaller blocks and computing their Fourier Transforms. Then, an estimate of H can be obtained from Jain text as the followings.

$$\ln |H| = \frac{1}{M} \sum_{k=1}^M \ln(|V_k| / |U_k|) \quad (15)$$

The above estimate for H can be approximated by using normal vectors as the following.

$$|H| = 1 + \frac{1}{M} \sum_{k=1}^M (|V_k| - |U_k|) \quad (16)$$

Then, H is used with S_{nn} and S_{xx} to compute the Wiener Filter. Notice that this method only computes Magnitude of H , so it is optimal in case of without phase filters. This is necessary condition about our filter design of a phase free h .

IV. Application

In this section, we show some implementation and results by typical restoration methods, and compare the proposed method with them. Before an inverse filter was applied, we had to threshold B .

$$B(i, j) = \begin{cases} B(i, j), & B(i, j) > n \\ n, & \text{otherwise} \end{cases} \quad (17)$$

Here n is set arbitrarily close to zero for noiseless cases. The following image shows the results for $n=0.2$. The MSE for the restored image is 2337.3 (See Fig 7. (c)). We can see that the restoration image has a lot of noise specs. The image is not closely like the original. Because an inverse filter is a high pass filter, it does not perform well in the presence of noise. Usually, the MSE for the restored image is very large. We can improve the many specs by increasing n . In general, the more noise we have in the image, the higher we set n . The higher the n , the less high pass the filter is which means that it extends noise less. In the iterative method, the first thing to do is to pick a parameter (l). It must satisfy the following condition: The bigger l is

the faster and will converge. However, picking too large a parameter may also make and diverge. The following image is noise free after 1500 iterations.

The images are usually listed with PSNR (peak signal-to-noise ratio) and MSE. The restored image is improved in terms of the visual performance, but MSE does not indicate this, the major reason is that MSE is not a good metric for de-convolution (see Fig 7. (d)). The parameter starts off at 0.1 and decreases by 10%, every fifty iterations. The MSE is 645.9822. But, the image is sharper than the blurred image although the MSE is more or less high.

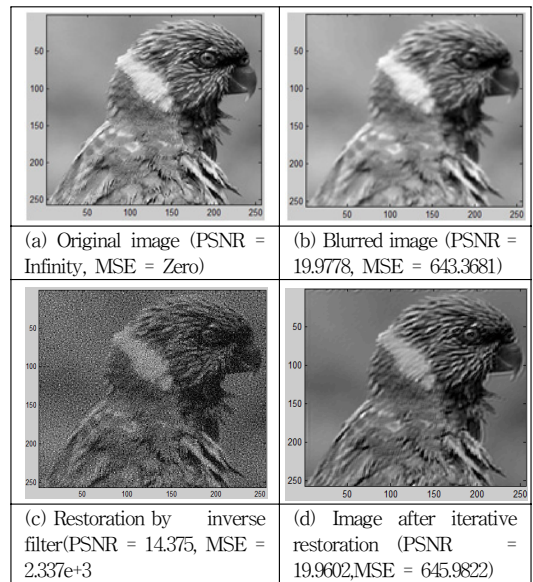


그림 7. 반복적인 방법으로 복원

Fig. 7. Restoration by the iterative method

To apply the Wiener filtering in image restoration, we used the standard 512 × 512 Lena test image. We blur the image with the low pass filter as the following.

Then, we put into the blurred image the additive white Gaussian noise of variance 100 (see Fig 8.). The Wiener filtering is applied to the original image with a cascade implementation of the noise smoothing and inverse filtering. The images are listed as the Fig 8. with the PSNR and MSE. Notice that the restored image is improved in terms of the visual performance, but the MSEs don't indicate this, the reason of which

is that MSE is not a reasonable measure for deconvolution.

For the Wavelet based image restoration, we applied Daubechies tap 8 wavelet transform to the corrupted image (see Fig 9.(a)). According to the visual performance and the mean square error, the algorithm improved a little the restoration performance (see Fig 9.(b)). However, the noise reduction step uses wavelet threshold technique to remove the noises, the image are blurred a little bit again, even though the MSE is improved.

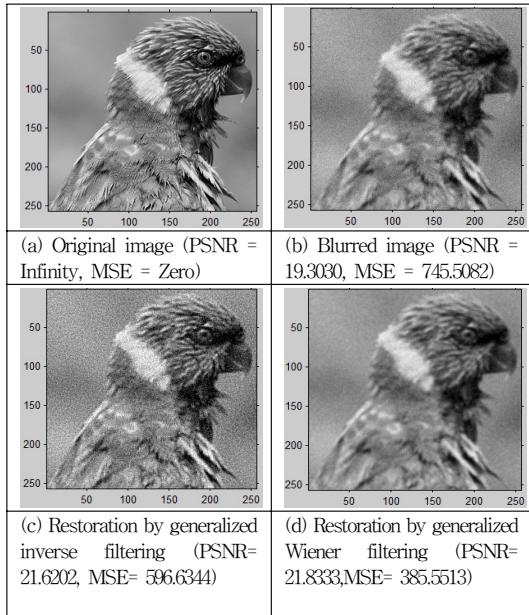


그림 8. 일반적인 위너 필터링으로 이미지 복원
 Fig. 8. Image restoration by the generalized Wiener filtering

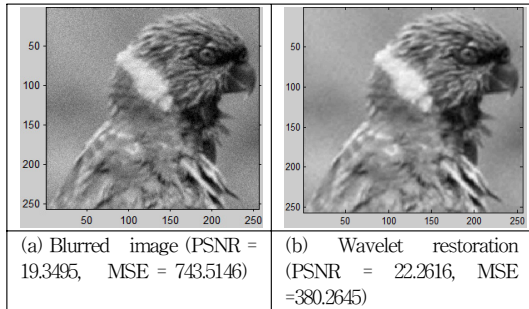


그림 9. 블릿 변환을 통한 이미지 복원
 Fig. 9. Image restoration by Wavelet transform

Finally, we examined the effectiveness of power spectral substitution. Note the similarity to Wiener Filtering, but we only used the magnitude of H . In general, the power spectrum equalization is not very robust because an exact representation of the noise is used. This is because the power spectrum equalization does not use inverse filtering, so the noise estimation is doubly important. S_{xx} is estimated as the magnitude squared of the Fourier Transform of the input image. Many blocks are due to the compression we used on the figure to store the images.

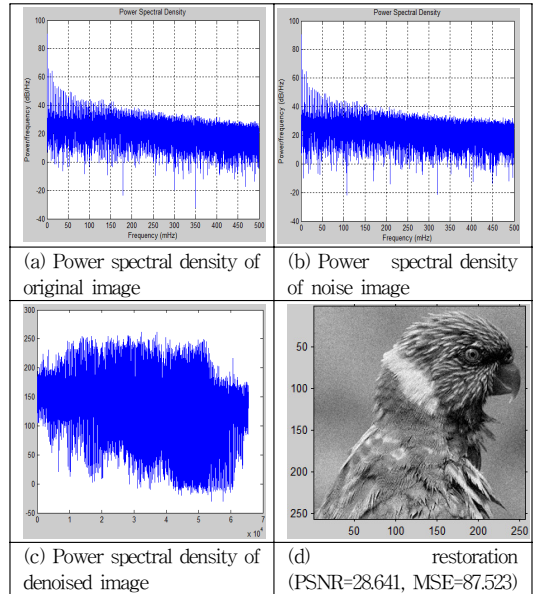


그림 10. 성능이 저하된 이미지 복원
 Fig. 10. Restoration of degraded image

The important things are the much reduction of MSE in Fig 10. (MSE of degraded image: 743.5146, MSE of restored image through Wiener filtering: 385.5513, MSE of restored image by suggested method: 87.5233). In general, the blurriness of the degraded image has been removed.

V. Conclusions

Inverse filtering is a very easy and accurate way to

restore an image provided that we know what the blurring filter is and that we have no noise. Because an inverse filter is a high pass filter, it tends to extend noise as was presented in our experiments. The inverse filtering with an iterative procedure is more or less an averaging method. It deals a little better with noise by averaging it out. But both methods do not deal well with noise. Thus, we used some method of restoration which would trade off inverse filtering with de-noising. Wiener filtering gives the optimal result between the inverse filtering and noise smoothing. It can be interpreted as an inverse filtering step followed by a noise attenuation step. However, to implement the Wiener filter we have to estimate the power spectrum of the original image from the corrupted observation. Since the Wiener filtering contains the inverse filtering step, it expands the noise when the blurring filter is not invertible. More importantly, the expanded noise is not attended. To remove the large noises, the best is to remove the noise using wavelet threshold. Wavelet noise elimination contains two separate steps, inverse filtering and wavelet de-noise. It has not control over the overall restoration performance. One new approach comprises FFT inversion by wavelet noise suppression. It uses the inverse filtering by introducing a parameter into the Wiener filtering, and the optimal substitution factor is chosen to minimize the overall MSE. The implementation of the Wiener filtering and inverse filtering order includes the estimation of the power spectrum of the original image as a new wavelet-based restoration. Since the wavelet coefficients of image are corresponding to estimate the power spectrum, we substituted the Wavelet transform into the order of inverse filtering. The methods we demonstrated all used zero phase degradation filters. Most methods that estimate phase are almost tricky.

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