

A Simple Method for Solving Type-2 and Type-4 Fuzzy Transportation Problems

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Abstract

In conventional transportation problem (TP), all the parameters are always certain. But, many of the real life situations in industry or organization, the parameters (supply, demand and cost) of the TP are not precise which are imprecise in nature in different factors like the market condition, variations in rates of diesel, traffic jams, weather in hilly areas, capacity of men and machine, long power cut, labourer's over time work, unexpected failures in machine, seasonal changes and many more. To counter these problems, depending on the nature of the parameters, the TP is classified into two categories namely type-2 and type-4 fuzzy transportation problems (FTPs) under uncertain environment and formulates the problem and utilizes the trapezoidal fuzzy number (TrFN) to solve the TP. The existing ranking procedure of Liou and Wang (1992) is used to transform the type-2 and type-4 FTPs into a crisp one so that the conventional method may be applied to solve the TP. Moreover, the solution procedure differs from TP to type-2 and type-4 FTPs in allocation step only. Therefore a simple and efficient method denoted by PSK (P. Senthil Kumar) method is proposed to obtain an optimal solution in terms of TrFNs. From this fuzzy solution, the decision maker (DM) can decide the level of acceptance for the transportation cost or profit. Thus, the major applications of fuzzy set theory are widely used in areas such as inventory control, communication network, aggregate planning, employment scheduling, and personnel assignment and so on.

Keywords: Fuzzy set, Trapezoidal fuzzy number, Type-2 fuzzy transportation problem, Type-4 fuzzy transportation problem, PSK method, Optimal solution

1. Introduction

The transportation problem is a special kind of linear programming problem that deals with shipping a homogenous product from any group of sources (Factories) to any group of destinations (Warehouses). The objective of the transportation problem is to determine the shipping schedule that minimizes the total transportation cost (or maximizes the total transportation profit) while satisfying supply and demand limits.

In literature, Hitchcock [1] originally developed the basic transportation problem. Arsham and Kahn [2] proposed a simplex type algorithm for general transportation problems. Charnes and Cooper [3] developed the stepping stone method which provides an alternative way of determining the simplex method information. Appa [4] discussed several variations of the transportation problem. An introduction to operations research Taha [5] deals the transportation problem.

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In today's real world problems such as in corporate or in industry many of the distribution problems are imprecise in nature due to variations in the parameters. To deal quantitatively with imprecise information in making decision, Zadeh [6] introduced the fuzzy set theory and has applied it successfully in various fields. The use of fuzzy set theory becomes very rapid in the field of optimization after the pioneering work done by Bellman and Zadeh [7]. The fuzzy set deals with the degree of membership (belongingness) of an element in the set. In a fuzzy set the membership value (level of acceptance or level of satisfaction) lies between 0 and 1 where as in crisp set the element belongs to the set represent 1 and the element not belongs to the set represent 0.

Due to the applications of fuzzy set theory, Dinagar and Palanivel [8] investigated the transportation problem in fuzzy environment using trapezoidal fuzzy numbers (TrFN). Pandian and Natarajan [9] proposed a new algorithm for finding a fuzzy optimal solution for fuzzy transportation Problem (FTP) where all the parameters are TrFNs. Mohideen and Kumar [10] did a comparative study on transportation problem in fuzzy environment. Gani and Razak [11] obtained a fuzzy solution for a two stage cost minimizing FTP in which availabilities and requirements are TrFNs using a parametric approach. Sudhakar and Kumar [12] proposed a different approach for solving two stage FTPs in which supplies and demands are TrFNs. Rani et al. [13] presented a method for unbalanced transportation problems in fuzzy environment taking all the parameters are TrFNs. Gani et al. [14] presented simplex type algorithm for solving FTP where all the parameters are triangular fuzzy numbers. Solaiappan and Jeyaraman [15] did a new optimal solution method for trapezoidal FTP. Basirzadeh [16] discussed an approach for solving FTP where all the parameters are TrFNs.

OhEigeartaigh [17] presented an algorithm for solving transportation problems where the availabilities and requirements are fuzzy sets with linear or triangular membership functions. Chanas et al. [18] presented a fuzzy linear programming model for solving transportation problems with fuzzy supply, fuzzy demand and crisp costs. Saad and Abass [19] proposed an algorithm for solving the transportation problems under fuzzy environment. Das and Baruah [20] discussed Vogel's approximation method to find the fuzzy initial basic feasible solution of FTP in which all the parameters (supply, demand and cost) are represented by triangular fuzzy numbers. De and Yadav [21] modified the existing method (Kikuchi [22]) by using TrFNs instead of triangular fuzzy numbers. Chanas et al. [23] formulated the FTPs in three different situations and proposed method

for solving the FTPs. Chanas and Kuchta [24] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution.

Chanas and Kuchta [25] developed a new method for solving fuzzy integer transportation problem by representing the supply and demand parameters as L-R type fuzzy numbers. Chiang [26] proposed a method to find the optimal solution of transportation problems with fuzzy requirements and fuzzy availabilities. Li et al. [27] proposed a new method based on goal programming approach for solving FTPs with fuzzy costs. Chen et al. [28] proposed the methods for solving transportation problems on a fuzzy network. Lin [29] used genetic algorithm for solving transportation problems with fuzzy coefficients. Kumar [30] formulated the FTPs in two different situations and proposed method namely PSK method for solving the formulated FTPs. Chen and Hsieh [31] discussed "Graded mean integration representation of generalized fuzzy numbers" which is used to compare any two TrFNs. Therefore, the number of authors has solved FTPs. To the best of our knowledge, some of the authors discussed a simple method for solving FTP but none of them proved mathematically the solution obtained by simple method for solving FTP is optimal. In addition to that all the existing literature deals with conversion of FTP into crisp TP and it finds only the occupied cells but they couldn't find out how to allot the maximum possible value (supply or demand) to the occupied cells. So, the existing literature offers solution for limited number of sources and destinations only through inspection. But in our real life there is always a fair chance to have large number of sources and destinations. To counter this difficulty, in this paper proposed a solution by using PSK method.

In this paper, a new method called PSK method is proposed to find the optimal objective value of type-2 and type-4 FTP in single stage. The existing ranking procedure of Liou and Wang [32] is used to transform the type-2 and type-4 FTP into a crisp one so that the conventional method may be applied to solve the transportation problem. The occupied cells of crisp TP that we obtained are as same as the occupied cells of type-2 and type-4 FTP, but the value of occupied cells for type-2 and type-4 FTP is the maximum possible value of supply (or fuzzy supply) and demand (or fuzzy demand). On the basis of this idea the solution procedure differs from TP to FTP in allocation step only. Therefore, the new method called PSK method and new multiplication operation on TrFN is proposed to find the optimal solution in terms of TrFNs. The necessary theorems

are proved for the solution obtained by PSK method for solving a type-2 and type-4 FTP with equality constraints is an optimal solution (or fuzzy optimal solution) for the FTP. The optimum object value obtained by proposed method is always positive i.e., all entries of the objective value is in positive. It is much easier to apply the proposed method when compared to all the existing methods.

Rest of the paper is organized as follows: Section 2 deals with some terminology and new multiplication operation, Section 3 consists of ranking procedure and ordering principles of TrFN. Section 4 provides the definition of type-2 and type-4 FTP and its mathematical formulation, Section 5 consists of the PSK method, Section 6 provides the numerical example, results and discussion, finally the conclusion is given in Section 7.

2. Preliminaries

In this section, some basic definitions and new multiplication operation is given.

Definition 2.1. Let A be a classical set $\mu_A(x)$ be a function from A to $[0, 1]$. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by, $A^* = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0, 1]\}$.

Definition 2.2. A normal fuzzy set is one whose membership function has at least one element X in the universe whose membership value is unity. i.e., there exists an $x \in X$ such that $\mu_A(x) = 1$.

Definition 2.3. A fuzzy set A is convex if for any λ in $[0, 1]$, $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$.

Definition 2.4. The fuzzy number A is an extension of a regular number in the sense that it does not refers to one single value but rather to a connected set of possible values, where each possible values has its own weight between 0 and 1. The weight (membership function) denoted by $\mu_A(x)$ that satisfies the following conditions.

- i) $\mu_A(x)$ is piecewise continuous.
- ii) $\mu_A(x)$ is a convex fuzzy subset.
- iii) $\mu_A(x)$ is the normality of a fuzzy subset, implying that for at least one element x_0 the membership grade must be 1 (i.e.,) $\mu_A(x_0) = 1$.

Definition 2.5. A real fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is a fuzzy subset from the real line R with the membership function $\mu_{\tilde{a}}(a)$ satisfying the following conditions.

- i) $\mu_{\tilde{a}}(a)$ is a continuous mapping from R to the closed interval $[0, 1]$.

- ii) $\mu_{\tilde{a}}(a) = 0$ for every $a \in (-\infty, a_1]$.
- iii) $\mu_{\tilde{a}}(a)$ is strictly increasing and continuous on $[a_1, a_2]$.
- iv) $\mu_{\tilde{a}}(a) = 1$ for every $a \in [a_2, a_3]$.
- v) $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_3, a_4]$.
- vi) $\mu_{\tilde{a}}(a) = 0$ for every $a \in [a_4, +\infty)$.

Definition 2.6. We define a ranking function $R : F(R) \rightarrow R$, which maps each fuzzy number into the real line, $F(R)$ represents the set of all trapezoidal fuzzy numbers. If R be any ranking function, then

$$R(\tilde{a}) = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

Definition 2.7. Defuzzification is the conversion of a fuzzy quantity into crisp quantity.

Particular Cases

Let $\tilde{A} = [a_1, a_2, a_3, a_4]$ be a TrFN. Then the following cases arise

Case 1: If $a_2 = a_3 = a_2$ (say) then \tilde{A} represent triangular fuzzy number (TFN).

It is denoted by $\tilde{A} = (a_1, a_2, a_4)$.

Case 2: If $a_1 = a_2 = a_3 = a_4 = m$ then \tilde{A} represent a real number m .

Definition 2.8. Let $\tilde{A} = [a_1, a_2, a_3, a_4]$ and $\tilde{B} = [b_1, b_2, b_3, b_4]$ be two trapezoidal fuzzy numbers then the arithmetic operations on \tilde{A} and \tilde{B} are as follows:

- a) Addition: $\tilde{A} \oplus \tilde{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4]$
- b) Subtraction: $\tilde{A} \ominus \tilde{B} = [a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1]$
- c) New multiplication: $\tilde{A} \otimes \tilde{B} = [a_1\mathfrak{R}(\tilde{B}), a_2\mathfrak{R}(\tilde{B}), a_3\mathfrak{R}(\tilde{B}), a_4\mathfrak{R}(\tilde{B})]$, if $\mathfrak{R}(\tilde{B}) \geq 0$.
 $\tilde{A} \otimes \tilde{B} = [a_4\mathfrak{R}(\tilde{B}), a_3\mathfrak{R}(\tilde{B}), a_2\mathfrak{R}(\tilde{B}), a_1\mathfrak{R}(\tilde{B})]$, if $\mathfrak{R}(\tilde{B}) < 0$.

Scalar multiplication:

- i. $c\tilde{A} = [ca_1, ca_2, ca_3, ca_4]$, for $c \geq 0$.
- ii. $c\tilde{A} = [ca_4, ca_3, ca_2, ca_1]$, for $c < 0$.

3. Comparison of TrFN

The following definitions are used to compare any two TrFNs.

Definition 3.1. A trapezoidal fuzzy number $\tilde{A} = [a_1, a_2, a_3, a_4]$ is said to be non negative trapezoidal fuzzy number if and only if $\mathfrak{R}(\tilde{A}) \geq 0$.

Definition 3.2. A trapezoidal fuzzy number $\tilde{A} = [a_1, a_2, a_3, a_4]$ is said to be zero trapezoidal fuzzy number if and only if $\mathfrak{R}(\tilde{A}) = 0$.

Definition 3.3. Two trapezoidal fuzzy numbers $\tilde{A} = [a_1, a_2, a_3, a_4]$ and $\tilde{B} = [b_1, b_2, b_3, b_4]$ are said to be equal trapezoidal fuzzy number if and only if $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$.

Definition 3.4 (Liou and Wang [32]). A ranking function is a function $\mathfrak{R} : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line.

Let $\tilde{A} = [a_1, a_2, a_3, a_4]$ be a trapezoidal fuzzy number then $\mathfrak{R}(\tilde{A}) = \left(\frac{a_1+a_2+a_3+a_4}{4}\right)$.

Let $\tilde{A} = [a_1, a_2, a_3, a_4]$ and $\tilde{B} = [b_1, b_2, b_3, b_4]$ be two TrFNs. Then,

- i. $\mathfrak{R}(\tilde{A}) > \mathfrak{R}(\tilde{B})$ iff $\tilde{A} \succ \tilde{B}$
- ii. $\mathfrak{R}(\tilde{A}) < \mathfrak{R}(\tilde{B})$ iff $\tilde{A} \prec \tilde{B}$
- iii. $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$ iff $\tilde{A} \approx \tilde{B}$,

where $\mathfrak{R}(\tilde{A}) = \left(\frac{a_1+a_2+a_3+a_4}{4}\right)$, $\mathfrak{R}(\tilde{B}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$.

Theorem 1. The ranking function $\mathfrak{R} : F(R) \rightarrow R$ is a linear function.

Proof. Let $\tilde{A} = [\phi_1, \phi_2, \phi_3, \phi_4]$ and $\tilde{B} = [\Psi_1, \Psi_2, \Psi_3, \Psi_4]$ be two TrFNs. Then, for all $c \geq 0$, we have

$$\mathfrak{R}(c\tilde{A} \oplus \tilde{B}) = \mathfrak{R}[c(\phi_1, \phi_2, \phi_3, \phi_4) \oplus (\Psi_1, \Psi_2, \Psi_3, \Psi_4)].$$

We know that (Wkt) $c\tilde{A} = [c\phi_1, c\phi_2, c\phi_3, c\phi_4]$, for $c \geq 0$.

$$\mathfrak{R}(c\tilde{A} \oplus \tilde{B}) = \mathfrak{R}((c\phi_1, c\phi_2, c\phi_3, c\phi_4) \oplus (\Psi_1, \Psi_2, \Psi_3, \Psi_4)).$$

$$\text{Wkt } \tilde{A} \oplus \tilde{B} = [\phi_1 + \Psi_1, \phi_2 + \Psi_2, \phi_3 + \Psi_3, \phi_4 + \Psi_4]$$

$$\mathfrak{R}(c\tilde{A} + \tilde{B}) = ((c\phi_1 + \Psi_1, c\phi_2 + \Psi_2, c\phi_3 + \Psi_3, c\phi_4 + \Psi_4))$$

$$\begin{aligned} \mathfrak{R}(c\tilde{A} \oplus \tilde{B}) &= \left(\frac{c\phi_1 + \Psi_1 + c\phi_2 + \Psi_2 + c\phi_3 + \Psi_3 + c\phi_4 + \Psi_4}{4}\right) \end{aligned}$$

$$\text{Since } \mathfrak{R}(\tilde{A}) = \left(\frac{\phi_1+\phi_2+\phi_3+\phi_4}{4}\right)$$

$$\begin{aligned} \mathfrak{R}(c\tilde{A} \oplus \tilde{B}) &= \left(\frac{c\phi_1 + c\phi_2 + c\phi_3 + c\phi_4 + \Psi_1 + \Psi_2 + \Psi_3 + \Psi_4}{4}\right) \end{aligned}$$

$$\begin{aligned} \mathfrak{R}(c\tilde{A} \oplus \tilde{B}) &= \left(\frac{c\phi_1 + c\phi_2 + c\phi_3 + c\phi_4}{4}\right) + \left(\frac{\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4}{4}\right) \end{aligned}$$

$$\mathfrak{R}(c\tilde{A} \oplus \tilde{B})$$

$$\begin{aligned} &= c \left(\frac{\phi_1 + \phi_2 + \phi_3 + \phi_4}{4}\right) + \left(\frac{\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4}{4}\right) \\ \mathfrak{R}(c\tilde{A} \oplus \tilde{B}) &= c \left(\tilde{A}\right) \oplus \left(\tilde{B}\right). \end{aligned}$$

Similarly it can be proved for $c < 0$. This implies that the ranking function is a linear function. \square

4. Fully Fuzzy Transportation Problem and Its Mathematical Formulation

The following basic terminologies used in this article are defined in this section.

Definition 4.1. If the transportation problem has at least one of the parameter (cost) or two of the parameters (supply and demand) or all of the parameters (supply, demand and cost) are in fuzzy numbers then the problem is called FTP.

- Further, FTP can be classified into four categories. They are
- Type-1 FTP
- Type-2 FTP
- Type-3 FTP (mixed FTP)
- Type-4 FTP (fully FTP)

Definition 4.2. A transportation problem having fuzzy availabilities and fuzzy demands but crisp costs is termed as fuzzy transportation problem of type-1.

Definition 4.3. A transportation problem having crisp availabilities and crisp demands but fuzzy costs is termed as fuzzy transportation problem of type-2.

Definition 4.4. The transportation problem is said to be the type-3 fuzzy transportation problem or mixed fuzzy transportation problem if all the parameters of the transportation problem (such as supplies, demands and costs) must be in the mixture of crisp numbers, triangular fuzzy numbers and trapezoidal fuzzy numbers.

Definition 4.5. The transportation problem is said to be the type-4 fuzzy transportation problem or fully fuzzy transportation problem if all the parameters of the transportation problem (such as supplies, demands and costs) must be in fuzzy numbers.

Definition 4.6. The transportation problem is said to be balanced fuzzy transportation problem if total fuzzy supply is equal to total fuzzy demand.

That is,

$$\sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j.$$

Definition 4.7. The transportation problem is said to be an unbalanced fuzzy transportation problem if total fuzzy supply

is not equal to total fuzzy demand.

That is,

$$\sum_{i=1}^m \tilde{a}_i \neq \sum_{j=1}^n \tilde{b}_j.$$

Definition 4.8. A set of fuzzy non negative allocations $\tilde{x}_{ij} > \tilde{0}$ satisfies the row and column restriction is known as fuzzy feasible solution.

Definition 4.9. Any feasible solution is a fuzzy basic feasible solution if the number of non negative allocations is at most $(m + n - 1)$ where m is the number of rows and n is the number of columns in the $m \times n$ transportation table.

Definition 4.10. If the fuzzy basic feasible solution contains less than $(m + n - 1)$ non negative allocations in $m \times n$ transportation table, it is said to be degenerate.

Definition 4.11. Any fuzzy feasible solution to a transportation problem containing m origins and n destinations is said to be fuzzy non degenerate, if it contains exactly $(m + n - 1)$ occupied cells.

Definition 4.12. The fuzzy basic feasible solution is said to be fuzzy optimal solution if it minimizes the total fuzzy transportation cost (or) it maximizes the total fuzzy transportation profit.

Mathematical Formulation Consider the transportation problem with m origins (rows) and n destinations (columns). Let $\tilde{c}_{ij} = [c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4]$ be the cost of transporting one unit of the product from i th origin to j th destination, $\tilde{a}_i = [a_i^1, a_i^2, a_i^3, a_i^4]$ be the quantity of commodity available at origin i , $\tilde{b}_j = [b_j^1, b_j^2, b_j^3, b_j^4]$ the quantity of commodity needed at destination j , $\tilde{x}_{ij} = [x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4]$ is the quantity transported from i th origin to j th destination, so as to minimize the total fuzzy transportation cost.

Mathematically fully fuzzy transportation problem can be stated as (FFTP)

(P) Minimize $\tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

Subject to,

$$\sum_{j=1}^n \tilde{x}_{ij} \approx \tilde{a}_i, \text{ for } i = 1, 2, \dots, m, \tag{1}$$

$$\sum_{i=1}^m \tilde{x}_{ij} \approx \tilde{b}_j, \text{ for } j = 1, 2, \dots, n, \tag{2}$$

$$\tilde{x}_{ij} \succcurlyeq \tilde{0}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n, \tag{3}$$

where m is the number of supply points, n is the number of demand points.

When the supplies, demands and costs are fuzzy numbers, then the total cost becomes the fuzzy number.

$\tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \tilde{x}_{ij}$. Hence it cannot be minimized directly. For solving the problem we convert the fuzzy supplies, fuzzy demands and the fuzzy costs into crisp ones by a fuzzy number ranking method.

Consider the transportation problem with m origins (rows) and n destinations (columns). Let c_{ij} be the cost of transporting one unit of the product from i th origin to j th destination, a_i be the quantity of commodity available at origin i , b_j the quantity of commodity needed at destination j , x_{ij} is the quantity transported from i th origin to j th destination, so as to minimize the total transportation cost.

(P*) Minimize $\Re(\tilde{Z}^*) = \sum_{i=1}^m \sum_{j=1}^n \Re(\tilde{c}_{ij}) \otimes \Re(\tilde{x}_{ij})$

Subject to,

$$\sum_{j=1}^n \Re(\tilde{x}_{ij}) \approx \Re(\tilde{a}_i), \text{ for } i = 1, 2, \dots, m, \tag{4}$$

$$\sum_{i=1}^m \Re(\tilde{x}_{ij}) \approx \Re(\tilde{b}_j), \text{ for } j = 1, 2, \dots, n, \tag{5}$$

$$\Re(\tilde{x}_{ij}) \succcurlyeq \Re(\tilde{0}), \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n. \tag{6}$$

Since $\Re(\tilde{c}_{ij}), \Re(\tilde{a}_i), \Re(\tilde{b}_j)$, all are crisp values, this problem (P*) is obviously the crisp transportation problem of the form (P) which can be solved by the conventional method namely the Zero Point Method, Modified Distribution Method or any other software package such as TORA, LINGO and so on. Once the optimal solution x^* of Model (P*) is found, the optimal fuzzy objective value \tilde{Z}^* of the original problem can be calculated as

$$\tilde{Z}^* = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \otimes \tilde{x}_{ij}^*,$$

where, $\tilde{c}_{ij} = [c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4], \tilde{x}_{ij}^* = [x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4]$.

The above FFTP and its equivalent crisp TP can be stated in the below tabular form in Tables 1 and 2.

5. Proposed Method (PSK Method)

This proposed method is used for finding the optimal basic feasible solution in fuzzy environment and the following step by step procedure is utilized to find out the same.

Step 1. Consider the TP having all the parameters such as supply, demand and costs must be in fuzzy numbers (This situation is known as FFTP).

Table 1. Tabular representation of FFTP

Source	Destination				Fuzzy availability a_i
	D_1	D_2	\dots	D_n	
S_1	\tilde{c}_{11}	\tilde{c}_{12}	\dots	\tilde{c}_{1n}	\tilde{a}_1
S_2	\tilde{c}_{21}	\tilde{c}_{22}	\dots	\tilde{c}_{2n}	\tilde{a}_2
\vdots	\vdots	\vdots	\dots	\vdots	\vdots
S_m	\tilde{c}_{m1}	\tilde{c}_{m2}	\dots	\tilde{c}_{mn}	\tilde{a}_m
Fuzzy demand \tilde{b}_j	\tilde{b}_1	\tilde{b}_2	\dots	\tilde{b}_n	$\sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j$

Table 2. Tabular representation of crisp TP

Source	Destination				Availability (\tilde{a}_i)
	D_1	D_2	\dots	D_n	
S_1	$\mathfrak{R}(\tilde{c}_{11})$	$\mathfrak{R}(\tilde{c}_{12})$	\dots	$\mathfrak{R}(\tilde{c}_{1n})$	$\mathfrak{R}(\tilde{a}_1)$
S_2	$\mathfrak{R}(\tilde{c}_{21})$	$\mathfrak{R}(\tilde{c}_{22})$	\dots	$\mathfrak{R}(\tilde{c}_{2n})$	$\mathfrak{R}(\tilde{a}_2)$
\vdots	\vdots	\vdots	\dots	\vdots	\vdots
S_m	$\mathfrak{R}(\tilde{c}_{m1})$	$\mathfrak{R}(\tilde{c}_{m2})$	\dots	$\mathfrak{R}(\tilde{c}_{mn})$	$\mathfrak{R}(\tilde{a}_m)$
Demand $\mathfrak{R}(\tilde{b}_j)$	$\mathfrak{R}(\tilde{b}_1)$	$\mathfrak{R}(\tilde{b}_2)$	\dots	$\mathfrak{R}(\tilde{b}_n)$	$\sum_{i=1}^m \mathfrak{R}(\tilde{a}_i) = \sum_{j=1}^n \mathfrak{R}(\tilde{b}_j)$

Step 2. Examine whether the total fuzzy supply equals total fuzzy demand. If not, introduce a dummy row/column having all its cost elements as fuzzy zero and fuzzy supply/fuzzy demand as the positive difference of fuzzy supply and fuzzy demand.

Step 3. After using Step 2, transform the FFTP into its equivalent crisp TP using the ranking procedure as mentioned in Section 3.

Step 4. Now, the crisp TP having all the entries of supply, demand and costs are integers then kept as it is. Otherwise at least one or all of the supply, demand and costs are not in integers then rewrite its nearest integer value.

Step 5. After using Step 4 of the proposed method, now solve the crisp TP by using any one of the existing methods (MODI, Zero Point Method) or software packages such as TORA, LINGO and so on. This step yields the optimal allocation and optimal objective value of the crisp TP (The optimal allotted cell in crisp transportation table is referred as occupied cells. The remaining cells are called unoccupied cells. The numbers of occupied cells in crisp TP which are exactly $m + n - 1$ and all have zero cost. Similarly in FFTP also have the same $m + n - 1$ number of occupied cells but its corresponding costs are fuzzy zeros). Now, construct the new fully fuzzy transportation table (FFTT) whose occupied

cells costs are fuzzy zeros and the remaining cells costs are its original cost. Subtract each row entries for the current table from the row minimum. Next subtract each column entries of the reduced table from the column minimum. Clearly, each row and each column of the resulting table has at least one fuzzy zero. The current resulting table is the allotment table.

Step 6. After using Step 5 of the proposed method, now we check the allotment table if one or more rows/columns having exactly one occupied cell (fuzzy zero) then allot the maximum possible value to that cell and adjust the corresponding supply or demand with a positive difference of supply and demand. Otherwise, if all the rows/columns having more than one occupied cells then select a cell in the α -row and β -column of the transportation table whose cost is maximum (If the maximum cost is more than one i.e., a tie occurs then select arbitrarily) and examine which one of the cells is minimum cost (If the minimum cost is more than one i.e., a tie occurs then select arbitrarily) among all the occupied cells in that row and column then allot the maximum possible value to that cell. In this manner proceeds selected row and column entirely. If the entire row and column of the occupied cells having fully allotted then select the next maximum cost of the transportation table and examine which one of the cells is minimum cost among all the occupied cells in that row and column then allot the maximum possible value to that cell. Repeat this process until all the fuzzy supply points are fully used and all the fuzzy demand points are fully received. This allotment yields the fully fuzzy solution to the given fully fuzzy transportation problem.

Remark 5.1. Allot the maximum possible value to the occupied cells in type-2 and type-4 FTP which is the most preferable row/column having exactly one occupied cell.

Remark 5.2. From the MODI method, we conclude that the TP have exactly $m + n - 1$ number of non-negative independent allocations.

Remark 5.3. From the zero point method, we can make exactly $m + n - 1$ number of zeros (zero referred to as zero cost) in the cost matrix. All these zeros (costs) are in independent positions.

Remark 5.4. From Remark 5.2 and Remark 5.3, we can directly replace the fuzzy zeros instead of original costs in the occupied cells in the original FTP. This modification does not affect the originality of the problem

Now, we prove the following theorems which are used to derive the solution to a fuzzy transportation problem obtained by the PSK method is a fuzzy optimal solution to the FFTP.

Theorem 5.1. Any optimal solution to the fully fuzzy transportation (P_1) where

(P₁) Minimize $\tilde{Z}^* = \sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \tilde{x}_{ij}$
 Subject to (1) to (3) are satisfied, where \tilde{u}_i (minimum of i th row of the newly constructed transportation table \tilde{c}_{ij}) and \tilde{v}_j (minimum of j th column of the resulting transportation table $[\tilde{c}_{ij} \ominus \tilde{u}_i]$) are some real TrFNs, is an optimal solution to the problem (P) where

(P) Minimize $\tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \otimes \tilde{x}_{ij}$
 Subject to (1) to (3) are satisfied.

Proof. Let \tilde{u}_i be the minimum of i th row of the newly constructed transportation table $[\tilde{c}_{ij}]$. Now, we subtract \tilde{u}_i^I from the i th row entries so that the resulting table is $[\tilde{c}_{ij} \ominus \tilde{u}_i]$. Let \tilde{v}_j be the minimum of j th column of the resulting table $[\tilde{c}_{ij} \ominus \tilde{u}_i]$. Now, we subtract \tilde{v}_j from the j th column entries so that the resulting table is $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j)$. It may be noted that $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \succcurlyeq \tilde{0}$, for all i and j . Further each row and each column having at least one fuzzy zero.

$$\begin{aligned} \text{Now, } \tilde{Z}^* &\approx \sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \tilde{x}_{ij} \\ \tilde{Z}^* &\approx \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \otimes \tilde{x}_{ij} \ominus \sum_{i=1}^m \sum_{j=1}^n \tilde{u}_i \otimes \tilde{x}_{ij} \ominus \sum_{i=1}^m \sum_{j=1}^n \tilde{v}_j \otimes \tilde{x}_{ij}, \\ \tilde{Z}^* &\approx \tilde{Z} \ominus \sum_{i=1}^m \tilde{u}_i \otimes \tilde{b}_j \ominus \sum_{j=1}^n \tilde{v}_j \otimes \tilde{a}_i. \end{aligned}$$

Since $\sum_{i=1}^m \tilde{u}_i \otimes \tilde{b}_j$ and $\sum_{j=1}^n \tilde{v}_j \otimes \tilde{a}_i$ are independent of \tilde{x}_{ij} , for all i and j , we can conclude that any optimal solution to the problem (P₁) is also a fuzzy optimal solution to the problem (P). Hence the theorem. \square

Theorem 5.2. If $\{\tilde{x}_{ij}^o, i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ is a feasible solution to the problem (P) and $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \succcurlyeq \tilde{0}$, for all i and j where \tilde{u}_i and \tilde{v}_j are some real TrFNs, such that the minimum $\sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \tilde{x}_{ij}$ subject to (1) to (3) are satisfied, is fuzzy zero, then $\{\tilde{x}_{ij}^o, i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ is a fuzzy optimal solution to the problem (P).

Proof. Let $\{\tilde{x}_{ij}^o, i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ be the feasible solution to the problem (P). Now, consider the problem (P) with $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \succcurlyeq \tilde{0}^I$, for all i and j denoted by problem (P₁). From the Theorem 5.1, Clearly, (P) is a original problem and (P₁) is a reduced problem of problem (P). Moreover, in a problem (P₁) there is no possibility to minimize the cost below fuzzy zero. Hence the theorem. \square

Now, we prove that the solution to the FFTP obtained by the PSK method is a fully fuzzy optimal solution to the FFTP.

Theorem 5.3. A solution obtained by the PSK method for a FFTP with equality constraints (P) is a fully fuzzy optimal solution for the fully fuzzy transportation problem (P).

Proof. Let us, now describe the PSK method in detail.

We construct the fully fuzzy transportation table $[\tilde{c}_{ij}]$ for the given fully intuitionistic fuzzy transportation problem and then, convert it into a balanced one if it is not balanced. Next, transform the FFTP into its equivalent crisp TP using the ranking procedure of Liou and Wang [32].

Now, the crisp TP having all the entries of supply, demand and costs are integers then kept as it is. Otherwise at least one or all of the supply, demand and costs are not in integers then rewrite its nearest integer value because decimal values in transportation problem has no physical meaning (such a transportation problem referred as crisp TP).

Now, solve the crisp TP by using any one of the existing methods (MODI, Zero Point Method) or software packages such as TORA, LINGO and so on. This process will yield the optimal allotment and optimal objective value of the crisp TP (The optimal allotted cells in crisp transportation table is referred to as occupied cells which are exactly $m + n - 1$. All the decision variables in this occupied cells are basic feasible with zero cost. Clearly, each row and each column have at least one zero cost which corresponds to the occupied cells. The remaining cells are called unoccupied cells. All the decision variables in these unoccupied cells are non basic. The value of decision variables in this unoccupied cells are at zero level).

By the definitions, occupied cells in crisp TP is same as that of occupied cells in FFTP but the value of occupied cells for FFTP is the maximum possible value of fuzzy supply and fuzzy demand. Therefore, we need not further investigate the occupied cells in FFTP. But only we claim that how much quantity (fuzzy supply, fuzzy demand) to allot the occupied cells subject to (1),(2) and (3) are satisfied. The occupied cells in crisp TP is exactly $m + n - 1$ and all are having zero cost. Similarly in FFTP also have the same $m + n - 1$ number of occupied cells but its corresponding cost is fuzzy zeros. Now, construct the new FFTT whose occupied cells costs are fuzzy zeros and the remaining cells costs are its original cost. Let \tilde{u}_i be the minimum of i th row of the current table $[\tilde{c}_{ij}]$. Now, we subtract \tilde{u}_i from the i th row entries so that the resulting table is $[\tilde{c}_{ij} \ominus \tilde{u}_i]$. Let \tilde{v}_j be the minimum of j th column of the resulting table $[\tilde{c}_{ij} \ominus \tilde{u}_i]$. Now, we subtract \tilde{v}_j from the j th column entries so that the resulting table is $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j)$. It may be noted that $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \succcurlyeq \tilde{0}$, for all i and j . Clearly, each row and each column have at least one fuzzy zero. The current resulting table is the allotment table.

Now, we check the allotment table if one or more rows/columns having exactly one occupied cell then allot the maximum possible value to that cell and adjust the corresponding

supply or demand with a positive difference of supply and demand. Otherwise, if all the rows/columns having more than one occupied cells then select a cell in the α -row and β -column of the transportation table whose cost is maximum (If the maximum cost is more than one i.e., a tie occurs then select arbitrarily) and examine which one of the cells is minimum cost (If the minimum cost is more than one i.e., a tie occurs then select arbitrarily) among all the occupied cells in that row and column then allot the maximum possible value to that cell. In this manner proceeds selected row/column entirely. If the entire row and column of the occupied cells having fully allotted then select the next maximum cost of the transportation table and examine which one of the cells is minimum cost among all the occupied cells in that row and column then allot the maximum possible value to that cell. Repeat this process until all the fuzzy supply points are fully used and all the fuzzy demand points are fully received. This step yields the optimum fuzzy allotment.

Clearly, the above process satisfies all the rim requirements (row and column sum restriction). If all the rim requirements are satisfied then automatically it satisfies, total fuzzy supply is equal to total fuzzy demand i.e., the necessary and sufficient condition for a FFTP is satisfied.

Finally, we have a solution $\{\tilde{x}_{ij}, i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ for the FFTP whose cost matrix is $[\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j]$ such that $\tilde{x}_{ij} \approx \tilde{0}$ for $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \succ \tilde{0}$ and $\tilde{x}_{ij} \succ \tilde{0}$ for $(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \approx \tilde{0}$.

Therefore, the minimum $\sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j) \tilde{x}_{ij}$ subject to (1) to (3) are satisfied, is fuzzy zero. Thus, by the Theorem 5.2, the solution $\{\tilde{x}_{ij}, i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$ is obtained by the PSK method for a fully fuzzy transportation problem with equality constraints is a fully fuzzy optimal solution for the fully fuzzy transportation problem. Hence the theorem. \square

The proposed method is illustrated by the following numerical examples.

6. Illustrative Example

6.1 Example 1

A firm has three factories $S_1, S_2,$ and S_3 that manufacture the same product of air coolers in three different places. The firm manager would like to transport air coolers from three different factories to four different warehouses D_1, D_2, D_3 and D_4 . All the factories are connected to all the warehouses by roads and air coolers are transported by lorries. The availability (availability of air coolers are depends on its production but production depends on men, machine, etc.) of air coolers are

not known exactly due to long power cut, labour’s over time work, unexpected failures in machine etc. The demand of air coolers is not known exactly due to seasonal changes (In sunny days the sale of air coolers are more when compared to rainy days). Similarly the transportation cost is not known exactly due to variations in rates of petrol, traffic jams, weather in hilly areas etc. So, all the parameters of the TP are in uncertain quantities which are given in terms of TrFN. The transportation costs (Rupees in hundreds) for an air cooler from different factories to different warehouses are given below from the past experience.

Table 3. Type-4 FTP

Factories	Warehouses				Fuzzy Availability \tilde{a}_i
	D_1	D_2	D_3	D_4	
S_1	[0, 1, 3, 4]	[0, 1, 3, 4]	[0, 1, 3, 4]	[0, 0, 2, 2]	[0, 2, 4, 6]
S_2	[4, 8, 12, 16]	[4, 7, 9, 12]	[2, 4, 6, 8]	[1, 3, 5, 7]	[2, 4, 9, 13]
S_3	[2, 4, 9, 13]	[0, 6, 8, 10]	[0, 6, 8, 10]	[4, 7, 9, 12]	[2, 4, 6, 8]
Fuzzy Demand \tilde{b}_j	[1, 3, 5, 7]	[0, 2, 4, 6]	[1, 3, 5, 7]	[1, 3, 5, 7]	

Find the optimal allocation which minimizes total fuzzy transportation cost.

Solution by proposed method:

Now using Step 2 we get, $\sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j = 60$, the given problem is balanced FFTP.

Now, using Step 3 of proposed method, in conformation to Model (2) FFTP can be transformed into its equivalent crisp transportation problem by using the ranking method $\mathfrak{R}(\tilde{A}) = (a_1 + a_2 + a_3 + a_4)/4$.

Table 4. Crisp version of type-4 FTP

Factories	Warehouses				Availability a_i
	D_1	D_2	D_3	D_4	
S_1	2	2	2	1	3
S_2	10	8	5	4	7
S_3	7	6	6	8	5
Demand b_j	4	3	4	4	

Since in transportation problems supplies, demands and costs

are not in decimal values because no physical meaning of fractional value. Therefore whenever the rank values occur in the form of decimal then we convert it into its nearest integer value. Otherwise, all the rank values of the FFTP is in integers then kept as it is.

After using Step 5 of the proposed method, the optimal allotment of the above problem is

Table 5. Crisp optimum table of type-4 FTP

Factories	Warehouses				Availability a_i
	D_1	D_2	D_3	D_4	
S_1	0 (3)	2	2	1	3
S_2	10	8	0 (3)	0 (4)	7
S_3	0 (1)	0 (3)	0 (1)	8	5
Demand b_j	4	3	4	4	

The optimum solution is $x_{11} = 3, x_{23} = 3, x_{24} = 4, x_{31} = 1, x_{32} = 3, x_{33} = 1$.

The minimum objective value $Z = (2 \times 3) + (5 \times 3) + (4 \times 4) + (7 \times 1) + (6 \times 3) + (6 \times 1) = 68$ (Rupees in hundreds).

After using Step 5 of the proposed method, now using Step 6, we get the optimal allotment directly for the FFTP is shown in the following table.

Table 6. Fuzzy optimum table of type-4 FTP

Factories	Warehouses				Fuzzy Availability \tilde{a}_i
	D_1	D_2	D_3	D_4	
S_1	[0, 2, 4, 6] [-2,-1, 1, 2]	[0, 1, 3, 4]	[0, 1, 3, 4]	[0, 0, 2, 2]	[0, 2, 4, 6]
S_2	[4, 8, 12, 16]	[4, 7, 9, 12]	[-5,-1, 6, 12] [1, 3, 5, 7]	[-2,-1, 1, 2] [1, 3, 5, 7]	[2, 4, 6, 8, 10, 12]
S_3	[-5,-1, 3, 7] [-2,-1, 1, 2]	[0, 2, 4, 6] [-2,-1, 1, 2]	[-11, -3, 6, 12] [1, 3, 5, 7]	[4, 7, 9, 12]	[2, 4, 6, 8]
Fuzzy demand \tilde{b}	[1, 3, 5, 7]	[0, 2, 4, 6]	[1, 3, 5, 7]	[1, 3, 5, 7]	

The fuzzy optimal solution in terms of trapezoidal fuzzy number is

$$\tilde{x}_{11} = [0, 2, 4, 6], \tilde{x}_{31} = [-5, -1, 3, 7], \tilde{x}_{23} = [-5, -1, 6, 12],$$

$$\tilde{x}_{32} = [0, 2, 4, 6], \tilde{x}_{24} = [1, 3, 5, 7], \tilde{x}_{33} = [-11, -3, 6, 12].$$

Hence, the total fuzzy transportation cost is

$$\begin{aligned} \text{Min } \tilde{Z} &= [0, 1, 3, 4] \otimes [0, 2, 4, 6] \oplus [2, 4, 6, 8] \\ &\otimes [-5, -1, 6, 12] \oplus [1, 3, 5, 7] \otimes [1, 3, 5, 7] \\ &\oplus [2, 4, 9, 13] \otimes [-5, -1, 3, 7] \oplus [0, 6, 8, 10] \\ &\otimes [0, 2, 4, 6] \oplus [0, 6, 8, 10] \otimes [-11, -3, 6, 12] \\ \text{Min } \tilde{Z} &= [0, 1, 3, 4] \otimes [3] \oplus [2, 4, 6, 8] \otimes [3] \oplus [1, 3, 5, 7] \\ &\otimes [4] \oplus [2, 4, 9, 13] \otimes [1] \oplus [0, 6, 8, 10] \otimes [3] \\ &\oplus [0, 6, 8, 10] \otimes [1] \\ \text{Min } \tilde{Z} &= [0, 3, 9, 12] \oplus [6, 12, 18, 24] \oplus [4, 12, 20, 28] \\ &\oplus [2, 4, 9, 13] \oplus [0, 18, 24, 30] \oplus [0, 6, 8, 10] \\ \text{Min } \tilde{Z} &= [12, 55, 88, 117] \text{ (Rupees in hundreds)} \\ \mathfrak{R}(\tilde{Z}) &= [12, 55, 88, 117] = \frac{12 + 55 + 88 + 117}{4} \\ &= 68 \text{ (Rupees in hundreds)} \end{aligned}$$

6.2 Example 2

There are three sources (rows) and four destinations (columns), all the sources are connected to all the destinations by roads and the goods are transported by trucks. The supply and demand of goods are well known crisp quantities but the transportation cost is not known exactly (due to variations in rates of petrol, traffic jams, weather in hilly areas etc). Hence crisp supply, crisp demand and unit transportation cost (given in terms of TrFNs) are given in the following table. Find the optimal allocation which minimizes total fuzzy transportation cost.

Table 7. Type-2 FTP

	D_1	D_2	D_3	D_4	Availability
O_1	[1, 2, 4, 5]	[1, 4, 6, 9]	[3, 6, 10, 13]	[5, 8, 16, 19]	3
O_2	[0, 2, 4, 6]	[2, 4, 10, 12]	[7, 10, 12, 15]	[1, 4, 8, 11]	5
O_3	[2, 4, 6, 8]	[4, 6, 10, 12]	[0, 10, 20, 30]	[4, 8, 12, 16]	12
Demand	5	4	3	8	

Solution by proposed method:

Now using Step 2, we get, $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = 20$, the given problem is balanced.

Now, using Step 3 and Step 4 of the proposed method, we get the following crisp TP (P).

After using Step 4 of the proposed method, now using Step 5

Table 8. Crisp version of type-2 FTP (P)

	D ₁	D ₂	D ₃	D ₄	Availability
O ₁	3	5	8	12	3
O ₂	3	7	11	6	5
O ₃	5	8	15	10	12
Demand	5	4	3	8	

we have

The optimal solution is $x_{13} = 3, x_{24} = 5, x_{31} = 5, x_{32} = 4, x_{33} = 0, x_{34} = 3$

The total transportation cost $\text{Min}Z = 8 \times 3 + 6 \times 5 + 5 \times 5 + 8 \times 4 + 15 \times 0 + 10 \times 3 = 141$.

Now, construct the new transportation table (P*) whose occupied cells costs are zeros and the remaining cells costs are its original cost. Subtract each row entries for the current table from the row minimum. Next subtract each column entries of the reduced table from the column minimum. Clearly, each row and each column of the resulting table has at least one fuzzy zero. The current resulting table is the allotment table.

Table 9. Crisp optimum table of type-2 FTP (P*)

	D ₁	D ₂	D ₃	D ₄	Availability
O ₁	3	5	0	12	3
O ₂	3	7	11	0	5
O ₃	0	0	0	0	12
Demand	5	4	3	8	

Now using Step 6 of the proposed method we get

Table 10. Crisp optimum allocation table of type-2 FTP

	D ₁	D ₂	D ₃	D ₄	Availability
O ₁	3	5	0 (3)	12	3
O ₂	3	7	11	0 (5)	5
O ₃	0 (5)	0 (4)	0 (0)	0(3)	12
Demand	5	4	3	8	

The total transportation cost $\text{Min}Z^* = 0 \times 3 + 0 \times 5 + 0 \times 5 + 0 \times 4 + 0 \times 0 + 0 \times 3 = 0$ (We could not minimize cost below zero).

Therefore, Any optimal solution of problem (P*) (subject to all the constraints are satisfied) is also an optimal solution of problem (P) (subject to all the constraints are satisfied).

Hence the solution obtained by PSK method for solving FTP is always optimal. The optimal allotment is

Table 11. Fuzzy optimum table of type-2 FTP

	D ₁	D ₂	D ₃	D ₄	Availability
O ₁	[1, 2, 4, 5]	[1, 4, 6, 9]	(3)	[5, 8, 16, 19]	3
O ₂	[0, 2, 4, 6]	[2, 4, 10, 12]	[7, 10, 12, 15]	(5)	5
O ₃	(5)	(4)	(0)	(3)	12
O ₃	[-2,-1, 1, 2]	[-2,-1, 1, 2]	[-2,-1, 1, 2]	[-2,-1, 1, 2]	
Demand	5	4	3	8	

$$\begin{aligned} \text{Min}\tilde{Z} &= [-2, -1, 1, 2] \times 3 \oplus [-2, -1, 1, 2] \times 5 \\ &\oplus [-2, -1, 1, 2] \times 5 \oplus [-2, -1, 1, 2] \times 4 \\ &\oplus [-2, -1, 1, 2] \times 0 \oplus [-2, -1, 1, 2] \times 3 \\ &= [-6, -3, 3, 6] \oplus [-10, -5, 5, 10] \oplus [-10, -5, 5, 10] \\ &\oplus [-8, -4, 4, 8] \oplus [0, 0, 0, 0] \oplus [-6, -3, 3, 6] \\ &= [-40, -20, 20, 40] = \tilde{0} \end{aligned}$$

(We could not minimize fuzzy cost below fuzzy zero).

Hence, the optimal solution is $x_{13} = 3, x_{24} = 5, x_{31} = 5, x_{32} = 4, x_{33} = 0, x_{34} = 3$.

The total fuzzy transportation cost is

$$\begin{aligned} \text{Min}\tilde{Z} &= [3, 6, 10, 13] \times 3 \oplus [1, 4, 8, 11] \times 5 \\ &\oplus [2, 4, 6, 8] \times 5 \oplus [4, 6, 10, 12] \times 4 \\ &\oplus [0, 10, 20, 30] \times 0 \oplus [4, 8, 12, 16] \times 3 \\ &= [9, 18, 30, 39] \oplus [5, 20, 40, 55] \oplus [10, 20, 30, 40] \\ &\oplus [16, 24, 40, 48] \oplus [0, 0, 0, 0] \oplus [12, 24, 36, 48] \\ &= [52, 106, 176, 230] = 141. \end{aligned}$$

Results and discussion

The minimum total fuzzy transportation cost (Rupees in hundreds) of problem 1 is

$$\tilde{Z}I = [12, 55, 88, 117]. \tag{7}$$

The result in (7) can be explained (Refer to Figure 1) as follows:

Table 12. Comparative study

Problem Number	Ranking method		Solution methods				
	Liou and Wang [32]	VAM	MODI	Zero Point Method	Fuzzy Zero Point Method	Fuzzy Modified Distribution Method	PSK Method
1	$\frac{a_1 + a_2 + a_3 + a_4}{4}$	68 (Rupees in hundreds)	68 (Rupees in hundreds)	68 (Rupees in hundreds)	[12,55,88,117] 68 (Rupees in hundreds)	[12,55,88,117] 68 (Rupees in hundreds)	[12,55,88,117] 68 (Rupees in hundreds)
2	$\frac{a_1 + a_2 + a_3 + a_4}{4}$	141	141	141	[52,106,176,230] 141	[52,106,176,230] 141	[52,106,176,230] 141

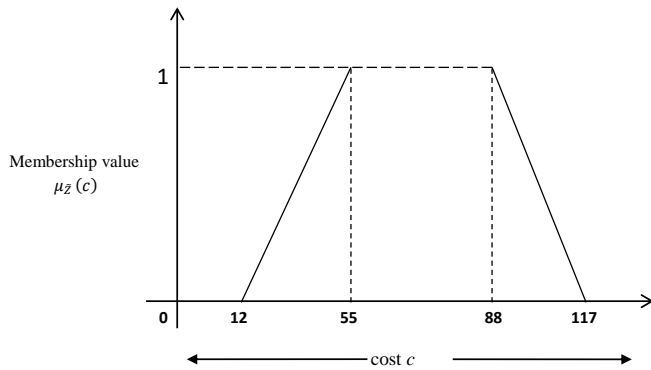


Figure 1. Graphical representation of type-4 fuzzy transportation cost.

- (a) Transportation cost (Rupees in hundreds) lies in [12, 117].
- (b) 100% experts are in favour that the transportation cost (Rupees in hundreds) is [55, 88] as $\mu_{\bar{z}}(c) = 1, x = 55$ to 88.
- (c) Assuming that $\mu_{\bar{z}}(c)$ is a membership value at c . Then $100\mu_{\bar{z}}(c)\%$ experts are in favour that the transportation cost (Rupees in hundreds) is c .

Values of $\mu_{\bar{z}}(c)$ at different values of c can be determined using equations given below.

$$\mu_{\bar{z}}(c) = \begin{cases} 0, & \text{for } c \leq 12, \\ \frac{c - 12}{43}, & \text{for } 12 \leq c \leq 55, \\ 1, & \text{for } 55 \leq c \leq 88, \\ \frac{117 - c}{29}, & \text{for } 88 \leq c \leq 117, \\ 0, & \text{for } c \geq 117 \end{cases}$$

Advantages of the proposed method

By using the proposed method a decision maker has the

following advantages:

- i. The optimum objective value of the type-2 and type-4 FTP is non-negative TrFN i.e., there is no negative part in the obtained TrFN.
- ii. The proposed method is computationally very simple and easy to understand. The solution obtained by PSK method for solving FTP is always optimal. Hence we need not check optimality criteria.

7. Conclusion

On the basis of the present study, it can be concluded that the FTP and FFTP which can be solved by the existing methods (Pandian and Natarajan [9], Dinagar and Palanivel [8], Rani et al. [13], Basirzadeh [16], Gani and Razak [11]) can also be solved by the proposed method. However, it is much easier to apply the proposed method as compared to all the existing methods. Also, new method and new multiplication operation on TrFN is proposed to compute the optimal objective values in terms of TrFN which are very simple and easy to understand and it can be easily applied by decision maker to solve type-2 and type-4 FTP. The solution obtained by this method the objective value of the FFTP remains always positive i.e., there is no negative part in the TrFN. Hence the proposed method is physically meaningful and computationally very simple when compared to all the existing methods.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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References

- [1] F. L. Hitchcock, “The distribution of a product from several sources to numerous localities,” *Journal of Mathematics and Physics*, vol. 20, no. 1, pp. 224-230, 1941. <http://dx.doi.org/10.1002/sapm1941201224>
- [2] H. Arsham and A. B. Kahn, “A simplex-type algorithm for general transportation problems: an alternative to stepping-stone,” *Journal of the Operational Research Society*, vol. 40, no. 6, pp. 581-590, 1989. <http://dx.doi.org/10.1057/jors.1989.95>
- [3] A. Charnes and W. W. Cooper, “The stepping stone method of explaining linear programming calculations in transportation problems,” *Management Science*, vol. 1, no. 1, pp. 49-69, 1954. <http://dx.doi.org/10.1287/mnsc.1.1.49>
- [4] G. M. Appa, “The transportation problem and its variants,” *Journal of the Operational Research Society*, vol. 24, no. 1, pp. 79-99, 1973. <http://dx.doi.org/10.1057/jors.1973.10>
- [5] H. A. Taha, *Operations Research: An Introduction*, 8th ed. Delhi, India: Pearson Education India, 2008.
- [6] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965. [http://dx.doi.org/10.1016/S0019-9958\(65\)90241-X](http://dx.doi.org/10.1016/S0019-9958(65)90241-X)
- [7] R. E. Bellman and L. A. Zadeh, “Decision-making in a fuzzy environment,” *Management Science*, vol. 17, no. 4, pp. B141-B164, 1970. <http://dx.doi.org/10.1287/mnsc.17.4.B141>
- [8] D. S. Dinagar and K. Palanivel, “The transportation problem in fuzzy environment,” *International Journal of Algorithms, Computing and Mathematics*, vol. 2, no. 3, pp. 65-71, 2009.
- [9] P. Pandian and G. Natarajan, “A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems,” *Applied Mathematical Sciences*, vol. 4, no. 2, pp. 79-90, 2010.
- [10] S. I. Mohideen and P. S. Kumar, “A comparative study on transportation problem in fuzzy environment,” *International Journal of Mathematics Research*, vol. 2, no. 1, pp. 151-158, 2010.
- [11] A. N. Gani and K. A. Razak, “Two stage fuzzy transportation problem,” *Journal of Physical Sciences*, vol. 10, pp. 63-69, 2006.
- [12] V. J. Sudhakar and V. N. Kumar, “A different approach for solving two stage fuzzy transportation problems,” *International Journal of Contemporary Mathematical Sciences*, vol. 6, no. 11, pp. 517-526, 2011.
- [13] D. Rani, T. R. Gulati, and A. Kumar, “A method for unbalanced transportation problems in fuzzy environment,” *Sadhana*, vol. 39, no. 3, pp. 573-581, 2014. <http://dx.doi.org/10.1007/s12046-014-0243-8>
- [14] A. N. Gani, A. E. Samuel, and D. Anuradha, “Simplex type algorithm for solving fuzzy transportation problem,” *Tamsui Oxford Journal of Information and Mathematical Sciences*, vol. 27, no. 1, pp. 89-98, 2011. <http://dx.doi.org/10.13140/2.1.1865.7929>
- [15] S. Solaiappan and K. Jeyaraman, “A new optimal solution method for trapezoidal fuzzy transportation problem,” *International Journal of Advanced Research*, vol. 2, no. 1, pp. 933-942, 2014.
- [16] H. Basirzadeh, “An approach for solving fuzzy transportation problem,” *Applied Mathematical Sciences*, vol. 5, no. 32, pp. 1549-1566, 2011.
- [17] M. OhEigeartaigh, “A fuzzy transportation algorithm,” *Fuzzy Sets and Systems*, vol. 8, no. 3, pp. 235-243, 1982. [http://dx.doi.org/10.1016/S0165-0114\(82\)80002-X](http://dx.doi.org/10.1016/S0165-0114(82)80002-X)
- [18] S. Chanas, W. Kolodziejczyk, and A. Machaj, “A fuzzy approach to the transportation problem,” *Fuzzy Sets and Systems*, vol. 13, no. 3, pp. 211-221, 1984. [http://dx.doi.org/10.1016/0165-0114\(84\)90057-5](http://dx.doi.org/10.1016/0165-0114(84)90057-5)
- [19] O. M. Saad and S. A. Abass “A parametric study on transportation problem under fuzzy environment,” *Journal of Fuzzy Mathematics*, vol. 11, no. 1, pp. 115-124, 2003.
- [20] M. K. Das and H. K. Baruah, “Solution of the transportation problem in fuzzified form,” *Journal of Fuzzy Mathematics*, vol. 15, no. 1, pp. 79-95, 2007.

- [21] P. K. De and B. Yadav, "Approach to defuzzify the trapezoidal fuzzy number in transportation problem," *International Journal of Computational Cognition*, vol. 8, pp. 64-67, 2010.
- [22] S. Kikuchi, "A method to defuzzify the fuzzy number: transportation problem application," *Fuzzy Sets and Systems*, vol. 116, no. 1, pp. 3-9, 2000. [http://dx.doi.org/10.1016/S0165-0114\(99\)00033-0](http://dx.doi.org/10.1016/S0165-0114(99)00033-0)
- [23] S. Chanas, M. Delgado, J. L. Verdegay, and M. A. Vila, "Interval and fuzzy extensions of classical transportation problems," *Transportation Planning and Technology*, vol. 17, no. 2, pp. 203-218, 1993. <http://dx.doi.org/10.1080/03081069308717511>
- [24] S. Chanas and D. Kuchta, "A concept of the optimal solution of the transportation problem with fuzzy cost coefficients," *Fuzzy Sets and Systems*, vol. 82, no. 3, pp. 299-305, 1996. [http://dx.doi.org/10.1016/0165-0114\(95\)00278-2](http://dx.doi.org/10.1016/0165-0114(95)00278-2)
- [25] S. Chanas and D. Kuchta, "Fuzzy integer transportation problem," *Fuzzy Sets and Systems*, vol. 98, no. 3, pp. 291-298, 1998. [http://dx.doi.org/10.1016/S0165-0114\(96\)00380-6](http://dx.doi.org/10.1016/S0165-0114(96)00380-6)
- [26] J. Chiang, "The optimal solution of the transportation problem with fuzzy demand and fuzzy product," *Journal of Information Science and Engineering*, vol. 21, pp. 439-451, 2005.
- [27] L. Li, Z. Huang, Q. Da, and J. Hu, "A new method based on goal programming for solving transportation problem with fuzzy cost," in *Proceedings of 2008 International Symposiums on Information Processing*, Moscow, Russia, 2008, pp. 3-8. <http://dx.doi.org/10.1109/ISIP.2008.9>
- [28] M. Chen, H. Ishii, and C. Wu, "Transportation problems on a fuzzy network," *International Journal of Innovative Computing, Information and Control*, vol. 4, no. 5, pp. 1105-1109, 2008.
- [29] F. T. Lin, "Solving the transportation problem with fuzzy coefficients using genetic algorithms," in *Proceedings of IEEE International Conference on Fuzzy Systems, 2009*, Jeju, Korea, 2009, pp. 1468-1473. <http://dx.doi.org/10.1109/FUZZY.2009.5277202>
- [30] P. S. Kumar, "PSK method for solving type-1 and type-3 fuzzy transportation problems," *International Journal of Fuzzy System Applications*, vol. 5, no. 4, pp. 121-146, <http://dx.doi.org/10.4018/IJFSA.2016100106>
- [31] S. H. Chen and C. H. Hsieh, "Graded mean integration representation of generalized fuzzy numbers," *Journal of the Chinese Fuzzy Systems Association*, vol. 5, no. 2, pp. 1-7, 1999.
- [32] T. S. Liou and M. J. J. Wang, "Ranking fuzzy numbers with integral value," *Fuzzy Sets and Systems*, vol. 50, no. 3, pp. 247-255, 1992. [http://dx.doi.org/10.1016/0165-0114\(92\)90223-Q](http://dx.doi.org/10.1016/0165-0114(92)90223-Q)



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