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Second Order Effect Induced by a Forced Heaving

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Abstract

In this paper, the 2nd order hydrodynamic force effect of heaving submerged circular cylinder is considered, with the linear potential theory. Boundary value problem (BVP) is expanded up to the 2nd order by using of the perturbation method and the 2nd order velocity potential is calculated by means of integral equation technique using the classical Green's function expressed in cylindrical coordinates. The method of solving BVP is based on eigenfunction expansions. With different cylinder heights and heaving frequencies, graphical results are presented. As a result of the study, the cause of oscillatory force pattern is analyzed with the occurrence of negative added mass when a top of the cylinder gets closer to the free surface.

Keywords: Second order force, Potential theory, Heaving cylinder, Eigen function expansions, Green function, Negative added mass.

1. Introduction

When the wave loads on the structure are calculated, velocity potential is calculated through the linearization of the nonlinear free surface boundary condition with a linear potential theory. However, a sudden increase in the demand of the fossil energy has brought an increase in the demand for the offshore structures with a low natural frequency such as Tension leg platforms (TLP) or Gravity base towers. Accordingly, approximated high order solutions using potential theory has been widely studied. 2nd order diffraction theory was developed with Lighthill (1979) and Molin (1979) as its starting point. Study that obtained 2nd order diffraction wave force acting on a vertical circular cylinder that is fixed to the bottom was performed by Molin & Marison (1986) and Eatock Taylor & Hung (1987). Furthermore, Malenica & Molin (1995) calculated even till the 3rd order at a high-frequency Diffraction wave force.

In this study, the 2nd order hydrodynamic force due to not the diffraction wave force performed in previous studies, but the heave motion of the cylinder, was calculated for the upper surface of the vertical circular cylinder with the top of it to be cut. In the study conducted by C. Tung (1979), the 1st order linear radiation force due to the sway movement was once obtained by targeting the underwater oil tank in the form of a circular cylinder with a top of it to be cut. However, 2nd order radiation force due to the motion of a body have not been researched yet. For the 2nd order radiation wave force calculated in this study, the 2nd order velocity potential was calculated by using the Green's function integral equation and eigenfunction expansions presented in Malenica & Molin (1995) and Chau & Eatock Taylor (1992). The results were graphically presented on the 2nd order force and heaving frequency by varying the ratio of the height of the cylinder and the distance of the free

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water surface.

When the upper side of the cylinder comes to close to the free water surface, an oscillatory wave force form appeared along the heave frequency, while wave force increased linearly in other cases. This is because of the influence of the negative added mass resulting from the 1st order. The negative added mass of the heaving cylinder had been carefully studied by Mciver & Evans (1983).

2. General theory

2.1 Definition of Problem

In this problem, a circular cylinder with a height of d and a radius of a is fixed to the bottom of the fluid with a depth of h_2 ($h_2 = h_1 + d$) and the origin exists on the upper side of the cylinder (z=0). The circular cylinder has a heave motion with an amplitude of ξ and an angular frequency of ω. The fluid is assumed to be inviscid and has irrotationality and satisfies the Laplace equation of $\nabla^2\Phi = 0$. In Fig. 1 and 2 below, S_F means the free water surface, S_B is the bottom surface, and S_∞ refers to the surface of the infinite radius $r \to \infty$.

Since the height of the region I ($r < a$) and II ($r > a$) in the fluid are different, their velocity potentials compose different eigenfunction expansions and unknown coefficients in each potential are calculated by using the orthogonality properties of the eigenfunctions in matching conditions. First, the governing equation of the area I, nonlinear kinetic free surface boundary condition, dynamic free surface boundary condition, and body boundary condition are as follows.

$$\nabla^2\Phi_I = 0 \quad in\ \Omega \qquad (r < a) \tag{1}$$

$$\frac{\partial \Xi_I}{\partial t} = \frac{\partial \Phi_I}{\partial z} - \nabla_0\Phi_I \cdot \nabla_0\Xi_I \quad on\ z = \Xi_I + h_1 \tag{2}$$

$$\frac{\partial^2 \Phi_I}{\partial t^2} + g\frac{\partial \Phi_I}{\partial z} + 2\nabla\Phi_I \cdot \nabla\frac{\partial \Phi_I}{\partial t} + \frac{1}{2}\nabla\Phi_I \cdot \nabla(\nabla\Phi_I \cdot \nabla\Phi_I) = 0 \quad z = \Xi_I + h_1 \tag{3}$$

$$\frac{\partial \Phi_I}{\partial z} = V \cdot n \quad on\ z = \ 0 \tag{4}$$

Φ_I means the velocity potential of the area I, Ξ_I is wave elevation, V is heaving speed, and n refers to a normal vector toward the outside of the object. Next, the governing equation of the area II, nonlinear kinetic free surface boundary condition, dynamic free surface boundary condition, bottom boundary condition, and radiation condition are as follows.

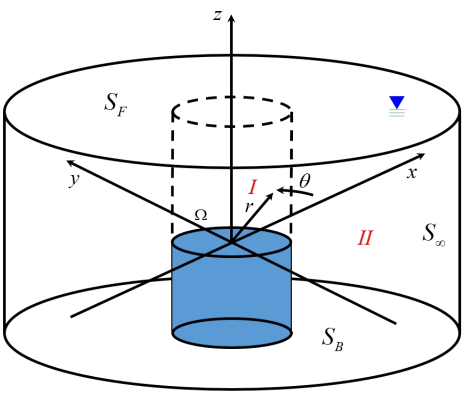

Fig. 1 3-D description of problem

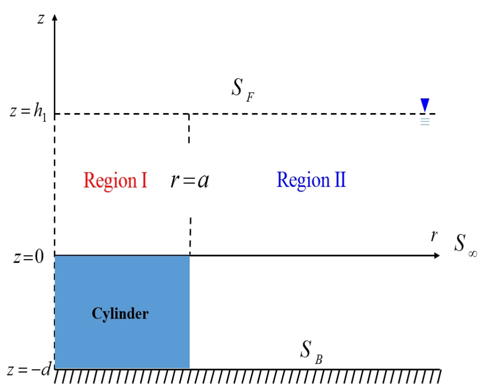

Fig. 2 2-D description of problem

$$\nabla^2\Phi_{II} = 0 \quad in\ \Omega \qquad (r > a) \tag{5}$$

$$\frac{\partial \Xi_{II}}{\partial t} = \frac{\partial \Phi_{II}}{\partial z} - \nabla_0\Phi_{II}\cdot\nabla_0\Xi_I \quad on\ z = \Xi_{II} + h_1 \tag{6}$$

$$\frac{\partial^2\Phi_{II}}{\partial t^2} + g\frac{\partial \Phi_{II}}{\partial z} + 2\nabla\Phi_{II}\cdot\nabla\frac{\partial\Phi_{II}}{\partial t} + \frac{1}{2}\nabla\Phi_{II}\cdot\nabla(\nabla\Phi_{II}\cdot\nabla\Phi_{II}) = 0 \quad z = \Xi_{II} + h_1 \tag{7}$$

$$\frac{\partial \Phi_{II}}{\partial z} = 0 \quad on\ z = -d \tag{8}$$

$$\lim_{kr\to\infty}\sqrt{kr}\left(\frac{\partial\Phi_{II}}{\partial r} - ik\Phi_{II}\right) = 0 \qquad ar\ r\to\infty \tag{9}$$

Φ_{II} means the velocity potential of the area II, Ξ_{II} is wave elevation, and k refers to a real wave number. Through the perturbation method, velocity potential and wave elevation of the each area are observed according to the order of the wave steepness ($\varepsilon = ka$). That the space and time terms of the velocity potential are divided with the assumption of ω harmonic motion is as follows.

$$\Phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + O(\varepsilon^3) = \mathrm{Re}\left\{-i\omega\xi\varphi^{(1)}e^{-i\omega t}\right\} + \overline{\varphi}^{(2)} + \mathrm{Re}\left\{\varphi^{(2)}e^{-2i\omega t}\right\} + O(\varepsilon^3) \tag{10}$$

$$\Xi = \varepsilon\Xi^{(1)} + \varepsilon^2\Xi^{(2)} + O(\varepsilon^3) = \mathrm{Re}\left\{-i\omega\xi\eta^{(1)}e^{-i\omega t}\right\} + \overline{\eta}^{(2)} + \mathrm{Re}\left\{\eta^{(2)}e^{-2i\omega t}\right\} + O(\varepsilon^3) \tag{11}$$

In this study, since only high frequency velocity potential component is considered, mean potential and wave elevation ($\overline{\varphi}^{(2)}, \overline{\eta}^{(2)}$) are ignored. Finally, pressure continuity condition (Eq. (12)) and normal velocity continuity condition (Eq. (13)) that are matching conditions to obtain the unknown coefficients of the velocity potential are as follows.

$$\Phi_I = \Phi_{II} \quad on \quad r = a \tag{12}$$

$$\frac{\partial \Phi_I}{\partial r} = \frac{\partial \Phi_{II}}{\partial r} \quad on \quad r = a \tag{13}$$

2.2 The 1st Order Potential

That the $O(\varepsilon)$ governing equation and boundary conditions of the each area are separated through the perturbation method and the free surface boundary conditions are combined together is as follows.

$$\nabla^2\varphi_I^{(1)} = 0 \quad in\,\Omega \qquad (r < a) \tag{14}$$

$$-\nu\,\varphi_I^{(1)} + \frac{\partial\varphi_I^{(1)}}{\partial z} = 0 \quad on\ z = h_1 \tag{15}$$

$$\frac{\partial\varphi_I^{(1)}}{\partial z} = 1 \quad on\ z = 0 \tag{16}$$

$$\nabla^2\varphi_{II}^{(1)} = 0 \quad in\,\Omega \qquad (r > a) \tag{17}$$

$$-\nu\,\varphi_{II}^{(1)} + \frac{\partial\varphi_{II}^{(1)}}{\partial z} = 0 \quad on\ z = h_1 \tag{18}$$

$$\frac{\partial\varphi_{II}^{(1)}}{\partial z} = 0 \quad on\ z = -d \tag{19}$$

$$\lim_{kr\to\infty}\sqrt{kr}\left(\frac{\partial\varphi_{II}^{(1)}}{\partial r} - ik\varphi_{II}^{(1)}\right) = 0 \qquad ar\ r\to\infty \tag{20}$$

The superscript $^{(1)}$ means the 1st order component, g is the acceleration of gravity, and ν refers to ω^2/g. Through the eigenfunction expansions, the 1st order velocity potential of each area is calculated as shown in Eq. (21) and Eq. (22). Since the body boundary condition of the area I is non-homogeneous linear differential equation, the potential of the area I consists of the sum of the homogeneous solution and non-homogenous solution. Also, the velocity potential is not the function of θ because the body is a circular cylinder that do an only heave motion.

$$\varphi_I^{(1)} = A_0^{(1)} f_0^{I,(1)}(z) J_0(\beta_0^{(1)} r) + \sum_{n=1}^{\infty} A_n^{(1)} f_n^{I,(1)}(z) I_0(\beta_n^{(1)} r) + z - h_1 + \frac{g}{\omega^2} \tag{21}$$

$$\varphi_{II}^{(1)} = B_0^{(1)} f_0^{II,(1)}(z) H_0^{(1)}(k_0^{(1)} r) + \sum_{n=1}^{\infty} B_n^{(1)} f_n^{II,(1)}(z) K_0(k_n^{(1)} r) \tag{22}$$

$A_n^{(1)}$ and $B_n^{(1)}$ mean the 1st order unknown coefficients of each area I and II and $\beta_n^{(1)}$ and $k_n^{(1)}$ refer to eigenvalues. The 1st order eigenfunction $f_I^{(1)}(z)$ and $f_{II}^{(1)}(z)$ are defined as follows.

$$f_0^{I,(1)}(z) = \frac{\cosh \beta_0^{(1)} z}{\cosh \beta_0^{(1)} h_1}, \qquad f_n^{I,(1)}(z) = \frac{\cos \beta_n^{(1)} z}{\cos \beta_n^{(1)} h_1} \tag{23}$$

$$f_0^{II,(1)}(z) = \frac{\cosh k_0^{(1)} z}{\cosh k_0^{(1)} h_2}, \qquad f_n^{II,(1)}(z) = \frac{\cos k_n^{(1)} z}{\cos k_n^{(1)} h_2} \tag{24}$$

The unknown coefficients $A_n^{(1)}$ and $B_n^{(1)}$ are obtained by establishing a systems of linear equations through the matching condition Eq. (12) and Eq. (13).

2.3 The 2nd Order Potential

When the boundary conditions of $O(\varepsilon^2)$ are separated by using the perturbation method, the each region's body boundary conditions ,which were the non-homogeneous in the 1st order theory, become the homogeneous differential equation. However, the free surface boundary condition of each area becomes non-homogenous differential equation in which forcing term $Q_I(\mathrm{r})$ and $Q_{II}(\mathrm{r})$ are on the right hand side, respectively. The 2nd order governing equation and boundary conditions of each area are as follows.

$$\nabla^2 \varphi_I^{(2)} = 0 \quad in\,\Omega \quad (r<a) \tag{25}$$

$$-4\nu\,\varphi_I^{(2)} + \frac{\partial \varphi_I^{(2)}}{\partial z} = Q_I(r) \quad on\, z = h_1 \tag{26}$$

$$\frac{\partial \varphi_I^{(2)}}{\partial z} = 0 \quad on\, z = 0 \tag{27}$$

$$\nabla^2 \varphi_{II}^{(2)} = 0 \quad in\,\Omega \quad (r>a) \tag{28}$$

$$-4\nu\,\varphi_{II}^{(2)} + \frac{\partial \varphi_{II}^{(2)}}{\partial z} = Q_{II}(r) \quad on\, z = h_1 \tag{29}$$

$$\frac{\partial \varphi_{II}^{(2)}}{\partial z} = 0 \quad on\, z = -d \tag{30}$$

$$\lim_{kr\to\infty} \sqrt{kr}\left(\frac{\partial \varphi_{II}^{(2)}}{\partial r} - ik\varphi_{II}^{(2)}\right) = 0 \quad ar\; r\to\infty \tag{31}$$

The forcing term $Q_I(\mathrm{r})$ and $Q_{II}(\mathrm{r})$ of the free surface boundary condition are given in the sum of the 1st order velocity potential products as follows.

$$Q_j(r) = -\frac{i\omega\xi^2}{g}\left(\nabla\varphi_j^{(1)} \cdot \nabla\varphi_j^{(1)} + \frac{1}{2}\varphi_j^{(1)}\left(\nu\frac{\partial \varphi_j^{(1)}}{\partial z} - \frac{\partial^2 \varphi_j^{(1)}}{\partial z^2}\right)\right), \quad j = I \text{ or } II \tag{32}$$

Since the free surface boundary condition is a non-homogeneous differential equation, the 2nd order velocity potential consists of the sum of homogenous solution and non-homogenous solution. The homogeneous solution was obtained through the eigenfunction expansions that is the same as the 1st order theory. Except for the eigen-values and functions, the 2nd order homogeneous solution is nearly the same as the form of the 1st order velocity potential. The homogeneous solution of each area is as follows.

$$\varphi_{I,h}^{(2)} = A_0^{(2)} f_0^{I,(2)}(z) J_0(\beta_0^{(2)} r) + \sum_{n=1}^{\infty} A_n^{(2)} f_n^{I,(2)}(z) I_0(\beta_n^{(2)} r) \tag{33}$$

$$\varphi_{II,h}^{(2)} = B_0^{(2)} f_0^{II,(2)}(z) H_0^{(1)}(k_0^{(2)} r) + \sum_{n=1}^{\infty} B_n^{(2)} f_n^{II,(2)}(z) K_0(k_n^{(2)} r) \tag{34}$$

$A_n^{(2)}$ and $B_n^{(2)}$ mean the 2nd order unknown coefficients of the area I and II and $\beta_n^{(2)}$ and $k_n^{(2)}$ are eigenvalues. The 2nd order eigenfunction $f_I^{(2)}(z)$ and $f_{II}^{(2)}(z)$ are defined as follows.

$$f_0^{I,(2)}(z) = \frac{\cosh \beta_0^{(2)} z}{\cosh \beta_0^{(2)} h_1}, \qquad f_n^{I,(2)}(z) = \frac{\cos \beta_n^{(2)} z}{\cos \beta_n^{(2)} h_1} \tag{35}$$

$$f_0^{II,(2)}(z) = \frac{\cosh k_0^{(2)} z}{\cosh k_0^{(2)} h_2}, \qquad f_n^{II,(2)}(z) = \frac{\cos k_n^{(2)} z}{\cos k_n^{(2)} h_2} \tag{36}$$

The non-homogeneous solution of the 2nd order velocity potential is not readily available because of the complicated form of the nonlinear forcing term. In this study, the non-homogeneous 2nd order velocity potential was obtained with an integral equation method by using Green's function. The Green's function as a form of being well-known that was used for calculation is as follows (e.g. Mei 1983). x and ζ represent field point (r,θ,z) and source point $(\rho,\vartheta,\varsigma)$, respectively.

$$G(x,\zeta) = \sum_{m=0}^{\infty} \varepsilon_m G_m(r,z;\rho,\varsigma) \cos m(\theta - \vartheta) \tag{37}$$

with

$$\begin{aligned} G_m(\mathrm{r},\mathrm{z};\rho,\varsigma) = &-\frac{i}{2} C_0 \begin{pmatrix} H_m^{(1)}(k_0^{(2)} r) J_m(k_0^{(2)} \rho) \\ J_m(k_0^{(2)} r) H_m^{(1)}(k_0^{(2)} \rho) \end{pmatrix} f_0^{(2)}(z) f_0^{(2)}(\varsigma) \\ &-\frac{1}{\pi} \sum_{n=1}^{\infty} C_n \begin{pmatrix} K_m(k_n^{(2)} r) I_m(k_n^{(2)} \rho) \\ I_m(k_n^{(2)} r) K_m(k_n^{(2)} \rho) \end{pmatrix} f_n^{(2)}(z) f_n^{(2)}(\varsigma) \qquad \begin{pmatrix} r > \rho \\ r < \rho \end{pmatrix} \end{aligned} \tag{38}$$

where C_0 and C_n are defined by

$$C_0 = \left(2\int_0^h \left(f_0^{(2)}(z) \right)^2 dz \right)^{-1}, \quad C_n = \left(2\int_0^h \left(f_n^{(2)}(z) \right)^2 dz \right)^{-1} \tag{39}$$

An integral equation can be made by using the Green's function of each area. The integral equation of the area I is shown in Eq. (41). In this equation, S_R refers to the surface of $r = a$ surrounding the area I.

$$\begin{pmatrix} \varphi_{I,p}^{(2)} \\ 0 \end{pmatrix} = -\iint_{S_F} G_I(\mathrm{x},\zeta) Q_I(\rho) dS + \iint_{S_R} \varphi_{I,p}^{(2)} \frac{\partial G_I(\mathrm{x},\zeta)}{\partial \rho} dS, \quad \begin{pmatrix} r<a \\ r>a \end{pmatrix} \tag{40}$$

Next, in order to obtain non-homogeneous velocity potential on the circular cylinder, the field point is placed to $r = a + \varepsilon\,(\mathrm{a} >> \varepsilon > 0)$ making the left hand side of the Eq. (40) 0. The non-homogeneous solution at $r = a + \varepsilon\,(\mathrm{a} >> \varepsilon > 0)$ in the form of the eigenfunction expansions is the same as the Eq. (41). Coefficients L_0 and L_n calculated by using the Orthogonality of the eigenfunctions are the same as Eq. (42).

$$\varphi_{I,p}^{(2)}(\mathrm{a,z}) = f_0^{(2)}(z) L_0 + \sum_{n=1}^{\infty} f_n^{(2)}(z) L_n \tag{41}$$

$$L_0 = \frac{2C_0^I \int_0^a J_0(\beta_0^{(2)} r) Q_I(r) r\, dr}{\beta_0^{(2)} a J_0'(\beta_0^{(2)} a)}, \qquad L_n = \frac{2C_n^I \int_0^a I_0(\beta_n^{(2)} r) Q_I(r) r\, dr}{\beta_n^{(2)} a I_0'(\beta_n^{(2)} a)} \tag{42}$$

If the velocity potential of any field point within the area I is calculated by applying the calculated potential on the circular cylinder to the integral equation, non-homogeneous solution in the complicated form below can be obtained.

$$\begin{aligned}
\varphi_{I,p}^{(2)}(\mathrm{r,z}) = {} & \pi i C_0^I f_0^{I,(2)}(\mathrm{z}) \left[H_0^{(1)}(\beta_0^{(2)} r) - Z_0^I J_0(\beta_0^{(2)} r) \right] \int_0^r J_0(\beta_0^{(2)} \rho) \mathrm{Q}_I(\rho) \rho d\rho \\
& + 2\sum_{n=1}^{\infty} C_n^I f_n^{I,(2)}(\mathrm{z}) \left[K_0(\beta_n^{(2)} r) - Z_n^I I_0(\beta_n^{(2)} r) \right] \int_0^r I_0(\beta_n^{(2)} \rho) \mathrm{Q}_I(\rho) \rho d\rho \\
& + \pi i C_0^I f_0^{I,(2)}(\mathrm{z}) J_0(\beta_0^{(2)} r) \int_r^a \left[H_0^{(1)}(\beta_0^{(2)} \rho) - Z_0^I J_0(\beta_0^{(2)} \rho) \right] \mathrm{Q}_I(\rho) \rho d\rho \\
& + 2\sum_{n=1}^{\infty} C_n^I f_n^{I,(2)}(\mathrm{z}) I_0(\beta_n^{(2)} r) \int_r^a \left[K_0(\beta_n^{(2)} r) - Z_n^I I_0(\beta_n^{(2)} r) \right] \mathrm{Q}_I(\rho) \rho d\rho
\end{aligned} \tag{43}$$

with

$$Z_0^I = \frac{H_0'^{(1)}(\beta_0 a)}{J_0'(\beta_0 a)}, \qquad Z_n^I = \frac{K_0'(\beta_n a)}{I_0'(\beta_n a)} \tag{44}$$

In the same way, the non-homogeneous velocity potential of the area II can be calculated. Malenica & Molin (1995) have proved that the integral over the infinite surface (S_∞) in the integral equation of the area II approached to 0. The calculated non-homogeneous velocity potential $\varphi_{II,P}^{(2)}$ is as follows.

$$\begin{aligned}
\varphi_{II,P}^{(2)}(\mathrm{r,z}) = {} & \pi i C_0^{II} f_0^{II,(2)}(\mathrm{z}) H_0^{(1)}(k_0^{(2)} r) \int_0^r \left[J_0(k_0^{(2)} \rho) - Z_0^{II} H_0^{(1)}(k_0^{(2)} \rho) \right] \mathrm{Q}_{II}(\rho) \rho d\rho \\
& + 2\sum_{n=1}^{\infty} C_n^{II} f_n^{II,(2)}(\mathrm{z}) K_0(k_n^{(2)} r) \int_0^r \left[I_0(k_n^{(2)} \rho) - Z_n^{II} K_0(k_n^{(2)} \rho) \right] \mathrm{Q}_{II}(\rho) \rho d\rho \\
& + \pi i C_0^{II} f_0^{II,(2)}(\mathrm{z}) \left[J_0(k_0^{(2)} r) - Z_0^{II} H_0^{(1)}(k_0^{(2)} r) \right] \int_r^a H_0^{(1)}(k_0^{(2)} \rho) \mathrm{Q}_{II}(\rho) \rho d\rho \\
& + 2\sum_{n=1}^{\infty} C_n^{II} f_n^{II,(2)}(\mathrm{z}) \left[I_0(k_0^{(2)} r) - Z_n^{II} K_0(k_0^{(2)} r) \right] \int_r^a K_0(k_n^{(2)} \rho) \mathrm{Q}_I \mathrm{I}(\rho) \rho d\rho
\end{aligned} \tag{45}$$

with

$$Z_0^{II} = \frac{J_0'(\beta_0 a)}{H_0'^{(1)}(\beta_0 a)}, \qquad Z_n^{II} = \frac{I_0'(\beta_n a)}{K_0'(\beta_n a)} \tag{46}$$

The final form of the 2nd order potential is presented as the sum of the homogeneous solution and non-homogeneous solution with Eq. (47). The unknown coefficient $A_n^{(2)}$ and $B_n^{(2)}$ were obtained by the matching conditions where the non-homogeneous velocity potential had assumed as locked wave. The integrals in the non-homogeneous potential were numerically calculated.

$$\varphi_j^{(2)} = \varphi_{j,h}^{(2)} + \varphi_{j,p}^{(2)}, \qquad j = I \text{ or } II \tag{47}$$

3. Hydrodynamic Forces

The 1st order and 2nd order hydrodynamic forces due to the heave motion of the cylinder were calculated. The area subjected to the hydrodynamic force was the upper side of the circular cylinder.

$$\bar{F}_1 = \mathrm{Re}\left\{F_1 e^{-i\omega t}\right\}, \qquad \bar{F}_2 = \mathrm{Re}\left\{F_2 e^{-2i\omega t}\right\} \tag{48}$$

where F_1 and F_2 are defined by

$$F_1 = \rho\omega^2 \xi \int_S \varphi_I^{(1)} \vec{n} dS \tag{49}$$

$$F_2 = \int_S \left(2i\omega\rho\varphi_I^{(2)} + \frac{1}{4}\omega^2\rho\xi^2 \nabla\varphi_I^{(1)} \cdot \nabla\varphi_I^{(1)} \right) \vec{n} dS + \frac{1}{4}\rho g \int_C \left(\eta_I^{(1)} \cdot \eta_I^{(1)} \right) \vec{n} dC \tag{50}$$

In the above equations, S is mean wetted surface of the body and C is mean water line.

3.1 Graphs

The graph of the heave frequency ω and absolute value of hydrodynamic force F_1 and F_2 was drawn by varying the ratio of h_1 to h_2. F_1, F_2 and ω were properly non-dimensionalized and k_2 means the 2nd order real waver number of the area II. The motion amplitude of a circular cylinder was defined as $\xi = h_1 / 10$ in the case of $h_1 / h_2 = 0.05$ and a / h_2 is fixed to 0.1.

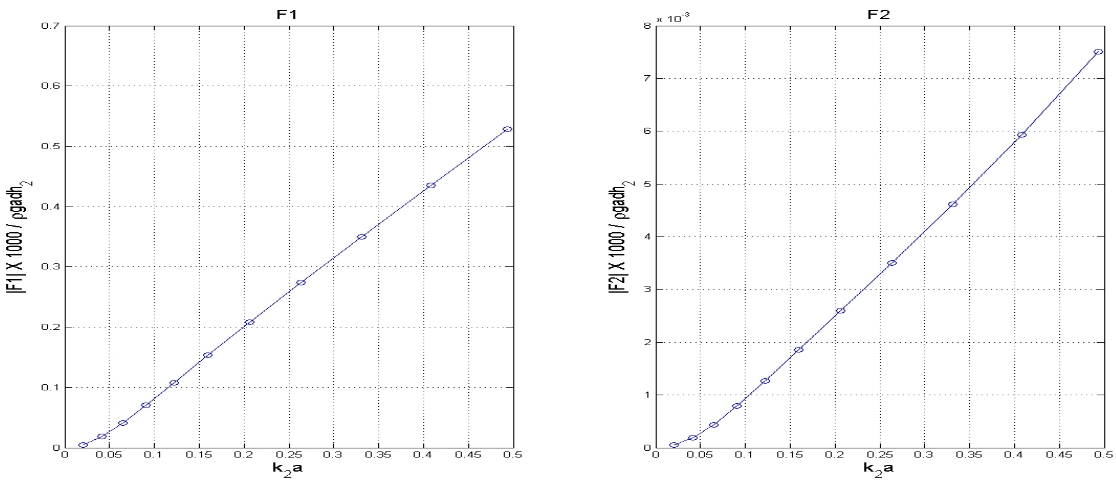

Fig. 3 Results of the 1st and 2nd order hydrodynamic force ($h_1 / h_2 = 0.9$)

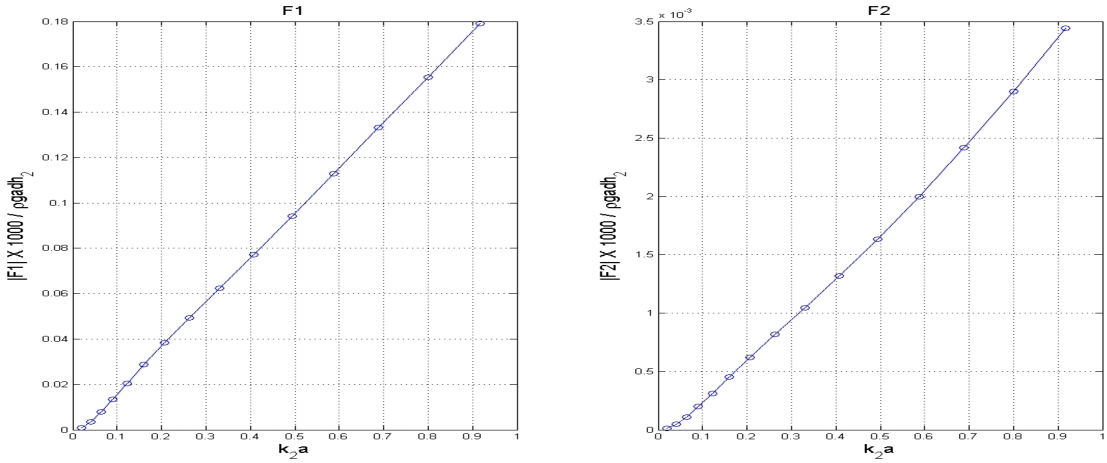

Fig. 4 Results of the 1st and 2nd order hydrodynamic force ($h_1 / h_2 = 0.5$)

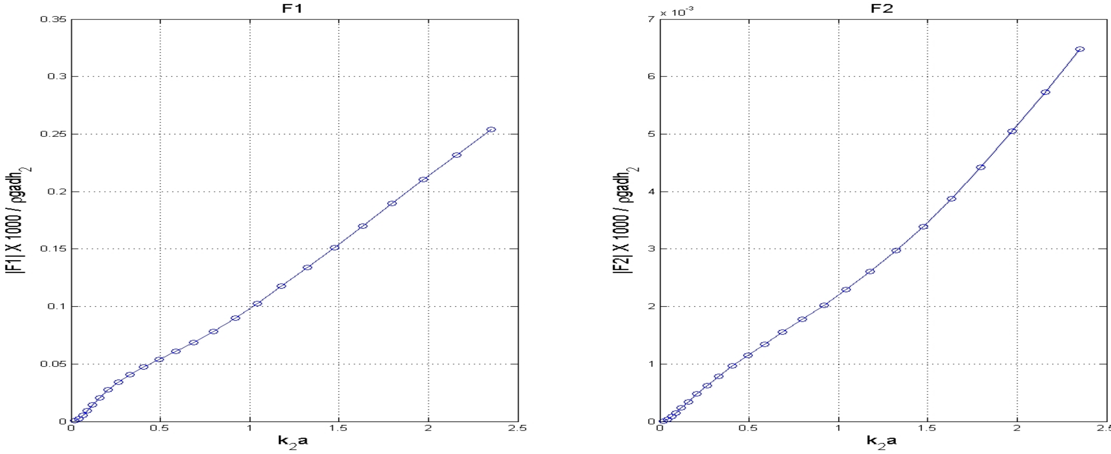

Fig. 5 Results of the 1st and 2nd order hydrodynamic force ($h_1 / h_2 = 0.2$)

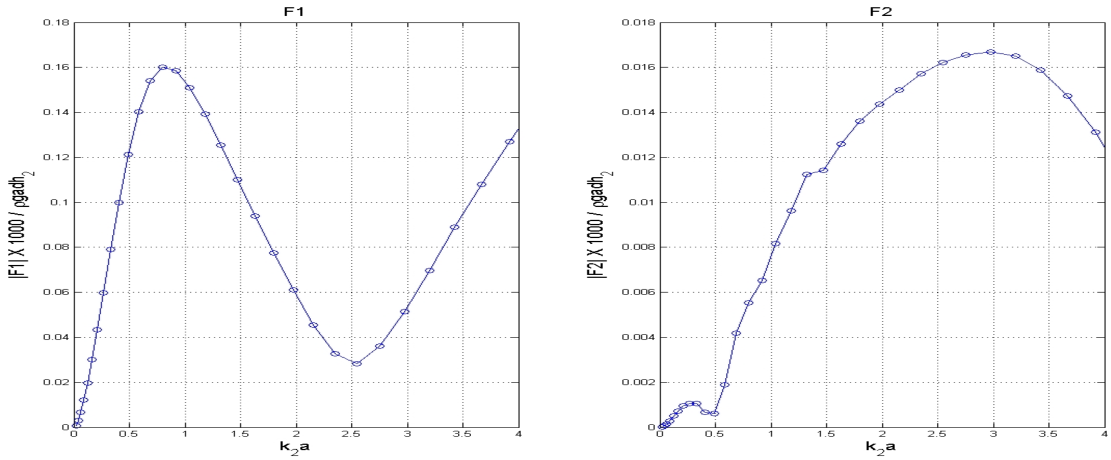

Fig. 6 Results of the 1st and 2nd order hydrodynamic force ($h_1 / h_2 = 0.05$)

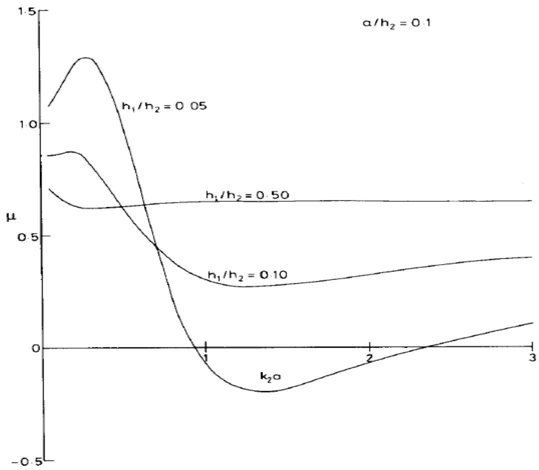

Fig. 7 Added mass μ as a function of the heaving frequency in Mciver & Evans (1983).

4. Conclusions

In this study, when a circular cylinder with the top of it to be cut does the heave motion, the radiation force acting on the upper surface of it was calculated till the 2nd order and the 1st order and 2nd order forces were compared with each other. The semi-analytic method using matching conditions at the boundary surface of different areas was utilized. The non-homogenous 2nd order velocity potential was derived by the use of the integral equation technique with Green's function. The summary of the research results is shown below.

✓ The 2nd order radiation force was calculated in consideration of the non-linear free surface boundary condition with the forcing term caused by the 1st order body motion. When h_1 / h_2 was relatively large, the 2nd order radiation force was negligible compared to the 1st order. When the upper surface of a circular cylinder was getting closer to the free surface, however, the ratio of the 2nd order radiation force to the 1st order radiation force was big enough not be ignored. This study indicated that the 2nd order force in a certain frequency accounted for about 60% of the 1st order force in the case of $h_1 / h_2 = 0.05$. This is because the 1st order force no longer increased linearly as ω increased when $h_1 / h_2 = 0.05$, but became oscillatory with the trough of the radiation force graph.

✓ The oscillation of the F_1 at small h_1 / h_2 results from the 1st order negative added mass. In the frequency rage where added mass is negative in Fig. 7, the 1st order radiation force decreases while the 2nd order force increases. Therefore, the importance of the 2nd order radiation force becomes relatively larger when the top of the cylinder is close to the free surface.

✓ Because of the non-linearity of the total radiation force ($F = F_1 + F_2$), the radiation force applied when an object moved up and down becomes asymmetry in the heave motion of the object.

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