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Rician-Nakagami 페이딩 채널에서 *M*-PSK와 *M*-DPSK 시스템에 대한 효과적인 점근적 심볼 에러 확률 성능 분석

Effective Asymptotic SER Performance Analysis for *M*-PSK and *M*-DPSK over Rician-Nakagami Fading Channels

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Abstract – Using the existing exact but quite complicated symbol error rate (SER) expressions for *M*-ary phase shift keying (*M*-PSK) and *M*-ary differential phase shift keying (*M*-DPSK), we derive effective and concise closed-form asymptotic SER formulas especially in Rician-Nakagami fading channels. The derived formulas can be utilized to efficiently verify the achievable error rate performances of *M*-PSK and *M*-DPSK systems for the Rician-Nakagami fading environments. In addition, by exploiting the modulation gains directly obtained from the asymptotic SER formulas, we also theoretically demonstrate that *M*-DPSK suffers an asymptotic SER performance loss of 3.01dB with respect to *M*-PSK for a given *M* in Rician-Nakagami fading channels at high signal-to-noise ratio (SNR).

Key Words : *M*-ary phase shift keying (*M*-PSK), *M*-ary differential phase shift keying (*M*-DPSK), asymptotic analysis, symbol error rate (SER), Rician-Nakagami fading channels, modulation gain, diversity order

1. Introduction

Performance evaluations of digital communication systems have been extensively carried out in particular over small-scale fading channels (e.g., Rayleigh, Rician, Hoyt, Nakagami-*m*, etc.) (e.g., [1], [2] and references therein). Additionally, in an effort to investigate the impacts of both small-scale (e.g., multipath) and large-scale (e.g., shadowing) fading channels on the error rate performance of various transmission techniques, Rician shadow fading models, such as Rician-Lognormal, Rician-Nakagami, and so on, have been considered [3-6]. Among the models, the Rician-Nakagami fading model is a line-of-sight (LOS) shadow fading model that assumes the Rician and Nakagami-*m* distributions for the multipath fading and shadowing, respectively, and is more mathematically tractable than the Rician-Lognormal fading model. Moreover, this model is also well-known to fit well to the land mobile satellite (LMS) channel data as well as provide good experimental fit to the underwater acoustic communication (UAC) channels [2, 7].

Recently, exact closed-form expressions for the average symbol error rate (SER) of various modulation schemes (e.g., *M*-ary phase shift keying (*M*-PSK) and *M*-ary differential phase shift keying (*M*-DPSK), etc.) over Rician-Nakagami fading channels were proposed in [6], where the formulas are expressed in terms of multivariate hypergeometric functions from the moment-generating function (MGF) based approach, as used in [2]. However, due to their computational complexity caused by utilizing the hypergeometric functions, it is hard to obtain explicit and useful insights into the achievable error rate performance (e.g., achievable modulation gain, asymptotic diversity order, etc.) from the exact (but complex) formulas for the transmission schemes over the fading environments. Therefore, in this paper, we attempt to derive more effective and simpler closed-form SER expressions for both *M*-PSK and *M*-DPSK modulation schemes, which are based on the existing formulas given in [6]. Furthermore, it is well-known that 3.01dB performance gain can be achieved by *M*-PSK over *M*-DPSK in Rayleigh fading channels [8]. Thus, by deriving the formulas for the achievable modulation gains of *M*-PSK and *M*-DPSK, we also evaluate the SER performance loss of *M*-DPSK with respect to *M*-PSK for any modulation order *M* at high signal-to-noise ratio (SNR) in Rician-Nakagami fading channels. Finally, we note that, more recently, some research results on the asymptotic SER performance comparison between *M*-PSK

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and M-DPSK were presented in [9], however, where the theoretical analysis was carried out for the typical fading channels, such as Rayleigh, Rician, Nakagami-m, Weibull, etc. (but not for the Rician-Nakagami fading, which is considered in this paper). Furthermore, while, in order to obtain asymptotic SER formulas, the authors in [9] adopted Taylor series expanded forms of the probability density function (PDF) for various fading types, we derive the asymptotic SER expressions by applying the high SNR approximation technique directly to the existing formulas.

The remainder of this paper is organized as follows. Section 2 presents the channel model for the Rician-Nakagami fading. In Section 3, the existing exact SER expressions are given for both M-PSK and M-DPSK. In Section 4, concise closed-form asymptotic SER formulas are derived, followed by some numerical results in Section 5. Finally, this paper is concluded in Section 6.

2. Rician-Nakagami Fading Channel Model

Let γ be the instantaneous SNR of the receiver output in Rician-Nakagami fading channels. Then, the corresponding PDF and MGF are given, respectively, as [1, 3, 6]

$$f(\gamma)=\left[\frac{\overline{\gamma_S}m}{\overline{\gamma_S}m+\overline{\gamma_L}}\right]\frac{1}{\overline{\gamma_S}}exp\left(-\frac{\gamma}{\overline{\gamma_S}}\right) \times {}_1F_1\left(m,1;\frac{\overline{\gamma_L}m}{\overline{\gamma_S}\left(\overline{\gamma_S}m+\overline{\gamma_L}\right)}\right),\ \gamma\ge 0, \tag{1}$$

$$M_\gamma(s)=\frac{\left(\overline{\gamma_S}m\right)^m\left(1+\overline{\gamma_S}s\right)^{m-1}}{\left[\left(\overline{\gamma_S}m+\overline{\gamma_L}\right)\left(1+\overline{\gamma_S}s\right)-\overline{\gamma_L}\right]^m}, \tag{2}$$

where $\overline{\gamma_L}=\Omega\overline{\gamma}$, $\overline{\gamma_S}=2b_0\overline{\gamma}$, $\overline{\gamma}=E_S/N_0$, E_S and N_0 are the transmitted symbol power and the noise power spectral density, respectively, m is a parameter to describe the severity of shadowing with $m\ge 0$, Ω is the average power of the specular LOS component, $2b_0$ is the average power of the scattered component, and ${}_1F_1(\bullet,\bullet;\bullet)$ denotes the confluent hypergeometric function [10, 11].

3. Existing Exact SER Expressions

3.1 SER of M-ary Phase Shift Keying

According to [1, 2], the exact SER formula for M-PSK in arbitrary fading channels is given, by MGF-based approach, as

$$\overline{P}_{M-PSK}=\frac{1}{\pi}\int_0^{\left(1-\frac{1}{M}\right)\pi}M_\gamma\left(\frac{g_{PSK}}{\sin^2\theta}\right)d\theta, \tag{3}$$

where $g_{PSK}=\sin^2(\pi/M)$. Then, by combining Eq. (2) and Eq. (3) and after some straightforward algebraic manipulations, we can obtain the following closed-form expression for M-PSK modulation schemes in Rician-Nakagami fading channels as [6, Eq. (8)]

$$\begin{aligned}\overline{P}_{M-PSK}=&\frac{M_\gamma(g_{PSK})}{4}F_1\left(\frac{1}{2},1-m,m;2;\frac{1}{1+\overline{\gamma_S}g_{PSK}},\frac{1}{m+\left(\overline{\gamma_S}m+\overline{\gamma_L}\right)g_{PSK}}\right)+\frac{\cos\left(\frac{\pi}{M}\right)M_\gamma(g_{PSK})}{\pi}\\&\times F_D^{(3)}\left(\frac{1}{2},-\frac{1}{2},1-m,m;\frac{3}{2};\cos^2\left(\frac{\pi}{M}\right),\frac{\cos^2\left(\frac{\pi}{M}\right)}{1+\overline{\gamma_S}\,g_{PSK}},\frac{m\cos^2\left(\frac{\pi}{M}\right)}{\left(\overline{\gamma_S}m+\overline{\gamma_L}\right)g_{PSK}}\right),\end{aligned} \tag{4}$$

where $F_D^{(n)}(\bullet,\bullet,...,\bullet;\bullet;\bullet,...\bullet)$ is the Lauricella multivariate hypergeometric function, which is defined by [6, 11] by

$$\begin{aligned}F_D^{(n)}(a,b_1,...,b_n;c;z_1,...,z_n)=&\frac{\Gamma(c)}{\Gamma(c-a)\Gamma(a)}\int_0^1 t^{a-1}t^{c-a-1}\\&\times\prod_{i=1}^{n}(1-tz_i)^{-b_i}dt,\\&Re(c)>Re(a)>0,\end{aligned} \tag{5}$$

and $F_1(a,b_1,b_2;c;x_1,x_2)=F_D^{(2)}(a,b_1,b_2;c;x_1,x_2)$ is the Appell hypergeometric function [11].

3.2 SER of M-ary Differential Phase Shift Keying

Using MGF-based approach, the exact SER formula for M-DPSK is given as [1, 2]

$$\overline{P}_{M-DPSK}=\frac{1}{\pi}\int_0^{\left(1-\frac{1}{M}\right)\pi}M_\gamma\left(\frac{\sin^2\left(\frac{\pi}{M}\right)}{1+\cos\left(\frac{\pi}{M}\right)\cos\theta}\right)d\theta. \tag{6}$$

Thus, by substituting Eq. (2) into Eq. (6) and after some manipulations, the exact SER formula for M-DPSK can be reformulated as [6, Eq. (18)]

$$\overline{P}_{M-DPSK}=\frac{2\cos\left(\frac{\pi}{2M}\right)M_{\gamma}(g_{DPSK})}{\pi}\left[I_1-I_2\right], \tag{7}$$

where $g_{DPSK}=2\sin^2(\pi/2M)$,

$$I_1=F_D^{(3)}\left(\frac{1}{2},\frac{1}{2},1-m,m;\frac{3}{2};\cos^2\left(\frac{\pi}{M}\right),\frac{\cos\left(\frac{\pi}{M}\right)}{1+\overline{\gamma}_S g_{DPSK}},\frac{m\cos\left(\frac{\pi}{M}\right)}{m+\left(\overline{\gamma}_S m+\overline{\gamma}_L\right)g_{DPSK}}\right),$$

and

$$I_2=\frac{\cos\left(\frac{\pi}{M}\right)}{3}F_D^{(3)}\left(\frac{3}{2},\frac{1}{2},1-m,m;\frac{5}{2};\cos^2\left(\frac{\pi}{2M}\right),\frac{\cos\left(\frac{\pi}{M}\right)}{1+\overline{\gamma}_S g_{DPSK}},\frac{m\cos\left(\frac{\pi}{M}\right)}{m+\left(\overline{\gamma}_S m+\overline{\gamma}_L\right)g_{DPSK}}\right).$$

4. Asymptotic SER Expressions

4.1 Asymptotic SER of *M*-ary Phase Shift Keying

In order to simplify the exact but quite complicated SER formula (i.e., Eq. (4)), from the high SNR approximation technique (i.e., $\overline{\gamma}\to\infty$), the concise closed-form asymptotic SER formula can be derived as

$$\overline{P}_{M-PSK}^{\infty}=\left[\frac{N^{\infty}(g_{PSK})}{4}+\frac{\cos\left(\frac{\pi}{M}\right)N^{\infty}(g_{PSK})}{2\pi}\times\left(\sin^{-1}\left(\cos\left(\frac{\pi}{M}\right)\right)\sec\left(\frac{\pi}{M}\right)+\left|\sin\left(\frac{\pi}{M}\right)\right|\right)\right]\frac{1}{\overline{\gamma}}, \tag{8}$$

where $N^{\infty}(s)=(2b_0)^{2m-1}m^m s^{m-1}/(4b_0 ms+2b_0\Omega s)^m$ is obtained from Eq. (2) with $\overline{\gamma}\to\infty$ and $F_D^{(3)}(a,b_1,b_2,b_3;c;0,0)={}_2F_1(a,b_1;c;x_1)$ is used, which can be readily obtained from the definition of the Lauricella hypergeometric functions in [11].

Based on the above simple equation (i.e., Eq. (8)) and from the fact that in general the asymptotic error rate can be expressed as $\overline{P}^{\infty}=\left(G_m^{\infty}\cdot\overline{\gamma}\right)^{-d^{\infty}}$ [1], where G_m^{∞} and d^{∞} stand for the achievable modulation gain and asymptotic diversity order, respectively, we can further evaluate another useful performance measure (i.e., achievable modulation gain) achieved by *M*-PSK signaling over Rician-Nakagami fading channels, which is given by

$$G_{M-DPSK}^{\infty}=\left[\frac{N^{\infty}(g_{PSK})}{4}+\frac{\cos\left(\frac{\pi}{M}\right)N^{\infty}(g_{PSK})}{2\pi}\times\left(\sin^{-1}\left(\cos\left(\frac{\pi}{M}\right)\right)\sec\left(\frac{\pi}{M}\right)+\left|\sin\left(\frac{\pi}{M}\right)\right|\right)\right]^{-1}. \tag{9}$$

In addition, Eq. (8) clearly indicates that the asymptotic diversity order achieved by *M*-PSK signaling over Rician-Nakagami fading channels is $d_{M-PSK}^{\infty}=1$.

4.2 Asymptotic SER of *M*-ary Differential Phase Shift Keying

As in the case of *M*-PSK signaling, the simple closed-form asymptotic SER formula for *M*-DPSK in Rician-Nakagami fading channels can be given as

$$\overline{P}_{M-DPSK}^{\infty}=\frac{\cos\left(\frac{\pi}{2M}\right)N^{\infty}(g_{DPSK})}{\pi}\left[2\sec\left(\frac{\pi}{2M}\right)\times\sin^{-1}\left(\cos\left(\frac{\pi}{2M}\right)\right)-\cos\left(\frac{\pi}{M}\right)\sec^3\left(\frac{\pi}{2M}\right)\times\left(\sin^{-1}\left(\cos\left(\frac{\pi}{2M}\right)\right)-\cos\left(\frac{\pi}{2M}\right)\times\left|\sin\left(\frac{\pi}{M}\right)\right|\right)\right]\frac{1}{\overline{\gamma}}. \tag{10}$$

Accordingly, the corresponding modulation gain is readily obtained as

$$G_{M-DPSK}^{\infty}=\left[\frac{\cos\left(\frac{\pi}{2M}\right)N^{\infty}(g_{DPSK})}{\pi}\left[2\sec\left(\frac{\pi}{2M}\right)\times\sin^{-1}\left(\cos\left(\frac{\pi}{2M}\right)\right)-\cos\left(\frac{\pi}{M}\right)\sec^3\left(\frac{\pi}{2M}\right)\times\left(\sin^{-1}\left(\cos\left(\frac{\pi}{2M}\right)\right)-\cos\left(\frac{\pi}{2M}\right)\times\left|\sin\left(\frac{\pi}{M}\right)\right|\right)\right]\right]^{-1}, \tag{11}$$

and the diversity order is given by $d_{M-DPSK}^{\infty}=1$.

4.3 Asymptotic SER Performance Loss of *M*-DPSK w.r.t. *M*-PSK

Although *M*-PSK is known as a fundamental modulation scheme that is widely applied in practical wireless communication systems, it requires channel state information (CSI) at the receiver to achieve its optimal performance. On the contrary, M-DPSK with differentially coherent detection can be adopted with a sacrifice of some performance without channel state information at the receiver. In addition, it is well-known that M-PSK has 3.01dB performance advantage over M-DPSK in Rayleigh fading channels [8]. Thus, to analytically evaluate the performance loss of M-DPSK with respect to M-PSK over Rician- Nakagami fading channels, we calculate the following asymptotic SER performance loss (in dB) of M-DPSK w.r.t. M-PSK, using Eq. (9) and Eq. (11) and after some algebraic manipulations, as

$$SNR^{\infty}_{M-DPSK\ w.r.t.\ M-PSK} = 10\log_{10}\left[\frac{G^{\infty}_{M-PSK}}{G^{\infty}_{M-DPSK}}\right] = 3.01\,\mathrm{dB} \qquad (12)$$

Hence, Eq. (12) obviously reveals that the asymptotic SER performance loss of M-DPSK w.r.t. M-PSK over Rician-Nakagami fading channels is exactly 3.01dB, as in the case of Rayleigh fading channels. Furthermore, we can also find that the performance loss is independent of channel parameters (e.g., m, Ω, and b_0) and modulation order M.

5. Numerical Results

In this section, we present some numerical results to evaluate the accuracy of the derived formulas. In addition, we also consider three different channel parameter sets for Rician-Nakagami fading channels, as in [3, Table III]; infrequent light shadowing ($m = 19.4$, $\Omega = 1.29$, $b_0 = 0.158$), frequent heavy shadowing ($m = 0.739$, $\Omega = 8.97 \times 10^{-4}$, $b_0 = 0.063$), and average shadowing ($m = 10.1$, $\Omega = 0.835$, $b_0 = 0.126$).

Figure 1-(a) and Figure 1-(b) present the SER performances of QPSK and 8-PSK, respectively, over Rician-Nakagami fading channels with various channel parameters. For comparison purpose, the curves obtained from the exact SER formula (i.e., Eq. (4)) are illustrated along with the lines from our derived asymptotic SER formula (i.e., Eq. (8)). From the figures, it is obvious that the lined from the proposed asymptotic formula yield an excellent agreement with the curves from the existing complicated expression in the high SNR regime.

For the case of M-DPSK with differentially coherent detection, we demonstrate the SER performances of 4-DPSK and 8-DPSK in Figure 2-(a) and Figure 2-(b), respectively. Similar to the case of M-PSK in Figure 1, we can easily observe in Figure 2 that the lines obtained from our derived asymptotic SER formula (i.g., Eq. (10)) for M-DPSK over various channel conditions are asymptotically converges to those from the exact SER formula (i.e., Eq. (7)) at high SNR.

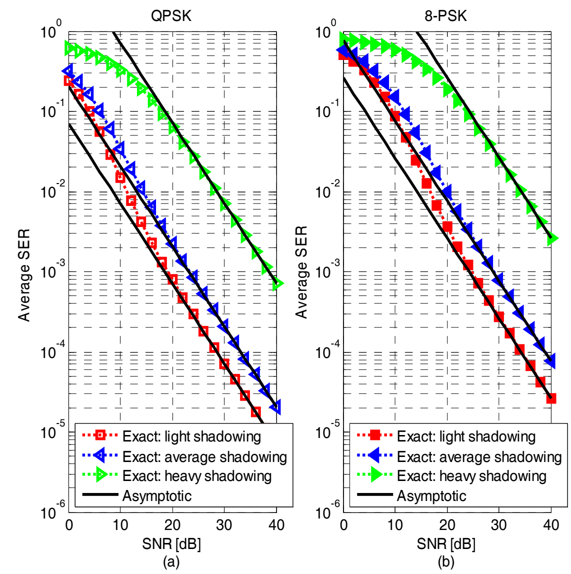

Fig. 1 Average SER vs. SNR of M-PSK (i.e., QPSK and 8-PSK) over Rician-Nakagami fading channels with various channel conditions

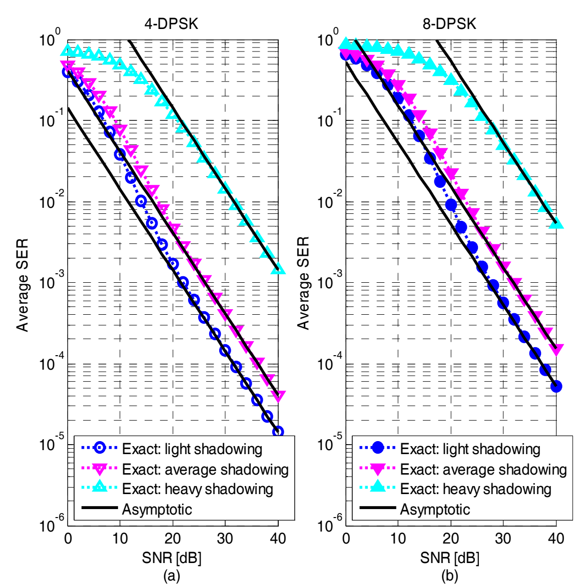

Fig. 2 Average SER vs. SNR of M-DPSK (i.e., 4-DPSK and 8-DPSK) over Rician-Nakagami fading channels with various channel conditions

In Figure 3, to investigate the SER performance loss of

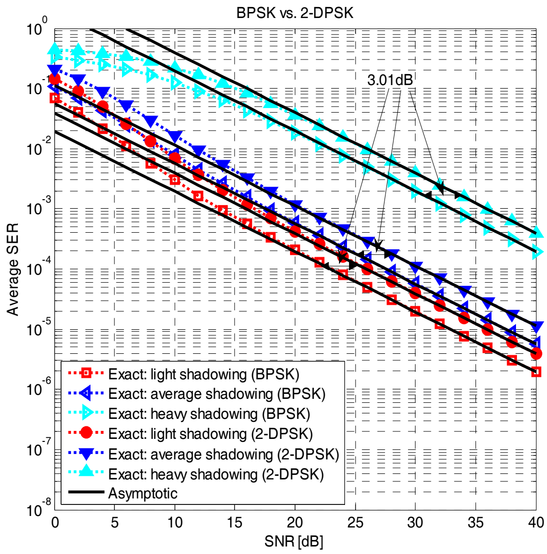

Fig. 3 Average SER vs. SNR of *M*-PSK and *M*-DPSK (i.e., BPSK and 2-DPSK) over Rician-Nakagami fading channels with various channel conditions

M-DPSK over *M*-PSK in Rician-Nakagami fading channels, we compare the SER performances, for example, of BPSK and 2-DPSK over aforementioned three different channel conditions. As evaluated in Eq. (12), we can apparently observe that the SER performance gap between BPSK and 2-DPSK is exactly 3.01dB, regardless of various channel parameters, such as m, Ω, and b_0.

6. Conclusion

In this paper, judiciously utilizing the existing exact SER expressions, we have derived simple and effective closed-form asymptotic SER formulas of both *M*-PSK and *M*-DPSK over Rician-Nakagami fading channels. In contrast to the existing but very complex SER expressions, the derived concise asymptotic SER formulas can be efficiently exploited to more intuitively and usefully evaluate the achievable SER performance in the high SNR regime. In addition, by using the corresponding modulation gains easily obtained from the derived asymptotic SER formulas, we have also demonstrated that the SER performance loss of *M*-DPSK with respect to *M*-PSK is constant with a value of 3.01dB, which coincides with the case of Rayleigh fading channels.

감사의 글

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References

[1] M. K. Simon and M.-S. Alouini, Digital Communications over Fading Channels: A Unified Approach to Performance Analysis, Wiley & Sons, Hoboken, NJ, 2005.

[2] A. Annamalai and C. Tellambura, “Error Rates for Nakagami-*m* Fading Multichannel Reception of Binary and *M*-ary Signals,” *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 58-68, Jan. 2001.

[3] A. Abdi, W. C. Lau, M.-S. Alouini, and M. Kaveh, “A New Simple Model for Land Mobile Satellite Channels: First- and Second-Order Statistics,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 3, pp. 519-528, May 2003.

[4] M. Uysal, “Pairwise Error Probability of Space-Time Codes in Rician-Nakagami Channels,” *IEEE Commun. Lett.*, vol. 8, no. 3, pp. 132-134, Mar. 2004.

[5] F. Xu, D.-W. Yue, F. C. M. Lau, and Q. F. Zhou, “Closed-Form Expressions for Symbol Error Probability of Orthogonal Space-Time Block Codes over Rician-Nakagami Channels,” *IET Commun.*, vol. 1, no. 4, pp. 655-661, Aug. 2007.

[6] X. Lei and P. Fan, “On the Error Performance of *M*-ary Modulation Schemes on Rician-Nakagami Fading Channels,” *Wireless Pers. Commun.*, vol. 53, no. 4, pp. 591-602, Jun. 2010.

[7] J. F. Paris, “Statistical Characterization of κ-μ Shadowed Fading,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 2, pp. 518-526, Feb. 2014.

[8] N. Ekanayake, “Performance of *M*-ary PSK Signals in Slow Rayleigh fading Channels,” *IET Electron. Lett.*, vol. 26, no. 10, pp. 618-619, May 1990.

[9] X. Song, F. Yang, J. Cheng, and M.-S. Alouini “Asymptotic SER Performance Comparison of MPSK and MDPSK in Wireless Fading Channels,” *IEEE Wireless Commun. Lett.*, vol. 4, no. 1, pp. 18-21, Feb. 2015.

[10] Wolfram Research: “The Wolfram functions site.” Available: http://functions.wolfram.com.

[11] H. Exton, Multiple Hypergeometric Functions and Applications, Wiley & Sons, New York, 1976.

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