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DYNAMIC CHARACTERISTICS OF A ROTATING TIMOSHENKO BEAM SUBJECTED TO A VARIABLE MAGNITUDE LOAD TRAVELLING AT VARYING SPEED

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ABSTRACT. In this study, the dynamic behaviour of a rotating Timoshenko beam when under the actions of a variable magnitude load moving at non-uniform speed is carried out. The effect of cross-sectional dimension and damping on the flexural motions of the elastic beam was neglected. The coupled second order partial differential equations incorporating the effects of rotary and gyroscopic moment describing the motions of the beam was scrutinized in order to obtain the expression for the dynamic deflection and rotation of the vibrating system using an elegant technique called Galerkin's Method. Analyses of the solutions obtained were carried out and various results were displayed in plotted curve. It was found that the response amplitude of the simply supported beam increases with an increase in the value of the foundation reaction modulus. Effects of other vital structural parameters were also established.

1. INTRODUCTION

Studies concerning vibrating bodies resting on an elastic foundation carrying moving loads are of considerable practical importance and have been a subject of numerous scientific investigations by different authors in past few years [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. In most of the studies available in literature, such as the work of Sadiku and Leipolz [21], Oni and Awodola [22], the scope of the problem of assessing the dynamic response of a structural member under the passage of moving load has been limited to that of thin beam or thick beam. Kolousek et al. [23] studied works on uniform thin beam. In their analysis the adopted a normal modes method, effects of damping and foundation were not

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included in their analysis. Kenny [24] worked on the problem of infinite elastic beam resting on elastic foundation and under the influence of a dynamic load moving with constant speed. Unlike Bernoulli-Euler beam or Rayleigh beam, forced vibrations of deep beam have received little attention for the past few years. Among few authors who have published scholarly article on the dynamic characteristics of a Timoshenko beam subjected moving loads are Oni [25] who considered the problem of a harmonic time-dependent concentrated force moving at constant velocity. The method of integral transform method was adopted. Series solution which converges is obtained for the deflection of simply supported beam. Travelling force on a Timoshenko beam has also been studied by Florence [26], Huang [27] studied the effect of rotary inertia and of shear deformation on the frequency and normal mode equations of uniforms with simple end conditions. Deterministic and random vibration of an axially loaded Timoshenko beam resting on an elastic foundation has been considered by Chang [28]. Vibration and reliability of a rotating beam with random properties under random excitation was presented by Hosseini and Khadem [29]. More recently, Omolofe et al. [30] studied the transverse motion of non-prismatic deep beam under the actions of variable magnitude moving loads.

In all these aforementioned studies, investigations are limited to the cases where the velocity of the travelling masses is held constant throughout its motion on the structure. However, situation arises when a travelling mass may accelerate by a forward force or decelerate, reduce speed and come to rest at any desired position on the beam thereby causing the friction between the mass and the beam to increase considerably. Wang [31] studied the dynamical analysis of a finite inextensible beam with an attached accelerating mass. He employed Galerkin procedure in conjunction with the method of numerical integration to tackle the partial differential equation which describes the transient vibration of the beam mass system. He concluded that the applied forward force amplifies the speed of the mass and the displacement of the beam. To the authors best of knowledge, studies concerning structural members where the effects of the rotary inertia correction factor and shear deformation are incorporated into the governing equation of motions are not common in literature and where they rarely exist the traversing load is assumed to travel with constant speed.

This present study therefore concerns the dynamic characteristics of a uniform beam resting on elastic foundation of the Winkler type and incorporating the effects of rotary inertia correction factor and shear deformation into the governing equation of motion. It is assumed that the speed at which the travelling load traverses the structural elements is time varying.

2. MATHEMATICAL FORMULATION

Consider a deep elastic beam having length L and resting on an elastic foundation with subgrade reaction modulus K which is directly proportional to beam deflection. If the system does not experience friction as the beam maintain contact with the subgtrade, the deflection $\phi(x,t)$ and rotation $\psi(x,t)$ is aptly described by the system of partial differential equations

$$\overline{m}\frac{\partial^2\phi\left(x,t\right)}{\partial t^2}-K^*GF\left[\frac{\partial^2\phi\left(x,t\right)}{\partial x^2}-\frac{\partial\psi\left(x,t\right)}{\partial x}\right]+K\phi\left(x,t\right)=P\left(x,t\right) \tag{2.1}$$

$$EJ\frac{\partial^2\psi(x,t)}{\partial x^2}+K^*GF\left[\frac{\partial\phi(x,t)}{\partial x}-\psi(x,t)\right]-JD\frac{\partial^2\psi(x,t)}{\partial t^2}=0 \tag{2.2}$$

Where $\overline{m}$ is the constant mass per unit length of the beam, K^* is a constant dependent on the shape of the cross section, G is the modulus of elasticity in shear, F is the cross sectional area, $P(x,t)$ is the harmonic force, E is the Young modulus of the beam, J is the constant moment of inertia of the beam cross section and D is the mass per unit volume.

Furthermore, at each of the boundary points there are two boundary conditions of the type

$$\phi(0,t)=0; \quad \psi(0,t)=0 \tag{2.3}$$

$$\frac{\partial\phi(0,t)}{\partial x}=0; \quad \frac{\partial\psi(L,t)}{\partial x}=0 \tag{2.4}$$

and the initial conditions are

$$\phi(x,0)=0=\frac{\partial\phi(x,0)}{\partial t} \tag{2.5}$$

$$\psi(x,0)=0=\frac{\partial\psi(x,0)}{\partial t} \tag{2.6}$$

The effect of gyroscopic moment $\dot{\psi}(x,t)$ is incorporated into the governing equations (2.1) and (2.2) to induce a displacement component perpendicular to the direction of the load. While $\ddot{\psi}(x,t)$ represents the effect of rotary inertia. Since the velocity of our moving force is non-uniform, the moving force $P(x,t)$ acting on the beam is chosen as

$$P(x,t)=P\cos\omega t\delta(x-f(t)) \tag{2.7}$$

where ω is the circular frequency of the harmonic load, $\delta(\cdot)$ and $f(t)$ is the distance covered by the moving load at any time t and is given by

$$f(t)=x_0+\gamma\sin\beta t \tag{2.8}$$

where x_0 is the equilibrium position of the longitudinal oscillating load, γ is the longitudinal amplitude of oscillation of the load and β is the longitudinal frequency of the load. Furthermore, for the variable elastic foundation function $K(x)$ we adopt the example in [30] and we have

$$K(x)=K_0\left(4x-3x^2+x^3\right) \tag{2.9}$$

3. SOLUTION TECHNIQUE AND PROCEDURE

To solve the beam problem stated above, we shall use an elegant solution technique called Galerkin's method. This method requires that the solutions of the deflection and the rotation of the coupled beam problems (2.1) and (2.2) be written as

$$\phi_{i_i}(x,t)=\sum_{i=1}^{n}V_i(t)U_i(x) \tag{3.1}$$

and

$$\psi_{i_i}(x,t)=\sum_{i=1}^{n}Y_i(t)X_i(x) \tag{3.2}$$

The function $U_i(x)$ and $X_i(x)$ are usually chosen to satisfy the pertinent boundary conditions. Thus, substituting equations (3.1) and (3.2) into the system of equations (2.1) and (2.2), yields

$$\overline{m}\sum_{i=1}^{n}\ddot{V}_i(t)\,U_i(x) - K^*GF\left[\sum_{i=1}^{n}V_i(t)\,U_i''(x) - \sum_{i=1}^{n}Y_i(t)\,X_i'(x)\right] + K(x)\sum_{i=1}^{n}V_i(t)\,U_i(x) = P\cos\omega t\delta\left[x-(x_0+\gamma\sin\beta t)\right] \tag{3.3}$$

and

$$EJ\sum_{i=1}^{n}Y_i(t)\,X_i''(x) + K^*GF\left[\sum_{i=1}^{n}V_i(t)\,U_i'(x) - \sum_{i=1}^{n}Y_i(t)\,X_i(x)\right] - JD\sum_{i=1}^{n}\ddot{Y}_i(t)\,X_i(x) = 0 \tag{3.4}$$

To determine the expression for $V_i(t)$ and $Y_i(t)$, the expression on the left hand side of the equations (3.3) and (3.4) are required to be orthogonal to functions $U_j(x)$ and $X_j(x)$ respectively. Thus,

$$\int_0^L\left[\sum_{i=1}^{n}\left\{\overline{m}\sum_{1=1}^{n}\ddot{V}_i(t)\,U_i(x) - K^*GF\left[V_i(t)\,U_i''(x) - Y_i(t)\,X_i'(x)\right] + K(x)\,V_i(t)\,U_i(x)\right\} - P\cos\omega t\delta\left[x-(x_0+\gamma\sin\beta t)\right]\right]\cdot U_j(x)\,dx = 0 \tag{3.5}$$

and

$$\int_0^L\left[\sum_{i=1}^{n}EJY_i(t)\,X_i''(x) + K^*GF\left[V_i(t)\,U_i'(x) - Y_i(t)\,X_i(x)\right] - JD\ddot{Y}_i(t)\,X_i(x)\right]\cdot X_j(x)\,dx = 0 \tag{3.6}$$

Equations (3.5) and (3.6) after some rearrangements yields,

$$\sum_{i=1}^{n}\left\{P_1(i,j)\,\ddot{V}_i(t) + P_2(i,j)\,V_i(t) + P_3(i,j)\,Y_i(t)\right\} = P\cos\omega t U_j(x_0+\gamma\sin\beta t) \tag{3.7}$$

and

$$\sum_{i=1}^{n}\left(Q_1(i,j)\,\ddot{Y}_i(t) + Q_2(i,j)\,V_i(t) + Q_3(i,j)\,Y_i(t)\right) = 0 \tag{3.8}$$

where

$$\begin{aligned}
P_1(i,j) &= \overline{m}\int_0^L U_i(x)\,U_j(x)\,dx,\\
P_2(i,j) &= \int_0^L \left[-K^*GFU_i'(x)\,U_j(x) + K(x)\,U_i(x)\,U_j(x)\right]dx\\
P_3(i,j) &= \int_0^L K^*GFX_i'(x)\,U_j(x)\,dx,\\
Q_1(i,j) &= JD\int_0^L X_i(x)\,U_j(x)\,dx\\
Q_2(i,j) &= \int_0^L K^*GFU_i'(x)\,U_j(x)\,dx,\\
Q_3(i,j) &= \int \left[EJX_i''(x)\,U_j(x) - K^*GFX_i(x)\,U_j(x)\right]dx
\end{aligned} \tag{3.9}$$

Since our beam is assumed to have simple supports at both ends $x = 0$ and $L = 0$, the mode functions $U_i(x)$ and $X_i(x)$, are chosen to be $\sin\dfrac{i\pi x}{L}$ and $\cos\dfrac{i\pi x}{L}$ respectively. Thus substituting these into integrals (3.9), one obtains

$$\begin{aligned}
&P_1(i,j) = \frac{\overline{m}L}{2}, \quad P_2(i,j) = K_0L^2\left(1 - \frac{L}{2} + \frac{L^2}{8}\right) + \frac{i^2\pi^2}{L^2}K^*GF\cdot\frac{L}{2},\\
&P_3(i,j) = \frac{i\pi}{L}K^*GF\cdot\frac{L}{2}, \quad Q_1(i,j) = -JD\frac{L}{2},\\
&Q_2(i,j) = \frac{i\pi}{L}K^*GF\cdot\frac{L}{2}, \quad Q_3(i,j) = \left[\frac{EJi^2\pi^2}{L^2} + K^*GF\right]\cdot\frac{L}{2}
\end{aligned} \tag{3.10}$$

Now considering only the ith concentrated moving force, equation (3.7) and (3.8) can be simplified further to give

$$\begin{aligned}
P_1(i,j)\,\ddot{V}_i(t) + P_2(i,j)\,V_i(t) + P_3(i,j)\,Y_i(t) = {} & P_0\cos\omega t\cos(\gamma\sin\beta t)\\
& + P_1\cos\omega t\sin(\gamma\sin\beta t)
\end{aligned} \tag{3.11}$$

and

$$Q_1(i,j)\,\ddot{Y}_i(t) + Q_2(i,j)\,V_i(t) + Q_3(i,j) = 0 \tag{3.12}$$

where

$$P_0 = P\sin\frac{j\pi x_0}{L} \quad , \quad P_1 = P\cos\frac{j\pi x_0}{L}, \quad \text{and} \quad \gamma = \frac{j\pi x}{L} \tag{3.13}$$

In order to further simplify equation (3.11), use is made of the following Bessel relations.

$$\sin(\gamma\sin\beta t) = 2\sum_{i=0}^{\infty} J_{2k-1}(\gamma)\sin([2k+]\,\beta t) \tag{3.14}$$

and

$$\cos(\gamma \sin \beta t) = J_0(\gamma) + 2\sum_{k=0}^{\infty} J_{2k}(\gamma) \cos(2k\beta t) \tag{3.15}$$

where

$$J_k(\gamma) = \sum_{m=0}^{\infty} (-1)^m \left(\frac{\gamma}{m}\right)^{k+2m} \frac{1}{m!\,(k+1)!} \tag{3.16}$$

is the modified Bessel function of the first kind of order k.

In view of the equation (3.14) and (3.15), equation (3.11) becomes

$$\begin{aligned} &P_1(i,j)\,\ddot{V}_i(t) + P_2(i,j)\,V_i(t) + P_3(i,j)\,Y_i(t) \\ &\qquad = P_0 J_0(\gamma)\cos\omega t + 2P_0 \sum_{k=1}^{\infty} J_{2k}(\gamma)\cos\omega t \cos(2k\beta t) \\ &\qquad\quad + 2P_1 \sum_{k=0}^{\infty} J_{2k-1}(\gamma)\cos\omega t \sin[(2k+1)\beta t] \end{aligned} \tag{3.17}$$

and

$$Q_1(i,j)\,\ddot{Y}_i(t) + Q_2(i,j)\,V_i(t) + Q_3(i,j)\,Y_i(t) = 0 \tag{3.18}$$

which can further be simplified to take the form

$$\begin{aligned} &P_1(i,j)\,\ddot{V}_i(t) + P_2(i,j)\,V_i(t) + P_3(i,j)\,Y_i(t) \\ &\qquad = P_0 J_0(\gamma)\cos\omega t + 2P_0 \sum_{k=1}^{\infty} J_{2k}(\gamma)[\cos\eta_1 t - \cos\eta_2 t] \\ &\qquad\quad + 2P_1 \sum_{k=0}^{\infty} J_{2k-1}(\gamma)[\sin\eta_3 t - \sin\eta_4 t] \end{aligned} \tag{3.19}$$

and

$$Q_1(i,j)\,\ddot{Y}_i(t) + Q_2(i,j)\,V_i(t) + Q_3(i,j)\,Y_i(t) = 0 \tag{3.20}$$

where

$$\eta_1 = \omega + 2k\beta, \quad \eta_2 = \omega - 2k\beta, \quad \eta_3 = \omega + (2k+1)\beta, \quad \eta_4 = \omega - (2k+1)\beta \tag{3.21}$$

Subjecting the system of ordinary differential equations (3.19) and (3.20), to a Laplace transform

$$(\tilde{\cdot}) = \int_0^{\infty} e^{-st} dt \tag{3.22}$$

In conjunction with the initial conditions defined in (2.5) and (2.6), one obtains the following algebraic simultaneous equations

$$\begin{aligned} &\left[P_1(i,j)\,S^2 + P_2(i,j)\right] V_i(S) + P_3(i,j)\,Y_i(S) \\ &\qquad = P_0 J_0(\gamma)\left[\frac{S}{S^2+\omega^2}\right] + P_0 \sum_{k=0}^{\infty} J_{2k}(\gamma)\left[\frac{S}{S^2+\eta_1^2} - \frac{S}{S^2+\eta_2^2}\right] \end{aligned}$$

$$+ P_1 \sum_{k=0}^{\infty} J_{2k+1}(\gamma) \left[\frac{\eta_3}{S^2+\eta_3^2} - \frac{\eta_4}{S^2+\eta_4^2} \right] \tag{3.23}$$

and

$$\left[Q_1(i,j) S^2 + Q_3(i,j) \right] Y_i(S) + Q_2(i,j) V_i(S) = 0 \tag{3.24}$$

In order to solve the above system, the following representations are made

$$\Omega_0 = \begin{vmatrix} P_1(i,j) S^2 + P_2(i,j) & P_3(i,j) \\ Q_2(i,j) & Q_1(i,j) + Q_3(i,j) \end{vmatrix} \tag{3.25}$$

$$\Omega_1 = \begin{vmatrix} \Omega_1(1,1) & P_3(i,j) \\ 0 & \left(Q_1(i,j) S^2 + Q_3(i,j) \right) \end{vmatrix} \tag{3.26}$$

where

$$\Omega_1(1,1) = \left(P_0 J_0(\gamma) \left[\frac{S}{S^2+\omega^2} \right] + P_0 \sum_{k=0}^{\infty} J_{2k}(\gamma) \left[\frac{S}{S^2+\eta_1^2} - \frac{S}{S^2+\eta_2^2} \right] + P_1 \sum_{k=0}^{\infty} J_{2k-1}(\gamma) \left[\frac{\eta_3}{S^2+\eta_3^2} - \frac{\eta_4}{S^2+\eta_4^2} \right] \right)$$

and

$$\Omega_2 = \begin{vmatrix} \left(P_1(i,j) S^2 + P_2(i,j) \right) & \Omega_2(1,2) \\ Q_2(i,j) & 0 \end{vmatrix} \tag{3.27}$$

where

$$\Omega_2(1,2) = \left(P_0 J_0(\gamma) \left[\frac{S}{S^2+\omega^2} \right] + P_0 \sum_{k=0}^{\infty} J_{2k}(\gamma) \left[\frac{S}{S^2+\eta_1^2} - \frac{S}{S^2+\eta_2^2} \right] + P_1 \sum_{k=0}^{\infty} J_{2k-1}(\gamma) \left[\frac{\eta_3}{S^2+\eta_3^2} - \frac{\eta_4}{S^2+\eta_4^2} \right] \right)$$

thus

$$V_i(S) = \frac{\left(Q_1(i,j) S^2 + Q_3(i,j) \right) \Omega_2(1,2)}{P_1(i,j) Q_1(i,j) S^4 + \left(P_1(i,) Q_3(i,j) + P_2(i,j) Q_1(i,j) \right) S^2 - P_3(i,j) Q_2(i,j)} \tag{3.28}$$

and

$$Y_i(S) = \frac{Q_2(i,j) \Omega_1(1,1)}{P_1(i,j) Q_1(i,j) S^4 + \left(P_1(i,) Q_3(i,j) + P_2(i,j) Q_1(i,j) \right) S^2 - P_3(i,j) Q_2(i,j)} \tag{3.29}$$

Furthermore, equation (3.28) can be re-written in the form

$$V_i(S) = \frac{\left(Q_1(i,j) S^2 + Q_3(i,j) \right) \Omega_2(1,2)}{B_1 \left[\left(S^2+\phi^2 \right) \left(S^2+\varphi^2 \right) \right]} \tag{3.30}$$

and

$$Y_i(S) = \frac{Q_2(i,j)\,\Omega_1(1,1)}{B_1\left[(S^2+\phi^2)(S^2+\varphi^2)\right]} \tag{3.31}$$

where

$$\begin{aligned}
&\phi = \sqrt{\frac{B_2}{2B_1} - \left(\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}\right)^{\frac{1}{2}}}, \quad \varphi = \sqrt{\frac{B_2}{2B_1} + \left(\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}\right)^{\frac{1}{2}}} \\
&B_1 = P_1(i,j)\,Q_i(i,j), \quad B_2 = P_1(i,)\,Q_3(i,j) + P_2(i,j)\,Q_1(i,j), \\
&B_3 = -P_3(i,j)\,Q_2(i,j)
\end{aligned} \tag{3.32}$$

Using partial fractions technique, equations (3.30) and (3.31) can further be rewritten as

$$\begin{aligned}
V_i(S) =& \frac{1}{B_1}\left(\left[\frac{Q_1(i,j)\,\phi^2 - Q_3(i,j)}{(\varphi^2-\phi^2)}\cdot\frac{1}{S^2+\varphi^2}\right] - \left[\left[\frac{Q_1(i,j)\,\phi^2 - Q_3(i,j)}{(\varphi^2-\phi^2)}\cdot\frac{1}{S^2+\phi^2}\right]\right]\right) \\
&\cdot\left(P_0 J_0(\gamma)\left[\frac{S}{S^2+\omega^2}\right] + P_0\sum_{k=0}^{\infty} J_{2k}(\gamma)\left[\frac{S}{S^2+\eta_1^2} - \frac{S}{S^2+\eta_2^2}\right]\right. \\
&\left.+P_1\sum_{k=0}^{\infty} J_{2k-1}(\gamma)\left[\frac{\eta_3}{S^2+\eta_3^2} - \frac{\eta_4}{S^2+\eta_4^2}\right]\right)
\end{aligned} \tag{3.33}$$

and

$$\begin{aligned}
V_i(S) =& \frac{1}{B_1}\frac{Q_2(i,j)}{(\varphi^2-\phi^2)}\left(\frac{1}{S^2+\phi^2} - \frac{1}{S^2+\phi^2}\right)\cdot\left(P_0 J_0(\gamma)\left[\frac{S}{S^2+\omega^2}\right]\right. \\
&\left.+P_0\sum_{k=0}^{\infty} J_{2k}(\gamma)\left[\frac{S}{S^2+\eta_1^2} - \frac{S}{S^2+\eta_2^2}\right] + P_1\sum_{k=0}^{\infty} J_{2k-1}(\gamma)\left[\frac{\eta_3}{S^2+\eta_3^2} - \frac{\eta_4}{S^2+\eta_4^2}\right]\right)
\end{aligned} \tag{3.34}$$

which after some simplifications and rearrangements gives,

$$
\begin{aligned}
V_i(S) =& \frac{P_0 J_0(\gamma)}{B_1}\left(\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\omega^2}\right.\\
&\left.-\left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\omega^2}\right)\\
&+\frac{P_0}{B_1}\sum_{k=1}^{\infty}J_{2k}(\gamma)\left(\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_1^2}\right.\\
&-\left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_1^2}\\
&-\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_2^2}\\
&\left.-\left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_2^2}\right)\\
&+\frac{P_1}{B_1}\sum_{k=0}^{\infty}J_{2k+1}(\gamma)\left(\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_3^2}\right.\\
&-\left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_3^2}\\
&-\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_4^2}\\
&\left.-\left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2}-\frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_4^2}\right)
\end{aligned}
\tag{3.35}
$$

and

$$
\begin{aligned}
Y_i(S) =& \frac{P_0 J_0(\gamma)\,Q_2(i,j)}{B_1(\varphi^2-\phi^2)}\left[\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\omega^2}-\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\omega^2}\right]\\
&+\frac{P_0 Q_2(i,j)}{B_1(\varphi^2-\phi^2)}\sum_{k=1}^{\infty}J_{2k}(\gamma)\left[\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_1^2}\right.\\
&\left.-\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_1^2}-\left(\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_2^2}-\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_2^2}\right)\right]\\
&+\frac{P_1 Q_2(i,j)}{B_1(\varphi^2-\phi^2)}\sum_{k=0}^{\infty}J_{2k+1}(\gamma)\left[\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_3^2}-\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_3^2}\right.\\
&\left.-\left(\frac{1}{S^2+\phi^2}\cdot\frac{S}{S^2+\eta_4^2}-\frac{1}{S^2+\varphi^2}\cdot\frac{S}{S^2+\eta_4^2}\right)\right]
\end{aligned}
\tag{3.36}
$$

In order to obtain the Laplace inversion of equations (35) and (36), the following representations are employed

$$\begin{aligned} g_1(S) &= \frac{S}{S^2+\omega^2}, \quad g_2(S) = \frac{S}{S^2+\eta_1^2}, \quad g_3(S) = \frac{S}{S^2+\eta_2^2}, \quad g_4(S) = \frac{S}{S^2+\eta_3^2} \\ g_5(S) &= \frac{S}{S^2+\eta_4^2}, \quad f_1(S) = \frac{\varphi}{S^2+\varphi^2}, \quad f_2(S) = \frac{\phi}{S^2+\phi^2} \end{aligned} \tag{3.37}$$

so that the Laplace inversion of (35) and (36) is the convolution of $g_j(s)$ and $f_i(s)$ defined as

$$f_i(S) * g_j(S) = \int_0^t f_i(t-u)\, g_j(u)\, du \quad i = 1,2 \quad \text{and} \quad j = 1,2,3,4,5 \tag{3.38}$$

Thus the Laplace inversions of (35) and (36) are respectively given as

$$\begin{aligned} V_i(t) = {} & \frac{P_0 J_0(\gamma)}{B_1}\left\{\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_1 - \left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_2\right\} \\ & + \frac{P_0}{B_1}\sum_{k=1}^{\infty} J_{2k}(\gamma)\left\{\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_3 - \left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_4\right. \\ & \left. - \left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_5 - \left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_6\right\} \\ & + \frac{P_1}{B_1}\sum_{k=0}^{\infty} J_{2k+1}(\gamma)\left\{\left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_7 - \left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_8\right. \\ & \left. - \left[\frac{Q_1(i,j)\,\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_9 - \left[\frac{Q_1(i,j)\,\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2}\right]\cdot H_{10}\right\} \end{aligned} \tag{3.39}$$

and

$$\begin{aligned} Y_i(t) = {} & \frac{P_0 J_0(\gamma)\, Q_2(i,j)}{B_1(\varphi^2-\phi^2)}\left[H_2 - H_1\right] + \frac{P_0 Q_2(i,j)}{B_1(\varphi^2-\phi^2)}\sum_{k=1}^{\infty} J_{2k}(\gamma)\left[H_4 - H_3 - H_6 + H_5\right] \\ & + \frac{P_1 Q_2(i,j)}{B_1(\varphi^2-\phi^2)}\sum_{k=0}^{\infty} J_{2k+1}(\gamma)\left[H_8 - H_7 - H_{10} + H_9\right] \end{aligned} \tag{3.40}$$

where

$$
\begin{aligned}
H_1 &= \frac{1}{\varphi}\int_0^L \sin\varphi\,(t-u)\cos\omega u du \quad H_2 = \frac{1}{\phi}\int_0^L \sin\phi\,(t-u)\cos\omega u du \\
H_3 &= \frac{1}{\varphi}\int_0^L \sin\varphi\,(t-u)\cos\eta_1 u du \quad H_4 = \frac{1}{\phi}\int_0^L \sin\phi\,(t-u)\cos\eta_1 u du \\
H_5 &= \frac{1}{\varphi}\int_0^L \sin\varphi\,(t-u)\cos\eta_2 u du \quad H_6 = \frac{1}{\phi}\int_0^L \sin\phi\,(t-u)\cos\eta_2 u du \\
H_7 &= \frac{1}{\varphi}\int_0^L \sin\varphi\,(t-u)\cos\eta_3 u du \quad H_8 = \frac{1}{\phi}\int_0^L \sin\phi\,(t-u)\cos\eta_3 u du \\
H_9 &= \frac{1}{\varphi}\int_0^L \sin\varphi\,(t-u)\cos\eta_4 u du \quad H_{10} = \frac{1}{\phi}\int_0^L \sin\phi\,(t-u)\cos\eta_4 u du
\end{aligned}
\tag{3.41}
$$

By trigonometric identities, it can be shown that

$$
\begin{aligned}
\frac{1}{\beta}\int_0^t \sin B\,(t-u)\sin A u du =& \frac{B\sin Bt}{B^2-A^2}\left[\sin At\sin Bt - \frac{A}{B}\left(\cos At\cos Bt - 1\right)\right] \\
&+ \frac{B\cos Bt}{B^2-A^2}\left[\sin At\cos Bt - \frac{A}{B}\cos At\sin Bt\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{1}{\beta}\int_0^t \sin B\,(t-u)\cos A u du =& \frac{A\sin Bt}{A^2-B^2}\left[\cos Bt\sin At - \frac{B}{A}\sin Bt\cos At\right] \\
&+ \frac{A\cos Bt}{A^2-B^2}\left[\sin Bt\sin At - \frac{B}{A}\left(\cos Bt\cos At - 1\right)\right]
\end{aligned}
\tag{3.42}
$$

Integrals (3.41), taking into account the identities (3.42) become,

$$
\begin{aligned}
H_1 =& \frac{\omega\sin\varphi t}{\varphi\left(\omega^2-\varphi^2\right)}\left(\cos\varphi t\sin\omega t - \frac{\varphi}{\omega}\sin\varphi t\cos\omega t\right) \\
&- \frac{\omega\cos\varphi t}{\varphi\left(\omega^2-\varphi^2\right)}\left(\sin\varphi t\sin\omega t - \frac{\varphi}{\omega}\left(\cos\varphi t\cos\omega t - 1\right)\right) \\
H_2 =& \frac{\omega\sin\phi t}{\phi\left(\omega^2-\phi^2\right)}\left(\cos\phi t\sin\omega t - \frac{\phi}{\omega}\sin\phi t\cos\omega t\right) \\
&- \frac{\omega\cos\phi t}{\phi\left(\omega^2-\phi^2\right)}\left(\sin\phi t\sin\omega t - \frac{\phi}{\omega}\left(\cos\phi t\cos\omega t - 1\right)\right) \\
H_3 =& \frac{\eta_1\sin\varphi t}{\varphi\left(\eta_1^2-\varphi^2\right)}\left(\cos\varphi t\sin\eta_1 t - \frac{\varphi}{\eta_1}\sin\varphi t\cos\eta_1 t\right) \\
&- \frac{\eta_1\cos\varphi t}{\varphi\left(\eta_1^2-\varphi^2\right)}\left(\sin\varphi t\sin\eta_1 t - \frac{\varphi}{\eta_1}\left(\cos\varphi t\cos\eta_1 t - 1\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
H_4 &= \frac{\eta_1 \sin\phi t}{\phi\left(\eta_1^2-\phi^2\right)}\left(\cos\phi t\sin\eta_1 t-\frac{\phi}{\eta_1}\sin\phi t\cos\eta_1 t\right)\\
&\quad-\frac{\eta_1\cos\phi t}{\phi\left(\eta_1^2-\phi^2\right)}\left(\sin\phi t\sin\eta_1 t-\frac{\phi}{\eta_1}\left(\cos\phi t\cos\eta_1 t-1\right)\right)\\
H_5 &= \frac{\eta_2 \sin\varphi t}{\varphi\left(\eta_2^2-\varphi^2\right)}\left(\cos\varphi t\sin\eta_2 t-\frac{\varphi}{\eta_2}\sin\varphi t\cos\eta_2 t\right)\\
&\quad-\frac{\eta_2\cos\varphi t}{\varphi\left(\eta_2^2-\varphi^2\right)}\left(\sin\varphi t\sin\eta_2 t-\frac{\varphi}{\eta_2}\left(\cos\varphi t\cos\eta_2 t-1\right)\right) \qquad (3.43)\\
H_6 &= \frac{\eta_2 \sin\phi t}{\phi\left(\eta_2^2-\phi^2\right)}\left(\cos\phi t\sin\eta_2 t-\frac{\phi}{\eta_2}\sin\phi t\cos\eta_2 t\right)\\
&\quad-\frac{\eta_2\cos\phi t}{\phi\left(\eta_2^2-\phi^2\right)}\left(\sin\phi t\sin\eta_2 t-\frac{\phi}{\eta_2}\left(\cos\phi t\cos\eta_2 t-1\right)\right)\\
H_7 &= \frac{\eta_3 \sin\varphi t}{\varphi\left(\eta_3^2-\varphi^2\right)}\left(\cos\varphi t\sin\eta_3 t-\frac{\varphi}{\eta_3}\sin\varphi t\cos\eta_3 t\right)\\
&\quad-\frac{\eta_3\cos\varphi t}{\varphi\left(\eta_3^2-\varphi^2\right)}\left(\sin\varphi t\sin\eta_3 t-\frac{\varphi}{\eta_3}\left(\cos\varphi t\cos\eta_3 t-1\right)\right)\\
H_8 &= \frac{\eta_3 \sin\phi t}{\phi\left(\eta_3^2-\phi^2\right)}\left(\cos\phi t\sin\eta_3 t-\frac{\phi}{\eta_3}\sin\phi t\cos\eta_3 t\right)\\
&\quad-\frac{\eta_3\cos\phi t}{\phi\left(\eta_3^2-\phi^2\right)}\left(\sin\phi t\sin\eta_3 t-\frac{\phi}{\eta_3}\left(\cos\phi t\cos\eta_3 t-1\right)\right)\\
H_9 &= \frac{\eta_4 \sin\varphi t}{\varphi\left(\eta_4^2-\varphi^2\right)}\left(\cos\varphi t\sin\eta_4 t-\frac{\varphi}{\eta_4}\sin\varphi t\cos\eta_4 t\right)\\
&\quad-\frac{\eta_4\cos\varphi t}{\varphi\left(\eta_4^2-\varphi^2\right)}\left(\sin\varphi t\sin\eta_4 t-\frac{\varphi}{\eta_4}\left(\cos\varphi t\cos\eta_4 t-1\right)\right)\\
H_{10} &= \frac{\eta_4 \sin\phi t}{\phi\left(\eta_4^2-\phi^2\right)}\left(\cos\phi t\sin\eta_4 t-\frac{\phi}{\eta_4}\sin\phi t\cos\eta_4 t\right)\\
&\quad-\frac{\eta_4\cos\phi t}{\phi\left(\eta_4^2-\phi^2\right)}\left(\sin\phi t\sin\eta_4 t-\frac{\phi}{\eta_4}\left(\cos\phi t\cos\eta_4 t-1\right)\right)
\end{aligned}
$$

Thus, in view of expression (3.1), and taking into account (3.39), one obtains for the this vibrating system,

$$\phi_i(x,t) = \sum_{i=1}^{n} \left\{ \left(\frac{P_0 J_0(\gamma)}{B_1} \left\{ \left[\frac{Q_1(i,j)\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_1 - \left[\frac{Q_1(i,j)\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_2 \right\} \right) \right.$$
$$+ \left(\frac{P_0}{B_1} \sum_{k=1}^{\infty} J_{2k}(\gamma) \left\{ \left[\frac{Q_1(i,j)\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_3 - \left[\frac{Q_1(i,j)\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_4 \right. \right.$$
$$\left. \left. - \left[\frac{Q_1(i,j)\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_5 - \left[\frac{Q_1(i,j)\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_6 \right\} \right)$$
$$+ \left(\frac{P_1}{B_1} \sum_{k=0}^{\infty} J_{2k+1}(\gamma) \left\{ \left[\frac{Q_1(i,j)\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_7 - \left[\frac{Q_1(i,j)\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_8 \right. \right.$$
$$\left. \left. - \left[\frac{Q_1(i,j)\varphi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_9 - \left[\frac{Q_1(i,j)\phi^2}{\varphi^2-\phi^2} - \frac{Q_3(i,j)}{\varphi^2-\phi^2} \right] \cdot H_{10} \right\} \right) \left. \right\} \sin \frac{i\pi x}{L}$$

which represents the response amplitude of a prismatic deep beam when under the actions of harmonic variable magnitude load travelling at time dependent speed.

Similarly, in view of expression (3.2), taking into account of (3.40) one obtains

$$\psi_{i_i}(x,t) = \sum_{i=1}^{n} \left(\frac{P_0 J_0(\gamma) Q_2(i,j)}{B_1(\varphi^2-\phi^2)} [H_2 - H_1] + \frac{P_0 Q_2(i,j)}{B_1(\varphi^2-\phi^2)} \sum_{k=1}^{\infty} J_{2k}(\gamma) [H_4 - H_3 - H_6 + H_5] \right.$$
$$\left. + \frac{P_1 Q_2(i,j)}{B_1(\varphi^2-\phi^2)} \sum_{k=0}^{\infty} J_{2k+1}(\gamma) [H_8 - H_7 - H_{10} + H_9] \right) \cdot \cos \frac{i\pi x}{L} \tag{3.44}$$

which represents the rotation of the dynamical system.

4. COMMENTS ON THE CLOSED FORM SOLUTION

It is well known that the displacement response of an engineering structure under excitation may grow without bound and when this happens it leads to the occurrence called resonance. This occurrence of resonance in structural and highway engineering is quite undesirable. This is so because, its effects on such dynamical system could be devastating. In particular, it causes cracks, permanent deformation and destruction in structures and makes the structural systems unsaved for its occupants. Thus, it is very pertinent at this juncture to establish the conditions under which this undesirable phenomenon may occur. Equations (3.42) and (3.44) clearly indicates that the vibrating system under discussion reaches a state of resonance whenever

$$[P_1(i,) Q_3(i,j) + P_2(i,j) Q_1(i,j)]^2 = -4 \cdot (P_1(i,j) Q_i(i,j)) \cdot P_3(i,j) Q_2(i,j),$$
$$\omega = -2k\beta, \quad \omega = 2k\beta, \quad \omega = -(2k+1)\beta, \quad \omega = (2k+1)\beta$$

$$
\begin{aligned}
\omega &= \left(\frac{B_2}{2B_1} - \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \quad \text{or} \quad \omega = \left(\frac{B_2}{2B_1} + \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \\
\eta_1 &= \left(\frac{B_2}{2B_1} - \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \quad \text{or} \quad \eta_1 = \left(\frac{B_2}{2B_1} + \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \\
\eta_2 &= \left(\frac{B_2}{2B_1} - \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \quad \text{or} \quad \eta_2 = \left(\frac{B_2}{2B_1} + \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \\
\eta_3 &= \left(\frac{B_2}{2B_1} - \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \quad \text{or} \quad \eta_3 = \left(\frac{B_2}{2B_1} + \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \\
\eta_4 &= \left(\frac{B_2}{2B_1} - \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}} \quad \text{or} \quad \eta_4 = \left(\frac{B_2}{2B_1} + \sqrt{\frac{B_2^2}{4B_1^2} - \frac{B_3}{B_1}}\right)^{\frac{1}{2}}
\end{aligned}
\tag{4.1}
$$

5. RESULTS AND DISCUSSIONS

For the purpose of illustration, we adopt the beam parameters and material properties defined in Eftekhar et al. [32]. These properties are Length $L = 50$ m. The modulus of elasticity E is 3.34×10^{10} N/m^2, moment of inertia $I = 1.042 \times 10^4 m^4$, Density $\rho = 2400 Kg/m^3$, the shear modulus is $1.34 \times 10^{10} N/m^2$ and cross-sectional area of $2m^2$.

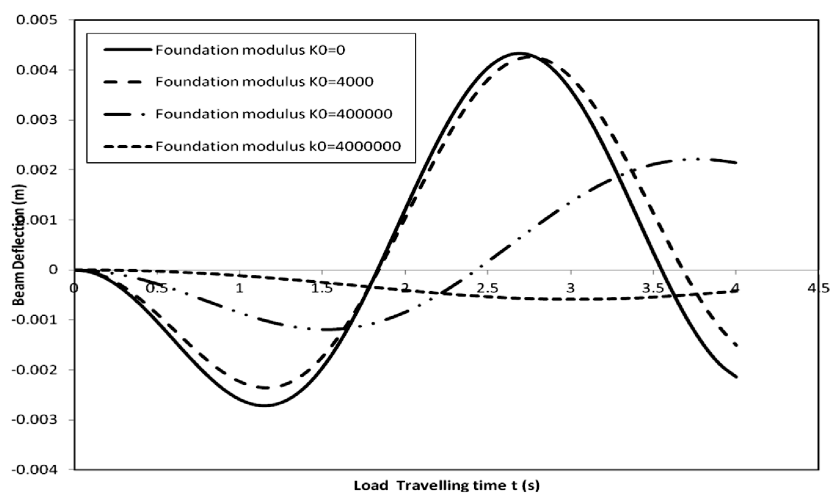

FIGURE 1. The transverse displacement response a uniform Timoshenko beam resting on variable elastic foundation and subjected to variable magnitude moving load for various values of foundation modulus K.

Figure 1 display the transverse displacement response of a uniform Timoshenko beam resting on variable elastic foundation when subjected to harmonic variable magnitude loads traveling with a variable velocity. It is deduced from this figure that for fixed values of other vital parameters, the transverse deflection of a uniform Timoshenko beam resting on variable elastic foundation and traversed by fast traveling masses decreases as the values of foundation reaction subgrade K_0 increases.

In figure 2 the deflection profile of a uniform Timoshenko beam resting on variable elastic foundation and subjected to variable magnitude moving load traveling at non uniform velocity is displayed for various load positions. It is clearly shown that the larger the value of the load positions the lower the deflection of the elastic beam.

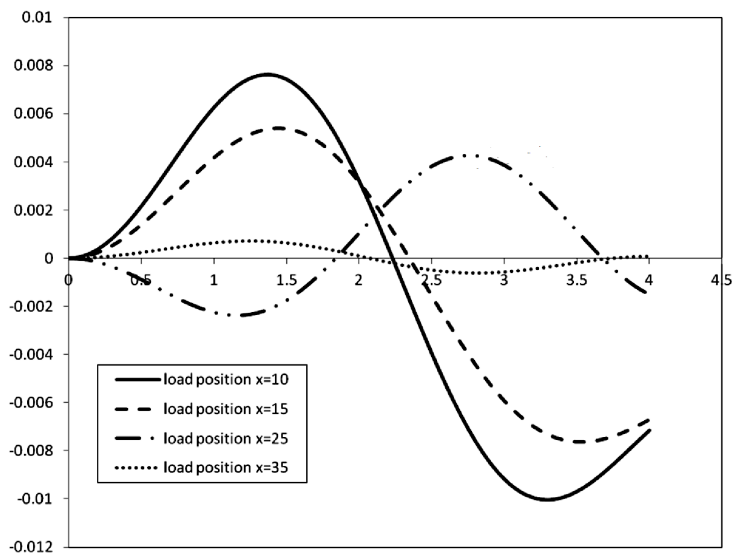

FIGURE 2. The deflection profile of a uniform Timoshenko beam resting on variable elastic foundation and subjected to variable magnitude moving load for various values of the load positions x.

The displacement response of a uniform Timoshenko beam resting on variable elastic foundation and under the actions of a variable magnitude moving load traveling at non uniform velocity is shown in figure 3. It is observed from this figure that higher values of the load longitudinal frequency β produce more stabilizing effects on the elastic beam.

Figure 4 depicts the deflection profile of the uniform Timoshenko beam resting on elastic foundation and subjected to fast traveling variable magnitude moving load. It is shown from the figure that for fixed value of foundation reaction K_0 and other structural parameters, the deflection of the beam reduces as the values of the circular frequency ω increases.

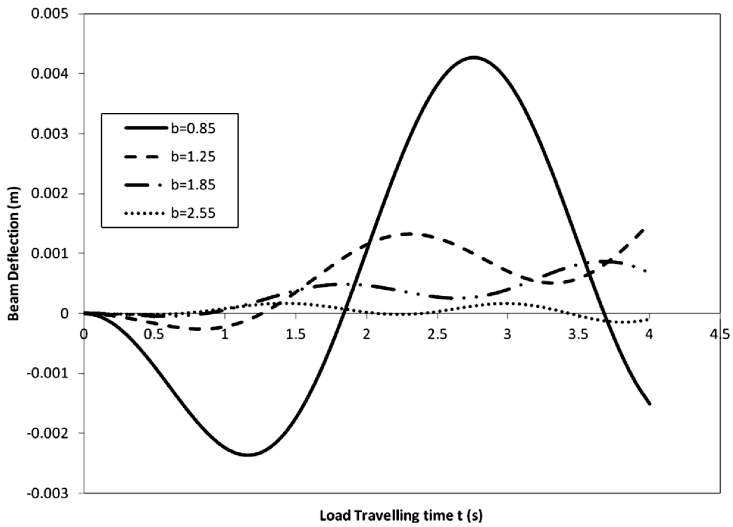

FIGURE 3. The displacement response of a uniform Timoshenko beam resting on variable elastic foundation and subjected to variable magnitude moving load for various values of longitudinal frequency of the load.

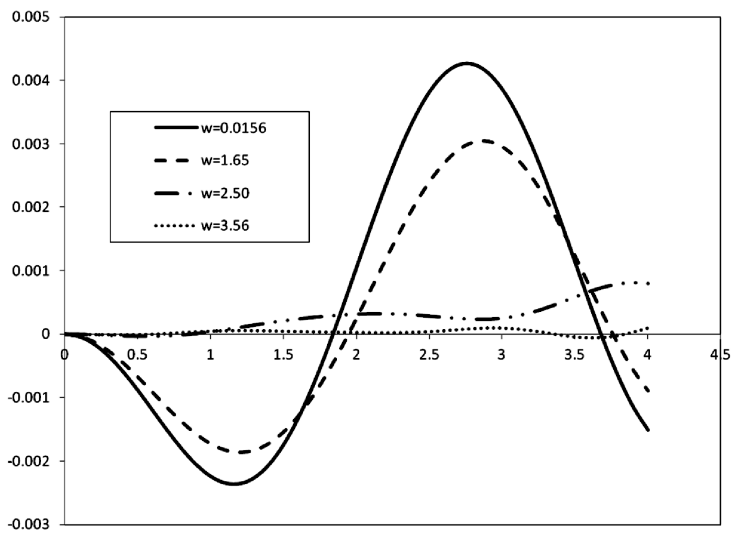

FIGURE 4. The response amplitude of a uniform Timoshenko beam resting on variable elastic foundation and under the actions of variable magnitude moving load for various values of circular frequency ω.

Figure 5 depicts the response amplitude of a uniform Timoshenko beam resting on variable elastic foundation when subjected to harmonic variable magnitude loads travelling with a variable velocity. It is clearly shown from the figure that as the values of the longitudinal amplitude of the oscillation of the travelling load increases, for fixed values of other parameters the dynamic deflection of the beam increases.

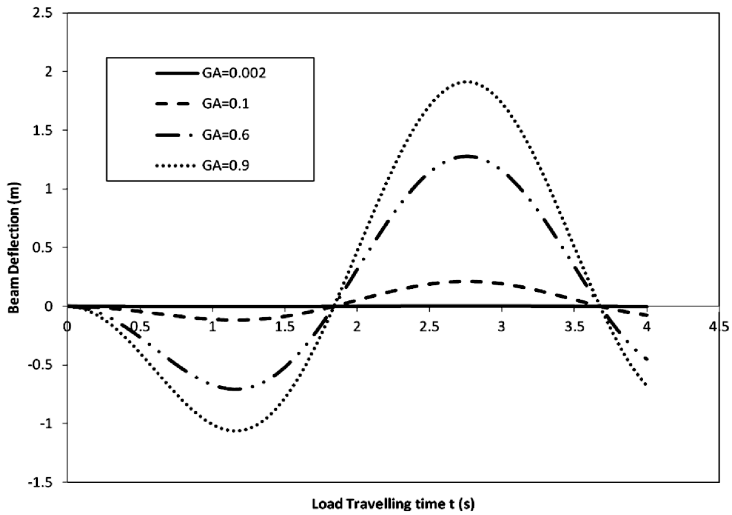

FIGURE 5. The response amplitude of a uniform Timoshenko beam resting on variable elastic foundation and under the actions of variable magnitude moving load for various values of longitudinal amplitude of oscillation of the load γ.

In figure 6, the transverse displacement response of a Timoshenko beam under the actions of harmonic magnitude load is shown. It is deduce from this figure that as the values of shear modulus Gincreases, the response amplitude of the beam increases.

6. CONCLUDING REMARKS

The dynamic behaviour of a simply supported Timoshenko beams resting on variable elastic foundation when carrying fast traveling concentrated loads of varying magnitudes has been investigated. The versatile analytical technique known as Galerkin's method has been employed in conjunction with integral transform method to obtain closed-form solution of this dynamical beam-load problem. Both analytical and numerical results presented in this paper are in perfect agreement with existing results. Results show that, as the values of foundation stiffness K_0 increases, the deflection profile of the uniform Timoshenko beam increases.

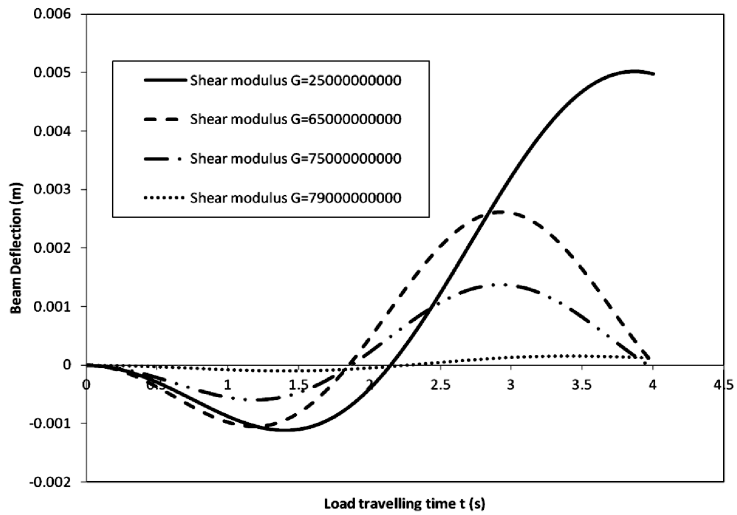

FIGURE 6. The response amplitude of a uniform Timoshenko beam resting on variable elastic foundation and under the actions of variable magnitude moving load for various values of shear modulus G.

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