Chaotic Behavior in Model with a Gaussian Function as External Force

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Abstract

In this paper, we propose a novel dynamical love model of Romeo and Juliet, which has an external force with a fuzzy membership function. The external force used in the model has the characteristics of a Gaussian function. The chaotic behavior in the model is demonstrated using time series and phase portraits.

Keywords: Chaotic phenomena, Nonlinear dynamic, Love model, Time series, Phase portrait, External force, Gaussian function

1.   Introduction

In general, the human society is modelled as a complex system. Complex systems possess the characteristics of a nonlinear dynamical system, which can be described by a differential equation. However, compared with linear systems, nonlinear systems are difficult to analyze, synthesize, or implement. All natural systems such as wind, weather, and human behavior are nonlinear systems, which include fuzzy systems, neural networks, chaotic systems, and complex systems. Among these nonlinear systems, chaotic systems have gained considerable interest over the last three decades among many researches in the field of mathematics, physics, chemistry, engineering, and social science.

In particular, studies on chaotic behaviors in the social sciences, including those related to habits and the human mind, such as addiction [1–4], happiness [5–8], and love models [8–13], involve an overlap of the fields of mathematics, biology, psychology, and social science.

Bae and his colleagues [1–4] proposed a mathematical model for addictions to digital leisure, internet, tobacco, and physical exercise. Further, they also verified the presence of nonlinear behaviors or chaotic phenomena by using time series and phase portraits.

Sprott [6, 8] proposed a basic dynamic equation for happiness and investigated its behavior. Bae [7] proposed a mathematical happiness model and a synchronization technique for the model.

Strogatz [14, 15] was the first to suggest modelling love affairs by using differential equations; for example, a simple model for the ill-fated romance of Romeo and Juliet can be modelled as a second-linear system. He also defined love affairs according to parameter values.

Inspired by Strogatz's research [14, 15], Sprott [8] proposed a love model based on Romeo and Juliet, which was represented by the linear differential equation; Sprott also described the

linear as well as nonlinear behaviors of the model.

Several models have been proposed for love: the Romeo and Juliet model [9–13], the Laura and Petrarch model [16, 17], the Adam and Eve model [18], and others [19–26]. Among these, the love model based on Romeo and Juliet is most commonly employed in the study of nonlinear dynamics.

The existence of periodic motion and chaotic behavior or motion in the love model based on Romeo and Juliet is represented through time series and phase portraits, with either the same or different time delays, an external force, and different external forces proposed by Bae [24, 25].

However, most previously published papers have not provided an exact time series or phase portrait to demonstrate chaotic phenomena or behavior, including periodic motion, in the love model of Romeo and Juliet.

In this paper, we propose a dynamic mathematical model of love with the Gaussian function as an external force. We also investigate the chaotic behaviors in the love models based on Romeo and Juliet with the Gaussian function as an external force, using time series and phase portraits. In order to describe the nonlinear phenomena using time series and phase portraits, we vary the magnitude of the Gaussian function as an external force. We also change the parameter values of the Romeo and Juliet model.

2. Love Model

There are many love models; we can classify these models as follows:

2.1 Basic Love Model

The basic love model proposed by Sprott [17] can be formulated as

$$\frac{dR}{dt} = aR + bJ,$$
$$\frac{dJ}{dt} = cR + dJ, \tag{1}$$

where a and b specify Romeo's romantic style, and c and d specify Juliet's style.

2.2 Alternative Love Model

An alternative love model, also proposed by Sprott [8], can be written as

$$\frac{dR}{dt} = aR + bJ(1 - |J|),$$

$$\frac{dJ}{dt} = cR(1 - |R|) + dJ, \tag{2}$$

where a and b specify Romeo's romantic style, and c and d specify Juliet's style.

2.3 Alternative Love Model with an External Force

Eqs.(1) and (2) cannot be used to model chaotic phenomena, because these represent second-order systems. To generate chaotic phenomena in a dynamic model, the model must involve a third-order system, and it must contain at least one nonlinear term. Because (1) and (2) for the Romeo and Juliet model do not satisfy the necessary conditions for a third-order system, they must be modified to satisfy the condition. An alternative love model proposed by Bae [24], which involves external forces, can be written as

$$\frac{dR}{dt} = aR + bJ(1 - |J|) + f(t),$$
$$\frac{dJ}{dt} = cR(1 - |R|) + dJ, \tag{3}$$

or

$$\frac{dR}{dt} = aR + bJ(1 - |J|),$$
$$\frac{dJ}{dt} = cR(1 - |R|) + dJ + y(t) \tag{4}$$

or

$$\frac{dR}{dt} = aR + bJ(1 - |J|) + f(t),$$
$$\frac{dJ}{dt} = cR(1 - |R|) + dJ + y(t), \tag{5}$$

where $f(t)$ and $y(t)$ are external forces applied to the Romeo and Juliet differential equations, respectively.

Bae et al. [24, 25] demonstrated the existence of chaotic phenomena when $f(t) = \sin\omega t$ and $y(t) = 0$ in (3), $f(t) = 0$ and $y(t) = \sin\omega t$ in (4), and $f(t) = \sin\omega t$ and $y(t) = \sin\omega t$ in (5), using time series and phase portraits, which include periodic motions and chaotic attractors.

3. Chaotic Behaviors in Love Model with Gaussian Function as an External Force

In this paper, we consider the chaotic behaviors in (3), (4), and (5) with the Gaussian function as an external force using computer simulation with MATLAB. In addition, we apply different parameters for (3), (4), and (5). Eq. (6) provides the
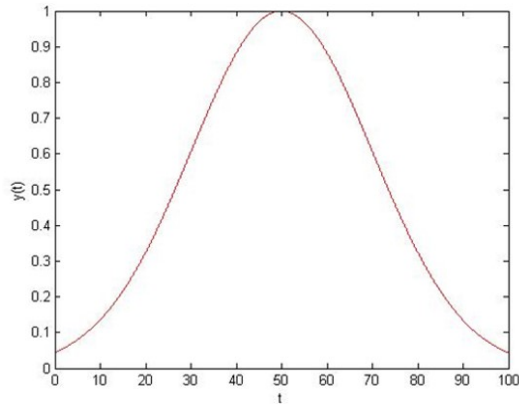
Figure 1. Gaussian function.

Gaussian function that is used as an external force, and Figure 1 shows its time series.

$$f(x) = \frac{1}{\sigma\sqrt{2}} \exp\left(-\frac{x-\mu}{2\sigma^2}\right). \qquad (6)$$

In the following section, the time series and phase portrait for (2) and (3)–(5) are reviewed with different parameter values, in different cases, when no external force is applied, and when external force is applied as a Gaussian function.

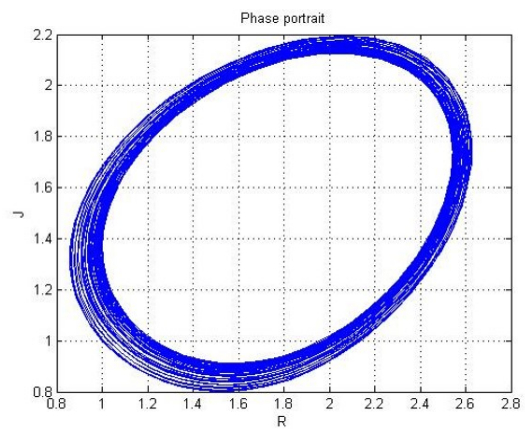## 3.1 Case $a = -1$, $b = -2$, $c = 1$, $d = 1$

First, we investigate the time series and phase portrait to show the behavior of love for Romeo and Juliet, when we set the parameters as $a = -1$, $b = -2$, $c = 1$, and $d = 1$ in (2), which has the characteristics of an alternative love model. In this case, because there is no external force applied to Romeo and Juliet, they are not affected by any influence from their parents, friends, and relatives. Figure 2 illustrates the results of the time series and phase portrait, when the alternative love equations are applied for Romeo and Juliet without external forces.

3.2 Case $a = -2$, $b = -2$, $c = 1$, $d = 1$

We investigate the time series and phase portrait to show the behavior of love for Romeo and Juliet when we set the parameters as $a = -2$, $b = -2$, $c = 1$, and $d = 1$ in (2), which has the characteristics of an alternative love model. Figure 3 shows the results of the time series and phase portrait, when we apply the alternative love equations for Romeo and Juliet without external forces.
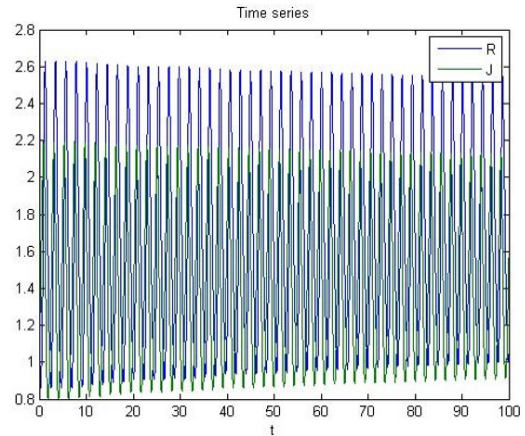
(a)

(b)

Figure 2. Results of (a) time series and (b) phase portrait when (2) is applied with $a = -1$, $b = -2$, $c = 1$, and $d = 1$ parameters.

3.3 Case $a = -1$, $b = -2$, $c = 1$, $d = 1$, $f(t) = Gaussian function$

We now investigate the chaotic attractor to discover the nature of the chaotic phenomena, when we set the parameters as $a = -1$, $b = -2$, $c = 1$, and $d = 1$ with the Gaussian function $f(t) = Gaussmf(t, (20, 50))$ as an external force in (3). Figure 4 illustrates the results of the time series and phase portrait with these parameters.

3.4 Case $a = -2$, $b = -2$, $c = 1$, $d = 1$, $f(t) = Gaussian function$

We now investigate the chaotic attractor to discover the nature of the chaotic phenomena, when we set the parameters as $a = -2$, $b = -2$, $c = 1$, and $d = 1$ with the Gaussian function $f(t) = Gaussmf(t, (20, 50))$ as an external force in (3). Figure 5
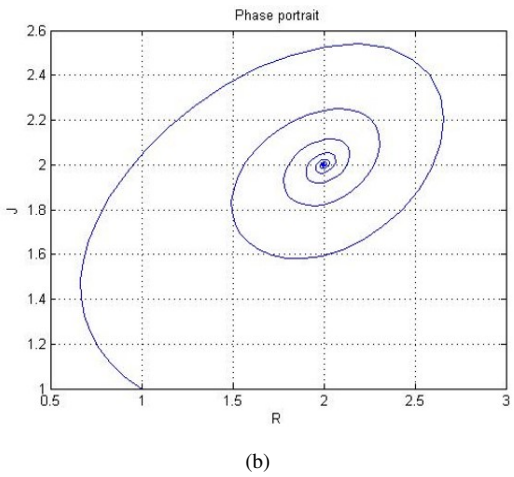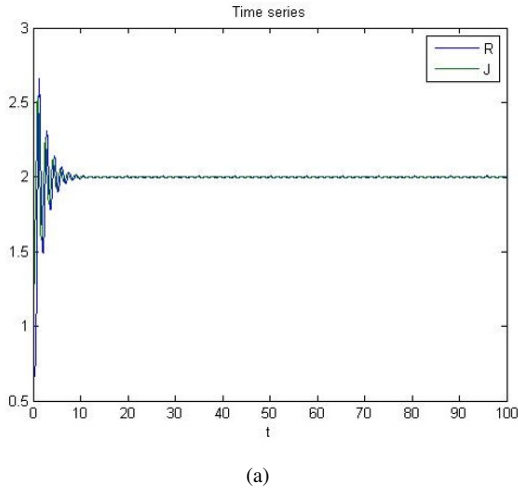
(a)

(b)

Figure 3. Results of (a) time series and (b) phase portrait when (2) is applied with $a = -2$, $b = -2$, $c = 1$, and $d = 1$ parameters.

(a)

(b)

Figure 4. Results of (a) time series and (b) phase portrait when external force is applied as $f(t) = Gaussmf(t, 20, 50)$ with $a = -1$, $b = -2$, $c = 1$, and $d = 1$.

illustrates the results of the time series and phase portrait with these parameters.

On comparing Figures 4 and 5, we recognize that there is a significant difference when the Gaussian function is applied as an external force $f(t)$ with different parameters.

3.5 Case $a = -1$, $b = -2$, $c = 1$, $d = 1$, $f(t) = 5 * Gaussian function$

We now investigate the chaotic attractor to discover the nature of the chaotic phenomena when we set the parameters as $a = -1$, $b = -2$, $c = 1$, and $d = 1$ with the Gaussian function $f(t) = 5 * Gaussmf(t, (20, 50))$ as an external force in (3). Figure 6 illustrates the results of the time series and phase portrait with these parameters.

By comparing Figures 5 and 6, it is observed that there is a clear difference when the magnitude of the Gaussian function
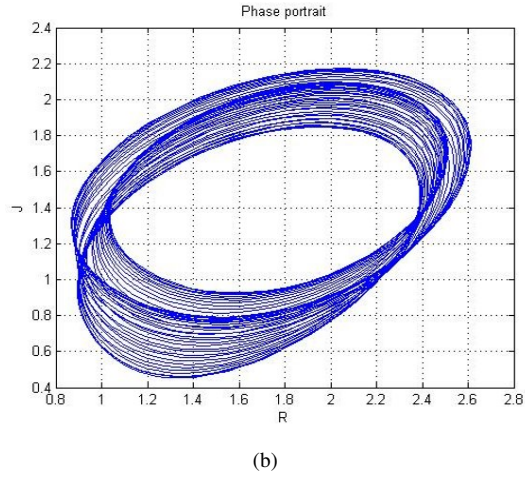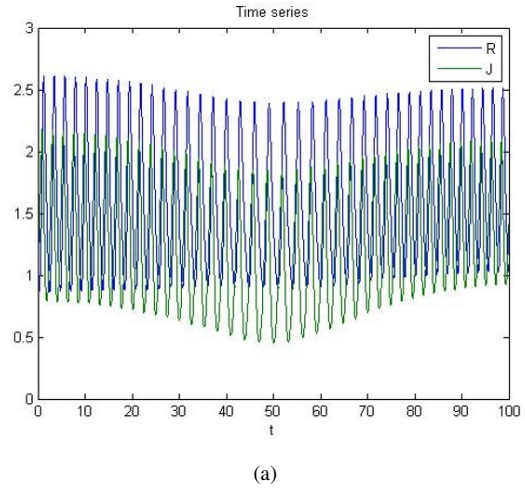
is varied as an external force. This means that intensity of an external force will affect the love behavior of Romeo and Juliet.

3.6 Case $a = -2$, $b = -2$, $c = 1$, $d = 1$, $f(t) = 5 * Gaussian function$

We now investigate the chaotic attractor to discover the nature of the chaotic phenomena when we set the parameters as $a = -2$, $b = -2$, $c = 1$, and $d = 1$ with the Gaussian function $f(t) = 5 * Gaussmf(t, (20, 50))$ as an external force in (3). Figure 7 illustrates the results of the time series and phase portrait with these parameters.

On comparing Figures 5-7, we recognize that there is a clear difference when we vary the magnitude of the external force. Therefore, we conjecture that the love between Romeo and Juliet is affected by many factors, including their own states of
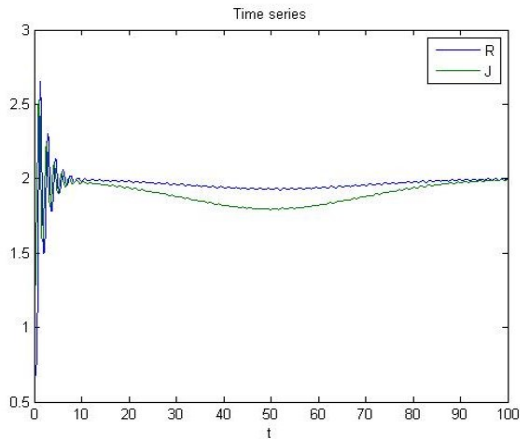
(a)
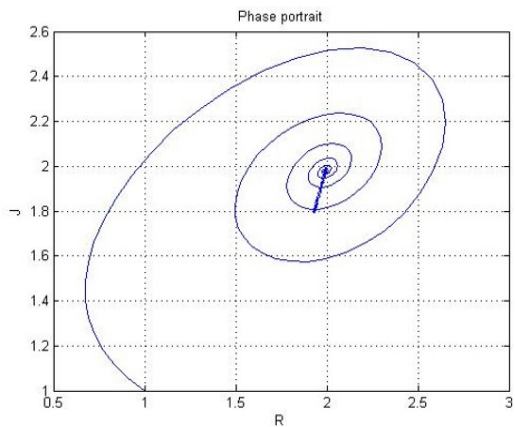
(b)

Figure 5. Results of (a) time series and (b) phase portrait when external force is applied as $f(t) = Gaussmf(t, 20, 50)$ with $a = -2, b = -2, c = 1$, and $d = 1$.

mind, and their current circumstances.

4. Conclusions and Remarks

In this paper, we proposed a dynamic mathematical model of love with a Gaussian function as an external force. We also investigated the chaotic behaviors in love models of Romeo and Juliet with the Gaussian function as an external force, using time series and phase portraits. In order to describe the nonlinear phenomena using time series and phase portraits, we vary the magnitude of the Gaussian function as an external force. We also change the parameter values of Romeo and Juliet's love model.

Two important facts were recognized from the magnitude of the external force. First, we recognize that there is a clear
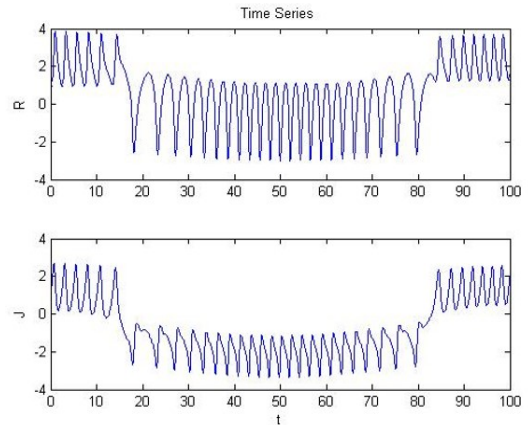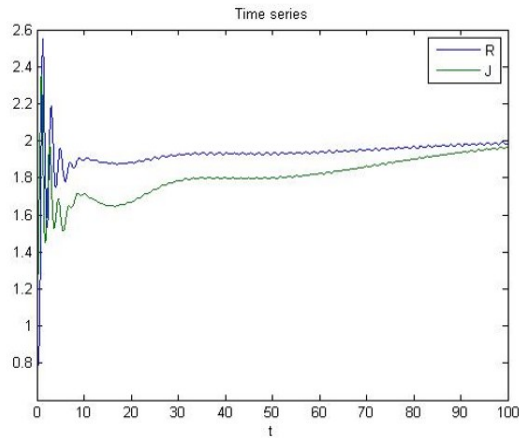

(a)
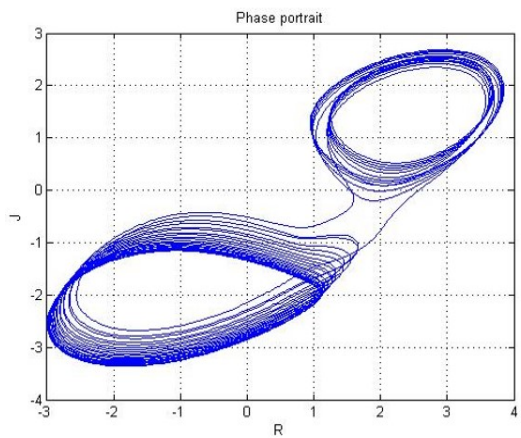
(b)

Figure 6. Results of (a) time series and (b) phase portrait when we apply the external forces as $f(t) = 5 * Gaussmf(t, 20, 50)$ with $a = -1, b = -2, c = 1$, and $d = 1$.

difference in the results as the external force $f(t)$ is varied. This implies that the status of love between Romeo and Juliet can be affected by external forces such as the opinions of parents, friends, and other family members. Second, there is a clear difference in the results in cases where both the parameters and external forces are changed. Therefore, we conjecture that the love between Romeo and Juliet is affected by many factors, such as their own states of mind and the present environment, including their friends, relatives, and parents.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

(a) Time series.

(b) Phase portrait.

Figure 7. Results of the time series and phase portrait when we apply the external forces as $f(t) = 5 * Gaussmf(t, 20, 50)$ with $a = -2$, $b = -2, c = 1$, and $d = 1$.
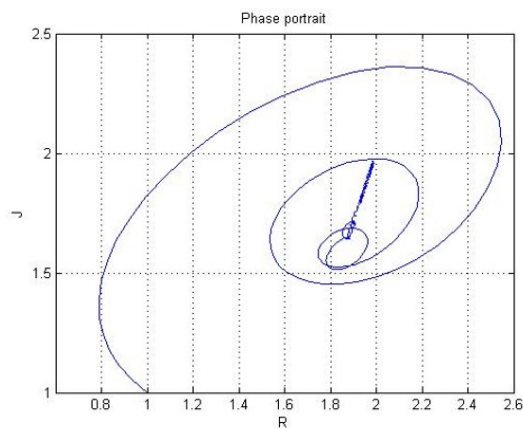
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