

Multi-Objective Soft Computing-Based Approaches to Optimize Inventory-Queuing-Pricing Problem under Fuzzy Considerations

Alireza Alinezhad*

Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

Amin Mahmoudi

Department of Industrial Engineering, Raja University, Qazvin, Iran

Vahid Hajipour

Industrial Engineering Department, Faculty of Engineering, Bu-Ali Sina University, Hamedan, Iran

(Received: November 15, 2016 / Revised: November 22, 2016 / Accepted: November 25, 2016)

ABSTRACT

Due to uncertain environment, various parameters such as price, queuing length, warranty, and so on influence on inventory models. In this paper, an inventory-queuing-pricing problem with continuous review inventory control policy and batch arrival queuing approach, is presented. To best of our knowledge, (I) demand function is stochastic and price dependent; (II) due to the uncertainty in real-world situations, a fuzzy programming approach is applied. Therefore, the presented model with goal of maximizing total profit of system analyzes the price and order quantity decision variables. Since the proposed model belongs to NP-hard problems, Pareto-based approaches based on non-dominated ranking and sorting genetic algorithm are proposed and justified to solve the model. Several numerical illustrations are generated to demonstrate the model validity and algorithms performance. The results showed the applicability and robustness of the proposed soft-computing-based approaches to analyze the problem.

Keywords: Inventory Control, Queuing Theory, Fuzzy Programming, Pricing, Computational Intelligence

* Corresponding Author, E-mail: alalinezhad@gmail.com

1. INTRODUCTION

Nowadays, combination of queuing theory and inventory control models has been widely investigated by many researchers. In this system, customers arrive to the system to take your service. In order to compliance the customer requirement, a product of inventory is required. After complete the customer service, they leave the system and on-hand inventory is reduced. Inventory is provided by an external supplier. A system with mentioned characteristic is well-known as queuing-inventory system (Schwarz *et al.*, 2006). In these models, satisfying

each demand needs on-hand inventory and involves a process or service that takes time. In these systems, an important issue is the reaction of inventory management to queuing of demands. It is also different from the traditional inventory management because the inventory is consumed at the serving rate rather than the customers' arrival rate when there are customers queued up for service (Teimoury *et al.*, 2010; Chang and Lu, 2010).

Berman and kim (1999) considered a queuing-inventory system with Poisson arrival rate and exponential service times under zero lead time. Berman and Sapna (2000) considered queuing-inventory systems with Pois-

son arrival and a conventional service rate and zero lead time. Berman and Sapna (2001) studied a capacitated queuing system under exponential arrival rate, service rate and lead time. Berman and Kim (2001) investigated a supply chain based on internet under Poisson arrivals, exponential service times and the Erlang lead times. The authors founded that the optimal ordering policy has a monotonic threshold structure. Berman and Kim (2004) studied a queuing-inventory system with Poisson arrival rate and exponential service rate and lead time. They obtained a replacement policy with maximizing profit. Dong and Chen (2005) developed a queuing-inventory system for analyzing performance of an integrated logistic. They considered ordering policy and lot size to use $G^r/G/1$ system under inventory control policy of (s, S) . Isotupa (2006) analyzed an (s, Q) Markovian inventory system with lost sales and two demand classes. She developed an efficient algorithm for minimizing replenishing, lost sales and inventory costs. Schwarz *et al.* (2006) considered $M/M/1$ queuing-inventory system with lost sales under various inventory management policies such as (r, Q) policy and (r, S) policy and derived stationary distributions of joint queue length and inventory processes in explicit product form. Schwarz and Daduna (2006) studied an $M/M/1$ queuing-inventory system with considering backordering. Krishnamoorthy *et al.* (2006) considered an (s, S) inventory system that server continues processing the products even in the absence of customers. Krishnamoorthy *et al.* (2006) introduced a new policy called N -policy in (s, S) inventory system. Hill (2007) considered continuous-review inventory models with no fixed order cost, lost-sales, and a Poisson demand process to minimize the long run total cost and explore alternative approaches which might offer better solutions. Manuel *et al.* (2007) and Manuel *et al.* (2008) investigated a perishable queuing-inventory system with Markovian arrival process. Haji *et al.* (2011) presented an integrated queuing-inventory system in a Two-level Supply Chain. Zhao and Lian (2011) considered a queuing-inventory system with two classes of customers with Poisson arrival and exponential service rate. Teimoury *et al.* (2012) studied decision making on Order Penetration Point strategy with a queuing concept. Order Penetration Point is boundary between Make-to-order and Make-to-stock. Saffari *et al.* (2013) considered an $M/M/1$ queuing system under (r, Q) inventory policy and in presence of lost sales. They derived the stationary distributions of the joint queue length (number of customers in the system) and on-hand inventory. Krishnamoorthy *et al.* (2015) considered a queuing-inventory system, with the item given with probability γ to a customer at his service completion epoch. They discussed two inventory policy of (s, S) and (s, Q) and obtained the joint distribution of the number of customers and the number of items in the inventory as the product of their marginal under the assumption that customers do not join when inventory level is zero. Alaghebandha and Hajipour (2015) proposed a multi-product continuous review inventory con-

trol problem within batch arrival queuing approach ($M^{Qr}/M/1$) to find the optimal quantities of maximum inventory. They proposed an efficient imperialist competitive algorithm (ICA) to solve the model. Baek and Moon (2014) investigated a queuing system integrated with a production-inventory system in which the stocks are delivered both by an outside supplier and an internal production. Güray Güler *et al.* (2014) modeled the inventory problem under $M/M/1$ queuing system by considering several classes of customers. Sajadi *et al.* (2015) proposed a mixed integer nonlinear programming model to address the location-inventory problem under queuing approach. They considered uncertain demand and lead-time, which follow Poisson and Exponential distributions, respectively. Baek and Moon (2016) extended a production-inventory model with an $M/M/c$ service queue and lost sales.

Most queuing-inventory models analyzed in certain situation while information in real-life cases are often uncertain and ambiguous. Therefore, in the mentioned literature, there was less attention to analyze the price in supply chain. Whereas often revenue management as maximizing the profit is one of the main aims in supply chain management. As price affects on demand quantity, and demand rate affects on queuing parameters, therefore analyzing the price variable can be an important task. In this paper, a queuing-inventory model is presented in uncertainty environment using fuzzy set theory introduced by Zadeh (1965) to formulate the problem in presence of increasing accuracy and validity of the problem. Moreover, arrival rate behaviors as Poisson and depends on price are considered, simultaneously. The goal is to maximize the total profit to determine optimal price as well as the order quantities. Since the proposed model is Non-deterministic Polynomial-time hard; therefore, non-dominated ranking genetic algorithm is presented and compared with the literature to solve and justified the proposed model.

Rest of the paper is organized as follow: Next section defines the problem, notations, decision variables, and proposed mathematical formulation of the problem. Section 3 described Pareto-based approaches based on GA to solve the model. Section 4 analyzed the results and comparisons. Finally, conclusions and some future researches are given.

2. PROBLEM FORMULATION

2.1 Problem Description

In this paper a system is considered that contain an external supplier, warehouse, retailer and customers. Retailer demands act as Poisson probability distribution. Retailer orders customer demands to the warehouse in a stochastic time interval. Demand of retailers causes to be provided some orders constantly with predetermined

time horizons into the warehouse. Warehouse provides his order from external supplier. It is also assumes that transportation time from warehouse to retailer and also lead time delivery of external supplier are constant. To make the model more realistic and increase accuracy in decision making, in this research, we have proposed a fuzzy mathematical model under uncertainty environment. Modeling and optimizing a model under uncertain environment with considering costs parameters is one of the contributions of this paper. It is valuable to mention that restrictions of warehouse space, number for the shortage, service level, and expected shortage are considered in mathematical formulation process of the problem.

2.2 The Assumptions

- The retailer faces with Poisson demand;
- The warehouse faces with stochastic demand;
- Unsatisfied demand by the retailer is as lost sales;
- Shortage is not allowed at the warehouse;
- Shortage is allowed at the retailer;
- There is no lot-splitting at the warehouse;
- The transportation time for an order to arrive at the retailer from the warehouse is as exponential distribution;
- The warehouse orders to an external supplier with infinite capacity;
- The time between two consecutive orders is as exponential distribution;
- The lead time for an order to arrive at the warehouse is constant;
- Cost coefficients in objective function are as fuzzy number;

2.3 The Mathematical Model

Consider a supply chain consisting of external supplier, warehouse, retailer, and customers. In this system, retailer is dealt with customer's demand as Poisson process. Arrival rate is $\lambda_j = \xi_j - \alpha_j p_j$ which is dependent on price. Retailer orders Q_r to warehouse in interval T . Moreover orders of warehouse are provided by external supplier. Transportation time from warehouse to retailer is as Exponential distribution and it is assumed that lead time delivery from external supplier is fixed. Since arrival of products to retailer is stockpile, therefore system is faced with batch arrivals with random amount of Q_r . Time between batch arrivals is Exponential distribution with parameter $\phi_j (\phi_j > 0)$. Whereas only one retailer is in considered system, therefore system has one server. Service rate of the server for product j is as an exponential random variable with parameter of $\mu_j (\mu_j > 0)$. Arrival time and service time are independent. As an order is delivered a time is required to service it, if in this moment service time of previous order has not been completed or time of service has been longer than next order; therefore, we have congestions in the system. Since pro-

ducts arrive to the retailer as batch arrivals; thus, $M^q / M / 1$ queuing system is considered. With using mentioned issue and considering continuous review inventory control as inventory control policy, average number of customers of queuing system in long time (ℓ) is used for calculating average inventory (\bar{I}). In order for better understanding, the process of modelling is explained in two following subsections including the investigation of the model in crisp environment and fuzzy one, respectively.

2.3.1 Crisp Model

In this paper, warehouse applies continuous review inventory control policy and retailer uses periodic review inventory control policy. In this system, retailer orders to warehouse are in random interval time. It means that demand of warehouse is established by orders of retailer and is also a random variable. Random demand for warehouse causes safety stock is increased and also inventory control system is established. The crisp model is formulated as follow:

$$\text{Maximize } TC_B = (\sum_{j=1}^n p_j \times D_j) - \sum_{j=1}^n (A_j / E[T_j] + h_j \bar{I}_j + \pi_j \bar{b}(R_j) / E[T_j]) - \sum_{j=1}^n A_{w_j} / m_j [] + h_{w_j} - \lambda_j \quad \lambda_j - \lambda_j + m_j - Q_{r_j}$$

Subject to:

$$\sum_{j=1}^n \bar{b}(R_j) \leq G \tag{1}$$

$$\sum_{y_j=0}^{R_j} \lambda_j e^{-\lambda_j y_j} (y_j + 1 / \lambda_j) \geq P_j; \quad j = 1, 2, \dots, n$$

$$\sum_{j=1}^n (\pi_j \times \sum_{y_j=R_j}^{\infty} \lambda_j e^{-\lambda_j y_j} (y_j + 1 / \lambda_j) (y_j - R_j)) \leq S$$

$$\lambda_j Q_{r_j} < T_{jr}; \quad j = 1, 2, \dots, n$$

$$\lambda_j - \bar{b}(R_j) = D_j; \quad j = 1, 2, \dots, n$$

$$R_j \geq 0$$

$$Q_{r_j} \text{ and } m_j \text{ are as positive integ}$$

$$P_j \geq 0$$

$$\lambda_j = \xi_j - \alpha_j p_j; \quad j = 1, 2, \dots, n$$

2.3.2 The Proposed Fuzzy Model

In this paper, a fuzzy mathematical model is presented for modeling uncertainty costs parameters in the proposed system. Therefore, shortage cost, holding cost, and ordering cost are considered as fuzzy number of $\tilde{A}, \tilde{h}, \tilde{\pi}$. To do so, Lia and Hwang (1992) method is applied to formulate fuzzy mathematical model. Consider following single objective model with fuzzy coefficients.

$$\text{Maximize } \tilde{C}X$$

S.t.

$$x \in X = \{x | Ax \leq b \text{ and } x \geq 0\}$$

where $\tilde{C} = (C_L, C_M, C_R)$ are triangular fuzzy numbers.

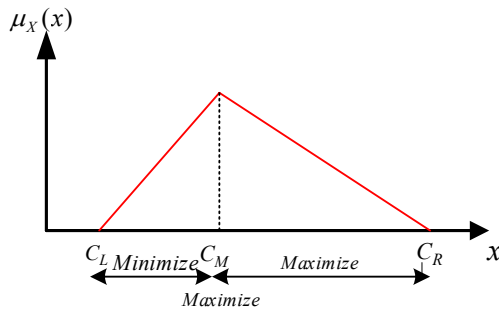


Figure 1. Objective function optimization based on triangular fuzzy number.

For this category of problem a multi-objective approach is presented. In order to maximize fuzzy objective function, Figure 1 indicates how single fuzzy objective is converted to three crisp objectives.

In order to maximize fuzzy objective function following steps are required:

- 1) Maximize C_M
- 2) Minimize interval between C_M and C_L
- 3) Maximize interval between C_M and C_R

Therefore, multi-objective mathematical model is

$$\text{Minimize } Z_1 = (C_M - C_L)X$$

$$\text{Maximize } Z_2 = C_M X$$

$$\text{Maximize } Z_3 = (C_R - C_M)X$$

S.t.

$$x \in X = \{x | Ax \leq b \text{ and } x \geq 0\}$$

Note that with considering $\tilde{A} = (A^L, A^M, A^R)$, $\tilde{h} = (h^L, h^M, h^R)$, $\tilde{\pi} = (\pi^L, \pi^M, \pi^R)$ in proposed model, we have following multi-objective model:

$$\begin{aligned} \text{Minimize } TC_1 = & \sum_{j=1}^n ((A_j^M - A_j^L) / E[T_j]) + (h_j^M - h_j^L) \bar{I}_j \\ & + (\pi_j^M - \pi_j^L) \bar{b}(R_j) / E[T_j] + \sum_{j=1}^n (A_{wj}^M - A_{wj}^L) / m_j E[T_j] \\ & + (h_{wj}^M - h_{wj}^L) ((-\ln \lambda_j / 2) / \lambda_j - \lambda_j + (m_j - 1) Q_{vj} / 2) \end{aligned} \quad (2)$$

$$\begin{aligned} \text{Maximize } TC_2 = & \sum_{j=1}^n (A_j^M / E[T_j]) + h_j^M \bar{I}_j + \pi_j^M \bar{b}(R_j) / E[T_j] \\ & + \sum_{j=1}^n A_{wj}^M / m_j E[T_j] + h_{wj}^M ((-\ln \lambda_j / 2) / \lambda_j - \lambda_j \\ & + (m_j - 1) Q_{vj} / 2) \end{aligned} \quad (3)$$

$$\begin{aligned} \text{Maximize } TC_3 = & \sum_{j=1}^n ((A_j^R - A_j^M) / E[T_j]) + (h_j^R - h_j^M) \bar{I}_j \\ & + (\pi_j^R - \pi_j^M) \cdot \bar{b}(R_j) / E[T_j] + \sum_{j=1}^n (A_{wj}^R - A_{wj}^M) / m_j E[T_j] \\ & + (h_{wj}^R - h_{wj}^M) ((-\ln \lambda_j / 2) / \lambda_j - \lambda_j + (m_j - 1) Q_{vj} / 2) \end{aligned} \quad (4)$$

Subject to:

$$\sum_{j=1}^n \bar{b}(R_j) \leq G \quad (5)$$

$$\sum_{y_j=0}^{R_j} \lambda_j e^{-\lambda_j y_j} (y_j + 1 / \lambda_j) \geq P_j; \quad j=1, 2, \dots, n \quad (6)$$

$$\sum_{j=1}^n (\pi_j \times \sum_{y_j=R_j}^{\infty} \lambda_j e^{-\lambda_j y_j} (y_j + 1 / \lambda_j) (y_j - R_j)) \leq S \quad (7)$$

$$\lambda_j Q_{vj} < T_{jr}; \quad j=1, 2, \dots, n \quad (8)$$

$$\lambda_j - \bar{b}(R_j) = D_j; \quad j=1, 2, \dots, n \quad (9)$$

$$R_j \geq 0$$

$$Q_{vj} \text{ and } m_j \text{ are as positive integer} \quad (10)$$

$$P_j \geq 0$$

$$\lambda_j = \xi_j - \alpha_j p_j; \quad j=1, 2, \dots, n \quad (11)$$

Objective functions (2-4) defuzzified objectives of propose model. Constraints (5-7) are upper limits of expected shortage of each product, lower limit of service level, and upper limit of total shortage costs, respectively. Constraints (8) indicate stationary of queuing system. Constraints (9) ensure demand of each product is satisfied. Constraints (10) determine ranges of variables and constraints (11) indicates arrival rate of customers which is dependent on price.

3. PARETO-BASED APPROACHES

Most exact methodologies such as Lagrangian relaxation (Fisher, 2004) and branch and bound (As'ad and Demirli, 2011) have been developed in the literature to solve less complicated inventory control models but they cannot be applied to find exact optimal solutions of the model. In this paper, a non-dominated sorting genetic algorithm (NSGA-II) and a non-dominated ranked genetic algorithm (NRGA) are presented to find Pareto solutions.

3.1 The Proposed NSGA-II

Non-dominated sorting genetic algorithm (NSGA), introduced by Deb *et al.* (2001), is one of the best-developed Pareto-based meta-heuristic algorithms. Using Pareto dominance solutions, it is a computationally efficient algorithm implementing the idea of a selection method based on classes of dominance of all the solutions. The NSGA-II includes five main steps: initialization, fast non-dominated sorting (FNDS), crossover, mutation, and the elitist crowded-comparison operator. The main steps are illustrated in details as follow:

3.1.1 Initialization

In this research, a chromosome represents different number of products ordered in different periods as presented by Alaghebandha and Hajipour (2015). A vector

is added to the chromosome structure to generate the prices. Therefore, upper bound on the price in each product is considered, in the initialization step of the proposed NSGA-II algorithm, a uniform integer random number is generated for each chromosome.

3.1.2 Fast Non-Dominated Dorting

The computational complexity of NSGA-II is $O(MR^2)$ where M and R are the number of objectives and the population size, respectively (Deb, 2001). In this step, the R populations that were generated in the previous step are compared and are sorted. To do this, all chromosomes in the first non-dominated front are first found. By concept of domination, in which a solution x_i is said to dominate solution x_j , if $\forall o \in \{1, 2\}$ we have $Z_o(x_i) \leq Z_o(x_j)$ and $\exists o \in \{1, 2\}$ such that $Z_o(x_i) < Z_o(x_j)$. In this case, we say x_i is the non-dominated solution within the solution set $\{x_i, x_j\}$. Otherwise, it is not. Then, in order to find the chromosomes in the next non-dominated front, the solutions of the previous fronts are disregarded temporarily. This procedure is repeated until all solutions are set into fronts.

3.1.3 Crowding Distance

Crowding distance operator evaluates solution fronts of populations in terms of the relative density of individual solutions. To do this, consider Z and f_k ; $k=1, 2, \dots, M$ the number of non-dominated solutions in a particular front (F) and the objective functions, respectively. Besides, let d_i and d_j be the value of crowding distance on the solution i and j . The crowding distance is obtained using the following steps:

- (1) Set $d_i = 0$ for $i = 1, 2, \dots, Z$
- (2) Sort all objective functions f_k ; $k = 1, 2, \dots, M$ in ascending order
- (3) The crowding distance for end solutions in each front (d_1 and d_z) are $d_1 = d_z = \infty$
- (4) The crowding distance for d_j ; $j = 2, 3, \dots, (Z-1)$ are $d_j = d_j + (f_{k_{j+1}} - f_{k_{j-1}})$

3.1.4 Selection Strategy

To choose individuals of the next generation, tournament selection operator “ \succ ” is applied (Deb, 2001). To do this, n individuals in the population is randomly selected. Then, non-dominated rank of each individual is obtained. Following this, CD of the solutions with equal non-dominated rank is computed. At end, the solutions with least rank are the selected ones. Moreover, if more than one individual share the least rank, the individual with highest crowding distance must be selected.

3.1.5 Crossover and Mutation Operators

In presented NSGA-II, we implemented continues uniform crossover and swap strategy for crossover and mutation operators, respectively (Hajipour *et al.*, 2014, Montazer-Gh and Mahmoodi-k, 2016). It should be mentioned that, after the crossover and mutation operations, the fitness function values are calculated for offspring.

3.1.7 Merging the Population

In this step, the parents and offspring population are merged to ensure the elitism. Since the combined population size is naturally greater than the original population size N , FNDS is performed. In fact, chromosomes with higher ranks are selected and added to the populations until the population size becomes N . The last front is also consisted of the population based on the crowding distance. The algorithm stops when a predetermined number of iterations are reached.

3.2 The Proposed NPGA

NPGA is a new multi-objective genetic algorithm to find feasible Pareto front solutions. NPGA is similar to NSGA-II with the difference that in the selection operation the roulette wheel strategy is employed (Jadaan *et al.*, 2008). In NPGA, a fitness value representing rank is assigned to each individual of the population. In this regard, two ranked based roulette wheel selection features including:

- (1) Select the fronts
- (2) Choose solutions from the fronts.

The selection probability of fronts, P_f , and the selection probability of solutions, P_{fs} , are obtained using Eq. (12) and (13).

$$P_f = \frac{2 \times Rank_f}{NF \times (NF + 1)}; f = 1, \dots, NF \quad (12)$$

$$P_{fs} = \frac{2 \times Rank_{fs}}{NS_f \times (NS_f + 1)}; f = 1, \dots, NF, s = 1, \dots, NS \quad (13)$$

Where NF and NS_f are the number of fronts and the number of solutions in front f , respectively. Eq. (12) ensures that a front with highest rank has the highest probability to be selected. Similarly, based on Eq. (13), solutions with more crowding distance are assigned higher selection probability. The roulette wheel selection is iterated until a desired number of solutions are selected. At the end, the algorithm stops when a predetermined number of iterations are reached. The process of NSGA-II and NPGA are represented in Figure 2.

4. RESULT ANALYSIS

To evaluate the performances of the proposed Pareto-based algorithms, five standard metrics of multi-objective algorithms are applied as follows (Zitzler and Thiele, 1998):

- Diversity: measures the extension of the Pareto front in which bigger value is better.
- Spacing: measures the standard deviation of the distances among solutions of the Pareto front in which smaller value is better.
- Number of found solutions (NOS): counts the num-

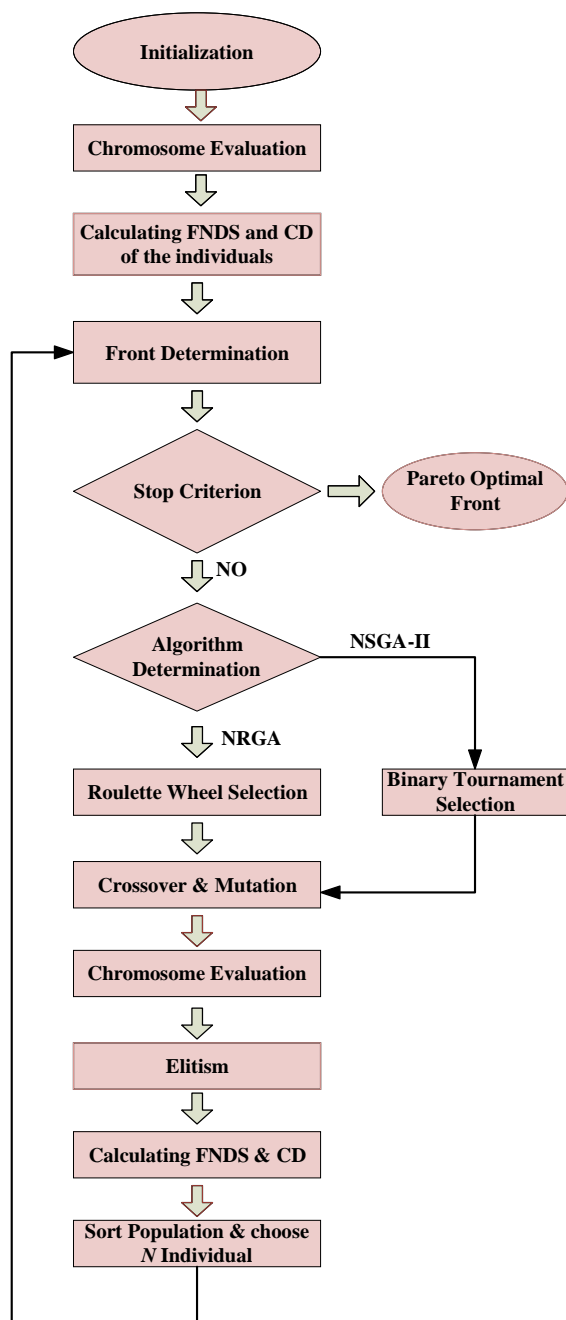


Figure 2. Flowchart of NSGA-II and NPGA.

ber of the Pareto solutions in Pareto optimal front in which bigger value is better.

- Mean ideal distance (MID): measures the convergence rate of Pareto fronts to a certain point (0, 0).
- The computational (CPU) time of running the algorithms to reach near optimum solutions

The experiments are implemented on 10 test problems. Furthermore, to eliminate uncertainties of the solutions obtained, each problem is used three times under different random environments. Then, the averages of

these three runs are treated as the ultimate responses. The NSGA-II as most applicable Pareto-based MOEAs in the literature is applied to demonstrate capability of the proposed algorithms to solve the multi-objective optimization problems. It required to be mentioned that the input parameters of the algorithms are reported in Table 1.

In order to analyze the performance of the proposed Algorithms, Table 2 first generate some test problems and then Table 3 and Table 4 report the multi-objective metrics amounts on the 10 test problems. It is noticed that the MATLAB Software has been used to code the proposed meta-heuristic algorithms, and the programs have been executed on a 2 GHz laptop with eight GB RAM.

Table 1. Values of algorithms parameters

Multi-Objective Algorithms	Algorithm Parameters	Optimum amount
NRGA And NSGA-II	Number of Population	25
	Crossover Probability	0.8
	Mutation Probability	0.2
	Maximum Number of Generation	100

Table 2. Generated test problems

Problem number	Number of Product	Number of shortage
1	3	1,000
2	5	1,000
3	10	4,500
4	15	6,000
5	20	8,000
6	35	12,000
7	50	18,000
8	80	26,000
9	120	54,000
10	320	145,000

Table 3. Computational results based on multi-objective metrics for NSGA-II

Problem No.	NS	SP	MID	CPUT	DIV
1	21	28.9×10 ⁴	65.7×10 ⁶	28.66	74.2×10 ⁵
2	18	34.1×10 ⁵	46.3×10 ⁶	34.74	42.1×10 ⁶
3	19	54.3×10 ⁴	83.8×10 ⁵	49.62	31.3×10 ⁵
4	24	34.2×10 ⁵	18.8×10 ⁵	59.73	95.4×10 ⁵
5	24	75.3×10 ⁴	23.1×10 ⁵	78.27	72.9×10 ⁶
6	21	50.1×10 ⁴	93.2×10 ⁵	106.63	33.8×10 ⁵
7	22	49.9×10 ⁵	92.3×10 ⁶	120.77	76.5×10 ⁵
8	18	54.4×10 ⁵	70.3×10 ⁶	167.37	29.9×10 ⁶
9	21	66.7×10 ⁵	57.5×10 ⁶	221.02	54.1×10 ⁶
10	19	51.8×10 ⁵	64.2×10 ⁶	267.34	70.8×10 ⁶

Table 4. Computational results based on multi-objective metrics for NREGA

Problem No.	NS	SP	MID	CPUT	DIV
1	23	34.2×10^5	44.3×10^5	30.77	74.2×10^5
2	24	43.5×10^4	59.1×10^6	33.71	42.1×10^6
3	20	63.2×10^4	66.8×10^6	46.83	31.3×10^5
4	19	54.3×10^5	28.9×10^5	61.88	95.4×10^5
5	22	59.9×10^4	40.2×10^6	76.72	72.9×10^6
6	21	49.6×10^5	57.8×10^5	98.33	33.8×10^5
7	21	39.3×10^4	97.6×10^5	112.49	76.5×10^5
8	19	69.4×10^5	21.8×10^6	154.75	29.9×10^5
9	22	73.7×10^5	18.5×10^6	189.61	54.1×10^6
10	20	55.8×10^5	58.7×10^5	253.74	70.8×10^6

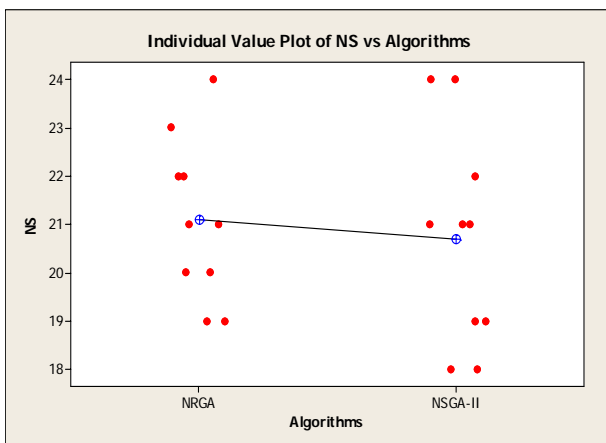


Figure 3. Statistical comparison of both algorithms based on *NOS* metric.

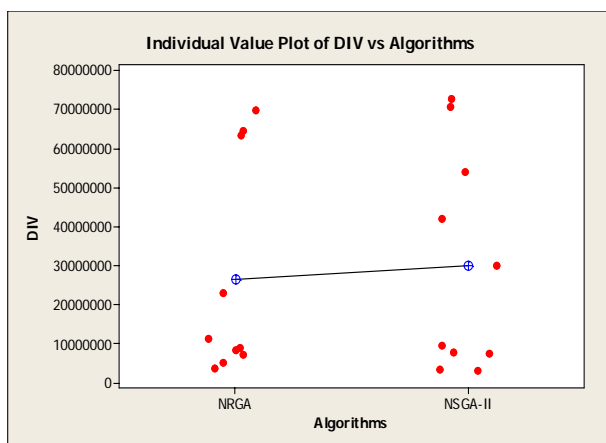


Figure 4. Statistical comparison of both algorithms based on *DIV* metric.

The algorithms are statistically compared based on the properties of their obtained solutions via the analysis of variance (ANOVA) test. In order to clarify our statistical results, individual value plots are represented in Figure 3-Figure 7.

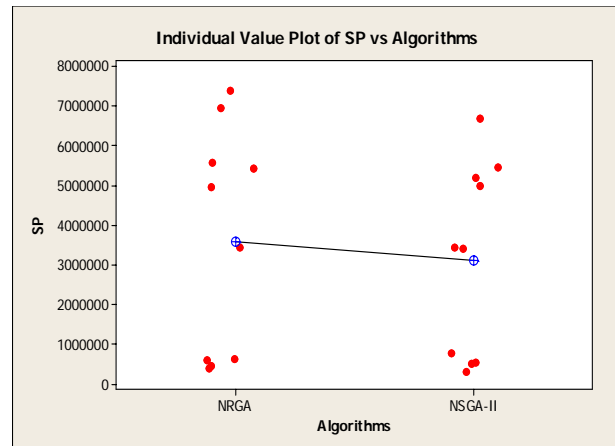


Figure 5. Statistical comparison of both algorithms based on *SP* metric.

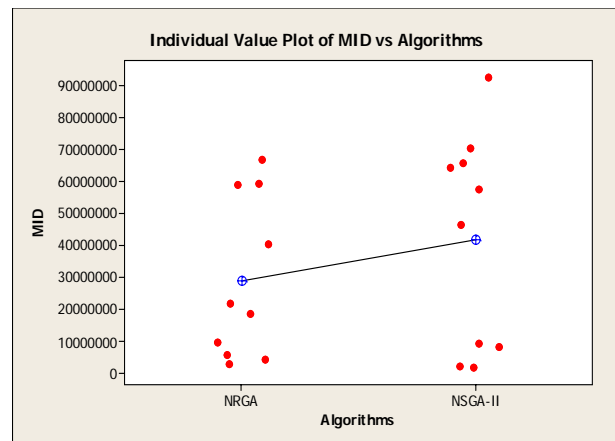


Figure 6. Statistical comparison of both algorithms based on *MID* metric.

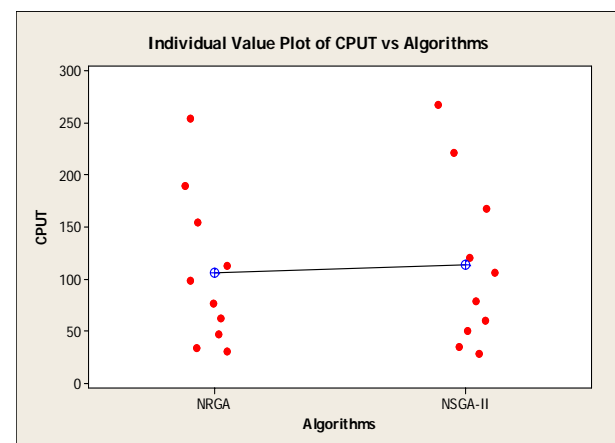


Figure 7. Statistical comparison of both algorithms based on *CPUT* metric.

The results show that both algorithms work same and both are capable to be selected as solving methodologies for our problem. However, NREGA works better in large-size problems.

5. CONCLUSION

In this paper, a fuzzy queuing-inventory system with continuous review inventory control policy and batch arrival queuing approach is presented. We assume that demand function is stochastic and price dependent. The goal was to maximize total profit of system. Since the problem was NP-hard, NREGA and NSGA-II are proposed and justified to solve the model. The results show that NREGA perform NSGA-II in large-size problems. For future research, one can develop the model into multi-echelon one.

REFERENCES

- Alaghebandha, M. and Hajipour, V. (2015), A soft computing-based approach to optimize queuing inventory control problem, *International Journal of Systems Science*, **46**(6), 1113-1130.
- Al Jadaan, O., Rao, C. R., and Rajamani, L. (2008), *Non-Dominated ranked genetic algorithm for solving Multi-Objective optimization problems: NREGA*, Journal of Theoretical and Applied Information Technology, 60-67.
- As'ad, R. and Demirli, K. (2011), A bilinear programming model and a modified branch-and-bound algorithm for production planning in steel rolling mills with substitutable demand, *International Journal of Production Research*, **49**(12), 3731-3749.
- Baek, J. W. and Moon, S. K. (2016), A production-inventory system with a Markovian service queue and lost sales, *Journal of the Korean Statistical Society*, **45**(1), 14-24.
- Baek, J. W. and Moon, S. K. (2014), The M/M/1 queue with a production-inventory system and lost sales, *Applied Mathematics and Computation*, **233**, 534-544.
- Berman, O. and Kim, E. (2004), Dynamic inventory strategies for profit maximization in a service facility with stochastic service, demand and lead time, *Mathematical Methods of Operations Research*, **60** (3), 497-521.
- Berman, O. and Kim, E. (2001), Dynamic order replenishment policy in internet-based supply chains, *Mathematical Methods of Operations Research*, **53** (3), 371-390.
- Berman, O. and Kim, E. (1999), Stochastic models for inventory management at service facilities, *Stochastic Models*, **15**(4), 695-718.
- Berman, O. and Sapna, K. P. (2000), Inventory management at service facilities for systems with arbitrarily distributed service times, *Communications in Statistics. Stochastic Models*, **16**(3/4), 343-360.
- Berman, O. and Sapna, K. P. (2001), Optimal control of service for facilities holding inventory, *Computers and Operations Research*, **28**(5), 429-441.
- Chang, K. H. and Lu, Y. S. (2010), Queueing analysis on a single-station make-to-stock/make-to-order inventory-production system, *Applied Mathematical Modelling*, **34**(4), 978-991.
- Deb, K. (2001), *Multi-objective optimization using evolutionary algorithms*, Wiley, Chichester, UK.
- Dong, M. and Chen, F. (2005), Performance modeling and analysis of integrated logistic chains: An analytic framework, *European Journal of Operational Research*, **162**, 83-98.
- Fisher, M. L. (2004), The Lagrangian relaxation method for solving integer programming problems, *Management science*, **50**(12), 1861-1871.
- Guray Guler, M., Bilgi, T., and Gullu, R. (2014), Joint inventory and pricing decisions when customers are delay sensitive, *Int. J. Production Economics*, **157**, 302-312.
- Haji, R., Haji, A., and Saffari, M. (2011), Queueing Inventory System in a Two-level Supply Chain with One-for-One Ordering Policy, *Journal of Industrial and Systems Engineering*, **5**, 52-62.
- Hajipour, V., Rahmati, S. H. A., Pasandideh, S. H. R., and Niaki, S. T. A. (2014), A multi-objective harmony search algorithm to optimize multi-server location-allocation problem in congested systems, *Computers and Industrial Engineering*, **72**, 187-197.
- Hill, R. (2007), Continuous-review, lost-sales inventory models with Poisson demand, a fixed lead time and no fixed order cost, *European Journal of Operational Research*, **176**, 956-963.
- Isotupa, K. (2006), An (s, Q) Markovian inventory system with lost sales and two demand classes, *Mathematical and Computer Modeling*, **43**, 687-694.
- Manuel, P., Sivakumar, B., and Arivarignan, G. (2008), A perishable inventory system with service facilities and retrial customers, *Computers and Industrial Engineering*, **54**(3), 484-501.
- Manuel, P., Sivakumar, B., and Arivarignan, G. (2007), A perishable inventory system with service facilities, MAP arrivals and PH-service times, *Journal of Systems Science and Systems Engineering*, **16**(1), 62-73.
- Montazeri-Gh, M. and Mahmoodi-K, M. (2016), Optimized predictive energy management of plug-in hybrid electric vehicle based on traffic condition, *Journal of Cleaner Production*, **139**, 935-948.
- Krishnamoorthy, A., Manikandan, R., and Lakshmy, B. (2015), A revisit to queuing-inventory system with positive service time, *Annals of Operations Research*, **233**(1), 221-236.
- Krishnamoorthy, A., Deepak, T. G., Narayanan, V. C., Vineetha, K. (2006), Effective utilization of idle time in an (s, S) inventory with positive service

- time, *Journal of Applied Mathematics and Stochastic Analysis*, 1-13.
- Krishnamoorthy, A., Narayanan, V. C., Deepak, T. G., and Vineetha, P. (2006), Control policies for inventory with service time, *Stochastic Analysis and Applications*, **24**(4), 889-899.
- Sadjadi, S. J., Makui, A., Ehsan Dehghani, E., and Pourmohammad, M. (2015), Applying queuing approach for a stochastic location-inventory problem with two different mean inventory considerations, *Applied Mathematical Modelling*, **40**(1), 578-596.
- Saffari, M. S. and Asmussen, R. H. (2013), The M/M/1 queue with inventory, lost sale, and general lead times, *Queueing Syst*, **75**(1), 65-77.
- Schwarz, M., Daduna, H. (2006), Queuing systems with inventory management with random lead times and with back ordering, *Mathematical Methods of Operations Research*, **64**(3), 383-414.
- Schwarz, M., Sauer, C., Daduna, H., Kulik, R., and Szekli, R. (2006), M/M/1 queueing systems with inventory, *Queueing Systems, Theory and Applications*, **54**(1), 55-78.
- Teimoury, E., Modarres, M., Ghasemzadeh, F., and Fathi, M. (2010), A queueing approach to production-inventory planning for supply chain with uncertain demands: Case study of PAKSHOO Chemicals Company, *Journal of Manufacturing Systems*, **29**(2), 55-62.
- Teimoury, E., Modarres, M., Khondabi, I. G., and Fathi, M. (2012), A queueing approach for making decisions about order penetration point in multi-echelon supply chains, *Int J Adv Manuf Technol*, **63**(1-4), 359-371.
- Zadeh, L. A. (1965), Fuzzy sets, *Information and control*, **8**(3), 338-353.
- Zhao, N. and Lian, Z. (2011), A queueing-inventory system with two classes of customers, *Int. J. Production Economics*, **129**, 225-231.
- Zitzler, E. and Thiele, L. (1998), *Multiobjective optimization using evolutionary algorithms-a comparative case study*, In Parallel problem solving from nature-PPSN V, Springer Berlin Heidelberg, 292-301.

APPENDIX

The Parameters

n	number of products
h_{wj}	Holding cost rate at the warehouse for product j
A_{wj}	The fixed cost of ordering related to the warehouse for product j
T_{wj}	Time interval between two consecutive orders of the warehouse for product j
h_{rj}	Holding cost rate at the retailer for product j
A_{rj}	The fixed cost of ordering related to the retailer for product j
T_{rj}	Time interval between two consecutive orders of the retailer for product j
Q_{wj}	Order quantity of the warehouse for product j
φ_j	Arrival rate of the customer for product j
μ_j	Service rate of the server for product j
ρ_j	Productivity coefficient for product j
\bar{I}_j	Average inventory level at the retailer between $(0, T)$ during lost sales period for product j which is equivalent with queue length for product j
π_j	Fixed shortage cost for product j
L_j	Length of lead time of product j is assumed to be constant
F	Available warehouse space for the retailer in all products
f_j	Space occupied by each unit of product j
G	Number of allowed shortage
P_j	Service level for product j
S	Expected allowable shortage cost in lost sales state
Γ_j	Maximum inventory at the warehouse for product j
SS_j	Safety stock for product j
$E[Q_{rj}]$	Average stockpile amount of product j

$(y_1)_j$	Random demand in period T for product j which acts as Poisson distribution $y_1 j \sim pp(\lambda_1 j)$
$(y_2)_j$	Random demand in period L for product j that which acts as Poisson distribution $y_2 j \sim pp(\lambda_2 j)$
$y_j = (y_1 + y_2)_j$	Random demand in period $L+T$ for product j which acts as Poisson distribution with parameter $\lambda_j = \lambda_1 j + \lambda_2 j$
$P(y_j)$	Demand probability density function
$\bar{b}_{(R)_j}$	Average shortage for product j
Ch_r	Expected holding cost per time unit at the retailer in the steady state
CB_r	Expected backorder cost per time unit at the retailer in the steady state
TC_r	Expected total cost per time unit at the retailer in the steady state.
TC_w	Expected total cost per time unit at the warehouse in the steady state.
TC_B	Expected total system (retailer and warehouse) cost per time unit in the steady state.
$K_1(R, T)$	Expected total cost per time unit at the retailer in the steady state in (R, T) system
D_j	demand of product j
ξ_j	Potential customer population of product j
α_j	Price sensitivity coefficient of product j

The decision variables

R_j	The maximum inventory position after order for product j
Q_{rj}	Stockpile amount random variable in batch arrival queuing system for product j which is equivalent of order quantity of the retailer for product j
P_j	Price of product j
m_j	The number of orders for product j