

An Improved Dempster-Shafer Algorithm Using a Partial Conflict Measurement

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Abstract

Multiple evidences based decision making is an important functionality for computers and robots. To combine multiple evidences, mathematical theory of evidence has been developed, and it involves the most vital part called Dempster's rule of combination. The rule is used for combining multiple evidences. However, the combined result gives a counterintuitive conclusion when highly conflicting evidences exist. In particular, when we obtain two different sources of evidence for a single hypothesis, only one of the sources may contain evidence. In this paper, we introduce a modified combination rule based on the partial conflict measurement by using an absolute difference between two evidences' basic probability numbers. The basic probability number is described in details in Section 2 "Mathematical Theory of Evidence". As a result, the proposed combination rule outperforms Dempster's rule of combination. More precisely, the modified combination rule provides a reasonable conclusion when combining highly conflicting evidences and shows similar results with Dempster's rule of combination in the case of the both sources of evidence are not conflicting. In addition, when obtained evidences contain multiple hypotheses, our proposed combination rule shows more logically acceptable results in compared with the results of Dempster's rule.

Keywords: Partial conflict measurement, Combination rule, Decision making

1. Introduction

In computer and robot, evidences can be obtained from noisy and complex environment. The evidences are mostly uncertain and incomplete. It is important to make a decision based on the evidences to interact with their environment.

In early days, Bayes rule was used to combine multiple evidences. However, this rule requires a prior probability for each hypothesis which leads to difficulty when the rule is applied in a practical situation.

Thus, mathematical theory of evidence has been developed in [1] and [2] for combining multiple evidences without requiring any prior probability. The heart of this theory is Dempster's rule of combination that combines multiple evidences. It is used as a dominant tool in information fusion.

In [3], the author argued that this combination rule draws a counterintuitive conclusion when combining highly conflicting evidences. There have been many researches [4–8] carried out to fix the counterintuitive conclusion. Some of these researches are modified the combination rule [4–6] and [8] with different approaches and other research including [7] was extended the general concepts of this theory to make a better combination rule.

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In this paper, we introduce the modified combination rule using the partial conflict measurement (PCM). The rule manages the conflict between multiple evidences. The results from the modified combination rule are: 1) it provides reasonable conclusions when combining highly conflicting evidences for a single hypothesis and 2) it also provides more logically acceptable conclusions when obtained evidences contain multiple hypotheses.

2. Mathematical Theory of Evidence

In this section, we give brief explanations of mathematical theory of evidence (or Dempster-Shafer theory). This theory was originated from Dempster [1] and has been developed by Shafer [2], it has promising abilities for handling uncertain information in expert systems [6].

2.1 Dempster’s Rule of Combination

According to [1] and [2], Θ is a finite set of possible answers to a given hypothesis H_i , called a frame of discernment. Each distinct evidence is represented by a basic probability assignment (bpa) function $m(\cdot)$. The bpa maps each element of $\Theta(m : 2^\Theta \rightarrow [0, 1])$ with the following two conditions:

$$m(\emptyset) = 0, \sum_{A \subseteq 2^\Theta} m(A) = 1. \tag{1}$$

A quantity of $m(A)$ is called a basic probability number (bpn). A belief function is represented as follows:

$$Bel(H) = \sum_{A \subseteq H} m(A), \quad \forall H \subseteq \Theta. \tag{2}$$

Dempster’s rule of combination is defined as follows:

$$m_{12}(A) = \frac{1}{1-K} \sum_{X_i \cap Y_j = A \neq \emptyset} m_1(X_i) m_2(Y_j), \tag{3}$$

where

$$K = \sum_{X_i \cap Y_j = \emptyset} m_1(X_i) m_2(Y_j). \tag{4}$$

In Eq. (3), K corresponds to total degree of conflict of the combining two evidences.

Table 1. Belief comparisons for two different sources of evidence

Hypotheses	Bel(H)	
	Dempster’s rule	Proposed rule
{ <i>cirr</i> }	0.42	0.2890
{ <i>hep, cirr</i> }	0.6	0.5560
{ <i>cirr, gall, pan</i> }	0.7	0.6110

2.2 Problems of Dempster’s Rule

In [3], the author first introduced highly conflicting evidence example which is given in Table 1 (case 1) and identified that Dempster’s rule cannot manage high conflicts. The example supposes that a patient P was examined by two doctors, X and Y for diseases *Tumor* (T), *Meningitis* (M) and *Concussion* (C). Frame of discernment for this example is $\Theta = \{H_1, H_2, H_3\} = \{T, M, C\}$.

In order to analyze the combination results, Dempster’s rule of combination is applied to the Zadeh’s example mentioned previously. The results are shown below:

$$m_{12}(T) = \frac{0.01 \times 0.01}{1-K} = \frac{0.0001}{0.0001} = 1,$$

$$m_{12}(M) = \frac{0 \times 0.99}{1-K} = 0,$$

$$m_{12}(C) = \frac{0.99 \times 0}{1-K} = 0,$$

$$K = 0.0099 + 0.99 = 0.9999.$$

As can be seen in the results above, the two problems were identified.

First problem was found in [3] that normalization part of Dempster’s rule of combination leads to a counterintuitive conclusion. According to the result of Dempster’s rule of combination, the beliefs for each hypothesis are $Bel(T) = 1$, $Bel(M) = 0$ and $Bel(C) = 0$. The conclusion is counterintuitive as pointed out in [3] and leads to a high risk.

Second problem of Dempster’s rule of combination is product operation that is used in numerator of Eq. (3).

This problem can be seen in calculation of above Zadeh’s example for combining evidences $m_{12}(M)$ and $m_{12}(C)$ respectively.

In a situation in which we observe the evidence $m_1(X_i)$ from source X for only a single hypothesis H_1, H_2 and H_3 with some of them have a zero element (i.e., $m_1(M) = 0$ given in Zadeh’s example and $m_1(T) = 0$ given in [4] and [5]), this evidence strongly affects to the further combination results

related to the hypothesis T as shown in Section 4.

In order to obtain reasonable conclusion by combining multiple evidences, we introduce a new concept referred to as the PCM in the modified combination rule. The combination rule can solve previously mentioned two problems of Dempster’s rule of combination and gives reasonable results.

3. Proposed Combination Rule

In this paper, we only concentrated on combination rule and proposed a modified combination rule for combining multiple evidences.

Dempster’s rule of combination uses total degree of conflict referred to K (which is defined in Eq. (4)) by collecting all conflicts between combining evidences.

Our proposed combination rule uses the partial conflict measurement referred to K_A , which is represented by the absolute difference between two evidences’ basic probability numbers.

The modified combination rule was defined as follows:

$$m_{12}(A)' = \frac{m_{12}(A) + K_A}{2}, \tag{5}$$

where

$$K_A = \sum_{X_i \cap Y_j = A \neq \emptyset} |m_1(X_i) - m_2(Y_j)|, \tag{6}$$

$$\hat{m}_{12}(A_i) = \frac{m_{12}(A_i)'}{\sum_{i=1} m_{12}(A_i)'}, \tag{7}$$

in Eq. (5), $m_{12}(A)$ is combination results of Dempster’s rule.

K_A is the partial conflict measurement of subset A_i ($A_i \neq A_j$, $i \neq j$), which is used in combination rule only if the evidences $m_1(X_i)$ and $m_2(Y_j)$ have $X_i \cap Y_j = A_i \neq \emptyset$.

The theoretical maximum and minimum values of $|m_1(X_i) - m_2(Y_j)|$ are presented in the following:

$$|m_1(X_i) - m_2(Y_j)| = \begin{cases} 0, & \text{if } m_1(X_i) = 0, \quad m_2(Y_j) = 0, \\ 0, & \text{if } m_1(X_i) = 1, \quad m_2(Y_j) = 1, \\ 1, & \text{if } m_1(X_i) = 1, \quad m_2(Y_j) = 0, \\ 1, & \text{if } m_1(X_i) = 0, \quad m_2(Y_j) = 1, \end{cases} \tag{8}$$

where

$$X_i \cap Y_j = A \neq \emptyset.$$

We attempt to give detailed explanations for the partial conflict measurement K_A .

When combining two evidences, we can measure the differences between combining evidences’ basic probability numbers by calculating the absolute difference between $m_1(X_i)$ and $m_2(Y_j)$.

If the two evidences (e.g., $m_1(X_i)$ and $m_2(Y_j)$) both have same quantities (or basic probability numbers) for a hypothesis or the two evidences have totally same opinions for the same hypothesis then $|m_1(X_i) - m_2(Y_j)| = 0$.

If two evidences have totally opposite opinions for the same hypothesis then $|m_1(X_i) - m_2(Y_j)| = 1$.

Our proposed combination rule uses the combination result of Dempster’s rule as can be seen in Eq. (5).

As we mentioned before, a numerator of Dempster’s rule of combination is represented by product operation. If one of the evidences for a single hypothesis has no evidence then the whole combination result becomes zero for the hypothesis.

Therefore, we used an addition operation, added the partial conflict measurement, and averaged. The partial conflict measurement Eq. (6), K_A is only related to two combining evidences (e.g., $m_1(X_i)$ and $m_2(Y_j)$), where: $X_i \cap Y_j = A \neq \emptyset$ at a time and it does not use information related to $m_1(X_k)$ and $m_2(Y_l)$ (if $X_i \cap X_k = \emptyset$, $Y_j \cap Y_l = \emptyset$, where: $i \neq k$, $j \neq l$, respectively).

In case of Dempster’s rule of combination, the total degree of conflict K , it is used in every combination process of each hypothesis as a normalization role.

We argue that when we combine two evidences for a single hypothesis which is obtained from different sources, the evidences conflict with each other partially as we referred to as the partial conflict measurement K_A in our proposed combination rule Eq. (5).

We apply our proposed combination rule into two different examples including highly conflicting example that contains single hypothesis and non-conflicting evidence example that contain multiple hypotheses.

On the basis of these examples, we aim to show how the partial conflict measurement is used and why it is important to consider.

3.1 Proposed Combination Rule in Zadeh’s Example

In this example, we give a general idea of how the partial conflict measurement is used and how it manages the high conflict between the two evidences. By applying our proposed combination rule into Zadeh’s example, it shows as follows:

$$K_T = |m_1(X_1) - m_2(Y_1)| = |0.01 - 0.01| = 0,$$

$$\begin{aligned}
 m_{12}(T)' &= \frac{m_{12}(T) + K_T}{2} = 0.5, \\
 K_M &= |m_1(X_2) - m_2(Y_2)| = |0.99 - 0| = 0.99, \\
 m_{12}(M)' &= \frac{m_{12}(M) + K_M}{2} = 0.49005, \\
 K_C &= |m_1(X_3) - m_2(Y_3)| = |0 - 0.99| = 0.99, \\
 m_{12}(C)' &= \frac{m_{12}(C) + K_C}{2} = 0.49005,
 \end{aligned}$$

where

$$X_1 \cap Y_1 = T, \quad X_2 \cap Y_2 = M \quad \text{and} \quad X_3 \cap Y_3 = C.$$

By applying Eq. (7) into the results (i.e., $m_{12}(T)'$, $m_{12}(M)'$ and $m_{12}(C)'$) of above example, we can obtain:

$$\hat{m}_{12}(T) = 0.3378, \quad \hat{m}_{12}(M) = 0.3311, \quad \hat{m}_{12}(C) = 0.3311.$$

After that, the beliefs for each single hypothesis are:

$$Bel(T) = 0.3378, \quad Bel(M) = 0.3311, \quad Bel(C) = 0.3311.$$

The conclusions drawn after using our proposed combination rule were reasonable. From the above procedures, we can obtain information about partial conflict measurements for each hypothesis:

First, the evidences observed from source X and Y for hypothesis T , have the partial conflict measurement $K_T = 0$, which means that, there is no partial conflict between the two evidences (i.e., $m_1(X_1) = 0.01$ and $m_2(Y_1) = 0.01$, $X_1 \cap Y_1 = T$) because these two evidences have exactly same values, 0.01.

Second, evidences observed from source X and Y for hypothesis M have the partial conflict measurement $K_M = 0.99$, which means that, there is partial conflict between two evidences (i.e., $m_1(X_2) = 0$ and $m_2(Y_2) = 0.9$, $X_2 \cap Y_2 = M$).

Third, evidences observed from source X and Y for hypothesis C have the partial conflict measurement $K_C = 0.99$, which means that partial conflict between two evidences (i.e., $m_1(X_3) = 0.99$ and $m_2(Y_3) = 0$, $X_3 \cap Y_3 = C$) are very high.

In this manner, the partial conflict measurement allows us to extract very useful information about the combined evidences.

3.2 Proposed Combination Rule in Non-conflicting Evidence Example

In this subsection, the example (i.e., example 5 and 6) presented in [9] is used for the reason to identify the importance of the partial conflict measurement in combining evidences while evidences contain multiple hypotheses. The example is in the following. Supposing a physician is considering a case of cholestatic jaundice for which there is a diagnostic hypothesis set of *hepatitis (hep)*, *cirrhosis (cirr)*, *gallstone (gall)* and *pancreatic cancer (pan)*. Then the frame of discernment is $\Theta = \{hep, cirr, gall, pan\}$. Based on these hypotheses, the evidences were observed by three different sources:

$$\begin{aligned}
 m_1(\{hep, cirr\}) &= 0.6, \quad m_1(\Theta) = 0.4, \\
 m_2(\{cirr, gall, pan\}) &= 0.7, \quad m_2(\Theta) = 0.3, \\
 m_3(\{hep\}) &= 0.8, \quad m_3(\Theta) = 0.2.
 \end{aligned}$$

Based on the example, we used Dempster's rule for the two different evidences (i.e., m_1 and m_2) for the hypotheses and obtained the following results accordingly.

$$\begin{aligned}
 K &= 0, \\
 m_{12}(\{cirr\}) &= 0.42, \\
 m_{12}(\{hep, cirr\}) &= 0.18, \\
 m_{12}(\{cirr, gall, pan\}) &= 0.28, \\
 m_{12}(\Theta) &= 0.12.
 \end{aligned}$$

The combination rule results on the two different evidences (i.e., m_{12} and m_3) for the hypotheses based on Dempster's rule show the following:

$$\begin{aligned}
 K &= 0.56, \\
 m_{123}(\{hep\}) &= 0.545, \\
 m_{123}(\{cirr\}) &= 0.191, \\
 m_{123}(\{hep, cirr\}) &= 0.82, \\
 m_{123}(\{cirr, gall, pan\}) &= 0.127, \\
 m_{123}(\Theta) &= 0.055.
 \end{aligned}$$

Based on the same example tested with Dempster's rule, the combination results of the two different evidences (i.e., m_1 and m_2) for the hypotheses by our proposed combination rule presents the following results:

$$K_{\{cirr\}} = |m_1(\{hep, cirr\}) - m_2(\{cirr, gall, pan\})|$$

$$\begin{aligned}
 &= |0.6 - 0.7| = 0.1, \\
 m_{12}(\{cirr\})' &= \frac{m_{12}(\{cirr\}) + K_{\{cirr\}}}{2} = 0.26, \\
 K_{\{hep,cirr\}} &= |m_1(\{hep,cirr\}) - m_2(\Theta)| \\
 &= |0.6 - 0.3| = 0.3, \\
 m_{12}(\{hep,cirr\})' &= \frac{m_{12}(\{hep,cirr\}) + K_{\{hep,cirr\}}}{2} \\
 &= 0.24, \\
 K_{\{cirr,gall,pan\}} &= |m_1(\Theta) - m_2(\{cirr,gall,pan\})| \\
 &= |0.4 - 0.7| = 0.3, \\
 m_{12}(\{cirr,gall,pan\})' &= \frac{m_{12}(\{cirr,gall,pan\})}{2} \\
 &\quad + \frac{K_{\{cirr,gall,pan\}}}{2} = 0.29, \\
 K_{\Theta} &= |m_1(\Theta) - m_2\Theta| = |0.4 - 0.3| = 0.1, \\
 m_{12}(\Theta)' &= \frac{m_{12}(\Theta) + K_{\Theta}}{2} = 0.11.
 \end{aligned}$$

After applying Eq. (7) into the above combination results, we can obtain the following:

$$\begin{aligned}
 \hat{m}_{12}(\{cirr\}) &= 0.2890, \\
 \hat{m}_{12}(\{hep,cirr\}) &= 0.2670, \\
 \hat{m}_{12}(\{cirr,gall,pan\}) &= 0.3220, \\
 \hat{m}_{12}(\Theta) &= 0.1220.
 \end{aligned}$$

In addition, combination results of the two different evidences (i.e., \hat{m}_{12} and m_3) for the hypotheses by our proposed combination rule demonstrate as follows:

$$\begin{aligned}
 K_{\{hep\}} &= |m_3(\{hep\}) - \hat{m}_{12}(\{hep,cirr\})| \\
 &\quad + |m_3(\{hep\}) - \hat{m}_{12}(\Theta)| \\
 &= |0.8 - 0.2670| + |0.8 - 0.1220| = 1.211, \\
 m_{123}(\{hep\})' &= \frac{m_{123}(\{hep\}) + K_{\{hep\}}}{2} = 0.8783, \\
 K_{\{cirr\}} &= |m_3(\{hep\}) - \hat{m}_{12}(\{cirr\})| = |0.2 - 0.2890| \\
 &= 0.890, \\
 m_{123}(\{cirr\})' &= \frac{m_{123}(\{cirr\}) + K_{\{cirr\}}}{2} = 0.1399, \\
 K_{\{hep,cirr\}} &= |m_3(\Theta) - \hat{m}_{12}(\{hep,cirr\})| \\
 &= |0.2 - 0.2670| = 0.0670, \\
 m_{123}(\{hep,cirr\})' &= \frac{m_{123}(\{hep,cirr\}) + K_{\{hep,cirr\}}}{2} \\
 &= 0.0742,
 \end{aligned}$$

Table 2. Combination result comparisons for two different sources of evidence

Hypotheses	Dempster's rule	Proposed rule
	m_{12}	\hat{m}_{12}
$\{cirr\}$	0.42	0.2890
$\{hep,cirr\}$	0.18	0.2670
$\{cirr,gall,pan\}$	0.28	0.3220
Θ	0.12	0.1220

Table 3. Combination result comparisons for three different sources of evidence

Hypotheses	Dempster's rule	Proposed rule
	m_{123}	\hat{m}_{123}
$\{hep\}$	0.545	0.6844
$\{cirr\}$	0.191	0.1090
$\{hep,cirr\}$	0.082	0.0580
$\{cirr,gall,pan\}$	0.127	0.0970
Θ	0.055	0.0516

$$\begin{aligned}
 K_{\{cirr,gall,pan\}} &= |m_3(\Theta) - \hat{m}_{12}(\{cirr,gall,pan\})| \\
 &= |0.2 - 0.3220| = 0.1220, \\
 m_{123}(\{cirr,gall,pan\})' &= \frac{m_{123}(\{cirr,gall,pan\})}{2} \\
 &\quad + \frac{K_{\{cirr,gall,pan\}}}{2} = 0.1247, \\
 K_{\Theta} &= |m_3(\Theta) - \hat{m}_{12}(\Theta)| = |0.2 - 0.1220| = 0.0780, \\
 m_{123}(\Theta)' &= \frac{m_{123}(\Theta) + K_{\Theta}}{2} = 0.0660.
 \end{aligned}$$

After applying Eq. (7) into the above combination results, we can obtain:

$$\begin{aligned}
 \hat{m}_{123}(\{hep\}) &= 0.6844, \\
 \hat{m}_{123}(\{cirr\}) &= 0.1090, \\
 \hat{m}_{123}(\{hep,cirr\}) &= 0.0580, \\
 \hat{m}_{123}(\{cirr,gall,pan\}) &= 0.0970, \\
 \hat{m}_{123}(\Theta) &= 0.0516.
 \end{aligned}$$

Overall, we calculated the combination results by the two different combination rules (Dempster's rule and our proposed combination rule) based on the same example. The combination results (m_{12} and \hat{m}_{12}) of two sources' evidences are shown in Table 2 and the combination results (m_{123} and \hat{m}_{123}) of three sources' evidences are shown in Table 3, respectively.

In Table 2, we can see that the combination results of the two different combination rules provide the significant different values. A possible reason for the different values is explained accordingly.

In combination result $\hat{m}_{12}(\{cirr\})$ for the hypothesis $\{cirr\}$ is significantly different by our proposed combination rule in comparison to the combination result $m_{12}(\{cirr\})$ of Dempster's rule. The reason behind this result is that the hypothesis $\{cirr\}$ exists in all subsets of combination results:

$$\begin{aligned} \{cirr\} &\subset \{hep, cirr\}, \\ \{cirr\} &\subset \{cirr, gall, pan\}, \\ \{cirr\} &\subset \Theta. \end{aligned}$$

According to Dempster-Shafer theory, at least one answer (that contains only single hypothesis) must be true. However, all the combination results (m_{12} shown in Table 2) contain the hypothesis $\{cirr\}$. Thus, the combination result obtained by our proposed combination rule decreases the result of $m_{12}(\{cirr\})$ from 0.42 to 0.2890.

From logical point of view, result of our proposed combination rule is acceptable. The other combination result, $\hat{m}_{12}(\{hep, cirr\})$ for the hypotheses $\{hep, cirr\}$ is greater than the result of Dempster's rule. This result is also logically correct. A possible reason is that the hypothesis $\{cirr\}$ appeared in only two subsets:

$$\begin{aligned} \{cirr\} &\subset \{hep, cirr\}, \\ \{cirr\} &\subset \Theta. \end{aligned}$$

In the same manner, the combination result, $\hat{m}_{12}(\{cirr, gall, pan\})$ for the hypotheses $\{cirr, gall, pan\}$ are shown in only two subsets, $\{cirr, gall, pan\}$ and Θ . The combination result $\hat{m}_{12}(\Theta)$ remains the same as the result of Dempster's rule.

In Table 3, the combination result obtained by our proposed combination rule assigns different values in compared with the results of Dempster's rule. More particularly, the combination result containing the hypothesis $\{hep\}$ is appeared in three subsets as presented in the first column of Table 3. The hypothesis $\{cirr\}$ is shown in four subsets. Therefore, the value of combination result containing the hypothesis $\{hep\}$ is greater than the value of combination result of Dempster's rule. Reversely, the combination result contain the hypothesis $\{cirr\}$ is smaller than the result of Dempster's rule.

Therefore, the combination result obtained by our proposed

combination rule produces more logical conclusion than the combination result obtained by Dempster's rule.

In Dempster-Shafer theory, we finally need to calculate the belief function for given hypotheses. Then we calculated the belief for each hypothesis using the combination results obtained by two different combination rules as follows:

Based on Dempster's rule of combination, the belief for hypotheses using the combination results (i.e., m_{12}) present the following:

$$\begin{aligned} Bel(\{cirr\}) &= m_{12}(\{cirr\}) = 0.42, \\ Bel(\{hep, cirr\}) &= m_{12}(\{hep, cirr\}) + m_{12}(\{cirr\}) \\ &= 0.18 + 0.42 = 0.6, \\ Bel(\{cirr, gall, pan\}) &= m_{12}(\{cirr, gall, pan\}) \\ &\quad + m_{12}(\{cirr\}) = 0.28 + 0.42 \\ &= 0.7. \end{aligned}$$

The belief for hypotheses of the combination results (i.e., m_{123}) show the following:

$$\begin{aligned} Bel(\{hep\}) &= m_{123}(\{hep\}) = 0.545, \\ Bel(\{cirr\}) &= m_{123}(\{cirr\}) = 0.191, \\ Bel(\{hep, cirr\}) &= m_{123}(\{hep, cirr\}) + m_{123}(\{cirr\}) \\ &\quad + m_{123}(\{hep\}) = 0.082 + 0.191 + 0.545 \\ &= 0.818, \\ Bel(\{cirr, gall, pan\}) &= m_{123}(\{cirr, gall, pan\}) \\ &\quad + m_{123}(\{cirr\}) = 0.127 + 0.191 \\ &= 0.318. \end{aligned}$$

Based on our proposed combination rule, the belief for the hypotheses of combination results (i.e., \hat{m}_{12}) are shown below:

$$\begin{aligned} Bel(\{cirr\}) &= \hat{m}_{12}(\{cirr\}) = 0.2890, \\ Bel(\{hep, cirr\}) &= \hat{m}_{12}(\{hep, cirr\}) + \hat{m}_{12}(\{cirr\}) \\ &= 0.2670 + 0.2890 = 0.5560, \\ Bel(\{cirr, gall, pan\}) &= \hat{m}_{12}(\{cirr, gall, pan\}) \\ &\quad + \hat{m}_{12}(\{cirr\}) = 0.3220 + 0.2890 \\ &= 0.6110. \end{aligned}$$

The belief for hypotheses of combination results (i.e., \hat{m}_{123}) are:

$$Bel(\{hep\}) = \hat{m}_{123}(\{hep\}) = 0.6844,$$

Table 4. Belief comparisons for three different sources of evidence

Hypotheses	Bel(H)	
	Dempster's rule	Proposed rule
{ <i>hep</i> }	0.545	0.6844
{ <i>cirr</i> }	0.191	0.1090
{ <i>hep, cirr</i> }	0.818	0.8514
{ <i>cirr, gall, pan</i> }	0.318	0.2060

$$\begin{aligned}
 Bel(\{cirr\}) &= \hat{m}_{123}(\{cirr\}) = 0.1090, \\
 Bel(\{hep, cirr\}) &= \hat{m}_{123}(\{hep, cirr\}) + \hat{m}_{123}(\{cirr\}) \\
 &\quad + \hat{m}_{123}(\{hep\}) \\
 &= 0.580 + 0.1090 + 0.6844 = 0.8514, \\
 Bel(\{cirr, gall, pan\}) &= \hat{m}_{123}(\{cirr, gall, pan\}) \\
 &\quad + \hat{m}_{123}(\{cirr\}) \\
 &= 0.0970 + 0.1090 = 0.2060.
 \end{aligned}$$

Tables 1 and 4 show comparisons of the beliefs for the hypotheses between Dempster's rule and our proposed combination rule.

First, in Table 1 the beliefs for the hypotheses {*cirr*}, {*hep, cirr*} and {*cirr, gall, pan*} assign different values based on the combination results by the two different combination rules. In particular, the belief $Bel(\{cirr\})$ for the hypothesis {*cirr*} was significantly reduced by our proposed combination rule due to the existence of the hypothesis {*cirr*} in subsets of all the three beliefs $Bel(\{cirr\})$, $Bel(\{hep, cirr\})$ and $Bel(\{cirr, gall, pan\})$. The beliefs for the hypotheses {*hep, cirr*} and {*cirr, gall, pan*} were reduced with small amounts. The reason is that the hypothesis {*hep*} exists only in $Bel(\{hep, cirr\})$ and the hypotheses {*gall, pan*} exists only in $Bel(\{cirr, gall, pan\})$. Therefore, the results of our proposed combination rule provide better logical results than Dempster's rule.

Second, in Table 4, the beliefs that contain the hypothesis {*hep*} obtained by our proposed combination rule were increased in compared with the belief obtained by Dempster's rule. However, the beliefs for the hypothesis {*cirr*} based on our proposed combination rule was reduced in compared with the beliefs from Dempster's rule. A possible reason behind these results is that the hypothesis {*hep*} exists only in $Bel(\{hep\})$ and $Bel(\{hep, cirr\})$, and the hypothesis {*cirr*} exists in $Bel(\{cirr\})$, $Bel(\{hep, cirr\})$ and $Bel(\{cirr, gall, pan\})$.

From the results of our proposed combination rule, we can

Table 5. Results of Dempster's rule and proposed combination rule in conflicting and non-conflicting evidence case

	<i>X</i>	<i>Y</i>	$Bel(H_i)$	Dempster's rule	Proposed rule
1	0.01	0.01	<i>T</i>	1	0.3356
	0.99	0	<i>M</i>	0	0.3322
	0	0.99	<i>C</i>	0	0.3322
2	0.45	0.45	<i>T</i>	1	0.4762
	0.55	0	<i>M</i>	0	0.2619
	0	0.55	<i>C</i>	0	0.2619
3	0.99	0.99	<i>T</i>	1	0.9804
	0.01	0	<i>M</i>	0	0.0098
	0	0.01	<i>C</i>	0	0.0098
4	0.7	0.7	<i>T</i>	0.9160	0.9160
	0.15	0.15	<i>M</i>	0.0420	0.0420
	0.15	0.15	<i>C</i>	0.0420	0.0420

conclude the following: 1) if a certain hypothesis occurs only in a few subsets that contain various hypotheses, the belief for the hypothesis increases, 2) if a certain hypothesis occurs in most subsets, the belief for the hypothesis decreases.

Therefore, the results found from the beliefs showed that our proposed combination rule provides better logical conclusions in contrast to the results found from Dempster's rule of combination.

4. Numerical Examples

In this section, three different examples are used to validate the result of our proposed combination rule against other combination rules.

The first fictitious example is shown in Table 5. This example contains four different cases with two conflicting and two non-conflicting evidences observed by two independent sources *X* and *Y*. Case 1 is Zadeh's highly conflicting evidence example, case 2 is also conflicting evidence example and case 3 and 4 are non-conflicting examples. On the basis of the four cases, Dempster's rule of combination and our proposed combination rule were compared. After identifying the combination results, we can calculate the belief Eq. (2) for each hypothesis using the results of the two combination rules.

The second fictitious example used in [4] has four different sources of evidences shown in Table 6 with three hypotheses, *A*, *B* and *C*. For consistency, we changed letters *A*, *B* and *C*

Table 6. Evidences from four different sources

$m_1(T) = 0.98$	$m_1(M) = 0.01$	$m_1(C) = 0.01$
$m_2(T) = 0$	$m_2(M) = 0.01$	$m_2(C) = 0.99$
$m_3(T) = 0.90$	$m_3(M) = 0.01$	$m_3(C) = 0.09$
$m_4(T) = 0.90$	$m_4(M) = 0.01$	$m_4(C) = 0.09$

Table 7. Results of combination and belief for each hypothesis by different combination rules

Combination rules	$Bel(H_i)$	m_{12}	m_{123}	m_{1234}
Dempster's rule	T	0	0	0
	M	0.01	0.0011	0.0001
	C	0.99	0.9989	0.9999
Rule proposed in [4]	T	0.4703	0.7431	0.8358
	M	0.01	0.01	0.01
	C	0.5197	0.2469	0.1542
Rule proposed in [5]	T	0.4924	0.7016	0.8075
	M	0.0051	0.0059	0.0068
	C	0.5025	0.2925	0.1857
Our proposed rule	T	0.3311	0.6515	0.7930
	M	0.0034	0.0031	0.0046
	C	0.6655	0.3454	0.2024

Table 8. Evidences from five different sources

$m_1(T) = 0.50$	$m_1(M) = 0.20$	$m_1(C) = 0.30$
$m_2(T) = 0$	$m_2(M) = 0.90$	$m_2(C) = 0.10$
$m_3(T) = 0.55$	$m_3(M) = 0.01$	$m_3(C) = 0.35$
$m_4(T) = 0.55$	$m_4(M) = 0.01$	$m_4(C) = 0.35$
$m_5(T) = 0.60$	$m_5(M) = 0.01$	$m_5(C) = 0.30$

into T , M and C respectively. The results of the four different combination rules in Table 7 were compared using the example below.

The third fictitious example used in [5] has five different sources of evidences presented in Table 8 with three hypotheses, A , B and C . For consistency, we changed letters A , B and C into T , M and C respectively.

The results of the four different combination rules in Table 9 were compared based on the example given in Table 8.

5. Discussion

In this paper, three different comparisons with three different examples were carried out to validate our proposed combination

Table 9. Results of combination and belief for each hypothesis by different combination rules

Combination rules	$Bel(H_i)$	m_{12}	m_{123}	m_{1234}	m_{12345}
Dempster's rule	T	0	0	0	0
	M	0.8571	0.6316	0.3288	0.1404
	C	0.1429	0.3684	0.6712	0.8596
Rule proposed in [4]	T	0.1543	0.4861	0.7773	0.8909
	M	0.7569	0.3481	0.0628	0.0086
	C	0.0988	0.1657	0.1600	0.1005
Rule proposed in [5]	T	0.2360	0.3888	0.4797	0.5588
	M	0.6032	0.3771	0.2375	0.1586
	C	0.1608	0.2341	0.2828	0.2816
Our proposed rule	T	0.2083	0.4010	0.5117	0.6453
	M	0.6488	0.3964	0.2613	0.1712
	C	0.1429	0.2026	0.2270	0.1835

rule accordingly.

First, the result of our proposed combination rule was compared with the result of Dempster's rule of combination on the basis of four different cases shown in Table 5. The cases include Zadeh's highly conflicting evidence example in case 1, another conflicting example in case 2, and non-conflicting evidences examples in case 3, 4 in Table 5 and the cases were compared.

Next, the belief for each hypothesis (i.e., T , C and M) were calculated based on the result from the two combination rules and compared in Table 5.

The counterintuitive conclusion (mentioned in the last part of Section 2 drawn by Dempster's rule for case 1 in Table 5 does not appear in the conclusion drawn by our proposed combination rule. In particular, the beliefs for each hypothesis are very similar due to unreliable sources of evidence. In case 2, Dempster's rule also demonstrates a counterintuitive conclusion when the sources of evidence are unreliable. However, the conclusions are drawn from our proposed combination rule is reasonable. The results of non-conflicting evidences for case 3, 4 in Table 5 observed by the two combination rules are both reasonable and provide similar conclusions.

Second, the evidence example used in [4], those are presented in Table 6, were combined by four different combination rules and the results were compared in Table 7. In the comparison, the results of Dempster's rule of combination also showed an unacceptable conclusion when one source provides no evidence for the hypothesis (i.e., $m_2(T) = 0$). Thus, the final result of combination for the hypothesis T became zero shown in the

second row of Table 7.

The results of our proposed combination rule and other two combination rules defined in [4] and [5] show reasonable conclusions for each hypothesis.

Third, the evidence example can be seen in Table 8 used in [5] were combined by four different combination rules and the results were compared in Table 9. The result identified from Dempster's rule of combination is shown in the second row of Table 9. According to the table, we can see the problem when one source provides no evidence for a hypothesis, it strongly affects to the final combination result for the hypothesis. The results of three combination rules, [4], [5], and our proposed combination rule showed reasonable and competitive results on the belief for each hypothesis.

None of the methods can be stated to be the best because utility of a particular method depends upon the problem under consideration [5].

6. Conclusion

In this paper, we introduced a modified combination rule using the partial conflict measurement.

The combination results found from our proposed combination rule performed more reliable results in contrast to Dempster's rule. In particular, the results of our proposed combination rule give reasonable conclusions when combining highly conflicting evidences that contain a single hypothesis which involves zero elements. The results of our proposed combination rule also provide more logical conclusions when combining evidences with multiple hypotheses.

Our proposed combination rule is computationally efficient that allow us to apply the rule into practical situations for information fusion. The rule is more conceptually clear and accurate in comparison to other combination rules.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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