

Nonlinear Behavior in Love Model with Discontinuous External Force

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Abstract

This paper proposes nonlinear behavior in a love model for Romeo and Juliet with an external force of discontinuous time. We investigated the periodic motion and chaotic behavior in the love model by using time series and phase portraits with respect to some variable and fixed parameters. The computer simulation results confirmed that the proposed love model with an external force of discontinuous time shows periodic motion and chaotic behavior with respect to parameter variation.

Keywords: Chaoss, Love model, Nonlinear behavior, Phase plane, Time series

1. Introduction

Over the last three decades, nonlinear dynamics including chaotic dynamics has been studied in many science fields, such as mathematics, physics, chemistry, engineering, and social science. In particular, studies of chaotic behavior in love models [1-5] have increased.

The dynamical love model has been defined in several forms, such as the Romeo and Juliet model [1, 3-7], the Laura and Petrarch model [8], and the Adam and Eve model [9]. Among these love models, the Romeo and Juliet model is mostly used in the study of nonlinear dynamics.

The love model based on Romeo and Juliet with a linear differential equation was first proposed by Sprott [1, 2], who described its linear and nonlinear behaviors. While the other love models have been the subject of some research [3-7], the Romeo and Juliet-based love model has been the most studied. Bae and his associates [3-7] proposed that the existence of periodic and chaotic behaviors based on the Romeo and Juliet love model can be represented through time series and phase portraits with identical and different time delays and an external force.

In this paper, we propose a love model for Romeo and Juliet with a discontinuous external force. The discontinuous external force closely reflects the real love between a man and woman. The discontinuous external force implies that the relation of love between Romeo and Juliet is not always the same. Sometimes they do not have thoughts about each other or sometimes they do not have concerns for each other. In addition, we investigated the periodic motion and chaotic behavior in the proposed Romeo and Juliet love model, with a discontinuous external force, by using time series and phase portraits.

2. Love Model

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2.1 Romeo and Juliet Love Model

The basic love model for Romeo and Juliet was proposed by Sprott [1] and can be described as the following Eq. (1):

$$\begin{aligned} \frac{dR}{dt} &= aR + bJ \\ \frac{dJ}{dt} &= cR + dJ, \end{aligned} \tag{1}$$

where a and b specify Romeo’s romantic style, and c and d specify Juliet’s style.

According to parameters $a, b, c,$ and d , we divided behavior into four types, as proposed by Strogatz [10]. When $a > 0$ and $b > 0$, Romeo is an eager beaver. That is, Romeo is encouraged by his and Juliet’s feelings. When $a > 0$ and $b < 0$, Romeo is a narcissistic nerd. This implies that Romeo wants more of what he feels but retreats from Juliet’s feelings. When $a < 0$ and $b > 0$, Romeo is a cautious or secure lover. That is, Romeo retreats from his own feelings but is encouraged by Juliet’s. Finally, when $a < 0$ and $b < 0$, Romeo is a hermit. That is, Romeo retreats from both his and Juliet’s feelings.

Eq. (1) can be rewritten as Eq. (2) by using an advanced form of Eq. (1), as follows:

$$\begin{aligned} \frac{dR}{dt} &= aR + bJ(|(1 - J)|) \\ \frac{dJ}{dt} &= cR(|(1 - R)|) + dJ. \end{aligned} \tag{2}$$

From Eq. (2), we can describe a novel love model with a discontinuous external force, as in the following Eq. (3):

$$\begin{aligned} \frac{dR}{dt} &= aR + bJ(|(1 - J)|) + f(t) \\ \frac{dJ}{dt} &= cR(|(1 - R)|) + dJ, \end{aligned} \tag{3}$$

where $f(t) = 5 \sin\pi t_1$ is a discontinuous external force. In this paper, we gives 2 seconds per every 10 seconds as t_1 to have intermittence.

3. Nonlinear Behavior Analysis

For Eq. (3), we investigated the chaotic behavior by using time series and phase portraits according to parameter variation.

3.1 $a = -7, b = -2, c = 1,$ and $d = 1$

With this parameter set, we obtained the time series and phase portrait shown in Figures 1 and 2, respectively.

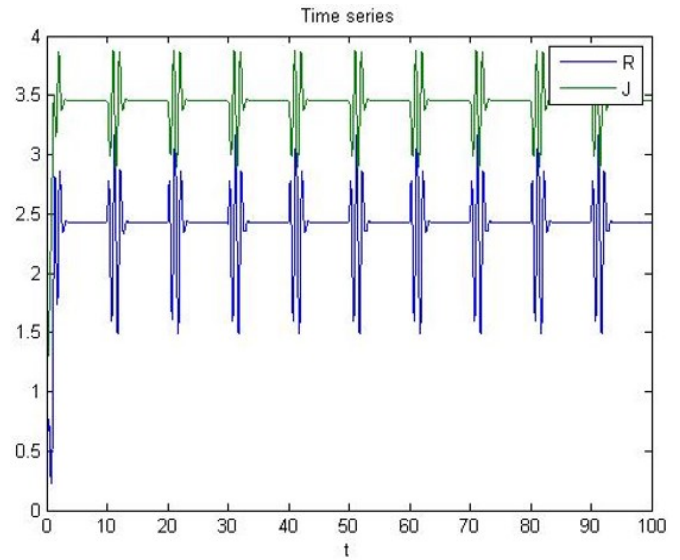


Figure 1. Time series of Romeo and Juliet model when $a = -7, b = -2, c = 1,$ and $d = 1.$

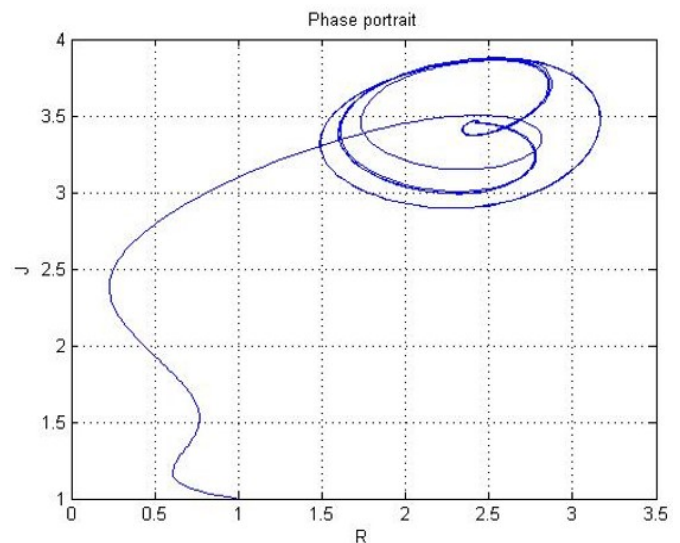


Figure 2. Phase portrait of Romeo and Juliet model when $a = -7, b = -2, c = 1,$ and $d = 1.$

Figures 1 and 2 confirm that the model has a 1-periodic motion.

3.2 $a = -3.5, b = -2, c = 1,$ and $d = 1$

The time series and phase portrait obtained using this parameter set are shown in Figures 3 and 4, respectively, which again confirm 1-periodic motion.

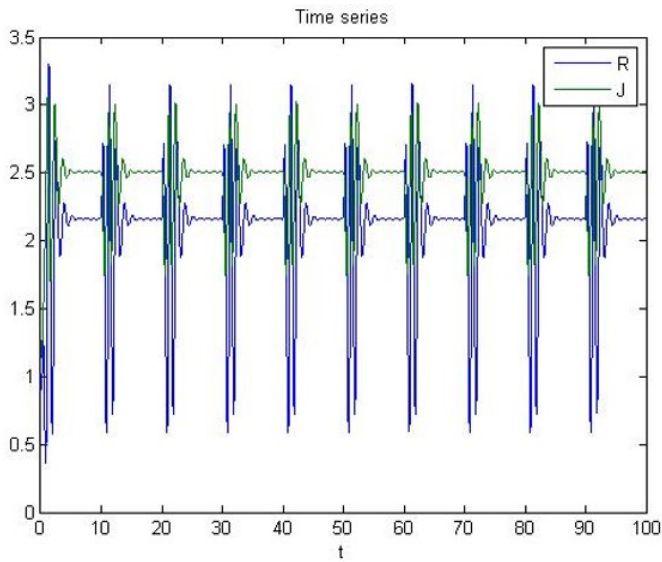


Figure 3. Time series of Romeo and Juliet model when $a = -3.5$, $b = -2$, $c = 1$, and $d = 1$.

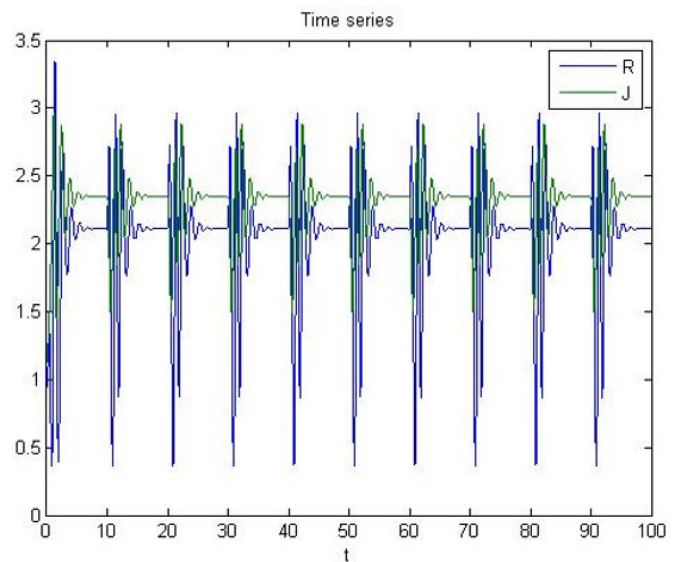


Figure 5. Time series of Romeo and Juliet model when $a = -3$, $b = -2$, $c = 1$, and $d = 1$.

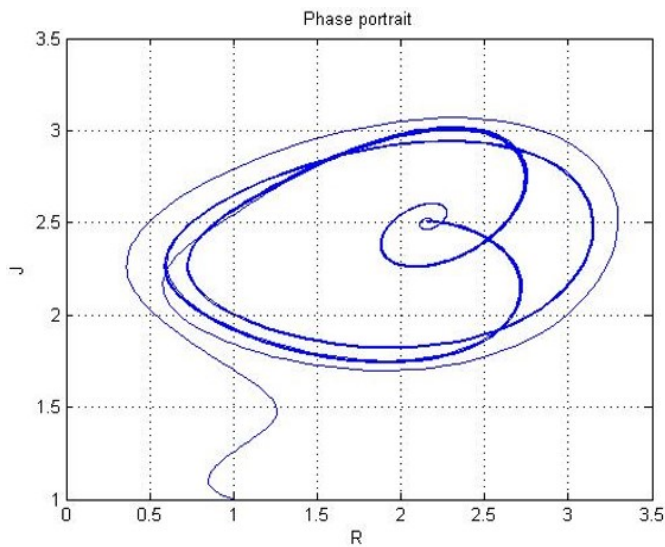


Figure 4. Phase portrait of Romeo and Juliet model when $a = -3.5$, $b = -2$, $c = 1$, and $d = 1$.

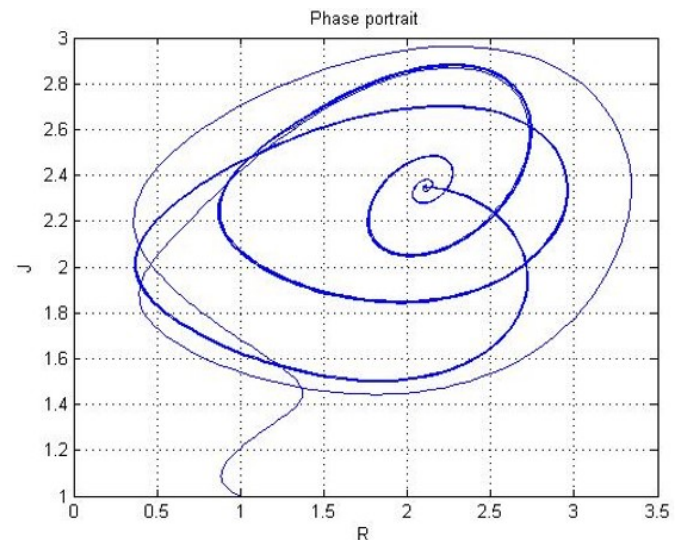


Figure 6. Phase portrait of Romeo and Juliet model when $a = -3$, $b = -2$, $c = 1$, and $d = 1$.

3.3 $a = -3, b = -2, c = 1$, and $d = 1$

The time series and phase portrait obtained using this parameter set are shown in Figures 5 and 6, respectively, which display 2-periodic motion.

3.4 $a = -2.5, b = -2, c = 1$, and $d = 1$

The time series and phase portrait obtained using this parameter set are shown in Figures 7 and 8, respectively, which indicate

that the model has quasi-periodic motion.

3.5 $a = -2, b = -2, c = 1$, and $d = 1$

The time series and phase portrait obtained using this parameter set are shown in Figures 9 and 10, respectively. The figures confirm quasi-periodic motion.

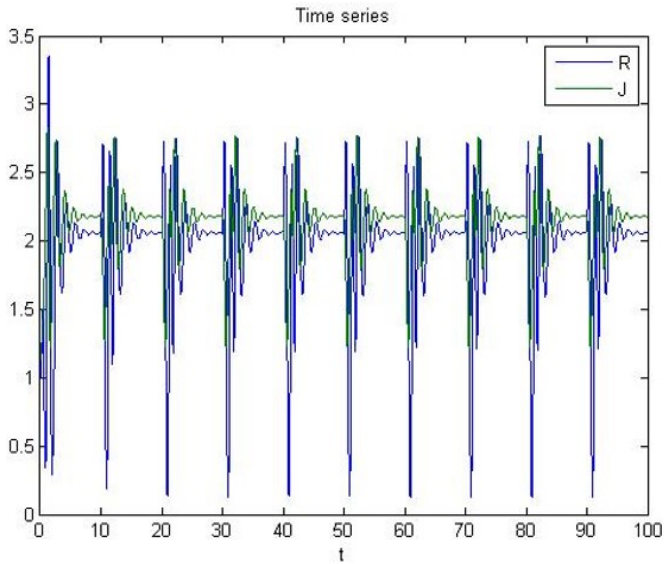


Figure 7. Time series of Romeo and Juliet model when $a = -2.5$, $b = -2$, $c = 1$, and $d = 1$.

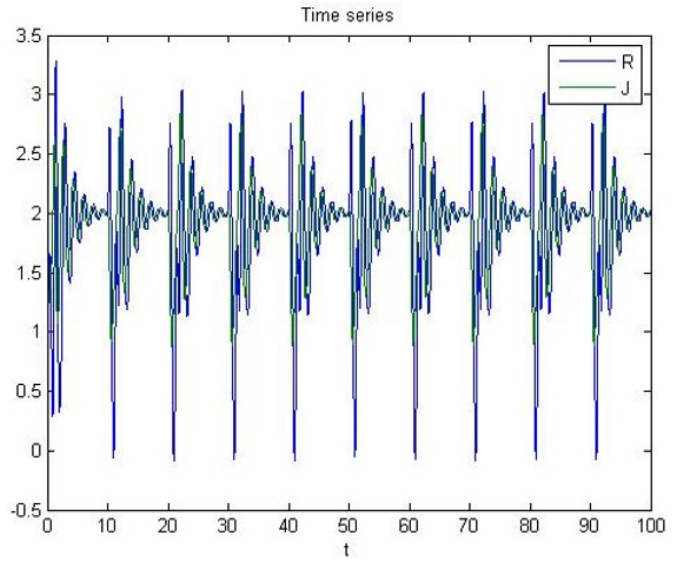


Figure 9. Time series of Romeo and Juliet model when $a = -2$, $b = -2$, $c = 1$, and $d = 1$.

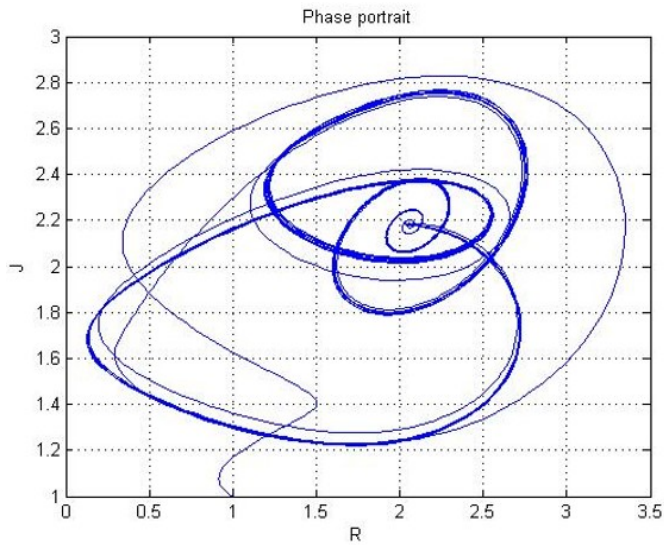


Figure 8. Phase portrait of Romeo and Juliet model when $a = -2.5$, $b = -2$, $c = 1$, and $d = 1$.

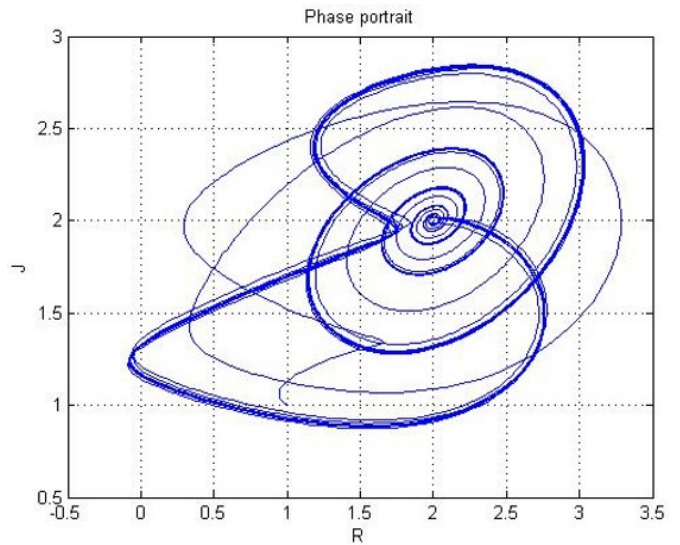


Figure 10. Phase portrait of Romeo and Juliet model when $a = -2$, $b = -2$, $c = 1$, and $d = 1$.

3.6 $a = -1.5, b = -2, c = 1, \text{ and } d = 1$

The time series and phase portrait of the model obtained using this parameter set are shown in Figures 11 and 12, respectively. The figures show a quasi-periodic motion or quasi-attractor.

3.7 $a = -1.4, b = -2, c = 1, \text{ and } d = 1$

The time series and phase portrait of the model obtained using this parameter set are shown in Figures 13 and 14, respectively, which show that the model has a chaotic attractor.

3.8 $a = -1.3, b = -2, c = 1, \text{ and } d = 1$

The time series and phase portrait of the model obtained using this parameter set are shown in Figures 15 and 16, respectively.

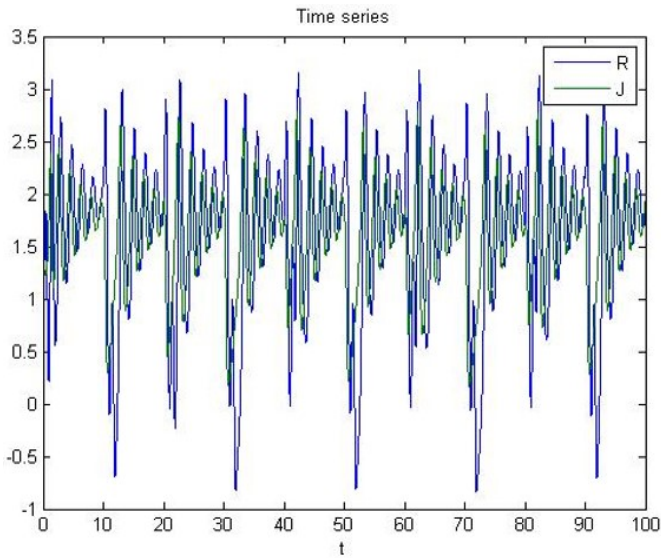


Figure 11. Time series of Romeo and Juliet model when $a = -1.5$, $b = -2$, $c = 1$, and $d = 1$.

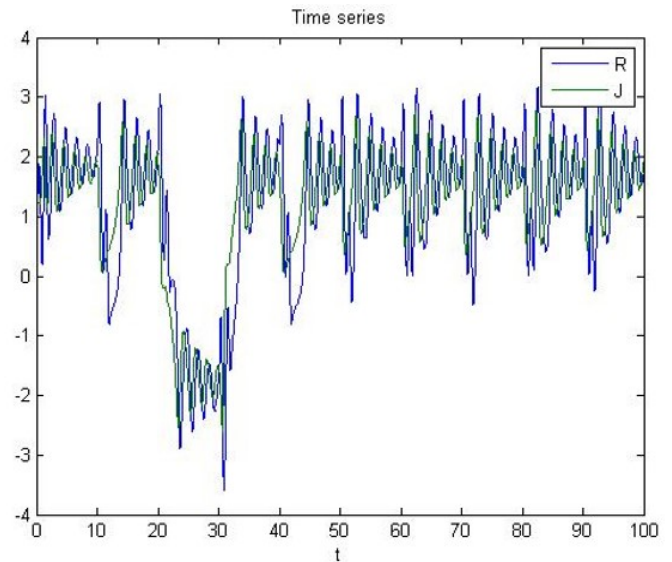


Figure 13. Time series of Romeo and Juliet model when $a = -1.4$, $b = -2$, $c = 1$, and $d = 1$.

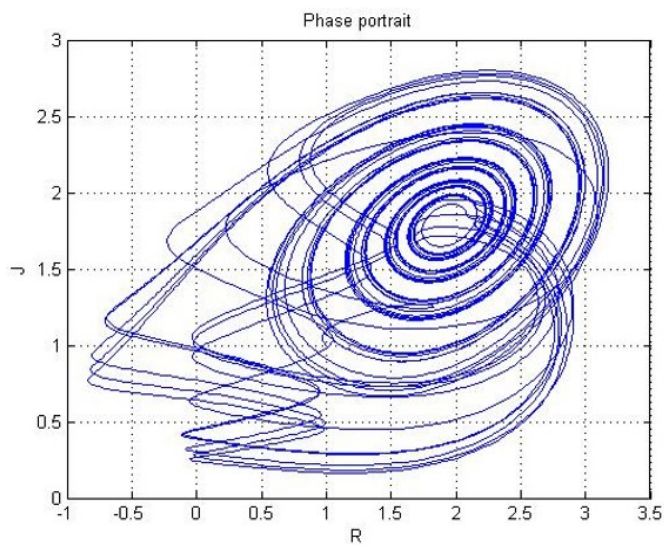


Figure 12. Phase portrait of Romeo and Juliet model when $a = -1.5$, $b = -2$, $c = 1$, and $d = 1$.

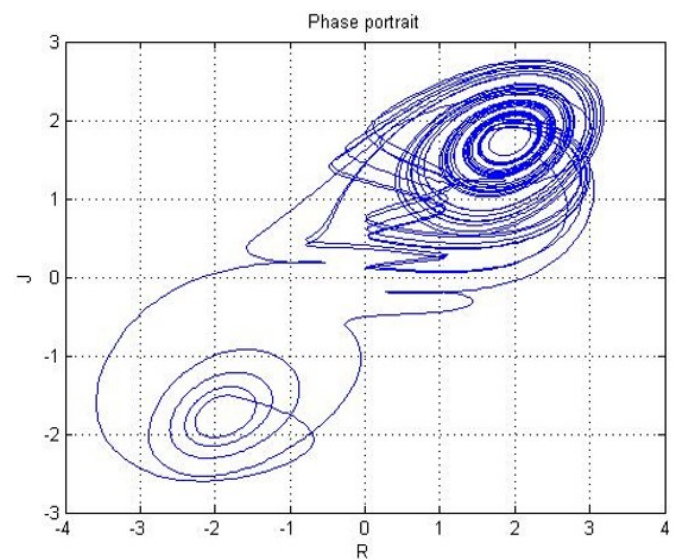


Figure 14. Phase portrait of Romeo and Juliet model when $a = -1.4$, $b = -2$, $c = 1$, and $d = 1$.

The figures confirm that the model has a chaotic motion.

3.9 $a = -1.1$, $b = -2$, $c = 1$, and $d = 1$

The time series and phase portrait of the model obtained using this parameter set are shown in Figures 17 and 18, respectively. These figures confirm a chaotic attractor with different type compare to other chaotic attractor such as Lorenz attractor, Chua attractor and Chen attractor.

3.10 $a = -1.0$, $b = -2$, $c = 1$, and $d = 1$

The time series and phase portrait obtained using this parameter set are shown in Figures 19 and 20, respectively. These figures confirm a quasi-periodic motion.

Figures 1–20 confirm that the behavior of the love model with a discontinuous external force changes from periodic motion to quasi-attractor, chaotic attractor, and quasi-periodic motion as confirmed by the various time series and phase portraits.

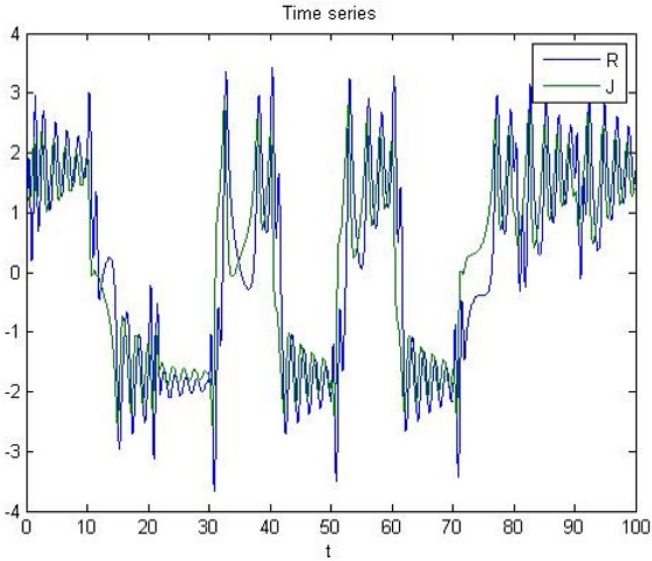


Figure 15. Time series of Romeo and Juliet model when $a = -1.3$, $b = -2$, $c = 1$, and $d = 1$.

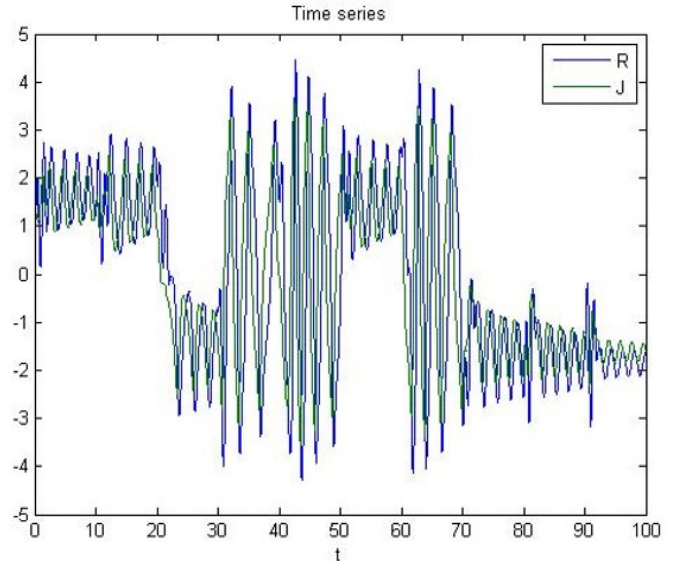


Figure 17. Time series of Romeo and Juliet model when $a = -1.1$, $b = -2$, $c = 1$, and $d = 1$.

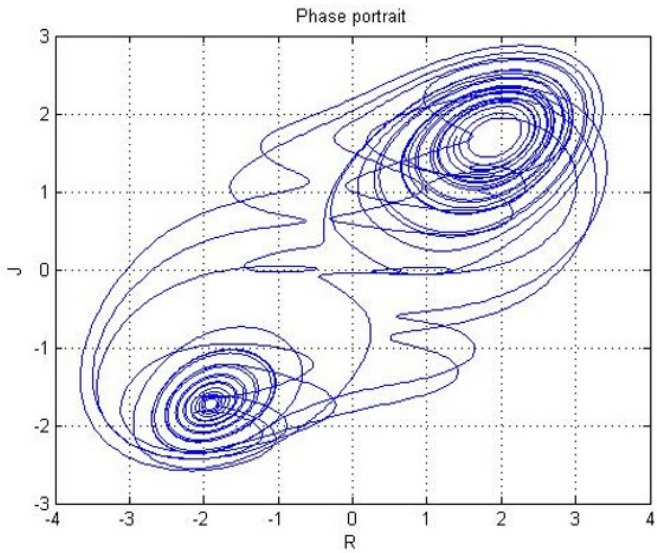


Figure 16. Phase portrait of Romeo and Juliet model when $a = -1.3$, $b = -2$, $c = 1$, and $d = 1$.

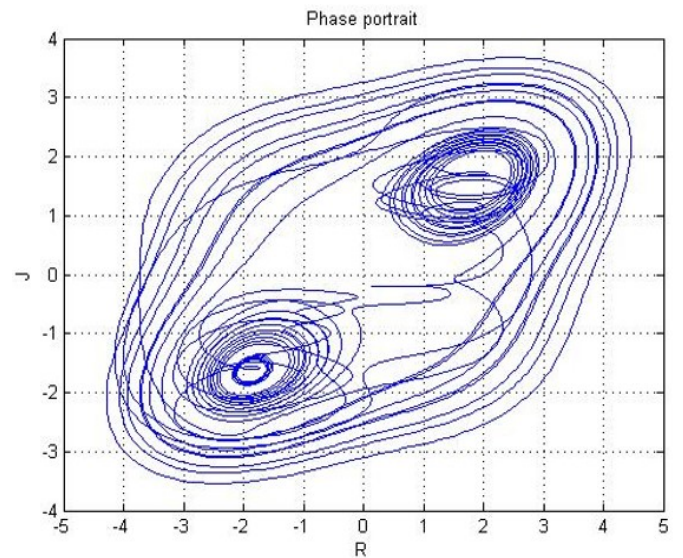


Figure 18. Phase portrait of Romeo and Juliet model when $a = -1.1$, $b = -2$, $c = 1$, and $d = 1$.

Thus, these results confirm that the system has a typical chaotic dynamics.

4. Conclusion

This paper proposed a love model for Romeo and Juliet that is organized by a discontinuous external force. From the proposed love model, we obtained periodic and chaotic motions by time

series and phase portrait graphs by varying the parameters.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

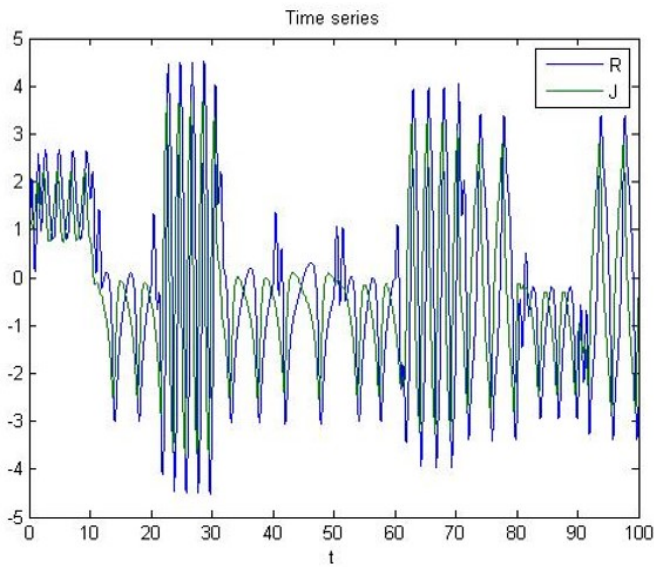


Figure 19. Time series of Romeo and Juliet model when $a = -1.0$, $b = -2$, $c = 1$, and $d = 1$.

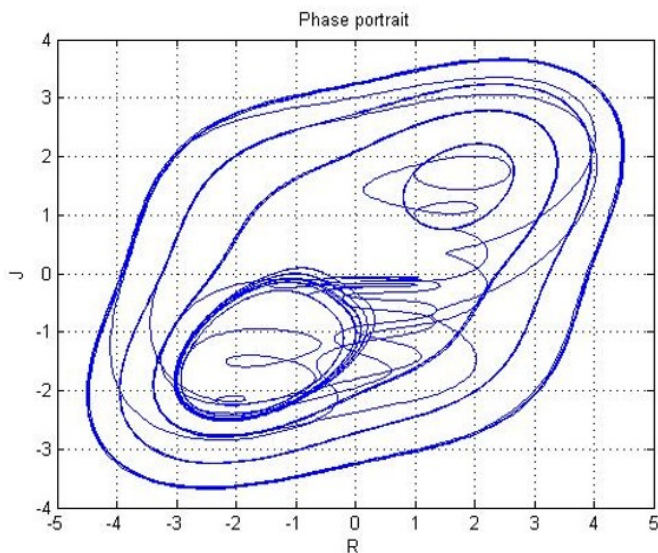


Figure 20. Phase portrait of Romeo and Juliet model when $a = -1.0$, $b = -2$, $c = 1$, and $d = 1$.

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