Industrial Engineering
& Management Systems
Vol 15, No 4, December 2016, pp.385-395
ISSN 1598-7248 │ EISSN 2234-6473 │
https://doi.org/10.7232/iems.2016.15.4.385

Modeling and Analyzing One Vendor-Multiple Retailers VMI SC Using Stackelberg Game Theory

Amir-Mohammad Golmohammadi*
Department of Industrial Engineering, University of Yazd, Yazd, Iran

Negar Jahanbakhsh Javid
Department of Industrial Engineering , University of Alzahra, Tehran, Iran

Lily Poursoltan
Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

Hamid Esmaeeli
Department of Industrial Engineering, North Tehran Branch, Islamic Azad University, Tehran, Iran

(Received: November 19, 2016 / Revised: November 23, 2016 / Accepted: November 29, 2016)

ABSTRACT

Game theory is a powerful tool for analyzing the Supply chain (SC) with different conflicting elements. Among them, the Stackelberg game is the one in which a player as leader has more power than the other ones as followers. Since in many SC systems one element has, in essence, more power than the others; the Stackelberg game has found many applications in SC studies. In this paper, we apply the Stackelberg game-theoretic approach and the corresponding equilibrium point to formulate and analyze a two echelon VMI SC. Comprehensive computational results on an experimental case are conducted to numerically analyze the performance of VMI system against three groups of critical parameters. Moreover, a critical comparison demonstrates the poorer performance of decentralized VMI system than centralized one. This naturally necessitates designing proper contracts between VMI partners in order to more effectively implement the realistic decentralized system.

Keywords: Supply Chain, Vendor Managed Inventory, Game Theory, Decentralized, Stackelberg

* Corresponding Author, E-mail: amir.m.golmohammadi@stu.yazd.ac.ir (A.M. Golmohammadi)

1. INTRODUCTION

Ideally, the decisions in SC can be made under the tight control of a core decision maker having perfect and exact information with the aim of optimizing system performance. In general, such a SC named as a centralized SC. But, in reality, neither suppliers nor retailers could monitor the entire SC. Each element of a given SC has its own objectives and priorities; therefore, it would naturally like to optimize its own performance rather than to optimize the performance of whole system. In the other words, under a separated structure, SC elements try to individually optimize their performances. Such SC referred to as a decentralized SC (Li and Wang, 2007) leads to the double marginalization and bullwhip effect lessening the SC performance (Wei and Choi, 2010; Lin *et al.*, 2010). Accordingly, to promote the performance of decentralized SC and bring up nearer to that of centralized one, designing appropriate contracts and strategies is as a vital task taking by many researchers into consideration. In this regard, vendor-managed inventory (VMI) is one of the well-known strategies used by many firms in the recent two decades.

VMI is a “pull” replenishment system designed to enable vendors to have a quick response to the actual demands. It represents a high level of partnership between vendor and retailer in which the vendor plays as the primary decision-maker in the order planning and inventory control processes. Under a VMI system, the supplier decides on the appropriate inventory levels of each item and the corresponding inventory policies to maintain those levels (Tyan and Wee, 2003). Thus, a VMI partnership has, in fact, two main characteristics: (1) Focusing mainly on the centralized inventory management by the vendor with the cooperation of retailers, and (2) Giving the right to vendor to have the whole information about the retailers' inventory and sales in order to properly implement VMI (Yu *et al.*, 2009a). In this manner, the vendor has indeed a direct view on the demand of final customers and can more accurately predict their consumption behaviors. Consequently, the bullwhip effect might be avoided to a large extent. Using more precise predictions, the vendor could significantly improve the production and distribution planning. Moreover, in such a system, there is only one control point in SC helping to reduce safety stock and to improve the customer service level. On the other hand, the retailers are exempted from all or some of costs related to the inventory control system; therefore, in long term, VMI could increase the profit of both sides-i.e., supplier and retailers (Yu H *et al.*, 2009).Many industries are widely implementing the VMI system. As a successful case of implementing VMI, Ortmeyer and Buzzell (1995) mention the cooperation between Wal-Mart and Proctor and Gamble in 1985 through which the on-time shipments of P&G as well as the sales of Wal-Mart are significantly increased (See for more examples, Challener, 2000; Shah, 2002; Yu and Huang, 2010).

2. STACKELBERG GAME AND ITS EQUILIBRIUM

Game theory has become as a powerful and essential tool for analyzing SCs with multiple entities often having conflicting objectives. It can work as an ideal choice for modeling and analysis when the decisions of each entity affect the payoff of the other ones. Game theory deals with interactive optimization problems (Cachon and Netessine, 2003).

Usually, games can be classified to static and dynamic games. In the former, players choose their strategies simultaneously while in the latter, each player chooses their strategies sequentially after the prior. The simplest dynamic game was introduced by Stackelberg. In Stackelberg game model, player 1 (the leader) chooses a strategy at first and then based upon, player 2 (the follower) designatesa suitable strategy choice. Since in most of existing SC models, the upstream entities (e.g. wholesalers) are stronger than typically smaller downstream ones (e.g. retailers), the Stackelberg equilibrium concept has considerably been applied in SC research works (Cachon and Netessine, 2003). When one element of SC holds greater channel power, it is modeled as a Stackelberg game where the element is the leader and the others are the followers. In contrast, the equal-power scenario is modeled as a simultaneous-decision game (Bichescu and Fry, 2009).

In this paper, Modeling and analyzing the decentralized SC is based on the Stackelberg game theory. To find the Stackelberg equilibrium, we need to solve a dynamic two-period problem via backward induction: first, player 2 (follower) selects the best strategy taking all possible strategies of the first player (leader) into consideration. Considering the best response of player 2, then, player 1 selects an appropriate strategy. If x_i and π_i are the selected strategy and payoff of player i, respectively, the Stackelberg equilibrium can be represented as follows (Cachon and Netessine, 2003, Yang *et al.*, 2015):

$$x_2^*(x_1): \frac{\partial \pi_2(x_2, x_1)}{\partial x_2} = 0,$$
$$\frac{\partial \pi_1(x_1, x_2^*(x_1))}{\partial x_1} = \frac{\partial \pi_1(x_1, x_2^*)}{\partial x_1} + \frac{\partial \pi_1(x_1, x_2)}{\partial x_2}\frac{\partial x_2^*}{\partial x_1} = 0 \tag{2}$$

In practice, however, applying the approach of Almehdawe and Mantin (2010) as follows is more convenient:

Step 1: Formulate the followers' optimization problem
Step 2: Formulate the Leader's optimization problem
Step 3: Derive the Karush-Kuhn-Tucker (KKT) conditions for the followers' optimization problem
Step 4: Involve the KKT conditions in the leader's optimization problem

The solution of final model in Step 4 gives the Stackelberg equilibrium. Noteworthily, in order to maintain their profit, none of the game players would like to deviate from the Stackelberg equilibrium point. If the game leader wants to deviate from this point, his/her profit decreases. As the followers' decisions are influenced by the leader's strategy, the followers also do not tend to alter their decisions in the equilibrium point.

In this paper, we considera single-item SC consisting of one vendor as a leader and multiple retailers as followers in which the VMI approach is applied. As usual, the demands for item in retailers' market arecharacterized by a decreasing function of the price. The item is procured by the vendor in fixed unit cost from an external supplier and is sold to retailers in the same prices. The centralized SC at first and decentralized SC after that will be described and formulated. The rest of the paper is organized as follows. In the next section, we provide a brief review on the most related and supportive recent studies. Then, we present the problem description, notations, cost functions and both developed models for centralized and decentralized SC. In order to analyze the sensitivity of VMI system performance into some critical parameters and to compare centralized with decentralized SC, several numerical experiments are designed and practical implications are highlighted. We also provide an example to clarify the Stackelberg equilibrium concept. At the end, we include concluding remarks and direction for future research.

2.1 Literature Review

In this section, we present some cases to prove the applications of game theory especially the Stackelberg game for analyzing VMI approach in SC systems. Cachon and Netessine (2003) are the first researchers that-widely survey game theory in SC analysis. They discuss both non-cooperative and cooperative games and their applications. Also, Nagarajan and Sosic (2008) and Fierstras *et al.* (2010) review and analyze the applications of game theory in SC management. Many authors use different aspects of this theory for analyzing SC in details. Yu *et al*. (2009a, 2009b) apply the Stackelberg game to examine a VMI SC with one manufacturer as leader and multiple retailers as followers. Using the Stackelberg game, Almehdawe and Mantin (2010) provide a comparative study of the VMI SC under two scenarios for game lea-der: (1) the manufacturer and (2) one of the retailers. Bechesco and Fry (2009) formulate the situation where in the supplier has the most power, as the Stackelberg game and compare the centralized and decentralized SC; but, they do not consider the VMI strategy.

Yu and Huang (2010) apply Nash game for analyzing VMI system in a three level SC with multiple suppliers, one manufacturer and multiple retailers. YuH *et al.* (2009) study the trend of VMI system using the evolutionary game theory and conclude that in the early stage of implementing VMI system, the supplier's profit may decrease; however, in long term all partners of this system are profited. Guan and Zhao (2010) apply bargaining process for analyzing two scenarios of implementing VMI system in a one vendor-one retailer SC. Kim and Park (2010) apply the system dynamic simulation and differential game theory to analyze the VMI SC. Su and Shi (2002) apply a two-stage game to formulate and discuss the return and discount policies in a one manufacturer-one retailer SC.

In this paper, we formulate and analyze the implementation of VMI in a one vendor-multiple retailers SC. Although such system is analyzed by Darvish and Odah (2010), our developed models involve the following extensions: (1) in addition to the centralized SC, decentralized VMI SCis formulated using the Stackelberg game theory, (2) the demands are practically assumed to be as a decreasing function of price, and (3) to attain a more practical formulation, some critical parameters such as transportation as well as back order costs are considered. Also, our paper has the following contributions from the numerical analysis view: (1) centralized and decentralized structures of SC are extensively compared by producing 100 random cases (2) A perfect and deep sensitivity analysis of VMI system is done.

3. PROBLEM FORMULATION

At first, we provide the parameters and decision variables used to develop the proposed models.

Parameters

- i Index for retailers, $i = 1, \cdots, n$
- e_i Price elasticity of demand rate for retailer i
- k_i Market scale for retailer i ($/unit)
- S_{bi} Fixed order cost for retailer i which is paid by the vendor ($/order)
- L_{bi} Back order cost for retailer i which is paid by the vendor ($/unit×time)
- H_{bi} Holding cost paid by the vendor to manage the inventory of retailer i ($/unit×time)
- ξ_i Inventory cost paid to the vendor by retailer i ($/unit ×time)
- Φ_i Transportation cost from the vendor to retailer i ($/unit)
- cm Vendor's Purchasing cost for each unit of item ($/unit)
- S_v Fixed order cost for vendor ($/order)
- H_v Holding cost at the vendor's site ($/unit×time)

Decision variables

- q_i Quantity transported to retailer i ($q = \sum_{i=1}^{n} q_i$)
- Q Vendor's order quantity
- p_i Item's retail price for retailer i ($/unit)
- cp Item's wholesale price ($/unit)
- $D_i(p_i)$ Demand rate for retailer i as a function of the retail price p_i which is a decision variable
- D Demand rate for the vendor $D = \sum_{i=1}^{n} D_i(p_i)$
- T_i Cycle time for retailer i
- T_R Common cycle time for retailers
- T Vendor's cycle time ($N = 1/T$ denotes the number of shipments received by a retailer)
- b_i Fraction of cycle time in which the demands of retailer i are postponed
- π_i Retailer i's profit ($/time)
- π_v Vendor's profit ($/time)
- π_t Total profit of VMI system ($/time)
- θ Ratio of VMI system's total profit under decentralized into centralized SC

Notably, the variables cp, b_i, q_i and N are determined by the vendor while p_i by the retailers. The other decision variables are quantified based upon the formers.

3.1 Problem Description

We consider a two-echelon SC consisting of one vendor and multiple retailers. The vendor procures item in a fixed unit price and order cost from an outside supplier with unlimited stock. The vendor's warehouse capacity is also assumed to be unlimited. The demands for this item in the retailers' markets are simulated by a decreasing and convex function of item's retail price using the Cobb-Douglas function:

$$D_i(p_i) = k_i p_i^{-e_i}; \quad i = 1, 2, \cdots, n; \; e_i > 1 \tag{2}$$

Cobb-Douglas function has frequently been used in the literature to show the relationship between the prices and demands(Lau *et al.*, 2007; Choi *et al.*, 2008; Yu *et al.*, 2009a; Yu *et al.*, 2009b).We assume that the retailers are independently act and do not compete to sell the item (for example, they are operating in distinct markets). Vendor applies the VMI strategy; so, he/she is naturally responsible for controlling the inventory in the whole SC-i.e., the retailers' and vendor's sites. Accordingly, the relationship between vendor and retailers is as leader-followers relationship. We practically assume that both parties (i.e., vendor and retailers) are interested in establishing a long-term relation. The VMI strategy reinforces such a relationship because retailers are inherently less likely to match with a different vendor due to high switching costs (Almehdawe and Mantin, 2010). In accordance with the VMI system, vendor replenishes all retailers at the same time, that is, $T_1 = \cdots = T_n = T_R$ (see Figure 1). This is a reasonable policy because the vendor makes the decisions regarding the replenishment time and amount (Darvish and Odah, 2010).

{Please insert figure 1 near here.}

3.2 The SC'scost Functions

Based on the assumptions presented in the previous subsection, we represent the cost functions of the considered SC. By subtracting such functions from the SC's revenues-either in centralized or in decentralized case-, we could determine the SC's profits. The SC's costs can be divided into direct and indirect costs. Direct costs are related to the procurement of item from the supplier and transportation from the vendor to the retailers. Indirect costs are the charges of inventory control systems for the vendor and retailers. The direct costs (*TDC*) can be determined by the following equation:

$$TDC = \sum_{i=1}^{n} D_i(p_i)(cm + \Phi_i) \tag{3}$$

It is clear that we have:

$$T_i = \frac{q_i}{D_i}; \quad i = 1, 2, \cdots, n \tag{4}$$

Therefore, as applying the same replenishment cycles for all retailers, we have:

$$T_R = \frac{q_1}{D_1} = \frac{q_2}{D_2} = \cdots = \frac{q_n}{D_n} \tag{5}$$

For the vendor's replenishment cycle, the following equation may be concluded:

$$T = N \cdot T_R = N \cdot \frac{q_1}{D_1} = N \cdot \frac{q_2}{D_2} = \cdots = N \cdot \frac{q_n}{D_n} \tag{6}$$

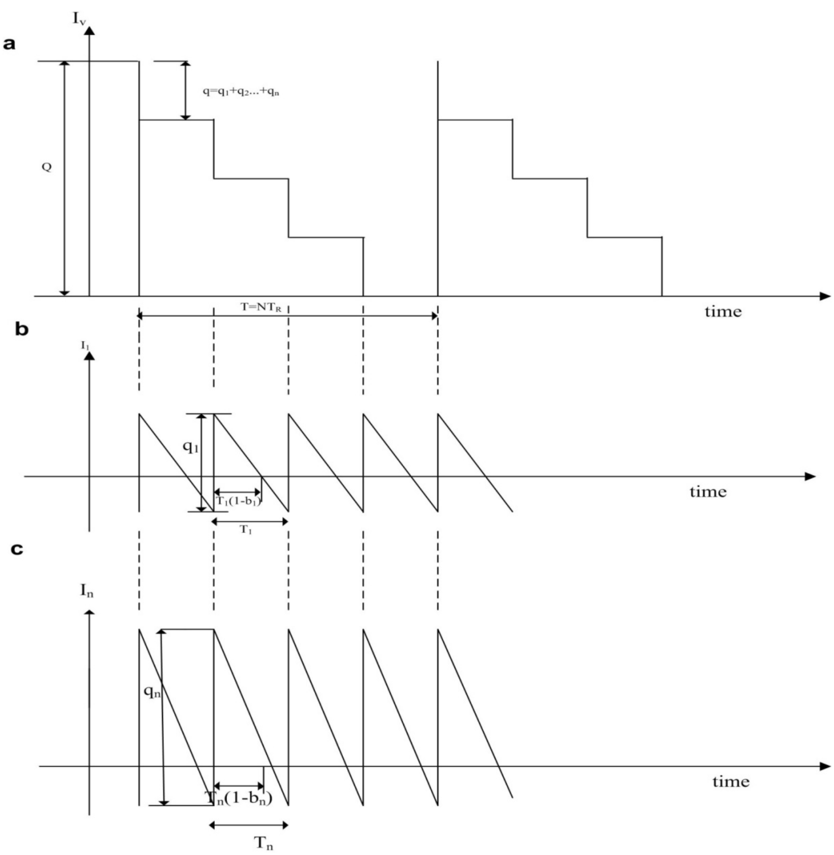

Figure 1. Inventory control diagram for (a) vendor (b) retailer 1 (c) retailer *n*.

Accordingly, calculating the indirect costs in the considered SC is expressed as follows:

The inventory holding cost in cycle T, incurred by the vendor, in his own warehouses, is as follows:

$$THC_v = H_v \cdot \left[(N-1) \cdot \frac{q^2}{D} + (N-2) \cdot \frac{q^2}{D} + \cdots + \frac{q^2}{D} \right]$$
$$= H_v \cdot \frac{N(N-1) \cdot q_1^2 \cdot D}{2[D_1(p_1)]^2} \tag{7}$$

We note that Eq. (7) is obtained by replacing $\frac{q}{D}$ with its equivalent expression (i.e., $\frac{q_1}{D_1(p_1)}$).

The total ordering costs in cycle T is equal to $S_v + N \cdot \sum_{i=1}^{n} S_{bi}$

Also, the inventory holding costs in warehouse of retailer i in one cycle T_R is quantified as follows:

$$THC_{bi} = \frac{D_i(p_i) \cdot T_R^2 (1-b_i)^2}{2} \cdot H_{bi} + \frac{D_i(p_i) \cdot T_R^2 b_i^2}{2} \cdot L_{bi} \tag{8}$$

Thus, the inventory holding cost in warehouse of retailer i can be expressed by the following equation:

$$THC_{bi} = \frac{q_1}{2D_1(p_1)} \cdot [D_i(p_i) \cdot (1-b_i)^2 \cdot H_{bi} + D_i(p_i) \cdot b_i^2 \cdot L_{bi}] \tag{9}$$

Finally, the total costs of inventory control systems (i.e., indirect costs) denoted as *TIC* is as follows:

$$TIC = \frac{q_1}{2D_1(p_1)} \cdot \left[\sum_{i=1}^{n} \left(D_i(p_i) \cdot (1-b_i)^2 \cdot H_{bi} + D_i(p_i) \cdot b_i^2 \cdot L_{bi} \right) \right]$$
$$+ H_v \cdot \frac{(N-1) q_1 \cdot D}{2D_1(p_1)} + (S_v + N \cdot \sum_{i=1}^{n} S_{bi}) \cdot \frac{D_1(p_1)}{N \cdot q_1} \tag{10}$$

3.3 Formulating Centralized SC as a Mixed-Integer Non-Linear Program

As stated before, in a centralized structure one core decision maker has the accessibility to all information of the whole SC; therefore, incurs the whole system costs and benefit the whole system profits. Although this is only an ideal case, we can use this structure as a benchmark to analyze the decentralized structure. Notably, for analyzing centralized structure, we did not need game theory. Considering the direct and indirect costs in equations (3) and (10), total profit for centralized SC is given as follows:

$$\pi_t = \sum_{i=1}^{n} p_i \cdot D_i(p_i) - TDC - TIC \tag{11}$$

Now, the optimal performance of centralized system can be determined by the following mixed-integer nonlinear program (MINLP):

Model L_1:

$$\text{Max} \quad \pi_t = \sum_{i=1}^{n} D_i(p_i) p_i - TDC - TIC$$

Subject to:

$$\begin{aligned} & p_i > cm; \quad \forall i \\ & 0 \le b_i \le 1; \quad \forall i \\ & N \ge 0; \text{ Integer} \end{aligned} \tag{12}$$

The first constraint ensures that the sales price in retailers' markets is more than the purchasing price. In the second constraint, the value of b_i is logically limited between zero and one.

3.4 Formulating Decentralized SC as a Mixed-Integer Non-Linear Program

Assuming the vendor as leader, we apply the Stackelberg game theory for modeling and analyzing the decentralized system. Retailer i, as a follower, optimizes the following problem:

Model F_i:

$$\text{Max} \quad \pi_i(p_i) = (p_i - cp - \xi_i) \cdot D_i(p_i)$$

Subject to:

$$p_i \ge cp + \xi_i; \quad \forall i \tag{13}$$

At bellow, we present a theorem on model F_i to better support and clarify the practical findings in the numerical experiments which will be established later for the analysis of the performance of the considered SC.

Theorem 1: The optimal value of p_i is equal to $p_i^* = \frac{(cp+\xi_i) \cdot e_i}{e_i - 1}$.

Proof:

$$\frac{d\pi_i}{dp_i} = (1-e_i) \cdot k_i \cdot p_i^{-e_i} + e_i \cdot (cp + \xi_i) . k_i \cdot p_i^{-(e_i+1)}; \quad \forall i \tag{14}$$

Also, $\frac{d^2\pi_i}{dp_i^2} < 0$ which means π_i is a strictly concave function of p_i; therefore, the optimal value of p_i is determined by setting the derivative in (14) to zero. We have:

$$\frac{d\pi_i}{dp_i} = 0 \Rightarrow p_i^* = \frac{(cp + \xi_i) \cdot e_i}{e_i - 1} \tag{15}$$

Since $e_i > 1$, the optimal value of p_i satisfies the constraint of model F_i. However, the retailers cannot directly determine p_i^* because it is a function of cp determined by the vendor. But, benefiting such an equation especially in numerical analysis of decentralized SC can be worthwhile. Q.E.D.

The Total revenue of vendor is calculated by the following equation:

$$\sum_{i=1}^{n} D_i(p_i)\cdot(cp+\xi_i) \tag{16}$$

Based on the VMI strategy, the vendor is responsible for inventory control in the whole system and should endure all of the corresponding costs in decentralized structure. So, the vendor as a game leader optimizes the following problem:

Model V:

$$Max\ \pi_v(N,\, cp,\, q_i,\, b_i) = \sum_{i=1}^{n} D_i(p_i)\cdot(cp+\xi_i) - TDC - TIC$$

Subject to:

$$\begin{aligned} & cp > cm; \\ & 0 \le b_i \le 1; \quad \forall i \\ & N \ge 0; \quad \text{Integer} \end{aligned} \tag{17}$$

To find the Stackelberg equilibrium point, the KKT conditions is derived from the retailers' models (F_i) and substituted in the vendor's model (V). The sequence of decisions is as follows. First, the vendor, as the Stackelberg leader, determines the item's wholesale price (cp), the quantity to be transported to each retailer (q_i), the fraction of backorder time in any cycle for each retailer (b_i), and the number of retailers' replenishments (N). Thereafter, the profit maximizing retailers, as the followers, determine the retail prices in their corresponding markets.

The KKT conditions for retailer i are formulated as follows:

$$\begin{aligned} & p_i - cp - \xi_i \ge 0 \perp r_i \ge 0; \\ & (1-e_i)\cdot k_i\cdot p_i^{-e_i} + e_i\cdot(cp+\xi_i)\cdot k_i\cdot p_i^{-(e_i+1)} + r_i \le 0 \perp p_i \ge 0 \end{aligned} \tag{18}$$

where r_i is the dual variable for each retailer's constraint (13), and the $\perp$ symbol is used to show the orthogonal relationship between the followers' complementary conditions.

Involving the KKT conditions (18) in the vendor's optimization problem (V) and penalizing (by big number M) the violation of the complementary conditions in the objective function, model L_2 is formulated as a new MINLP:

Model L_2:

$$\begin{aligned} Max\ \pi_v(N,\, cp,\, q_i,\, b_i,\, p_i) &= \sum_{i=1}^{n} D_i(p_i)\cdot(cp+\xi_i) \\ & -TDC - TIC - M\cdot\sum_{i=1}^{n} r_i\cdot(p_i - cp - \xi_i) \\ & +M\cdot\sum_{i=1}^{n} p_i\cdot[(1-e_i)\cdot k_i\cdot p_i^{-e_i} \\ & +e_i\cdot(cp+\xi_i)\cdot k_i\cdot p_i^{-(e_i+1)} + r_i] \end{aligned}$$

Subject to:

$$\begin{aligned} & cp > cm; \\ & 0 \le b_i \le 1; \quad \forall i \\ & p_i \ge cp + \xi_i; \quad \forall i \\ & (1-e_i)\cdot k_i\cdot p_i^{-e_i} + e_i\cdot(cp+\xi_i)\cdot k_i\cdot p_i^{-(e_i+1)} + r_i \le 0 \\ & N \ge 0; \quad \text{Integer} \end{aligned} \tag{19}$$

As it can be seen, we try to provide a closed equation for determining the optimal value of b_i in models L_1 and L_2. We apply theorem 2 as a useful tool in numerical analysis.

Theorem 2: The optimal value of b_i in models L_1 and L_2 is equal to $\frac{H_{bi}}{H_{bi}+L_{bi}}$.

Proof:

For model L_2, we have:

$$\frac{\partial \pi_v}{\partial b_i} = \frac{q_i}{D_i(p_i)}\cdot[D_i(p_i)\cdot H_{bi}\cdot(1-b_i) - D_i(p_i)\cdot L_{bi}\cdot b_i]$$

$$\frac{\partial^2 \pi_v}{\partial b_i^2} = -\frac{q_i}{D_i(p_i)}\cdot[D_i(p_i)\cdot H_{bi} + D_i(p_i)\cdot L_{bi}] \le 0 \tag{20}$$

Therefore, the function π_v is strictly concave with respect to b_i and the optimal value of b_i can be determined by setting the above derivative to zero:

$$\frac{\partial \pi_v}{\partial b_i} = 0 \Rightarrow b_i^* = \frac{H_{bi}}{H_{bi}+L_{bi}} \tag{21}$$

Obviously, b_i^* satisfies the constraints of model L_2.

The proof for model L_1 is exactly the same as the one for model L_2.Q.E.D.

3.5 An Example for the Stackelberg Equilibrium

The Stackelberg equilibrium point is the stable point of the game. In fact, none of the players in the equilibrium point will change their decisions and strategies. Table 1 gives the parameters of a one vendor-three retailers case used for sensitivity analysis purposes. Notably, to generate the input data of our case in Table 1, we try to appropriately consider the suggestions of the other research work son VMI systems (Yu *et al.*, 2009a, 2009b; Almehdawe and Mantin, 2010) and some properties of Cobb-Douglas function in equation 2. Table 2 indicates an example of Stackelberg equilibrium point by changing the wholesale price. In the equilibrium state, the purchasing price is 265 and it can be seen that for the prices being less or more than 265, the vendor's profit is reduced. Therefore, the vendor sells to the retailers at 265 currency units and retailers determine their strategy according to this price. Noteworthily, each retailer only determines its own price and based on the theorem 1, the retail prices are directly related to the wholesale price. Accordingly, since the vendor does not change his/her strategy in the equilibrium point; retailers thus do not so.

4. NUMERICAL ANALYSIS OF DECENTRALIZED VMI SC

In this section, we conduct some numerical analysis to gain some insights regarding the outcomes of modelL_2.We assess the sensitivity of results to changes in critical parameters of both parties (i.e., the vendor and retailers). To do this, three groups of parameters are taken into account including those related to the vendor system (i.e., purchasing price and transportations cost), the retailers' markets (i.e., price elasticity and market scale) and the inventory control system (i.e., holding, backorder and ordering costs). All numerical analysis in this section is conducted for case 1 in Table 1. The mathematical models were developed by Lingo 13 optimization package. Notably, in some reports, only the results for one retailer are illustrated for the sake of compression while they are correct also for the other retailers.

4.1 Vendor's Parameters

Hereby, we report the analytical results regarding the variations in parameters related to the vendor system (i.e., purchasing price and transportations cost).

Vendor's purchasing Price (cm)

Table 3 shows the effect of changes in vendor's purchasing price on the performance of VMI system partners. As it can be seen, by increasing *cm*, the profits of vendor, all retailers, and consequently the whole system will be decreased. Also, by increasing *cm*, the wholesale price increases. This increase, according to theorem1, leads to an increment in retailers' prices and consequently a decrement in retailers' demands. Considering the reverse relation between the order quantities and the demands (i.e., Eq. (4)), it can be concluded that by increasing *cm*, the quantities transferred to each retailer will be decreased. So, in general, we have:

$$cm\uparrow \Rightarrow \begin{cases} cp,\ p_i\ \uparrow \\ D_i(p_i),\ q_i,\ \pi_i,\ \pi_v,\ \pi_t\ \ \downarrow \end{cases} \tag{22}$$

Transportation Cost (Φ_i)

Table 4 illustrates the impact of transportation cost on the performance of VMI system. The Transportation cost for each retailer is increased from the base value given in Table 1 by the values in the Table 3. As observed, by increasing the transportation cost, the wholesale and retailers' prices are increased (based on theorem 1). In contrast, the profits of vendor, retailers and the whole system will be decreased. Also, the value of transfer lot to retailer 1 is decreased because of an increment in retailers' prices and a decrement in retailers' demands. In brief, we can conclude that:

Table 1. Parameters of the one vendor-three retailers case

k_i (×10^4)	Retailers' data						Vendor's data		
	e_i	ξ_i	Φ_i	H_{bi}	L_{bi}	S_{bi}	H_v	S_v	cm
390	1.2	5	6	6	500	100	1	150	50
160	1.5	8	5	8	300	120			
320	1.4	5	11	7	200	110			

Table 2. Example of Stackelberg game equilibrium

cp	100	150	200	265[a]	300	350
π_l(×10^4\$)	89.53962	82.82933	78.32535	74.10536	72.34244	70.17904
π_v(×10^4\$)	13.01044	16.32817	17.31864	17.58411	17.5436	17.39264
π_t(×10^4\$)	129.9998	122.4762	116.3841	110.1570	107.4451	104.0487

[a] Equilibrium point.

Table 3. The Influence of vendor's purchasing price on the VMI system performance

cm	N	q_1	cp	p_3	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
Base	2	200	265	946	14.7431	17.5841	110.157
75	2	162	387	1,371	12.7098	15.9454	100.56
100	2	139	509	1,800	11.3988	14.8544	94.121
125	2	121	658	2,319	10.3012	13.916	88.548

$$\Phi_i \uparrow \Rightarrow \begin{cases} cp,\ p_i \ \uparrow \\ D_i(p_i), q_i,\ \pi_i,\ \pi_v,\ \pi_t \quad \downarrow \end{cases} \tag{23}$$

4.2 Retailers' Markets Parameters

In this subsection, we demonstrate the analytical results regarding the variations in Retailers' markets parameters (i.e., price elasticity and market scale).

Price Elasticity

Table 5 and Figure 2 show the effect of change in e_1 (i.e., the price elasticity of retailer 1) on VMI system performance. As pointed, increments in e_1 result in a reduction in the prices and profits of both vendor and retailer 1. Although raising e_1 diminishes also the prices of the other retailers, it can increase their profits instead. We note that the retailers' markets are independent; however, because parameter e_1 affects the vendor performance, the outcomes of the other retailers may naturally be influenced indirectly. According to Eq. (1), we cannot assess the direction of changes in the demands of retailer 1. Finally the following observations may be deduced:

$$e_i \uparrow \Rightarrow \begin{cases} cp,\ p_i,\ \pi_i,\ \pi_v,\ \pi_t,\ p_j\,(j \neq i) \downarrow \\ D_j(p_j),\ q_j,\ \pi_j\,(j \neq i) \quad \uparrow \end{cases} \tag{24}$$

Market Scale

In Table 6, we try to draw and analyze the impact of k (i.e., the market scale) on the system outcomes. Based on theorem 1, it can be seen that increasing k_2 leads to (1) the decrease in the wholesale and retail prices, (2) the increase in vendor's and retailer 2's profits.

Table 4. The impact of transportation cost on VMI system performance

Φ	N	q_1	cp	p_3	$\pi_3(\times 10^4\$)$	$\pi_v(\times 10^4\$)$	$\pi_t(\times 10^4\$)$
Base	2	200	265	946	14.7431	17.5841	110.157
5	2	192	286	1,021	14.3051	17.239	108.129
10	2	182	314	1,116	13.8029	16.8337	105.773

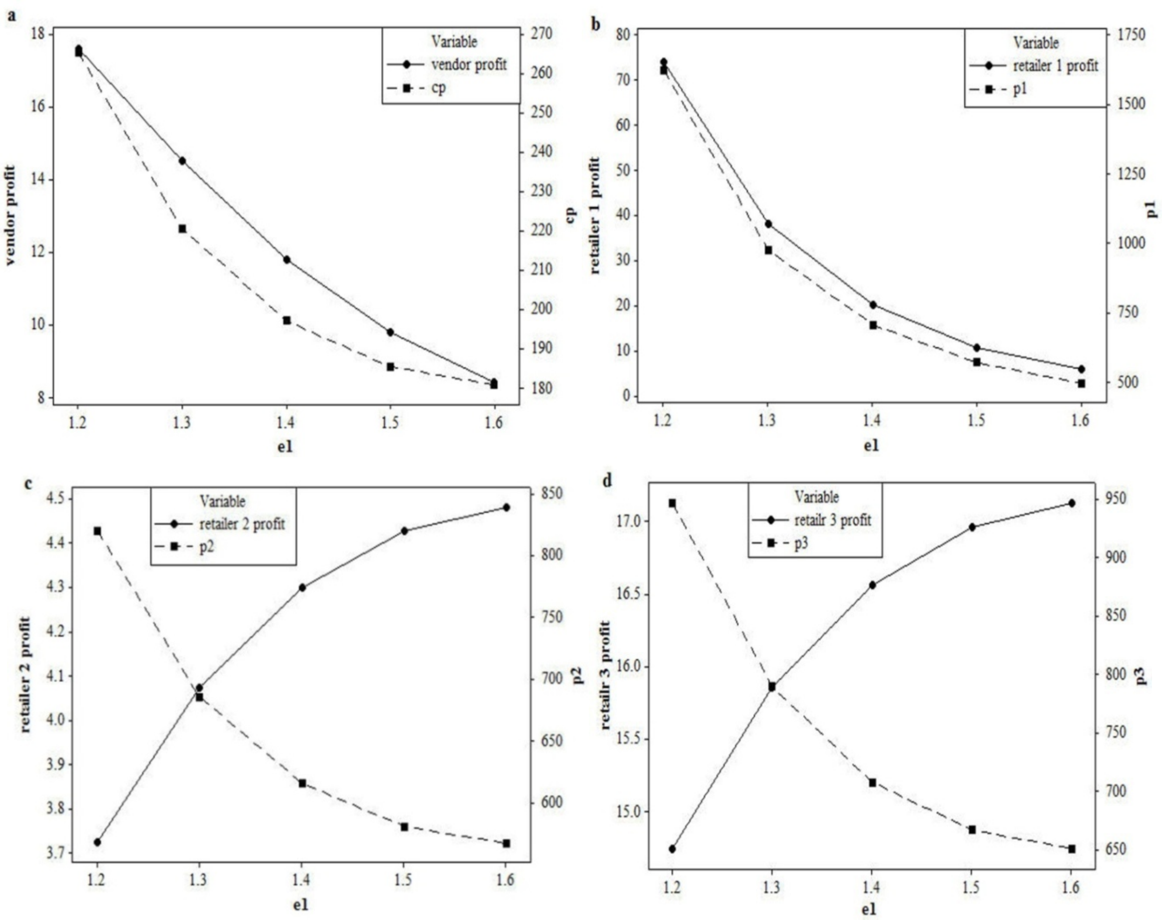

Figure 2. The impact of price elasticity on system performance (a) vendor (b) retailer 1(c) retailer 2 (d) retailer 3.

Table 5. Influence of price elasticity on VMI system performance

e_1	N	q_1	cp	p_1	p_2	p_3	π_1 (×10^4\$)	π_2 (×10^4\$)	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
Base	2	200	265	1622	820	946	74.1054	3.72446	14.7431	17.5841	110.157
1.3	2	179	221	977	686	789	38.028	4.07361	15.8526	14.5196	72.474
1.4	2	144	197	708	616	708	20.1834	4.29908	16.5607	11.8121	52.855
1.5	2	107	186	572	581	667	10.874	4.42643	16.9581	9.8198	42.078

Table 6. The influence of market scale on the VMI system performance

k_2	N	q_1	cp	p_1	p_2	p_3	π_1 (×10^4\$)	π_2 (×10^4\$)	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)
base	2	200	265	1622	820	946	74.1054	3.72446	14.7431	17.5841	110.157
210	2	202	260	1593	806	929	74.3733	4.93217	14.8499	18.0466	112.202
260	2	205	256	1567	792	914	74.6247	6.15764	14.9504	18.5111	114.244
310	2	210	252	1542	780	899	74.8608	7.39936	15.0452	18.9773	116.283

Table 7. The impact of holding cost in retailer side on the VMI system performance

H_{bi}	N	q_1	p_3	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)	cp
base	2	200	946	14.7431	17.5841	110.157	265
10	2	155	952	14.7035	17.521	109.942	267
20	3	123	957	14.6726	17.4719	109.775	268

Table 8. The effect of ordering cost in vendor side on the VMI system performance

S_v	N	q_1	p_3	π_3 (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)	cp
base	2	200	946	14.7431	17.5841	110.157	265
100	1	221	945	14.749	17.5936	110.189	265

Though, the changes in the profits of the other retailers are almost inconsiderable. This is because the demands of the other retailers are only influenced by the decrease in the wholesale and retail prices while those of retailer 2 are also under effect of the market scale. The-refore, the changes in the demands of retailer 2 are definitely much more than those of the other retailers.
So, in general, we can say that (Wei and Choi, 2010):

$$k_i \uparrow \Rightarrow \begin{cases} cp,\ p_i,\ p_j\,(j \neq i) \downarrow \\ D_i,\ q_i,\ \pi_i,\ \pi_v,\ \pi_t,\ D_j,\ q_j\,(j \neq i) \quad \uparrow \\ \pi_j\ (j \neq i) \quad \leftrightarrow \end{cases} \tag{25}$$

Generally, the retailers' market parameters and particularly the price elasticity have a significant effect on the system performance.

4.3 Inventory Control Parameters

Our analysis indicates that parameters related to the inventory control system have less impact on the values of profits and prices in VMI system. However, these parameters sometimes could influence on the policy of inventory control system applied by the vendor. Table 7 gives the effect of inventory holding costs in the warehouse of retailerson the system performance. The value of H_{bi} is increased from the base value denoted in Table 1. We can see that when the holding costs in the retailers' sites are increased from the base value by 20 currency units, the number of replenishments increases from 2 to 3. Also, table 8 illustrates the effect of S_v (i.e., the ordering costs) on the system performance. Again, we observe that decreasing S_v from 150 to 100 leads to a reduction in N-i.e., from 2 to 1.

4.4 Comparing Centralized and Decentralized Systems

In order to compare the performance of centralized and decentralized SCs, we use the proposed approach of Guan and Zhao (2010). If π_c and π_d is assumed to be the profit of centralized and decentralized systems, respectively, then parameter θ is defined as follows:

$$\theta = \frac{\pi_d}{\pi_c} \tag{26}$$

The value of θ is between 0 and 1. The more the value of θ, the closer the profit of decentralized system to centralized one proving better performance of decentralized VMI system. For comparing purposes, 100 numerical problems were generated at random based on the suggestions of the other researchers in their works on VMI system (Yu *et al.*, 2009a; 2009b, Almehdawe and Mantin, 2010). Table 9 gives the domain and step of variations of randomly generated data.

In Table 10, as an example, we illustrate the result of comparison for case 1 in Table 1. As observed, the transfer lot for retailer 1 in centralized system is 482 units while in decentralized one is only 199 units. Also, retailer 1 sells at 339 currency units in centralized system whereas in decentralized system at 1,622. Similar comparisons can be established for the other retailers. Finally, as a critical criterion, the ratio of decentralized profit to centralized profit (i.e., θ) is 0.81 for this case.

Thereafter, both models L_1 and L_2were solved to obtain the value of θ for all 100 random numerical tests. The result of comparison indicates that the performance of considered SC in decentralized state is very different than centralized state. In fact, in decentralized state, in addition to a decrease in system's profit, the selling prices in retailers' market are also very different from centralized state. For example, selling price of retailer 1 in decentralized state is, in average, fourfold of the one in centralized state. The results of 100 random cases indicate that the average and minimum value of θ is 0.79 and 0.55, respectively.

According to the reported numerical results and the previous studies on different VMI systems established by the other researchers (Guan and Zhao, 2010; Almehdawe and Mantin, 2010), it seems that the considered decentralized VMI SC has a poor performance compared to the centralized one. So, it seems that a critical area of research is to design the suitable contracts between vendor and retailers for improving decentralized system performance. This falls in the scope of designing contract for the VMI system and will be an interesting direction for future researches.

5. CONCLUSION

The proposed SC in this article was a two echelon SC, which consists of one vendor and multiple retailers. Assuming the vendor as a leader, we used the Stackelberg game theory for modeling and analyzing this system. In addition to the modeling of centralized structure, the decentralized SC was also modeled based on the Stackelberg game theory to analyze the system performance. Also, comprehensive analytical results for the VMI system performance were provided through a near to real numerical case inspired by the outcomes of this and the previous researches on VMI SC. Results of our analysis indicate that some parameters like price elasticity has a significant effect on the VMI system's performance while some other parameters like the inventory control parameters do not show such significant effects. Comparing the performance of the centralized and decentralized SC through on hundreds of randomly generated test problems indicates that the decentralized state inherently has a poor performance because both the profits and prices of all elements (i.e., vendor and retailers) are significantly different from centralized state. As a result, an interesting direction for future developments of this research is to investigate and design different proper contracts between SC elements and VMI partners, in order to improve the performance of decentralized system.

REFERENCES

Almehdawe, E. and Mantin, B. (2010), Vendor Managed inventory with capacitated manufacturer and multiple retailers: Retailer versus manufacturer leadership, *Internatinal Journal of Production Economics*, **128**, 292-302.

Bichescu, B. C. and Fry, M. J. (2009), A numerical analysis of SC performance under split decision rights, *Omega*, **37**, 358-379.

Table 9. Domain and step of variations for randomly generated parameters

Parameter	k_i (×10^4)	e_i	ξ_i	H_{bi}	L_{bi}	S_{bi}	Φ_i	H_v	S_v	cm
Domain of variations	150-400	1.2-1.7	2-15	5-10	100-500	20-150	2-15	1-4	100-200	50-200
Step of variations	1000	0.1	1	1	100	10	1	1	10	10

Table10. Comparing the performance of centralized and decentralized systems

	N	D_i	q_i	cp	b_i	p_i	π_t (×10^4\$)	π_v (×10^4\$)	π_t (×10^4\$)	θ
Centralized	2	3,590	482	-	0.011858	339	-	-	135.9806	-
		742	99		0.025974	167				
		1,733	233		0.033816	215				
Decentralized	2	548	199	265	0.011858	1622	74.10536	17.58411	110.157	0.81
		68	25		0.025974	820	3.724461			
		218	79		0.033816	946	14.74306			

Cachon, P. G. and Netessine, S. (2003), *Game Theory in SC Analysis, Handbook of SC Analysis in the eBusiness Era*. Kluwer Academic Publisher, USA.

Challener, C. (2000), Tacking the VMI step to collaborative commerce, *Chemical market Reporter*, **258**, 11-12.

Darvish, M. A. and Odah, O. M. (2010), Vendor managed inventory for single-vendor multi retailer supply chians, *European Journal of Operationl Research*, **204**, 473-484.

Fiestras, M. G., Garcia-Jurado, I., Meca, A., and Mosquera, M. A. (2010), Cooperative game theory and inventory management, *European Journal of Operational Research*, **210**, 459-466.

Guan, R. and Zhao, X. (2010), On contract for VMI program with continuous review (r, Q) policy, *European Journal of Operational Research*, **207**, 656-667.

Kim, B. and Park, C. (2010), Coordinating decisions by SC partners in a vendor-managed inventory relationship, *Journal of Manufacturing systems*, **29**, 71-80.

Lau, A., Lau, H., and Zhou, Y. (2007), A stochastic and asymmetric-information framework for a dominant-manufacturer SC, *European Journal of Operational Research*, **176**, 295-316.

Li, X. and Wang, Q. (2007), Coordination mechanisms of SC systems, *European Journal of Operational Research*, **179**, 1-16.

Lin, Z., Cai, C., and Xu, B. (2010), SC coordination with insurance contract, *European Journal of Operational Research*, **205**, 339-345.

Nagarajan, M. and Sosic, G. (2008), Game-theoretic analysis of cooperation among SC agents: Review and extensions, *European Journal of Operational Research*, **187**, 719-745.

Ortmeyer, R. D. and Buzzell, G. (1995), Channel partnerships streamline distribution, *Sloan Management Review*, **36**, 85.

Shah, B. J. (2002), ST, HP VMI program hitting its stride, *Electronics Business News*, **1309**, 42-43.

Su, C.-T. and Shi, C.-S. (2002), A manufacturer's optimal quantity discout strategy and return policy through game-theoretic approach, *Journal of the Operational Research Socity*, **53**, 922-926.

Tyan, J. and Wee, H. M. (2003), Vendor managed inventory: a survey of the Taiwanes grocery industry, *Journal of Purchasing and Supply Management*, **9**, 11-18.

Wei, Y. and Choi, T. M. (2010). Mean-variance analysis of SCs under wholesale pricing and profit sharing schemes, *European Journal of Operational Research*, **204**, 255-262.

Yang, D., Jiao, J., Ji, Y., Du, G., Helo, P., and Valente, A. (2015), Joint optimization for coordinated configuration of product families and supply chains by a leader-follower Stackelberg game, *European Journal of Operational Research*, **246**, 263-280

Yu, H., Zeng, A. Z., and Zhao, L. (2009), Analyzing the evolutionary stability of the vendor-managed inventory SCs, *Computers and industrial Engineering*, **56**, 274-282.

Yu, Y., Chu, F., and Chen, H. (2009a), A stackelberg game and its improvment in a VMI system with a manufacturing vendor, *European Journal of Operational Research*, **192**, 929-948.

Yu, Y., Huang, G. Q., and Liang, L. (2009b), Stackelberg game-theoretic model for optimizing advertising, pricing and inventory policies in vendor managed inventory (VMI) production SCs, *Computers and Industrial Engineering*, **57**, 368-382.

Yu, Y. and Huang, Q. G. (2010), Nash game model for optimizing market strategies, configuration of platform products in a Vendor Managed Inventory (VMI) SC for product family, *European Journal of Operational Research*, **206**, 361-373.