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THE PRICING OF QUANTO OPTIONS UNDER THE VASICEK'S SHORT RATE MODEL

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Abstract. We derive a closed-form expression for the price of a European quanto call option when both foreign and domestic interest rates follow the Vasicek's short rate model.

1. Introduction

A quanto is a type of financial derivative whose pay-out currency differs from the natural denomination of its underlying financial variable. A quanto option is a cash-settled, cross-currency derivative whose underlying asset has a payoff in one currency, but the payoff is converted to another currency when the option is settled. For that reason, the correlation between underlying asset and currency exchange rate plays an important role in pricing quanto option. Quanto options in this paper have both the strike price and the underlying asset price denominated in foreign currency. At exercise, the value of the option is calculated as the options intrinsic value in foreign currency, which is then converted to the domestic currency at the fixed exchange rate. This allows investors to obtain exposure to foreign assets without the corresponding foreign exchange risk.

Pricing quanto options based on the classical Black-Scholes [1] model, on which most of the research on quanto options has focused, has a weakness of assuming a constant volatility and constant interest rates. To overcome such weakness, in valuing quanto option, it is natural to consider a stochastic volatility or stochastic interest rate models. Despite its importance, very few researches have been done on finding analytic solutions of quanto option prices under a stochastic volatility model primarily due to the sophisticated stochastic processes and inability to obtain the general closed form. However, by assuming constant interest rates, Giese [4] provided a closed-form expression for the price of a quanto option in the Stein-Stein stochastic volatility model, and then Y. Lee et al. [5] got a closed-form expression for the price of a European quanto

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call option in the double square root stochastic volatility model with a specific restriction of parameters.

In this paper, we assume stochastic interest rates and constant volatility to obtain a closed-form expression for the price of a quanto option without any restriction of parameters. Indeed, we use the short rate model of Vasicek [6] to describe the stochastic processes of both domestic and foreign interest rates. Along with the Cox-Ingersoll-Ross (CIR) [3] model, the Vasicek's model is the most popular one-factor equilibrium model of short rate. The Vasicek's model was the first one to capture a mean reversion, an essential characteristic of the interest rate that sets it apart from other financial security prices. The Vasicek's model is also easy to manipulate and provides closed-form expressions for its conditional and steady state density functions.

We specify dynamics of the processes of underlying asset and short rate under the quanto measure in Section 2. Then, in Section 3, we drive a closed-form expression of a quanto option price under the model specified in the previous section. We mostly use the same notations as those in [5]. Theorem 3.4 is the main result of the paper.

2. A model specification

For a dividend paying asset with the dividend yield q and a constant volatility σ_S, the process of the asset price S_t may be assumed to be denominated in foreign currency X and to have the following dynamics:

$$dS_t = \left(r_t^X - q\right) S_t dt + \sigma_S S_t dB_t^{\mathbb{Q}^X}, \tag{1}$$

$$dr_t^X = \kappa^X \left(\theta^X - r_t^X\right) dt + \xi^X dW_t^{\mathbb{Q}^X}, \tag{2}$$

$$dr_t^Y = \kappa^Y \left(\theta^Y - r_t^Y\right) dt + \xi^Y d\hat{W}_t^{\mathbb{Q}^Y}, \tag{3}$$

where $B_t^{\mathbb{Q}^X}$ and $W_t^{\mathbb{Q}^X}$ are two standard Brownian motions under the foreign risk-neutral probability measure $\mathbb{Q}^X$, and $\hat{W}_t^{\mathbb{Q}^Y}$ is a standard Brownian motions under the domestic risk-neutral probability measure $\mathbb{Q}^Y$. Also, r_t^X and r_t^Y denote the foreign and domestic interest rates, respectively, which follow stochastic short rate processes of Vasicek [6] for the asset price S_t with constant parameters κ^i, θ^i and ξ^i for each $i = X, Y$. Furthermore, it may be assumed an investor whose domestic currency is Y and who wishes to obtain exposure to the asset price S_t without carrying the corresponding foreign exchange risk.

Let $Z_t^{Y/X}$ denote the price of one unit of currency Y in units of currency X and $Z_t^{Y/X}$ follows the standard Black-Scholes type dynamics under $\mathbb{Q}^X$ such as

$$Z_t^{Y/X} = \left(r_t^X - r_t^Y\right) Z_t^{Y/X} dt + \sigma_{\mathrm{FX}} Z_t^{Y/X} d\hat{B}_t^{\mathbb{Q}^X},$$

where $\hat{B}_t^{\mathbb{Q}^X}$ is a standard Brownian motion under $\mathbb{Q}^X$ and σ_{FX} is the constant volatility of the foreign exchange rate $Z_t^{Y/X}$. This model allows three constant correlation ρ, ν and β satisfying

$$dB_t^{\mathbb{Q}^X} dW_t^{\mathbb{Q}^X} = \rho dt, \quad dB_t^{\mathbb{Q}^X} d\hat{B}_t^{\mathbb{Q}^X} = \nu dt, \quad dW_t^{\mathbb{Q}^X} d\hat{B}_t^{\mathbb{Q}^X} = \beta dt.$$

Using the change of measure from $\mathbb{Q}^X$ to the domestic risk-neutral probability measure $\mathbb{Q}^Y$ with the Radon-Nikodým derivative

$$\left.\frac{d\mathbb{Q}^Y}{d\mathbb{Q}^X}\right|_{\mathcal{F}_t} = e^{-\frac{1}{2}\sigma_{\mathrm{FX}}^2 t + \sigma_{\mathrm{FX}} \hat{B}_t^{\mathbb{Q}^X}},$$

the Girsanov's theorem implies that the new processes $B_t^{\mathbb{Q}^Y}$, $W_t^{\mathbb{Q}^Y}$ and $\hat{B}_t^{\mathbb{Q}^Y}$ defined by

$$\begin{aligned} dB_t^{\mathbb{Q}^Y} &= dB_t^{\mathbb{Q}^X} - \nu\sigma_{\mathrm{FX}} dt, \\ dW_t^{\mathbb{Q}^Y} &= dW_t^{\mathbb{Q}^X} - \beta\sigma_{\mathrm{FX}} dt, \\ d\hat{B}_t^{\mathbb{Q}^Y} &= d\hat{B}_t^{\mathbb{Q}^X} + \sigma_{\mathrm{FX}} dt \end{aligned}$$

are again standard Brownian motions under the domestic risk-neutral probability measure $\mathbb{Q}^Y$, so called the *quanto measure*. Thus, the foreign exchange rate $Z_t^{X/Y}$ denoting the price in foreign currency X per unit of the domestic currency Y follows

$$Z_t^{X/Y} = \left(r_t^Y - r_t^X\right) Z_t^{X/Y} dt + \sigma_{\mathrm{FX}} Z_t^{X/Y} d\hat{B}_t^{\mathbb{Q}^Y}.$$

From (1) and (2), we also obtain the following dynamics of S_t and r_t^X under the quanto measure $\mathbb{Q}^Y$:

$$dS_t = \left(r_t^X + \nu\sigma_{\mathrm{FX}}\sigma_S - q\right) S_t dt + \sigma_S S_t dB_t^{\mathbb{Q}^Y}, \tag{4}$$

$$dr_t^X = \kappa^X \left(\hat{\theta}^X - r_t^X\right) dt + \xi^X dW_t^{\mathbb{Q}^Y} \tag{5}$$

with $\hat{\theta}^X = \theta^X + \frac{\beta\sigma_{\mathrm{FX}}\xi^X}{\kappa^X}$. We notice that (5) maintains the same form as (2). We may also assume that two standard Brownian motions $W_t^{\mathbb{Q}^Y}$ and $\hat{W}_t^{\mathbb{Q}^Y}$ are independent.

3. A closed-form expression

In this section, by using the model specified in the previous section, we will drive a closed-form expression for the price of a European quanto call option. The following three lemmas are about some special conditional expectations under the quanto measure $\mathbb{Q}^Y$, all of which are important ingredients to the main result of the paper.

Lemma 3.1. *Under the assumption of* (3)*, we get the following equality*:

$$\mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T r_u^Y du} \middle| \mathcal{F}_t \right] = e^{A(t,T) - B(t,T) r_t^Y}, \tag{6}$$

where

$$B(t,T) = \frac{1 - e^{-\kappa^Y (T-t)}}{\kappa^Y},$$

$$A(t,T) = \{B(t,T) - (T-t)\}\left(\theta^Y - \frac{\xi^Y}{2\kappa^Y}\right) - \frac{{\xi^Y}^2}{4\kappa^Y} B(t,T)^2.$$

Proof. Let us define

$$y^Y\left(t, r_t^Y\right) = \mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T r_u^Y du} \middle| \mathcal{F}_t \right].$$

Then according to the Feynman-Kač formula, y^Y is the solution of the following partial differential equation:

$$\frac{{\xi^Y}^2}{2}\frac{\partial^2 y^Y}{\partial {r^Y}^2} + \kappa^Y\left(\theta^Y - r^Y\right)\frac{\partial y^Y}{\partial r^Y} - r^Y y + \frac{\partial y^Y}{\partial t} = 0$$

with the terminal condition

$$y^Y\left(T, r_T^Y\right) = 1.$$

Now, it is straightforward to check that the expression for $y^Y\left(t, r_t^Y\right)$ given in (6) satisfies the above partial differential equation. See the Appendix B.1 of [2]. □

We shall now denote the price at time t of a zero-coupon bond in currency Y that matures a nominal value of 1 at time T by

$$P^Y(t,T) = \mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T r_u^Y du} \middle| \mathcal{F}_t \right].$$

In places, we call this the T-*(zero-coupon) bond in currency* Y. Moreover, we will compute the value of a quanto forward contract $\mathbb{E}_{\mathbb{Q}^Y}\left[S_T | \mathcal{F}_t \right]$ from the risk-neutral valuation method.

Lemma 3.2. *Under the assumptions of* (4) *and* (5)*, we get the following equality*:

$$\mathbb{E}_{\mathbb{Q}^Y}\left[S_T | \mathcal{F}_t \right] = S_t e^{\left(\nu\sigma_{FX}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2} - \frac{\rho\sigma_S\kappa^X\hat{\theta}^2}{\xi^X}\right)(T-t) - \frac{\rho\sigma_S}{\xi^X} r_t^X}$$
$$\times\, \mathbb{E}_{\mathbb{Q}^Y}\left[e^{c_1\int_t^T r_u^X du + c_2 r_T^X} \middle| \mathcal{F}_t \right]$$

with constants

$$c_1 = 1 + \frac{\rho\sigma_S\kappa^X}{\xi^X}, \quad c_2 = \frac{\rho\sigma_S}{\xi^X}.$$

Proof. Applying the Itô formula to (4) together with the tower property, we obtain

(7)

$$\mathbb{E}_{\mathbb{Q}^Y}\left[S_T | \mathcal{F}_t \right] = S_t e^{\left(\nu\sigma_{\mathrm{FX}}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2}\right)(T-t)} \mathbb{E}_{\mathbb{Q}^Y}\left[e^{\int_t^T r_u^X du + \rho\sigma_S\left(W_T^{\mathbb{Q}^Y} - W_t^{\mathbb{Q}^Y}\right)} \middle| \mathcal{F}_t \right],$$

where we may write $B_t^{\mathbb{Q}^Y}$ as $B_t^{\mathbb{Q}^Y} = \rho W_t^{\mathbb{Q}^Y} + \sqrt{1-\rho^2} W_t$ with W_t being a $\mathbb{Q}^Y$-standard Brownian motion independent of $W_t^{\mathbb{Q}^Y}$. From (5), we have

$$W_T^{\mathbb{Q}^Y} - W_t^{\mathbb{Q}^Y} = \frac{1}{\xi^X}\left\{ r_T^X - r_t^X - \kappa^X \hat{\theta}^X (T-t) + \kappa^X \int_t^T r_s^X ds \right\}. \tag{8}$$

Substituting (8) into (7), we obtain

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}^Y}\left[S_T \middle| \mathcal{F}_t \right] = S_t e^{\left(\nu\sigma_{\mathrm{FX}}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2} - \frac{\rho\sigma_S\kappa^X\hat{\theta}^2}{\xi^X}\right)(T-t) - \frac{\rho\sigma_S}{\xi^X} r_t^X} \\ \times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{c_1 \int_t^T r_u^X du + c_2 r_T^X} \middle| \mathcal{F}_t \right]\end{aligned}$$

with

$$c_1 = 1 + \frac{\rho\sigma_S\kappa^X}{\xi^X}, \quad c_2 = \frac{\rho\sigma_S}{\xi^X}.$$

□

Now, we need the following lemma to get the value of $\mathbb{E}_{\mathbb{Q}^Y}\left[S_T \middle| \mathcal{F}_t \right]$.

Lemma 3.3. *Under the assumption of* (5) *together with constants* c_1 *and* c_2, *we get the following equality*:

$$\mathbb{E}_{\mathbb{Q}^Y}\left[e^{c_1 \int_t^T r_u^X du + c_2 r_T^X} \middle| \mathcal{F}_t \right] = e^{C(t,T) - D(t,T) r_t^X}, \tag{9}$$

where

$$\begin{aligned} C(t,T) &= -c_1 \frac{1 - e^{-\kappa^X (T-t)}}{\kappa^X} - c_2 e^{-\kappa^X (T-t)}, \\ D(t,T) &= c_1 \hat{\theta}^X \left\{ T - t - \frac{1 - e^{-\kappa^X (T-t)}}{\kappa^X} \right\} + c_2 \hat{\theta}^X \left\{ 1 - e^{-\kappa (T-t)} \right\} \\ &\quad + c_1^2 \frac{{\xi^X}^2}{4{\kappa^X}^3} \left\{ 2\kappa^X (T-t) - 3 + 4 e^{-\kappa^X (T-t)} - e^{-2\kappa^X (T-t)} \right\} \\ &\quad + c_1 c_2 \frac{{\xi^X}^2}{2{\kappa^X}^2} \left\{ 1 - e^{-\kappa^X (T-t)} \right\}^2 + c_2^2 \frac{{\xi^X}^2}{4\kappa^X} \left\{ 1 - e^{-\kappa^X (T-t)} \right\}. \end{aligned}$$

Proof. In the same way as the proof of Lemma 3.1, let us define

$$y^X\left(t, r_t^X\right) = \mathbb{E}_{\mathbb{Q}^Y}\left[e^{c_1 \int_t^T r_u^X du + c_2 r_T^X} \middle| \mathcal{F}_t \right].$$

Then according to the Feynman-Kač formula, y^X is the solution of the following partial differential equation:

$$\frac{{\xi^X}^2}{2} \frac{\partial^2 y^X}{\partial {r^X}^2} + \kappa^X \left(\hat{\theta}^X - c_1 r^X \right) \frac{\partial y^X}{\partial r^X} - r^X y^X + \frac{\partial y^X}{\partial t} = 0$$

with the terminal condition

$$y^X\left(T, r_T^X\right) = c_2.$$

Once again, it is straightforward to check that the expression for $y^X\left(t, r_t^X\right)$ given in (9) satisfies the above partial differential equation. □

Using the results obtained in Lemma 3.1, Lemma 3.2 and Lemma 3.3, we can obtain a closed-form expression for the price of a European quanto call option. The following is the main theorem in this section. For convenience, we here put the predetermined fixed exchange rate to 1.

Theorem 3.4. *Let us denote the log-asset price by $x_t = \ln S_t$. Under the assumption of* (4)*, the price of a European quanto call option in domestic currency Y with foreign strike price K and maturity T is given by*

$$C_q^Y(t, S_t) = P^Y(t,T)\left\{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\,\mathcal{F}_t\right]P_1 - KP_2\right\},$$

where P_1, P_2 are defined by

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathbf{Re}\left[\frac{e^{i\phi\ln K} f_j(\phi)}{i\phi}\right]d\phi$$

for $j = 1, 2$, in which

$$\begin{aligned} f_1(\phi) = &\frac{e^{(1+i\phi)\left\{x_t + \left(\nu\sigma_{FX}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2} - \frac{\rho\sigma_S\kappa^X\hat{\theta}^X}{\xi^X}\right)(T-t) - \frac{\rho\sigma_S}{\xi^X} r_t^X\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\,\mathcal{F}_t\right]} \\ &\times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{m_1\int_t^T r_u^X du + m_2 r_T^X}\Big|\,\mathcal{F}_t\right] \end{aligned}$$

with

$$m_1 = (1+i\phi)\left(1 + \frac{\rho\sigma_S\kappa^X}{\xi^X}\right), \quad m_2 = (1+i\phi)\frac{\rho\sigma_S}{\xi^X}$$

and

$$\begin{aligned} f_2(\phi) = &\frac{e^{i\phi\left\{x_t + \left(\nu\sigma_{FX}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2} - \frac{\rho\sigma_S\kappa^X\hat{\theta}^X}{\xi^X}\right)(T-t) - \frac{\rho\sigma_S}{\xi^X} r_t^X\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\,\mathcal{F}_t\right]} \\ &\times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{n_1\int_t^T r_u^X du + n_2 r_T^X}\Big|\,\mathcal{F}_t\right] \end{aligned}$$

with

$$n_1 = i\phi\left(1 + \frac{\rho\sigma_S\kappa^X}{\xi^X}\right), \quad n_2 = i\phi\frac{\rho\sigma_S}{\xi^X}.$$

Proof. From the risk-neutral valuation method, the price $C_q^Y(t, S_t)$ of a European quanto call option in currency Y with foreign strike price K and maturity T is given by

$$C_q^Y(t, S_t) = \mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T r_u^Y du}\max(S_T - K, 0)\Big|\,\mathcal{F}_t\right].$$

For a new risk-neutral probability measure $\tilde{\mathbb{Q}}^Y$, the Radon-Nikodým derivative of $\tilde{\mathbb{Q}}^Y$ with respect to $\mathbb{Q}^Y$ is defined by

$$\frac{d\tilde{\mathbb{Q}}^Y}{d\mathbb{Q}^Y} = \frac{S_T}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\,\mathcal{F}_t\right]}.$$

Thus, the price of a European quanto call option can be rewritten as

$$C_q^Y(t, S_t) = P^Y(t,T)\,\mathbb{E}_{\mathbb{Q}^Y}\left[S_T\mathbb{1}_{\{S_T>K\}} - K\mathbb{1}_{\{S_T>K\}}\big|\,\mathcal{F}_t\right]$$

$$= P^Y(t,T)\left\{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\mathcal{F}_t\right]\tilde{\mathbb{Q}}^Y(S_T>K) - K\mathbb{Q}^Y(S_T>K)\right\}$$
$$= P^Y(t,T)\left\{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\mathcal{F}_t\right]P_1 - KP_2\right\}$$

with the risk-neutralized probabilities P_1 and P_2. Now, putting $x_t = \ln S_t$, the corresponding characteristic functions f_1 and f_2 can be represented as

$$\begin{aligned} f_1(\phi) &= \mathbb{E}_{\tilde{\mathbb{Q}}^Y}\left[e^{i\phi x_T}\middle|\mathcal{F}_t\right] \\ &= \frac{1}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\mathcal{F}_t\right]}\mathbb{E}_{\mathbb{Q}^Y}\left[e^{(1+i\phi)x_T}\middle|\mathcal{F}_t\right], \\ f_2(\phi) &= \mathbb{E}_{\mathbb{Q}^Y}\left[e^{i\phi x_T}\middle|\mathcal{F}_t\right]. \end{aligned}$$

On the other hand, applying the Itô formula to (4), we have

$$dx_t = \left(r^X + \nu\sigma_{\mathrm{FX}}\sigma_S - q - \frac{\sigma_S^2}{2}\right)dt + \rho\sigma_S dW_t^{\mathbb{Q}^Y} + \sqrt{1-\rho^2}\sigma_S dW_t.$$

From (8), we obtain

$$\begin{aligned} f_1(\phi) = &\frac{e^{(1+i\phi)\left\{x_t + \left(\nu\sigma_{\mathrm{FX}}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2} - \frac{\rho\sigma_S\kappa^X\hat{\theta}^X}{\xi^X}\right)(T-t) - \frac{\rho\sigma_S}{\xi^X}r_t^X\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\mathcal{F}_t\right]} \\ &\times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{m_1\int_t^T r_u^X du + m_2 r_T^X}\middle|\mathcal{F}_t\right] \end{aligned}$$

with

$$m_1 = (1+i\phi)\left(1+\frac{\rho\sigma_S\kappa^X}{\xi^X}\right), \quad m_2 = (1+i\phi)\frac{\rho\sigma_S}{\xi^X}$$

and

$$\begin{aligned} f_2(\phi) = &\frac{e^{i\phi\left\{x_t + \left(\nu\sigma_{\mathrm{FX}}\sigma_S - q - \frac{\rho^2\sigma_S^2}{2} - \frac{\rho\sigma_S\kappa^X\hat{\theta}^X}{\xi^X}\right)(T-t) - \frac{\rho\sigma_S}{\xi^X}r_t^X\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T|\mathcal{F}_t\right]} \\ &\times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{n_1\int_t^T r_u^X du + n_2 r_T^X}\middle|\mathcal{F}_t\right] \end{aligned}$$

with

$$n_1 = i\phi\left(1+\frac{\rho\sigma_S\kappa^X}{\xi^X}\right), \quad n_2 = i\phi\frac{\rho\sigma_S}{\xi^X}.$$

Here, each value of the above risk-neutral expectation was obtained in previous lemmas.

By having closed-form expressions for the characteristic functions f_1 and f_2, the Fourier inversion formula allows us to compute the probabilities P_1 and P_2 as follows:

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathbf{Re}\left[\frac{e^{i\phi\ln K}f_j(\phi)}{i\phi}\right]d\phi$$

for $j = 1, 2$. □

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