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Effect of Slot Opening on the Cogging Torque of Fractional-Slot Concentrated Winding Permanent Magnet Brushless DC Motor

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Cogging torque will affect the performance of a permanent magnet Brushless DC Motor (BLDCM), thus the reduction of cogging torque is key for BLDCM optimization. In this paper, the phase shifting of cogging torque for a fractional-slot concentrated winding BLDCM is analyzed using the Maxwell tensor method. Moreover, a 9-slot 10-pole concentrated winding BLDCM driven by ideal square waveform is studied with the finite element method (FEM). An effective method to reduce the cogging torque is obtained by adjusting the slot opening. In addition, the influences of different slot openings on back electromotive force (back-EMF), air gap flux density and flux linkage are investigated and experimentally validated using the prototype BLDCM.

Keywords : BLDCM, cogging torque, concentrated winding, finite element method, fractional-slot, slot opening

1. Introduction

The permanent magnet Brushless DC Motor (BLDCM) was developed rapidly due to the enhancement of permanent magnet material and the easy control method. However, cogging torque produced by the attraction between the permanent magnet and the iron core with the slot, is one of the unavoidable disadvantages of the permanent magnet BLDCM [1, 2], and potentially changes the magnetic field energy, bringing in torque ripple, noise and vibration, and thereby decreasing control accuracy. Therefore, the calculation and reduction of cogging torque for BLDCM has attracted much attention all over the world for the last few decades [3, 4].

Due to the existence of the slot and teeth in the stator core, the permeance in the air gap varies to result in the cogging torque. Thus, direct methods for reducing the cogging torque are either to adopt a slotless core, or to decrease the slot opening. As the manufacturing process of the former is more complex, the cost of the machine is higher. Therefore, the use of a slot opening is the primary method to reduce the cogging torque of the fractional-slot concentrated winding BLDCM.

In [5], a simple analytical method to synthesize the cogging torque of BLDCM regarding a single slot is presented. The result illustrates that part of the cogging torque harmonic of single-slot BLDCM contributes to the result of multi-slot versions. The cogging torque of BLDCM can be significantly reduced by adjusting the slot opening width when the pole pair number is approximately equal to the slot number. In [6], the permeance model of the air gap is derived. The cogging torque can be reduced by controlling the square of the air gap permeance function and the flux density function, which is related to the order component of the least common multiple of the number of poles and slots. In [7], an analytical method is presented to predict the cogging torque by calculating the air gap field distribution, which behaves with good accuracy for the peak value calculation of the cogging torque. In [8], an analytical model of the air gap magnetic field and the permeance of the air gap are built and derived respectively. The Fourier coefficients, which have influence on the cogging torque, are calculated to obtain the optimal slot opening width. In [9], the reduction of the cogging torque of BLDCM is realized by distributing the teeth and magnets nonuniformly, and a simple Gradient Descent method is used to optimize the design of the machine.

In this paper, the effect of slot opening on the cogging torque of the fractional-slot concentrated winding permanent magnet BLDCM is investigated. Considering a 9-

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slot 10-pole BLDCM, the phase-shift of the fractional-slot machine is analyzed by building a single-slot machine model. It was found that the torques at a special slot opening width can offset each other to distort the waveform periodically, so as to reduce the cogging torque significantly. The influences of slot opening on the cogging torque and other performances of BLDCM are studied, using finite element analysis to obtain optimal results. Finally, an experiment is carried out to verify the correctness of the theoretical analysis.

2. Analytical Method of Cogging Torque

Assuming that the permeability of the stator core is infinite in BLDCM, under the no-load condition, the energy storage in the machine can be expressed as the sum of the energies in the air gap and in the permanent magnets, namely [10-12],

$$W = W_{airgap} + W_{PM} = \frac{1}{2\mu_0}\int_V B^2 \,\mathrm{d}V \tag{1}$$

The magnetic energy is determined by the structural dimensions, the performance of the permanent magnet, and the position between the stator and the rotor. The air gap flux density distribution along the armature surface can be expressed as,

$$B(\theta,\alpha) = B_r(\theta)\frac{h_m(\theta)}{h_m(\theta)+\delta(\theta,\alpha)} \tag{2}$$

where, $B_r(\theta)$ is the flux density distribution of the remanence of the permanent magnet; $\delta(\theta, \alpha)$ is the effective length of the air gap; and $h_m(\theta)$ is the circumferential length along the magnetization direction of the permanent magnet. Thus, (2) can be rewritten as,

$$W = \frac{1}{2\mu_0}\int_V B_r^2(\theta)\left[\frac{h_m(\theta)}{h_m(\theta)+\delta(\theta,\alpha)}\right]^2 \mathrm{d}V \tag{3}$$

where, $B_r^2(\theta)$ can be expanded by Fourier function as,

$$B_r^2(\theta) = B_{r0} + \sum_{n=1}^{\infty} B_{rn}\cos 2np\theta \tag{4}$$

where, $B_{r0} = \alpha_p B_r^2$; $B_{rn} = 2B_r^2\sin(n\alpha_p\pi)/n\pi$; B_r is the remanence of the permanent magnet; and α_p is the pole-arc coefficient.

The Fourier expansion for $[h_m(\theta)/(h_m(\theta)+ \delta(\theta, \alpha))]^2$ of (3) is,

$$\left[\frac{h_m(\theta)}{h_m(\theta)+\delta(\theta,\alpha)}\right]^2 = G_0 + \sum_{n=1}^{\infty} G_n\cos nz\theta \tag{5}$$

where, $G_0 = \left[\frac{h_m(\theta)}{h_m(\theta)+\delta(\theta,\alpha)}\right]^2$, and $G_n = \frac{2}{n\pi}\left[\frac{h_m(\theta)}{h_m(\theta)+\delta(\theta,\alpha)}\right]^2$ $\sin\left(n\pi - \frac{nz\theta_{s0}}{2}\right)$. θ_{s0} is the radian of the slot opening width and z is the slot number.

Therefore, the expression of the cogging torque by neglecting the skewing slot is,

$$T_{cog} = \frac{\pi z L_a}{4\mu_0}(R_2^2 - R_1^2)\sum_{n=1}^{\infty} nG_n B_{r(nz/2p)}\sin(nz\alpha) \tag{6}$$

where, L_a is the axial length of the stator core; R_1 and R_2 are the outer and inner radius of the stator core, respectively; and n is required to keep integral $nz/2p$.

For a fractional slot machine, the period of the fundamental cogging torque waveform γ can be expressed by z and the highest common factor (HCF) of the pole number N_m as,

$$\gamma = \frac{2pz}{N_m} \tag{7}$$

Defining the least common multiple (LCM) N_c with z and $2p$, the relationship between N_c and N_m is $2pz = N_m N_c$. Thus, (7) can be rewritten as,

$$\gamma = LCM[z, 2p] \tag{8}$$

In this paper, a 9-slot 10-pole fractional slot BLDCM is analyzed, and the period number of the fundamental cogging torque waveform is $\gamma = 90$.

3. Finite Element Analysis

3.1. Phase shift of cogging torque

The Fourier progression of the single-slot cogging torque without skewing is,

$$T_{sc} = \sum_{i=1}^{n} T_{sci}\sin 2\pi\theta \tag{9}$$

where T_{sci} is the ith harmonic of the cogging torque in a single-slot machine model. The total cogging torque including z slot numbers is,

$$T_{cog} = \sum_{k=1}^{n}\sum_{i=1}^{n} T_{sci}\sin 2\pi\left(\theta + \frac{2\pi}{z}(k-1)\right) \tag{10}$$

In this paper, the studied 9-slot 10-pole fractional slot concentrated winding BLDCM is shown in Fig. 1. And the main parameters are listed in Table 1.

Figure 2 shows the magnetic field distributions of single-slot and 9-slot models, respectively. According to the calculation, the cogging torque of the 9-slot machine

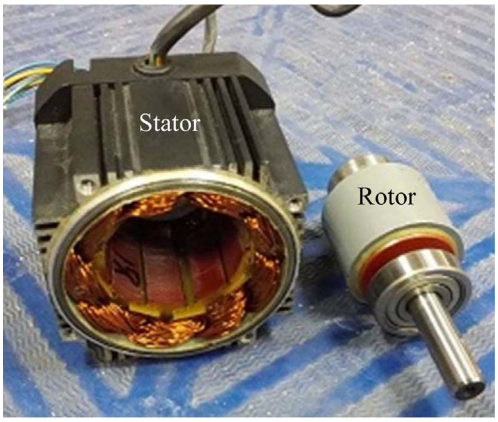

Fig. 1. (Color online) 9-slot, 10-pole fractional slot with concentrated winding BLDCM.

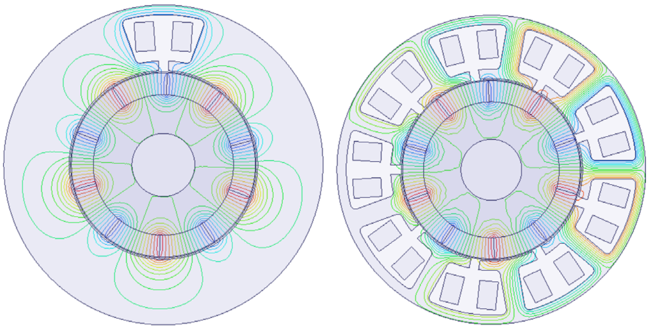
(a) (b)

Fig. 2. (Color online) Magnetic field distributions of a machine with the single-slot and 9-slot. (a) single-slot. (b) 9-slot.

can be obtained by adding the single-slot one, as shown in Fig. 3. From (9) and (10), the flux density in each slot are different to the fractional-slot machine, which causes the phase shift of the cogging torque. Therefore, the *i*th order harmonic of the cogging torque for a single slot will be offset in a 9-slot one.

3.2. Influence of slot opening width on cogging torque

According to (1), the total magnetic energy in BLDCM includes the magnetic energy both in PMs and an air gap. From (4) and (5), G_n is the Fourier coefficient of the square of the air gap permeance, while $B_{r(nz/2p)}$ is the Fourier coefficient of the square of the air gap flux density produced by PMs. Both of G_n and $B_{r(nz/2p)}$ are relevant to slot opening. Therefore, the slot opening width is a key factor affecting the cogging torque. Through calculation, the amplitudes of the cogging torque with different slot opening widths were obtained and shown in Fig. 4, which illustrates that the amplitude of the cogging torque of BLDCM is non-monotonous with a fractional slot. The optimal slot opening exists in fractional slot BLDCM.

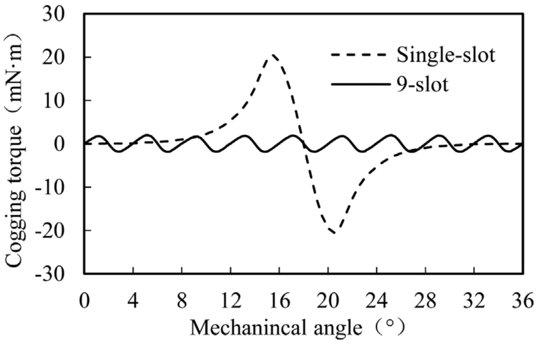

Fig. 3. Cogging torque waveforms for single-slot and 9-slot machines.

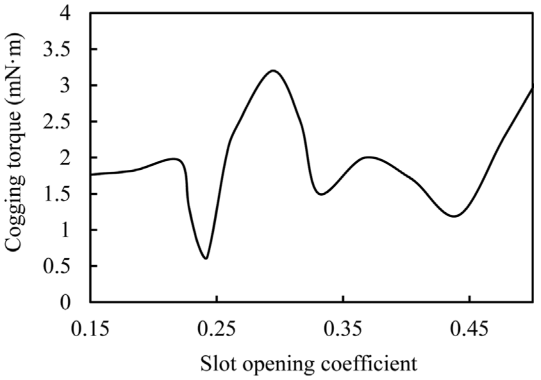

Fig. 4. Amplitude of cogging torque with different slot opening coefficients.

Table 1. Main parameters of 9-slot 10-pole concentrated winding BLDCM.

Parameters	Values	Parameters	Values
Outer diameter of stator core	66 mm	Air gap radial length	0.5 mm
Inner diameter of stator core	39 mm	Coercivity of PM	425 kA/m
Outer diameter of rotor	38 mm	Remanence	0.655T
Inner diameter of rotor	29 mm	Type of silicon steel	50DW470
PM height	4.5 mm	Type of PM	SNP-10
Axial length	33 mm	Rated power	20W
Rated voltage	24 V	Rated speed	1350 r/min

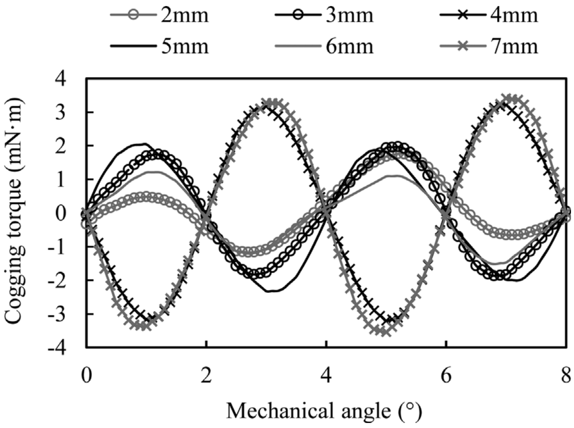

Fig. 5. Cogging torque waveforms with slot opening width from 2 mm to 7 mm.

Considering the manufacturing of the coils, the suitable slot opening width interval should to be considered. Figure 5 presents the cogging torque waveforms of different slot opening widths from 2 mm to 7 mm. It illustrates that the cogging torque of fractional-slot machine shows phase reversing phenomena. With the increase of the slot opening width, the cogging torque increases first and then decreases to cause the waveform distortion; it then increases again in reverse. According to the phase shift principle, the minimal values of the cogging torque exist with the slot opening widths between 3 mm to 4 mm, 4 mm to 5 mm, and 6 mm to 7 mm.

Therefore, the lower the slot opening width, especially the slotless or magnetic slot wedges can effectively reduce the cogging torque. Generally, the optimal region of the slot opening width of this prototype is 3.1 mm to 3.5 mm. The cogging torques with a slot opening width from 3.1 mm to 3.5 mm are shown in Fig. 6. It is found that the cogging torque reaches the minimum value when the slot opening width is 3.3 mm, which is in accordance with the analytical method result.

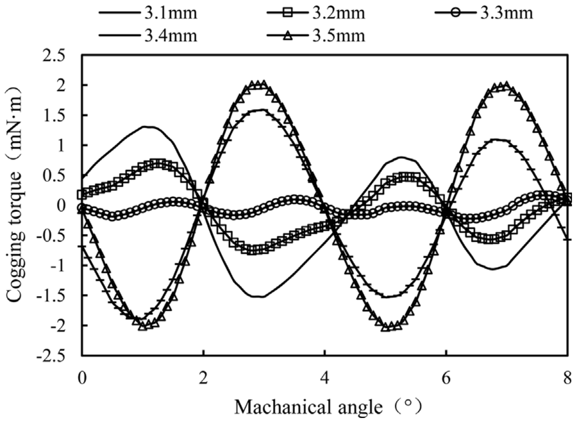

Fig. 6. Cogging torque waveforms with slot opening width from 3.1 mm to 3.6 mm.

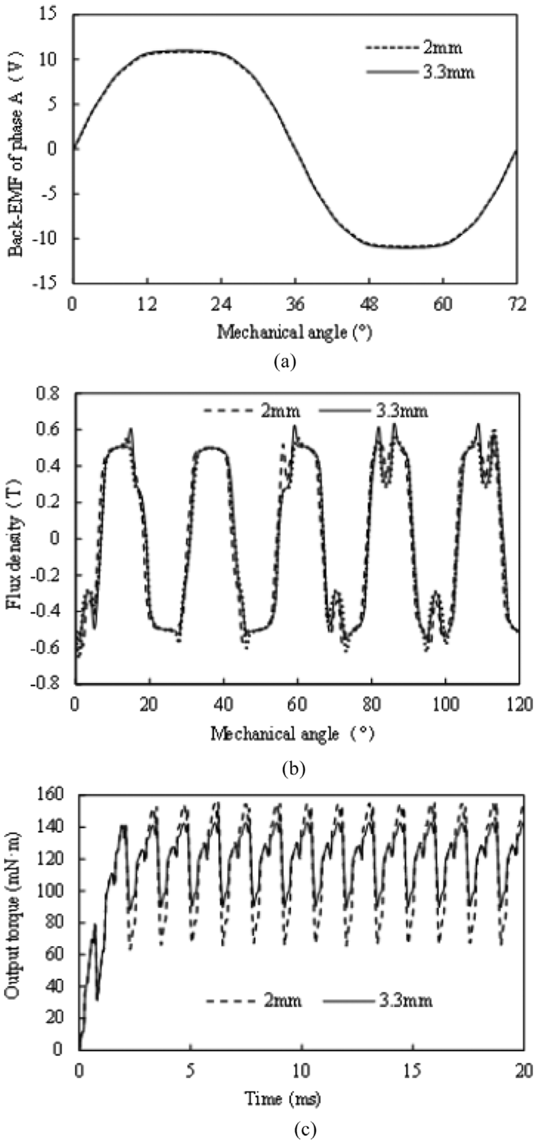

Fig. 7. Other performances under the conditions of the slot opening width a 2 mm and 3.3 mm. (a) Back-EMF. (b) air gap flux density. (c) output torque.

3.3. Influence of slot opening width on other performance

Besides the cogging torque, the slot opening width also influences other performance, such as back-EMF, the air gap flux density and the output torque, etc. Figures 7(a), (b) and (c) demonstrate back-EMF, the air gap flux density and the output torque waveforms with the slot opening at an initial 2 mm and optimal 3.3 mm, respectively. The results show that back-EMF and the air gap flux density can be affected rarely under the conditions of

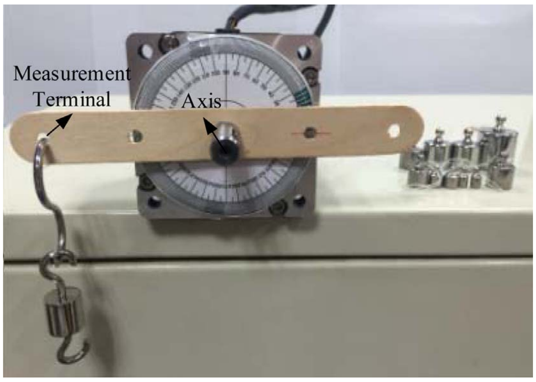

Fig. 8. (Color online) Experimental platform of cogging torque measurement.

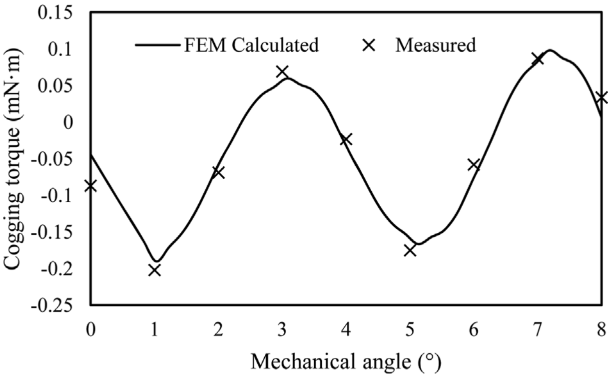

Fig. 9. Experimental and FEM calculated results of cogging torque.

these two slot opening widths. The amplitude of the back-EMF is about 11 V. The maximum value of the air gap flux density is about 0.55T. Moreover, the output torque ripple decreases when the optimal slot opening width is adopted. The fluctuation ratio changes from an initial 25 % to an optimized 16 %.

4. Experiment

The cogging torque experimental platform for this fractional slot BLDCM is built as shown in Fig. 8. The calculation of cogging torque can be expressed as,

$$T_{cog} = T_m - T_f = MgL\cos\theta - T_f \tag{11}$$

where, M is the counterbalance mass; g is the gravity acceleration; L is the horizontal length between the axis and the measurement terminal; T_f is the frictional torque and θ is the rotation angle, respectively.

The measured torque T_m contains the cogging torque and the frictional torque. The latter one can be changed hardly with different rotor positions, while the former one always changes periodically by theoretical analysis. Thus, the frictional torque can be obtained by calculating the average value of maximum and minimum measured torque, which is about 10 mN·m in this prototype. The cogging torque calculated by FEM and measured through an experiment with an optimal slot opening width of 3.3 mm, are depicted in Fig. 9.

According to Fig. 9, the measured cogging torque is very consistent with the calculation results. The error between these two methods is mainly caused by the precision of weights and the rotor friction.

5. Conclusion

This paper focuses on the influence of the slot opening width on the cogging torque for a 9-slot, 10-pole fractional slot concentrated winding BLDCM. The cogging torque expression is obtained through a virtual displacement method. The phase shift of the cogging torque for the single-slot machine model is studied, which illustrates that the cogging torque of fractional slot machines can be reduced significantly and the minimal value exist at some special slot opening widths. In addition, other performances of this prototype are analyzed by adjusting the slot opening coefficient. The experimental results verify the correctness and the effectiveness of the theoretical analyses.

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