JOURNAL OF THE
CHUNGCHEONG MATHEMATICAL SOCIETY
Volume **29**, No. 1, 2016
http://dx.doi.org/10.14403/jcms.2016.29.1.13

# DOUBLE $(r,s)(u,v)$-PREOPEN SETS

Eun Pyo Lee* and Seung On Lee**

Abstract. We introduce the concepts of double $(r,s)(u,v)$-preopen sets, double $(r,s)(u,v)$-preclosed sets and double pairwise $(r,s)(u,v)$-precontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

## 1. Introduction

Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and M. Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of double $(r,s)(u,v)$-preopen sets, double $(r,s)(u,v)$-preclosed sets and double pairwise $(r,s)(u,v)$-precontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

---

Received August 10, 2015; Accepted January 15, 2016.

2010 Mathematics Subject Classification: Primary 54A40, 03E72.

Key words and phrases: double $(r,s)(u,v)$-preopen sets, double $(r,s)(u,v)$-pre closed sets, double pairwise $(r,s)(u,v)$-precontinuous.

Correspondence should be addressed to Seung On Lee, solee@chungbuk.ac.kr.

This work was supported by the research grant of Chungbuk National University in 2014.

## 2. Preliminaries

Let $I$ be the unit interval $[0,1]$ of the real line. A member $\mu$ of $I^X$ is called a fuzzy set of $X$. For any $\mu \in I^X$, $\mu^c$ denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on $X$ with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let $X$ be a nonempty set. An *intuitionistic fuzzy set* $A$ is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set $\mu$ on $X$ is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

DEFINITION 2.1. [1] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on $X$. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let $f$ be a mapping from a set $X$ to a set $Y$. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of $X$ and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of $Y$. Then:

(1) The image of $A$ under $f$, denoted by $f(A)$, is an intuitionistic fuzzy set in $Y$ defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of $B$ under $f$, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in $X$ defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

An *intuitionistic fuzzy topology* on $X$ is a family $T$ of intuitionistic fuzzy sets in $X$ which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all $i$, then $\bigcup A_i \in T$.

The pair $(X, T)$ is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of $X$ and let $I \otimes I$ be the set of the pair $(r, s)$ such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. [12] Let $X$ be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ on $X$ is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I(\mathcal{T}^{\mu}, \mathcal{T}^{\gamma} : I(X) \to I)$ which satisfies the following properties:

(1) $\mathcal{T}^{\mu}(0_{\sim}) = \mathcal{T}^{\mu}(1_{\sim}) = 1$ and $\mathcal{T}^{\gamma}(0_{\sim}) = \mathcal{T}^{\gamma}(1_{\sim}) = 0$.
(2) $\mathcal{T}^{\mu}(A \cap B) \geq \mathcal{T}^{\mu}(A) \wedge \mathcal{T}^{\mu}(B)$ and $\mathcal{T}^{\gamma}(A \cap B) \leq \mathcal{T}^{\gamma}(A) \vee \mathcal{T}^{\gamma}(B)$.
(3) $\mathcal{T}^{\mu}(\bigcup A_i) \geq \bigwedge \mathcal{T}^{\mu}(A_i)$ and $\mathcal{T}^{\gamma}(\bigcup A_i) \leq \bigvee \mathcal{T}^{\gamma}(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma}) = (X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense.* Also, we call $\mathcal{T}^{\mu}(A)$ a *gradation of openness* of $A$ and $\mathcal{T}^{\gamma}(A)$ a *gradation of nonopenness* of $A$.

Let $A$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ and $(r, s) \in I \otimes I$. Then $A$ is said to be

(1) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* $(r, s)$*-open* set if $\mathcal{T}^{\mu}(A) \geq r$ and $\mathcal{T}^{\gamma}(A) \leq s$,
(2) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* $(r, s)$*-closed* set if $\mathcal{T}^{\mu}(A^c) \geq r$ and $\mathcal{T}^{\gamma}(A^c) \leq s$.

Let $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$*-fuzzy* $(r, s)$*-closure* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$*-fuzzy* $(r, s)$*-interior* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-open}\}.$$

Lemma 2.3. [9] For an intuitionistic fuzzy set $A$ in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ and $(r, s) \in I \otimes I$, we have:

(1) $\mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s)^c = \mathcal{T}^{\mu\gamma}\text{-int}(A^c, r, s)$.
(2) $\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s)^c = \mathcal{T}^{\mu\gamma}\text{-cl}(A^c, r, s)$.

A system $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ consisting of a set $X$ with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on $X$ is called a *double bitopological space.*

Definition 2.4. [9] Let $A$ be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then $A$ is said to be

(1) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy $(r,s)$-open set $B$ in $X$ such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B,u,v)$,
(2) $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen if there is an $\mathcal{U}^{\mu\gamma}$-fuzzy $(u,v)$-open set $B$ in $X$ such that $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(B,r,s)$,
(3) $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy $(r,s)$-closed set $B$ in $X$ such that $\mathcal{U}^{\mu\gamma}\text{-int}(B,u,v) \subseteq A \subseteq B$,
(4) $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed if there is an $\mathcal{U}^{\mu\gamma}$-fuzzy $(u,v)$-closed set $B$ in $X$ such that $\mathcal{T}^{\mu\gamma}\text{-int}(B,r,s) \subseteq A \subseteq B$.

## 3. Double $(r,s)(u,v)$-preopen sets

Definition 3.1. Let $A$ be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then $A$ is said to be

(1) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preopen* set if
$A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v),r,s)$,
(2) an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preopen* set if
$A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s),u,v)$,
(3) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* $(r,s)(u,v)$-*preclosed* set if
$A \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}((\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v),r,s)$,
(4) an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double* $(u,v)(r,s)$-*preclosed* set if
$A \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}((\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s),u,v)$.

Theorem 3.2. Let $A$ be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent:

(1) $A$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.
(2) $A^c$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set.

*Proof.* It follows from Lemma 2.3. □

Corollary 3.3. Let $A$ be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent:

(1) $A$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.
(2) $A^c$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set.

Theorem 3.4. Let $A$ be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$.

(1) If $A$ is $\mathcal{T}^{\mu\gamma}$-fuzzy $(r,s)$-open of $(X,\mathcal{T}^{\mu\gamma})$, then $A$ is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

(2) If $A$ is $\mathcal{T}^{\mu\gamma}$-fuzzy $(r,s)$-closed of $(X,\mathcal{T}^{\mu\gamma})$, then $A$ is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

*Proof.* (1) Let $A$ be a $\mathcal{T}^{\mu\gamma}$-fuzzy $(r,s)$-open set of $(X,\mathcal{T}^{\mu\gamma})$. Then $A=\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)$. Clearly, we have $A\subseteq\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v)$ and hence

$$A=\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)\subseteq\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A,u,v),r,s).$$

Thus $A$ is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

(2) Let $A$ be a $\mathcal{T}^{\mu\gamma}$-fuzzy $(r,s)$-closed set of $(X,\mathcal{T}^{\mu\gamma})$. Then $A=\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s)$. Clearly, we have $A\supseteq\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v)$ and hence

$$A=\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s)\supseteq\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A,u,v),r,s).$$

Thus $A$ is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$. □

COROLLARY 3.5. Let $A$ be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$.

(1) If $A$ is $\mathcal{U}^{\mu\gamma}$-fuzzy $(u,v)$-open of $(X,\mathcal{U}^{\mu\gamma})$, then $A$ is $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.
(2) If $A$ is $\mathcal{U}^{\mu\gamma}$-fuzzy $(u,v)$-closed of $(X,\mathcal{U}^{\mu\gamma})$, then $A$ is $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed of $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$.

But the converses in the above theorem and corollary need not be true which is shown by the following example.

EXAMPLE 3.6. Let $X=\{x,y\}$ and let $A_1,A_2,A_3$ and $A_4$ be intuitionistic fuzzy sets of $X$ defined as

$$A_1(x)=(0.2,0.4),\quad A_1(y)=(0.6,0.3);$$
$$A_2(x)=(0.4,0.3),\quad A_2(y)=(0.7,0.1);$$
$$A_3(x)=(0.5,0.2),\quad A_3(y)=(0.1,0.8);$$

and

$$A_4(x)=(0.5,0.3),\quad A_4(y)=(0.2,0.4).$$

Define $\mathcal{T}^{\mu\gamma}:I(X)\to I\otimes I$ and $\mathcal{U}^{\mu\gamma}:I(X)\to I\otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A)=(\mathcal{T}^{\mu}(A),\mathcal{T}^{\gamma}(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{5}) & \text{if } A=A_1,\\ (0,1) & \text{otherwise;}\end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A)=(\mathcal{U}^{\mu}(A),\mathcal{U}^{\gamma}(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{3},\frac{1}{4}) & \text{if } A=A_2,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on $X$. The intuitionistic fuzzy set $A_3$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preopen set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-open set and $A_3^c$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preclosed set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-closed set. Also $A_4$ is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-preopen set which is not an $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3}, \frac{1}{4})$-open set and $A_4^c$ is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-preclosed set which is not an $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3}, \frac{1}{4})$-closed set.

Lemma 3.7. That $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-semiopen ($(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(u, v)(r, s)$-semiopen) and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-preopen ($(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(u, v)(r, s)$-preopen) are independent notions is shown by the following example.

Example 3.8. Let $X = \{x, y\}$ and let $A_1, A_2, A_3, A_4, A_5$ and $A_6$ be intuitionistic fuzzy sets of $X$ defined as

$$A_1(x) = (0.2, 0.7), \quad A_1(y) = (0.5, 0.3);$$
$$A_2(x) = (0.5, 0.4), \quad A_2(y) = (0.2, 0.6);$$
$$A_3(x) = (0.5, 0.3), \quad A_3(y) = (0.4, 0.2);$$
$$A_4(x) = (0.3, 0.6), \quad A_4(y) = (0.5, 0.2);$$
$$A_5(x) = (0.8, 0.1), \quad A_5(y) = (0.1, 0.7);$$

and

$$A_6(x) = (0.6, 0.2), \quad A_6(y) = (0.2, 0.5).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^{\mu}(A), \mathcal{U}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on $X$. The intuitionistic fuzzy set $A_3$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preopen set which is not a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiopen set and $A_4$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiopen set which is not a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-preopen set. Also $A_5$ is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})$ $(\frac{1}{2}, \frac{1}{5})$-preopen set which is not an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-semiopen set and $A_6$ is an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-semiopen set which is not an $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-fuzzy $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$-preopen set.

THEOREM 3.9. Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v)\in I\otimes I$.

(1) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen sets of $X$, then $\bigcup A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.
(2) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen sets of $X$, then $\bigcup A_k$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.
(3) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed sets of $X$, then $\bigcap A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set.
(4) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed sets of $X$, then $\bigcap A_k$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set.

*Proof.* (1) Let $\{A_k\}$ be a collection of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen sets. Then for each $k$, $A_k\subseteq\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A_k,u,v),r,s)$. So

$$\begin{aligned}\bigcup A_k &\subseteq \bigcup\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A_k,u,v),r,s)\\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\bigcup A_k,u,v),r,s).\end{aligned}$$

Thus $\bigcup A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set.

(2) Let $\{A_k\}$ be a collection of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen sets. Then for each $k$, $A_k\subseteq\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A_k,r,s),u,v)$. So

$$\begin{aligned}\bigcup A_k &\subseteq \bigcup\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A_k,r,s),u,v)\\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(\bigcup A_k,r,s),u,v).\end{aligned}$$

Thus $\bigcup A_k$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set.

(3) It follows from (1) using Theorem 3.2.

(4) It follows from (2) using Corollary 3.3 □

DEFINITION 3.10. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space $X$ to a double bitopological space $Y$ and $(r,s),(u,v)\in I\otimes I$. Then $f$ is called *double pairwise* $(r,s)(u,v)$-*precontinuous* if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set of $X$ for each $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-open set $A$ of $Y$ and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of $X$ for each $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-open set $B$ of $Y$.

THEOREM 3.11. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s),(u,v)\in I\otimes I$. Then the following statements are equivalent:

(1) $f$ is a double pairwise $(r,s)(u,v)$-precontinuous mapping.
(2) $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set of $X$ for each $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-closed set $A$ of $Y$ and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of $X$ for each $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-closed set $B$ of $Y$.

(3) For each intuitionistic fuzzy set $A$ of $Y$,

$$f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)) \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s)$$

and

$$f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)) \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(A),r,s),u,v).$$

(4) For each intuitionistic fuzzy set $C$ of $X$,

$$\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s) \supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v),r,s)$$

and

$$\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v) \supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(C,r,s),u,v).$$

*Proof.* (1) $\Rightarrow$ (2) Let $A$ be any $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-closed set and $B$ any $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-closed set of $Y$. Then $A^c$ is a $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-open set and $B^c$ is a $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-open set of $Y$. Since $f$ is double pairwise $(r,s)(u,v)$-precontinuous, $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set and $f^{-1}(B^c)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of $X$. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of $X$.

(2) $\Rightarrow$ (1) Let $A$ be any $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-open set and $B$ any $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-open set of $Y$. Then $A^c$ is a $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-closed set and $B^c$ is a $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-closed set of $Y$. By (2), $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(B^c)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of $X$. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preopen set and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preopen set of $X$. Thus $f$ is a double pairwise $(r,s)(u,v)$-precontinuous mapping.

(2) $\Rightarrow$ (3) Let $A$ be any intuitionistic fuzzy set of $Y$. Then $\mathcal{V}^{\mu\gamma}$-cl$(A,r,s)$ is a $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-closed set and $\mathcal{W}^{\mu\gamma}$-cl$(A,u,v)$ is a $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-closed set of $Y$. By (2), $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v))$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of $X$. Thus

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)) &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)),u,v),r,s) \\ &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)) &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)),r,s),u,v) \\ &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(A),r,s),u,v). \end{aligned}$$

$(3) \Rightarrow (4)$ Let $C$ be any intuitionistic fuzzy set of $X$. Then $f(C)$ is an intuitionistic fuzzy set of $Y$. By (3),

$$\begin{aligned} f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s)) &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}f(C),u,v),r,s) \\ &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v),r,s) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v)) &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}f(C),r,s),u,v) \\ &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(C,r,s),u,v). \end{aligned}$$

Hence

$$\begin{aligned} \mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s) &\supseteq ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s)) \\ &\supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v),r,s)) \end{aligned}$$

and

$$\begin{aligned} \mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v) &\supseteq ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v)) \\ &\supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(C,r,s),u,v)). \end{aligned}$$

$(4) \Rightarrow (2)$ Let $A$ be any $\mathcal{V}^{\mu\gamma}$-fuzzy $(r,s)$-closed set and $B$ any $\mathcal{W}^{\mu\gamma}$-fuzzy $(u,v)$-closed set of $Y$. Then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy sets of $X$. By (4),

$$\begin{aligned} A = \mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s) &\supseteq \mathcal{V}^{\mu\gamma}\text{-cl}(ff^{-1}(A),r,s) \\ &\supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s)) \end{aligned}$$

and

$$\begin{aligned} B = \mathcal{W}^{\mu\gamma}\text{-cl}(B,u,v) &\supseteq \mathcal{W}^{\mu\gamma}\text{-cl}(ff^{-1}(B),u,v) \\ &\supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(B),r,s),u,v)). \end{aligned}$$

So

$$\begin{aligned} f^{-1}(A) &\supseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s)) \\ &\supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(A),u,v),r,s) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(B) &\supseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(B),r,s),u,v)) \\ &\supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(B),r,s),u,v). \end{aligned}$$

Therefore $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-preclosed set and $f^{-1}(B)$ is an $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-preclosed set of $X$. □

## References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87-96.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182-190.

[3] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness : Fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237-242.

[4] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81-89.

[5] D. Çoker and M. Demirci, *An introduction to intuitionistic fuzzy topological spaces in Šostak's sense*, BUSEFAL **67** (1996), 67-76.

[6] D. Çoker and A. Haydar Eş, *On fuzzy compactness in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. **3** (1995), 899-909.

[7] H. Gürçay, D. Çoker, and A. Haydar Eş, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. **5** (1997), 365-378.

[8] A. Kandil, *Biproximities and fuzzy bitopological spaces*, Simon Stevin **63** (1989), 45-66.

[9] E. P. Lee and S. O. Lee, *Double semiopen sets on double bitopological spaces*, Journal of The Chungcheong Mathematical Society **26** (2013), 691-702.

[10] E. P. Lee and S. O. Lee, *Double pairwise* $(r, s)(u, v)$*-semicontinuous mappings*, Journal of The Chungcheong Mathematical Society **27** (2014), 603-614.

[11] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371-375.

[12] A. P. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Janos Palermo, Sr. II **11** (1985), 89-103.

*

Department of Clinical Laboratory Science
Seonam University
Namwon 590-711, Republic of Korea
*E-mail*: eplee@seonam.ac.kr

**

Department of Mathematics
Chungbuk National University
Cheongju 361-763, Republic of Korea
*E-mail*: solee@chungbuk.ac.kr