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A Geometric Compression Method Using Dominant Points for Transmission to LEO Satellites

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Abstract

In the operation of a low earth orbit satellite, a series of antenna commands are transmitted from a ground station to the satellite within a visibility window (i.e., the time period for which an antenna of the satellite is visible from the station) and executed to control the antenna. The window is a limited resource where all data transmission is carried out. Therefore, minimizing the transmission time for the antenna commands by reducing the data size is necessary in order to provide more time for the transmission of other data. In this paper, we propose a geometric compression method based on B-spline curve fitting using dominant points in order to compactly represent the antenna commands. We transform the problem of command size reduction into a geometric problem that is relatively easier to deal with. The command data are interpreted as points in a 2D space. The geometric properties of the data distribution are considered to determine the optimal parameters for a curve approximating the data with sufficient accuracy. Experimental results demonstrate that the proposed method is superior to conventional methods currently used in practice.

Key words: Data Compression, Antenna Commands, B-spline Curve Fitting, Dominant Points, LEO Satellite

1. Introduction

A mission of a low earth orbit (LEO) satellite is performed by executing a series of commands transmitted from the ground station. For a timely transmission without loss of data, two conditions need to be satisfied. The first condition is that the satellite and the ground station should be able to see each other. The time interval in which this condition holds is called the visibility window, which is a limited resource during which the satellite and the ground station can send and receive the necessary data and commands. The visibility window can be accurately determined from the orbit information, current position, and velocity of the satellite. The second condition is that the antenna in the satellite should be oriented toward the ground station when the satellite enters the visibility window. Commands for the proper orientation of the antenna are prepared before the satellite enters the visibility window [1, 2].

Minimizing the transmission time of antenna commands is necessary to reserve more time for the transmission of other data within the visibility window. Antenna commands, which are used to orient the antenna of a satellite toward the ground station for efficient transmission of various data, are uploaded to the satellite within the visibility window. The transmission time of antenna commands depends on the mission that the satellite should perform. Specifically, if a complicated antenna control is required, the size of antenna commands

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would increase, and subsequently, more time would be assigned for the command transmission within the visibility window. It is only after the commands are transmitted to the satellite and executed, that the measurement data including images and other satellite commands are transmitted. Since the visibility window is a limited resource, minimizing the transmission time for antenna commands is advantageous because more time within the visibility window could be assigned for transmission of other data.

Such minimization can be greatly facilitated by reducing the size of antenna commands. In practice, power-basis polynomial functions of time with a set of appropriate coefficients, called TPF, are considered [3]. TPF stands for the tracking profile file that contains information of antenna commands for time. This approach is useful for commands with a profile that changes monotonically because a polynomial-based function can approximate such a profile with sufficient accuracy. However, if the profile shows a complicated pattern with rapid changes of curvature, or with a lot of humps and hills, polynomial-based functions cannot accurately approximate such a pattern. Instead, the number of polynomial coefficients needs to be increased to achieve an accurate representation of the pattern. Therefore, there is no advantage in reducing the data size by using the power-basis polynomial method. Moreover, the power-basis polynomial method cannot reconstruct antenna commands because increasing the degree of the polynomial may result in an undesirable oscillatory behavior. Such oscillation will produce antenna commands that may direct oscillatory movement of the antenna, and induce disturbances to the satellite.

To overcome the drawbacks of the polynomial-based approach, the input data set is subdivided into subsets, each of which contains a lesser number of commands. Each subset is given as input to the polynomial-based approach. Then, a low-degree (less than seven) polynomial is considered for each segment while satisfying the continuity conditions at the junction points among segments using the boundary condition method (BCM) [3]. The BCM is not optimal because satisfying the continuity conditions and the accuracy requirement of approximation would require either a higher-degree polynomial or a greater reduction in the segment size, which would result in an increased amount of information for data representation. Therefore, no benefit can be expected from this process.

In this study, a new method based on the concept of B-spline curve fitting is proposed, with emphasis on a practical application to the LEO satellite communication. The problem of data compression can be formulated as a curve-fitting problem, which has been an active research topic in geometric modeling and computer graphics communities; substantial literature on this subject can be found in [4–14]. The geometric properties of the given data points can be fully utilized in the fitting process, making it possible to represent the data in a more compact way than the conventional method. More compression can be achieved by using dominant points in the curve-fitting process than the standard B-spline-based fitting method [15, 16]. Here, the dominant points are points that characterize the shape of a given point set, typically including points of the local maximum curvature on a curve represented by the point set. Curve fitting using dominant points can improve the fitting accuracy with lesser control points than those not using them [17]. Although the geometric approach is useful for data compression in graphics and signal processing [15], it has not been used for data compression in satellite data transmission. The approach cannot be directly applied to satellite communication because of the complexity of its implementation and the different constraints required.

The data compression method proposed in this study can reduce the time for transmitting a set of antenna commands. Therefore, more sets of commands can be sent to the satellite within the same period. For missions requiring high agility, many sets of tracking commands are generated to represent such antenna movements accurately. Hence, it is necessary to allocate more time for transmission of the commands within the visibility time, leaving less time for other data transmissions such as images and other critical data. By using the B-spline-based compression method proposed in this study, we can reduce the size of antenna commands for transmission by more than 90%, thus, minimizing the time for transmission and maximizing the time for other operations within the limited visibility window.

The contributions of this study are twofold. First, the geometric approach based on the B-spline curve fitting using dominant points is applied to the LEO satellite data communication. Second, a new automatic process is proposed for the preparation of the command data representation for transmission. For the boundary condition method, the operator must determine the subdivision numbers manually through a couple of iterations of evaluation, computation, and subdivision, leading to a sub-optimal subdivision and increased preparation time. However, the proposed scheme determines the optimal result automatically and efficiently without any manual work.

2. Technical Background

In this section, the command structure and the BCM are presented to provide the background information.

2.1 Command Structure

The command for orienting an antenna of a satellite consists of the azimuth (θ_{azi}) and elevation (θ_{ele}) angles at time t, which yields a tuple of t, θ_{azi}, and θ_{ele}. The azimuth and elevation angles are represented as functions of time, respectively, producing two planar curves. Because time increases monotonically, each trajectory is simple, i.e., there exists no loop or self-intersection.

2.2 Boundary Condition Method

Consider a set of points in 2D space, $p_i = (s_i, a_i)$ $(1 \le i \le m)$, which are organized to form a curve with no loop or self-intersection. Assume that there is a polynomial f as follows:

$$f(s) = \sum_{j=0}^{n} b_j s^j \tag{1}$$

where s is a variable, and b_j are the coefficients of the polynomial. Then, the following equation should hold at each point:

$$a_i = f(s_i) = \sum_{j=0}^{n} b_j s_i^j, \ \ 1 \le i \le m. \tag{2}$$

Here, b_j are unknown coefficients. The input data set is subdivided into n_μ segments, each of which is approximated using a separate low-degree polynomial. For the continuity over the μ^{th} and $\mu+1^{\text{th}}$ segments, in addition to the equations given in (2) for each segment, the following conditions should hold at the boundary points:

$$\begin{aligned} f_\mu(s_m^\mu) &= f_{\mu+1}(s_1^{\mu+1}), \\ f'_\mu(s_m^\mu) &= f'_{\mu+1}(s_1^{\mu+1}), \\ f''_\mu(s_m^\mu) &= f''_{\mu+1}(s_1^{\mu+1}). \end{aligned} \tag{3}$$

Here, the prime indicates the derivative with respect to s, s^μ is the variable value for the μ^{th} segment, and f_μ is a fitting function for the μ^{th} segment. The first equation in (3) indicates the condition that the two segments have the same value at the junction point. The second and the third equations indicate the continuous first and second derivative values over the μ^{th} and $\mu+1^{\text{th}}$ segments. The continuity conditions for the first and the second derivatives imply continuous velocity and acceleration changes of the antenna movement over the two segments. With these conditions imposed, the antenna can be operated without a sudden change in its orientation, resulting in minimal disturbances to the satellite motion. Combining Equations (2) and (3) yields a system of linear equations, which can be solved using the singular value decomposition [16].

3. B-spline Method Using Dominant Points

3.1 B-spline Curve Fitting

The B-spline-based reduction method is used to approximate the input data by the B-spline curve fitting. We assume that the reader is familiar with the concepts of B-spline curves [6, 7, 10]. A parametric B-spline curve c*(s)* of degree p is defined by

$$\mathbf{c}(s) = \sum_{j=0}^{n} N_{j,p}(s)\mathbf{b}_j \tag{4}$$

where b_j (j = 0,.., n) are control points, and $N_{j,p}(s)$ are the normalized B-spline functions of degree p defined on a knot vector U={u_0, u_1, ..., u_{n+p}, u_{n+p+1}}. The knot vector U consists of non-decreasing real-value knots, and mostly its first and last knots are repeated with multiplicity equal to a degree p as follows: u_0=...=u_p and u_{n+p}=...=u_{n+p+1}=1.

Given a set of ordered points p_i (i = 0,..., m), we can obtain a B-spline curve c(s) approximating the points in three steps: parameterization, knot placement, and computation of control points. In the parameterization, we determine the parameters $\bar{s}_i$ corresponding to the points p_i (*i.e*,. p_i=c($\bar{s}_i$)). In the knot placement, we compute the knots u_i according to the degree p of the B-spline and the number n of unknown control points (n≤m). In the computation of the control points, we determine the control points p_i (j = 0,..., n) by minimizing an objective function, which is defined as follows:

$$E(\mathbf{b}_0,..,\mathbf{b}_n) = \sum_{i=0}^{m} \left|\mathbf{c}(\bar{s}_i) - \mathbf{p}_i\right|^2 \tag{5}$$

The minimization problem of Equation (5) is transformed into solving a linear system. The quality of the curve c(s) approximating p_i is affected by the selection of the parameters and the knots. However, the interaction between those properties is not formally defined.

3.2 Parameter Estimation and Knot Vector Determination

There are several methods of determining the parameters: uniform, chord length, and centripetal methods. The parameters $\bar{s}_i$ at the points p_i can be defined as follows [7, 10]:

$$\bar{s}_i = \bar{s}_{i-1} + \tfrac{1}{d}\left|\mathbf{p}_{j+1} - \mathbf{p}_j\right|^e \qquad (i = 1,..,m) \tag{6}$$

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where $\bar{s}_0 = 0$ and $d = \sum_{j=0}^{m-1} \left| \mathbf{p}_{j+1} - \mathbf{p}_j \right|^e$. It reduces to the uniform method with e=0, to the chord length method with e=1, and to the centripetal method with e=1/2.

According to the parameters $\bar{s}_i$, the number of control points, n, number of given points, m, and knots, u_i, must be selected at least to guarantee that a fitted curve exists, and to make the fitted curve as close as possible to the optimum. The Schoenberg–Whitney condition [18, 19] is a necessary and sufficient condition for a fitted B-spline curve to exist. It requires that there exists at least one parameter, $\bar{s}_i$ (i = 0,.., m), satisfying the following condition: $u_j \le \bar{s}_i \le u_{j+p+1}$ (j = 0,.., n). When the condition is violated, the system matrix becomes singular, and the objective function cannot be evaluated. Various approaches have been proposed to determine good knots. If the number and location of the knots are the unknowns to be determined, the knot placement in itself can be considered as a multivariate and multimodal nonlinear optimization problem [7, 10, 12]. When the number of knots is given, the knots can be determined by taking simple estimation approaches mostly based on the distribution of the parameters [7, 10]. The knots can be determined as follows [10]:

$$u_{p+i} = \begin{cases} \frac{1}{p} \sum_{j=i}^{i+p-1} \bar{s}_j & \Leftarrow m = n \\ (1-r)\bar{s}_{k-1} + r\bar{s}_k & \Leftarrow m > n \end{cases} \tag{7}$$

where k=int(i×d), d=(m+1)/(n-p+1), and r=i×d-k. This is called the averaging technique (AVG) for m=n, and the knot placement technique (KTP) for m>n. Piegl and Tiller [20] suggested another knot placement technique (NKTP), which is a generalization of the AVG technique.

3.3 Adaptive Knot Vector Determination Using Dominant Points

The aforementioned estimation approaches select the knots in a simple and trivial manner such that each knot span contains almost the same number of parameter values, which makes it difficult to achieve an adaptive fitting. To overcome this shortcoming, adaptive knot placement methods have been proposed [17, 21, 22]. Particularly, Park and Lee [17] presented a knot placement approach using the dominant points. It progressively selects the dominant points governing the overall shape of the given point data, and then determines the knots by averaging the parameters of the dominant points. With the dominant points, d_j (j = 0,.., n), selected among the given points, p_k (k = 0,.., m), the interior knots, u_i, can be computed as follows:

$$u_{p+i} = \frac{1}{p} \sum_{j=i}^{i+p-1} \bar{s}_{g_j} \qquad (i = 1,..,n-p) \tag{8}$$

where g_j denotes the index of the point, p_k, corresponding to a dominant point, d_j (i.e., k=g_j).

In the B-spline-based reduction method, we adopt the B-spline curve fitting using the dominant points [17], which is completed in four steps: parameterization, selection of dominant points, knot placement, and computation of control points. The time parameter, t_i, in each command (t_i, $\theta_{azi,\ i}$, $\theta_{ele,\ i}$) is replaced by $\bar{s}_i = \frac{(t_i - t_0)}{(t_m - t_0)}$. In this study, we consider that $\bar{s}_i$ are uniform as the commands are sampled at uniform intervals (i.e., t_i are uniform). The dominant points, d_j, and the knots, $\bar{s}_i$, are computed as described by Park and Lee [17]. Then, the control points, b_j, are easily computed by minimizing the objective function in Equation (5). The B-spline curve fitting using the dominant points yields a nonsingular system matrix during the minimization of the objective function in Equation (5), which assures the existence and the uniqueness of a B-spline curve that fits on the given points. Moreover, it can achieve an adaptive knot placement, which places fewer knots at flat regions but more at complex regions, providing a high probability of producing a better curve approximation compared with the approaches based on the trivial knot placement.

3.4 Accuracy Test

In this study, we consider the B-spline curve c(s) and the points p_i to be given as c(s)=(t(s), f(s)) and p_i=(t_i, f_i), respectively. Here, t(s) and f(s) denote the time and angle functions for the parameter s, and f_i denotes the angle at time t_i. As the function t(s) is increasing at the interval, the curve c(s) is monotonic. The error of the computed B-spline curve should be checked against a given constraint. This error checking is a task that the ground station should perform before the data transmission to the satellite begins. The constraint is given as an error range, within which the deviation of the fitted curve to the data points should lie, i.e., $0 \le e_i \le Tol$ where $e_i = \left|\mathbf{d}_i\right| = \left|\mathbf{c}(\bar{s}_i) - \mathbf{p}_i\right|$ and Tol is the user-defined tolerance.

The constraint, however, is associated with each data point. Therefore, it is necessary to impose the constraint between the points where no data are available. This condition assumes that two consecutive points are connected linearly, which is reasonable because the distance between two consecutive points is sufficiently small. Therefore, the approximated curve shape between the two points is almost linear. Consider the two polygons drawn in dotted lines, which connect the upper and lower boundary points of each error range, as shown in Fig. 1. As the dotted lines indicate the error bounds between two consecutive points, the test should verify that the approximation curve lies within the polygons.

Suppose that the lower and upper boundary points are $\mathbf{Q}_{l(i)} = (\bar{s}_i, f_i - Tol)$ and $\mathbf{Q}_{u(i)} = (\bar{s}_i, f_i + Tol)$, respectively. Then, the upper and lower lines, $L_u(s)$ and $L_l(s)$, for $s_{i-1} \le s \le s_i$ connecting the boundary points are given as

$$\mathbf{L}_u(s) = \left(1 - \frac{s - s_{i-1}}{s_i - s_{i-1}}\right)\mathbf{Q}_{u(i-1)} + \left(\frac{s - s_{i-1}}{s_i - s_{i-1}}\right)\mathbf{Q}_{u(i)},$$
$$\mathbf{L}_l(s) = \left(1 - \frac{s - s_{i-1}}{s_i - s_{i-1}}\right)\mathbf{Q}_{l(i-1)} + \left(\frac{s - s_{i-1}}{s_i - s_{i-1}}\right)\mathbf{Q}_{l(i)}. \tag{9}$$

Therefore, the approximated curve should satisfy $L_l(s) \le c(s) \le L_u(s)$. To verify this condition, the convex hull property of the B-spline curve, which is a property that B-spline curves always have, is employed. The approximated curve is subdivided into smaller curve segments for the range (s_{i-1}, s_i) by inserting knot values [10]. If the control points of the subdivided curve segments are inside the corresponding boundary polygon defined in Equation (9), the curve is enclosed in the polygons, i.e., the condition is guaranteed to be satisfied. This test is extended to all remaining ranges of s. When the curve satisfies the condition, the error between the curve and the given data points is evaluated to check if the constraints at each point are satisfied. The error measure e_i is computed at s_i.

4. Application to Satellite Communication

4.1 Data Necessary for Encoding Commands

The azimuth and elevation commands can be defined as functions of time, and therefore, they can be given as tuples of $\theta_{azi}(t)$, and $\theta_{ele}(t)$, which could be approximated by a single planar B-spline curve in 2D space formed by two axes, $\theta_{azi}(t)$ and $\theta_{ele}(t)$. However, this representation is not suitable because time is a fixed scalar value associated with a command. As the trace of the tuples may have self-intersection or a corner where the first derivative is not continuous, the corresponding curve fitting may not be correctly performed. Moreover, as the time is fixed for each pair of commands, it cannot be adjusted for better accuracy

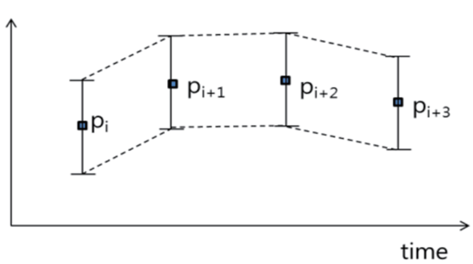

Fig. 1. Illustration of Error Ranges at Each Point

in approximation.

To solve this problem, the commands are separately defined by $c_{azi}(s) = (t(s), f_{azi}(s))$ and $c_{ele}(s) = (t(s), f_{ele}(s))$, where $c_{azi}(s)$ and $c_{ele}(s)$ are the B-spline curves for the azimuth and elevation angles, respectively. In this method, the parameter value s for the given time t should be computed to find the commands for t by solving the following equation:

$$t = t(s) \tag{10}$$

The parameter shall be computed numerically because solving the equation analytically is difficult, in general. The Newton–Raphson method, which efficiently finds the solution with an accuracy provided by the user, is a popular choice. However, this method poses a few problems in the satellite application. The method needs a good initial value for convergence to a correct root, which cannot always be guaranteed [23]. Failure to do so may lead to an incorrect orientation of the antenna resulting in failure of communication. Another issue relates to the command compression ratio, which is the ratio of the reduced data size to the data size before reduction. Equation (10) requires n+1 control points, which are essential for the definition of time, to represent time alone. Consequently, the amount of reduction may not be satisfactory. To avoid such problems, we use a simple linear function given as

$$s^* = \frac{t - t_s}{t_e - t_s} \tag{11}$$

where t_s and t_e are the starting and ending time values. A monotonic increase of time in a uniform manner is useful for data reduction. Specifically, time values can be represented as a line with t_s and t_e as the starting and ending values. Any intermediate time value can be obtained by evaluating Equation (11). Therefore, the data that should be transmitted are t_s and t_e, not the entire time values. For example, for a given time t, a parameter s^* is used instead of s in Equation (10). If the error between the actual and approximated time using s* is within the tolerance, then the value s^* is a valid parameter for evaluating the azimuth or elevation angle. Otherwise, the procedure increases the number of control points for $c_{azi}(s)$ or $c_{ele}(s)$. Compared with the method of solving Equation (10) directly, the use of Equation (11) may require additional iterations to satisfy the accuracy constraint. However, this effort is negligible because only one or two additional iterations are necessary in most cases. The robustness of the method is guaranteed because Equation (11) can always be computed analytically.

As time is represented using Equation (11), the following data are necessary for the proposed method to be used in satellite communication:

- The number of control points, n_{azi} and n_{ele}, of the B-spline curves, $c_{azi}(s)$ and $c_{ele}(s)$.
- The control points, $\{b_{azi, i}\}$ and $\{b_{ele, i}\}$, for $c_{azi}(s)$ and $c_{ele}(s)$.
- The degree p of the B-spline curve.
- The starting and ending time values, t_s and t_e, for the commands.
- The knot vectors, U_{azi} and U_{ele}, for $c_{azi}(s)$ and $c_{ele}(s)$.

As we use the knot vector U in the form of {0, 0, 0, 0, u_{p+1}, ..., u_n, 1, 1, 1, 1}, where n is the number of control points, the interior knots, u_{j+p} (j = 1,.., n-p), have to be transmitted to the satellite. However, the transmission of a knot vector can be avoided by computing it in the satellite using a simple rule based on the parameters of the points.

When Equation (8) is used for the knot placement (DOM), we can easily compute the knot vector in the satellite; however, we need to transmit the list of indices $\{g_i\}$ indicating which points are dominant points (i.e., $d_i=p_{g_i}$, where $0 \le g_i \le m$, i=0, ..., n). In order to reduce the amount of data transmitted to the satellite, we can modify the control points to encode the indices, $\{g_i\}$, into them. Given the control points $b_{angle, i}$ (i = 0,..,n) of a B-spline curve for the angle (azimuth or elevation), their modified control points $B_{angle, i}$ are defined as follows:

$$B_{angle,i} = g_i + v_i \qquad (i = 0,..,n) \tag{12}$$

The value v_i denotes the normalized control point defined as $v_i=(b_{angle, i}-b_{angle, min})/(b_{angle, max}-b_{angle, min})$, where $b_{angle,\min} = \min_k b_{angle,k}$ and $b_{angle,\max} = \max_k b_{angle,k}$. It should be noted that $0 \le v_i \le 1$ and $\frac{0}{0}=0$. Using the encoding scheme in Equation (12), the following five values are transmitted with the modified control points, $\{B_{azi, i}\}$ and $\{B_{ele, i}\}$, instead of the knot vectors, U_{azi} and U_{ele}:

- The number of commands, m.
- The minimum and maximum values of the azimuth, $b_{azi, \max}$ and $b_{azi, \min}$.
- The minimum and maximum values of the elevation, $b_{ele, \max}$ and $b_{ele, \min}$

In this study, cubic B-splines are used for curve fitting (p = 3), and the parameters of the points are uniform since the time parameters are sampled at uniform intervals. As the indices $\{g_k\}$ are decoded and used to compute the knot vectors by using Equation (8), the knot vectors need not be transmitted to the satellite. The time function $c_t(s)$ is obtained using t_s and t_e only. Therefore, the only elements that must be transmitted to the satellite for an accurate command reconstruction are n_{azi}, n_{ele}, $\{B_{azi, i}\}$, $\{B_{ele, i}\}$, t_s, t_e, m, $b_{azi, \max}$, $b_{azi, \min}$, $b_{ele, \max}$, and $b_{ele, \min}$. On the other hand, when Equation (7) is used for knot placement (KTP), the knot vectors can be generated in the satellite without the four values of $b_{azi, \max}$, $b_{azi, \min}$, $b_{ele, \max}$, and $b_{ele, \min}$. However, the KTP-based approach tends to require relatively more control points than the DOM-based approach [17].

4.2. Decoding Method for Command Reconstruction

A. Knot vector generation with decoding of control points

The control points are decoded, and the knot vectors can be generated in the satellite using the data transmitted to the satellite. The parameters $\bar{s}_i$ of the commands are given as $\bar{s}_i = i/m$. When the KTP in Equation (7) is used for the knot placement, the decoding of control points is not necessary. When the DOM in Equation (8) is used for the knot placement, the indices g_i of the dominant points are determined as $g_i=[B_{angle, i}]$, where $[a]$ is the largest integer, which is not less than the value of a. Then, the knot vectors are determined by Equation (8). In addition, the control points $b_{angle, i}$ are decoded as follows:

$$b_{angle,i} = b_{angle,\min} + v_i (b_{angle,\max} - b_{angle,\min}) \tag{13}$$

where $v_i = B_{angle,i} - \lceil B_{angle,i} \rceil = B_{angle,i} - g_i$.

B. Parameter estimation

To use the proposed method, the elevation and azimuth angles at an arbitrary time must be computed. Given a specific time, a parameter value corresponding to the time is computed using Equation (11), which is then provided to the B-spline representation for evaluation of the elevation and azimuth angles in order to yield the correct angle values at that time. It is not necessary to evaluate the error in the satellite because the validity of the command representation is performed in the ground station before transmission.

5. Analysis of Data Size for Transmission

In this section, the proposed method is compared with the boundary condition method in terms of data size for transmission.

5.1 Data Size

A. Boundary condition method

For one segment, the boundary condition method requires the starting and average times of each subdivided segment. Let us consider that the degrees of the polynomials for the elevation and azimuth angles are k_{ele} and k_{azi}, respectively. The numbers of coefficients for each polynomial are k_{ele}+1 and k_{azi}+1. Therefore, to represent the elevation and azimuth angles with respect to time for one segment, k_{ele}+k_{azi}+4 values are required as well as two values that define the starting and average times for each segment. If the number of segments is n_μ, then the total number of values is $n_\mu(k_{ele}+k_{azi}+4)$.

B. B-spline method

When Equation (7) is used for the knot placement (KTP), the total number of values for transmission is $n_{azi}+n_{ele}+5$. When Equation (8) is used for knot placement (DOM), the total number is $n_{azi}+n_{ele}+9$. The amount of data for the transmission of the boundary condition method is linearly dependent on the number of segments, whereas that of the B-spline method is a function of the number of control points of the azimuth and elevation angles.

5.2 Comparison

In terms of power consumption, the B-spline method is comparable to the boundary condition method because a B-spline function is a piecewise polynomial function. However, in terms of data size for communication, the B-spline method can perform better than the boundary condition method because the B-spline scheme can represent a data set more compactly.

The boundary condition method requires $n_\mu(k_{ele}+k_{azi}+4)$ of data, whereas the B-spline method needs $n_{azi}+n_{ele}+9$ at most. If the degrees are fixed to be three ($k_{ele}=k_{azi}=3$), the boundary condition method requires $10n_\mu$. Specifically, the proposed method requires $O(n_{azi}+n_{ele}+9)$, while the boundary condition method needs $O(n_\mu(k_{ele}+k_{azi}+4))$. It turns out that $n_\mu(k_{ele}+k_{azi})$ is linearly proportional to $n_{azi}+n_{ele}$ because the number of control points is an indirect indication of the number of knot values, which are associated with the inherent subdivided curve segments. However, we have experimentally found that $n_\mu \geq n_{azi}$, and $n_\mu \geq n_{ele}$. This means that the size complexity of the boundary condition method is larger than that of the proposed method. This trend becomes clearer as the shape of the curve gets more complicated.

6. Examples

In this section, the performance of the proposed B-spline method over the boundary condition method is presented using actual satellite command examples. In all the examples, the degree used for the boundary condition method is five, and the B-spline method uses a basis function of degree three. The allowable error is 1°.

Case 1

A set of 310 commands for the azimuth and elevation angles, which is shown in Fig. 2, is considered. The boundary condition method subdivides each of the profiles into six segments to satisfy the accuracy condition. The number of data for the approximation is 48 in both of the two commands. On the other hand, the DOM-based B-spline method only requires 17 and 13 data for the azimuth and elevation angles, respectively, while the tolerance is satisfied.

Case 2

The second example considers 263 commands with profiles shown in Fig. 3. The boundary condition method subdivides each profile into eight segments and approximates each profile using 64 data to satisfy the accuracy constraint. On the other hand, the DOM-based B-spline method requires 17 and 18 data for the azimuth and elevation angles, respectively, within the specified tolerance.

Case 3

The third example is the most complicated case, which the current satellite can possibly encounter in terms of the complexity of the trajectory that the commands represent. The number of such sets of commands can be more than one depending on the mission. In this example, however, one set of such commands is selected for demonstration.

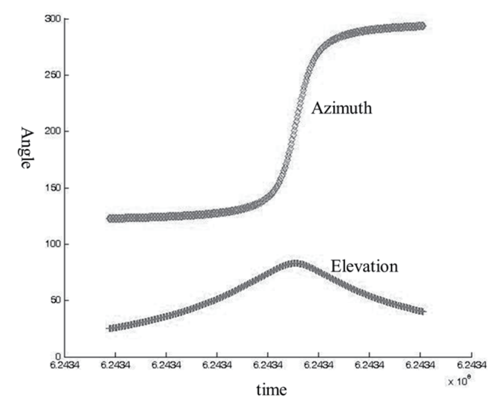

Fig. 2. Example 1 of Azimuth and Elevation Profiles

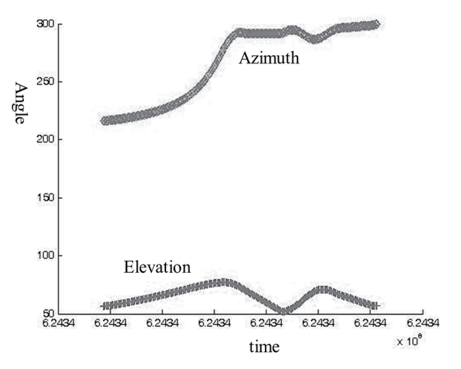

Fig. 3. Example 2 of Azimuth and Elevation Profiles

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It consists of 512 commands, which are plotted in a 2D plane, as shown in Fig. 4. The boundary condition method subdivides each profile into 16 segments, yielding 128 data for the approximation of each profile. On the other hand, the DOM-based B-spline method requires 28 and 30 data for the azimuth and elevation angles, respectively.

As summarized in Table 1, the B-spline method can represent the given commands with less information, which demonstrates the potential of the method for satellite applications. As shown in the table, the proposed method can reduce the data size by more than 90% compared with the original raw commands and from 50% to 75% compared with the boundary condition method. The third example is taken to analyze the convergence of the approximation accuracy using the B-spline method with respect to the number of coefficients. Although the error oscillates, it decreases with the number of coefficients as shown in Fig. 5.

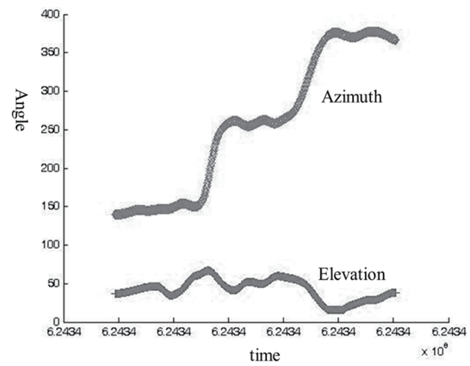

Fig. 4. Example 3 of Azimuth and Elevation Profiles

Table 1. Comparison of Data Reduction. A: Set of Commands, B: Boundary Condition Method, and C: Proposed Method

	Case 1			Case 2			Case 3		
	A	B	C	A	B	C	A	B	C
θ_{azi}	310	48	17	263	64	17	512	128	28
	0%	85%	95%	0%	76%	94%	0%	75%	95%
θ_{ele}	310	48	13	263	64	18	512	128	30
	0%	85%	96%	0%	76%	93%	0%	75%	94%

7. Conclusions

In this study, we have addressed the problem of reducing the size of commands for a two-axis Gimbal antenna control in an LEO satellite for an efficient data transmission and proposed a method for solving the problem based on a geometric approach. The proposed method considers the commands to be points in a 2D space and reduces their size using the B-spline curve-fitting scheme. Experimental and analytical results have demonstrated its efficiency in the reduction of the antenna command size for LEO satellites. The real examples showed the potential of the proposed method for satellite operation because the reduction rate is superior compared with that of the existing method. Using the proposed method, the transmission time of the antenna command data to the satellite can be minimized, saving time for the transmission of other critical data. This reduction is beneficial because the amount of power used to send and receive data and the waiting time of the satellite during the data transmission can be reduced, providing more available time for other operations.

The proposed method requires a thorough evaluation with the actual satellite hardware before it can be implemented in the satellite operation. The evaluation of the proposed method and its extension to multiple satellites should be considered in future studies.

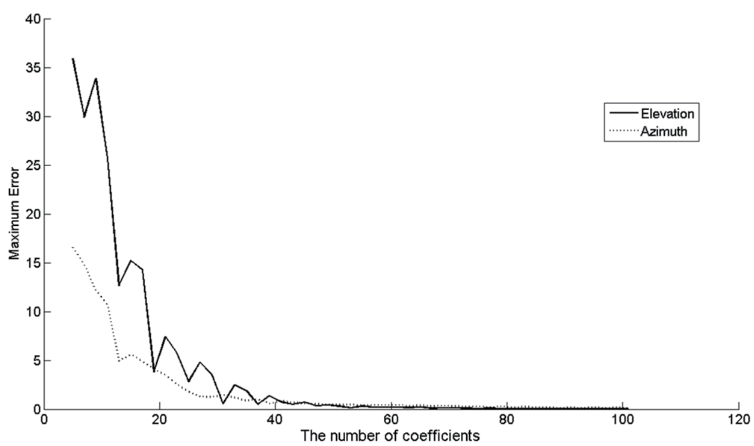

Fig. 5. Plots of Maximum Errors vs. Number of Coefficients for Azimuth and Elevation Angles

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