

# Hybrid design method for air-core solenoid with axial homogeneity

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## Abstract

In this paper, a hybrid method is proposed to design an air-core superconducting solenoid system for 6 T axial uniform magnetic field using Niobium Titanium (NbTi) superconducting wire. In order to minimize the volume of conductor, the hybrid optimization method including a linear programming and a nonlinear programming was adopted. The feasible space of solenoid is divided by several grids and the magnetic field at target point is approximated by the sum of magnetic field generated by an ideal current loop at the center of each grid. Using the linear programming, a global optimal current distribution in the feasible space can be indicated by non-zero current grids. Furthermore the clusters of the non-zero current grids also give the information of probable solenoids in the feasible space, such as the number, the shape, and so on. Applying these probable solenoids as the initial model, the final practical configuration of solenoids with integer layers can be obtained by the nonlinear programming. The design result illustrates the efficiency and the flexibility of the hybrid method. And this method can also be used for the magnet design which is required the high homogeneity within several ppm (parts per million).

*Keywords:* air-core solenoid, axial uniformity, hybrid method, linear programming, nonlinear programming

## 1. INTRODUCTION

Many methods have been used in the solenoid design. The classical approach is to specify the dimension of solenoid with the maximum Fabry factor [1]. However, it is not suitable to deal with the complex constraints of the practical design. Other methods such as Genetic Algorithm can involve the nonlinear problem and have the obvious advantage at the engineering design [2]. The requirements of design can be easily described as nonlinear inequality or equality constraints, and it enhances the flexibility and readability of optimization problem. The nonlinear optimization methods are powerful but relatively slow. There is often no guarantee that the computed solution is the most optimal. Recently, several methods using the linear optimization have been applied to solve the solenoid design problem [3]. These methods are very efficient and can get an optimal solution theoretically. Whereas the rounding layers and turns of winding are considered, the constraints of optimization may be violated seriously.

In this paper, a hybrid optimization method was presented to design a 6 T solenoid magnet to generate a uniform magnetic field of 0.4% within 600 mm along z-axis. The solenoid can be used in an electron beam ion source (EBIS) to compress the electron beam from an electron gun [4]. The hybrid optimization method is composed of a linear programming and a nonlinear programming [5]. The linear programming is employed to obtain a global optimal current distribution in a feasible space of solenoid. And the nonlinear programming is used to improve the current distribution

to satisfy the requirement of design and revise its size to practical solenoids with rectangular cross-section. The process of hybrid optimization method was described in section 2. Using the hybrid optimization method, the design result of superconducting solenoid magnet was presented in section 3.

## 2. OPTIMIZATION METHOD

### 2.1. Process of optimization method

The purpose of design in this paper is to obtain a minimum volume electromagnet, which can generate uniform magnetic field on a straight line. Air-core solenoid is used in the design. And the whole process of the hybrid optimization method can be divided into three stages:

#### A. Analysis stage:

Solenoids in the design are located in a cylindrical coordinates  $(\rho, \varphi, z)$  and symmetric about the z-axis. For the sake of convenience, it is assumed that the current density is uniform across the cross-section of solenoid. The designed magnetic field at center point is  $B_0$ , and the uniformity is better than  $\varepsilon$  within a certain range  $L_{max}$  on the z-axis.

Beside the constraints of the magnetic field at center point and the uniformity, the sizes of solenoids are restricted by the bobbins, the gap, and other equations. When the superconducting wire is used in solenoid design, the critical current depending on the maximum magnetic field also should be considered in the design.

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### B. Linear programming stage

The feasible space of solenoids is divided into many grid elements. The magnetic field generated by each grid element can be approximated to the field generated by an ideal current loop which is located at the center of grid.

Using the linear programming, a global optimal current distribution in several grid elements can be obtained. The clusters of non-zero current grids can indicate the number and size of solenoids.

### C. Nonlinear programming stage

The result of linear programming provides an initial point for the nonlinear programming. Using the nonlinear programming, all clusters of non-zero current grids are revised into practical solenoids with rectangular cross-section.

#### 2.2. Linear programming

Assume the inner radius of solenoid should be greater than  $\rho_{min}$  and the outer radius should be less than  $\rho_{max}$ , the height of solenoid should be less than  $2Z_{max}$ . Considering in the design, the solenoid is symmetric about the plane  $z = 0$ , only 1/2 feasible space is calculated in the optimization as shown in Fig. 1.

The feasible space is divided into  $M_s \times N_s$  grid elements, which  $M_s$ ,  $N_s$  are the number of segments on the radial direction and the axial direction, respectively. Each grid element represents an ideal current loop  $I_i$  which is located at center point  $(\rho_i, z_i)$  of the grid element.

The design variables for linear programming are the current flowing through each grid,  $[I_1, I_2, \dots, I_{M_s \times N_s}]$ . The maximum current  $I_{max}$  in each grid can be calculated by

$$I_{max} = J \frac{\rho_{max} - \rho_{min}}{M_s} \times \frac{z_{max}}{N_s} \quad (1)$$

where,  $J$  is the current density in each grid.

For the simplification, the design variables are normalized by  $I_{max}$  and became  $[e_1, e_2, \dots, e_{M_s \times N_s}]$ ,

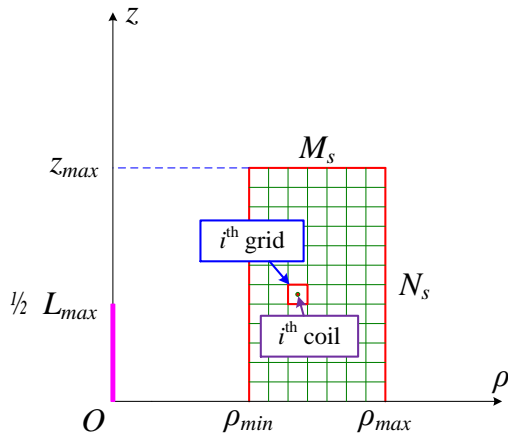


Fig. 1. Half of feasible space of solenoids and current grid elements.

where  $e_i = \frac{I_i}{I_{max}}$ ,  $i = 1, 2, \dots, M_s \times N_s$ . Then all design variables are between 0 and 1.

The objective function is the volume of solenoids, and it can be calculated by

$$Vol = 2\pi \frac{I_{max}}{J} \sum_{i=1}^{M_s \times N_s} \rho_i e_i \quad (2)$$

where  $I_{max}$  is the maximum current in each grid,  $J$  is the current density,  $\rho_i$  is the radius of the center point of  $i^{\text{th}}$  grid, and  $e_i$  is the normalized current in  $i^{\text{th}}$  grid.

Considering the axial component of magnetic field is much larger than the radial component in the solenoid, the uniformity of magnetic field can be verified by the  $B_z$  component of magnetic field at  $N_t$  target points in the specific range on the  $z$ -axis. According to the Biot-Savart law, the magnetic flux density at target point  $(\rho_j, z_j)$  generated by the ideal current loop  $I_i$  can be calculated by

$$B_z(\rho_j, z_j) = \frac{\mu_0 I_i}{2\pi} \frac{1}{\sqrt{(\rho_i + \rho_j)^2 + (z_j - z_i)^2}} \left[ K(k) - \frac{\rho_j^2 - \rho_i^2 + (z_j - z_i)^2}{(\rho_j - \rho_i)^2 + (z_j - z_i)^2} E(k) \right] \quad (3)$$

$$B_\rho(\rho_j, z_j) = \frac{\mu_0 I_i}{2\pi} \frac{z_j - z_i}{\rho_j \sqrt{(\rho_i + \rho_j)^2 + (z_j - z_i)^2}} \left[ \frac{\rho_j^2 + \rho_i^2 + (z_j - z_i)^2}{(\rho_j - \rho_i)^2 + (z_j - z_i)^2} E(k) - K(k) \right] \quad (4)$$

where  $\rho_i$  and  $z_i$  are the coordinates of  $i^{\text{th}}$  ideal current loop,  $\rho_j$  and  $z_j$  are the coordinates of  $j^{\text{th}}$  target point, the function  $K$  and  $E$  are the first and the second kind complete elliptic integrals, and

$$k = \sqrt{\frac{4\rho_i \rho_j}{(\rho_i + \rho_j)^2 + (z_j - z_i)^2}} \quad (5)$$

Furthermore, according to the characteristic of superconducting wire, for a given temperature, the highest magnetic field in the solenoid should be under the critical field  $B_c$ . Considering the maximum axial component of magnetic field in solenoid appears on the inner surface of solenoid, the magnetic fields at  $M_t$  target points  $(\rho_m, z_m)$  on the inner surface can be calculated by (3).

The linear programming problem can be expressed as

$$\text{Objective function: } Vol = 2\pi \frac{I_{max}}{J} \sum_{i=1}^{M_s \times N_s} \rho_i e_i \quad (6)$$

$$\text{Subject to } \begin{cases} |B_z(0, z_j) - B_0| \leq \frac{\epsilon}{2} |B_0|, j = 1, 2, \dots, N_t \\ |B(\rho_m, z_m)| \leq B_c, m = 1, 2, \dots, M_t \\ 0 \leq e_i \leq 1, i = 1, 2, \dots, M_s \times N_s \end{cases} \quad (7)$$

Solving the problem, the non-zero current grid elements are obtained by the result. These grids give a global optimal result for the solenoid configuration.

However, it is normally not able to produce a really solenoid magnet. Because the clusters of these grids are not rectangular and the magnetic field calculated in the linear programming is an approximate value.

### 2.3. Nonlinear programming

According to the optimal result from the linear programming, the number of solenoids in the feasible space,  $N_c$ , can be determined by the number of the clusters of the non-zero current grid elements. And the dimension of solenoids can be approximated by the size of clusters.

The design variables for nonlinear optimization are the dimension of solenoids,  $[a_{1(p)}, a_{2(p)}, b_{1(p)}, b_{2(p)}]$ , where  $a_{1(p)}$  and  $a_{2(p)}$  are the inner and outer radius of the  $p^{\text{th}}$  solenoid,  $b_{1(p)}$  and  $b_{2(p)}$  are the bottom and top height,  $p = 1, 2, \dots, N_c$ . Considering the current in the solenoid can be changed to satisfy the constraint of uniformity, the operating current also should be a design variable in the nonlinear optimization.

The objective function is still the volume of solenoids and can be calculated by

$$Vol = 2\pi \sum_{i=1}^{N_c} (a_{2(p)}^2 - a_{1(p)}^2) (b_{1(p)} - b_{2(p)}) \quad (8)$$

The magnetic flux density at the target points can be calculated with the controllable accuracy using  $r_zBI()$  program [6]. Considering solenoids are wound by the superconducting wire, the thickness and the height of each solenoid should be the integer multiple of the diameter of wire,  $D_w$ .

The nonlinear programming problem can be expressed as

$$\text{Objective function: } Vol = 2\pi \sum_{i=1}^{N_c} (a_{2(p)}^2 - a_{1(p)}^2) (b_{1(p)} - b_{2(p)}) \quad (9)$$

$$\text{Subject to } \begin{cases} \max B_z(0, z_j) - \min B_z(0, z_j) \leq \varepsilon B_0 \\ \max B(\rho_m, z_m) \leq B_c \\ \rho_{min} \leq a_{1(p)} \leq a_{2(p)} \leq \rho_{max} \\ 0 \leq b_{1(p)} \leq b_{2(p)} \leq z_{max} \\ (a_{2(p)} - a_{1(p)})/D_w \in \text{integer} \\ (b_{2(p)} - b_{1(p)})/D_w \in \text{integer} \end{cases} \quad (10)$$

The nonlinear programming problem can be solved by several methods, such as genetic algorithm, simulated annealing, evolution strategy, and so on. In the design, the optimization result is obtained using the genetic algorithm.

## 3. CASE STUDY

### 3.1. Analysis of design

In the case study, we want to design a 6 T solenoid system to generate a uniform magnetic field within 600 mm along z-axis. The uniformity should be better than 0.4% and can be calculated by

$$\text{Uniformity} = \frac{\max B_z - \min B_z}{B_0} \quad (11)$$

TABLE I  
REQUIREMENTS OF DESIGN.

Item		Value
Central Field	$B_0$	6 T
Uniformity	$\varepsilon$	$\leq 0.4\%$
Uniform range	$L_{max}$	$\geq 600$ mm
Minimum inner radius	$\rho_{min}$	$\geq 100$ mm
Maximum height	$2z_{max}$	$\leq 1000$ mm

TABLE II  
SPECIFICATIONS OF NbTi SUPERCONDUCTING WIRE.

Item		Value
Type		54S43
# Filaments		54
Cu:SC		1.3
Filament Diameter ( $\mu\text{m}$ )		85
Diameter (mm)	Bare	0.95
	Insulated	1.000
Critical Current (Amps @ 4.2 K) at Field (T)	5 T	780
	7 T	480
	9 T	165

TABLE III  
PARAMETERS FOR LINEAR PROGRAMMING.

Item	Value
$M_s$	20
$N_s$	100
$N_t$	31
$M_t$	51

where  $B_z$  is the axial component of magnetic flux density in the uniform range,  $B_0$  is the magnetic flux density at the center point of solenoid. Here, the value of  $B_0$  was set as 6 T.

Considering the diameter of warm bore, the thickness of bobbin, and some gaps between equipment, the inner radius of designed solenoid should be greater than 100 mm, and the total height of solenoid should be less than 1000 mm.

The requirements for the designed solenoid have been listed in Table I.

The solenoid is made with NbTi superconducting wire. And the specifications of wire are listed in Table II.

### 3.2. Result of optimization

The parameters for the linear programming are listed in Table III.

The result of linear programming is plotted in Fig. 2. The grid with red point means the non-zero current grid element. The normalized currents in the non-zero current grid elements are plotted in Fig. 3.

As shown in Fig. 2 and Fig. 3, the non-zero current grid elements became two clusters in the feasible space. Thus there are two probable solenoids in the space. The size of probable solenoids can be obtained from the figures and it can be an initial point for the nonlinear programming. Considering the edge of one solenoid is on the  $\rho$ -axis, the bottom height of this solenoid can be fixed to 0. Similarly, the inner radius of two solenoids can be fixed to 100 mm. Thus, the number of design variables in the nonlinear optimization can be reduced.

The parameters for nonlinear programming are listed in Table IV. And the result of nonlinear programming is plotted in Fig. 4. The magnetic flux density at the center

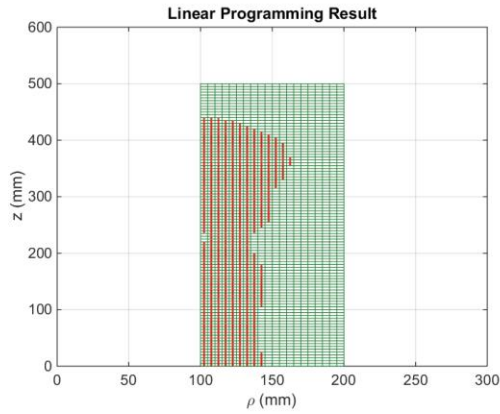


Fig. 2. The optimization result of linear programming.

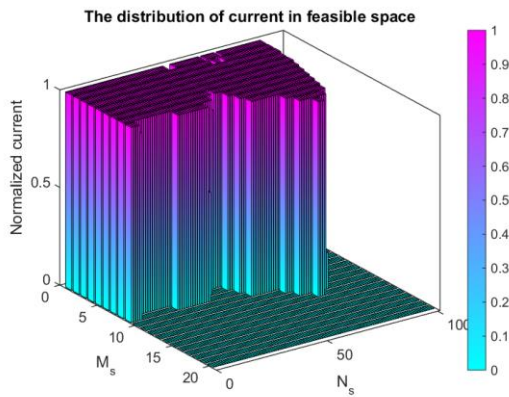


Fig. 3. The normalized current in each grid in the feasible space.

point is 5.9993 T, and the uniformity along the z-axis is less than 0.38% within 600 mm. At last, we can get the parameters of designed solenoids. The design result is listed in Table V.

TABLE IV  
PARAMETERS FOR NONLINEAR OPTIMIZATION.

Item	Value
$N_c$	2
$D_w$	1.000 mm

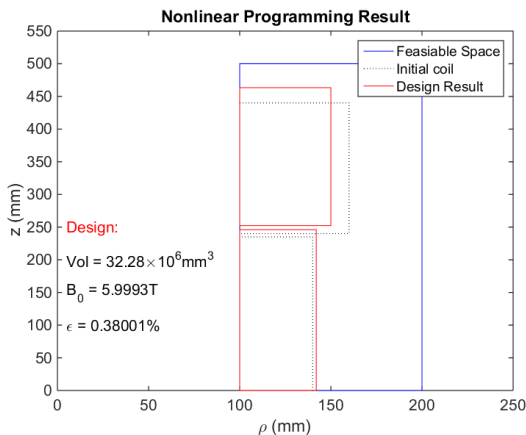


Fig. 4. The optimization result of nonlinear programming.

TABLE V  
PARAMETERS OF DESIGNED SOLENOID.

Item	Coil	
	1	2
$a_1$ (mm)	100	100
$a_2$ (mm)	142	150
$b_1$ (mm)	0	252.35
$b_2$ (mm)	246	463.35
Layers in radial direction	42	50
Layers in axial direction	246	211
Diameter of wire (mm)	1.000	
Operating current (A)	116.02	
Central field (T)	5.9993	
Maximum field (T)	6.2589	
Uniformity along z-axis	0.380%	
Length of wire (km)	32.28	

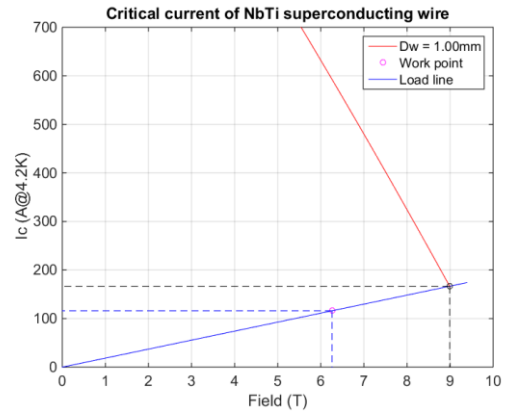


Fig. 5. Dependence of critical current of NbTi wire on applied magnetic field and the load line of designed solenoid.

When the operating current is 116.02 A, the maximum magnetic field appears in the coil 2 and is equal to 6.2589 T. However, according to the superconducting wire's critical characteristic, we can find that the maximum allowable current is about 166.64 A when the magnetic field in the coil is increased to 8.99 T as shown in Fig. 5. The operating current in the designed solenoid almost has a 30.38% margin of safety.

#### 4. CONCLUSION

Using the hybrid design method, a superconducting solenoid system for a 6 T uniform magnetic field is provided. In the hybrid design method, the linear programming can obtain a global optimal result of current distribution in the feasible space and give the initial value for the nonlinear programming. Then the nonlinear programming can give an optimization result for the practical solenoid.

#### ACKNOWLEDGMENT

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**REFERENCES**

- [1] D. B. Montgomery, *Solenoid Magnet Design*, Wiley-Interscience a Division of John Wiley & Sons, New York, 1969.
- [2] N. R. Shaw and R. E. Ansorge, "Genetic algorithm for MRI magnet design," *IEEE Transactions on Applied Superconductivity*, vol. 12, no. 1, pp. 733-736, 2002.
- [3] H. Xu, *et al.*, "Homogeneous magnet design using linear programming," *IEEE Transactions on Magnetics*, vol. 36, no. 2, pp. 476-483, 2000.
- [4] S. Kondrashev, *et al.*, "Development of electron beam ion source charge breeder for rare isotopes at Californium Rare Isotope Breeder Upgrade," *Rev. Sci. Instrum.*, vol. 83, pp. 02A902, 2012.
- [5] Z. Ni, *et al.*, "Globally optimal algorithm for design of 0.7 T actively shielded whole-body open MRI superconducting magnet system," *IEEE Transactions on Applied Superconductivity*, vol. 23, no. 3, pp. 4401104, 2013.
- [6] L. Huang and S. Lee, "A study on the effect of the condition number in the magnetic field mapping of the Air-Core solenoid," *Progress in Superconductivity and Cryogenics*, vol. 17, no. 2, pp. 31-35, 2015.