I. Introduction

Absorbency plays an important role in many textile applications, i.e., design and manufacture of comfort textiles as well as in various processes such as dyeing and finishing. To improve comfort-ability in active sportswear, intimate apparel, or foot wear, the sweat excreted from human body should be minimized on the skin without uncomfortable feeling of an inner fabric’s clinging to the skin. For the purpose, the apparels are designed to quickly transport sweat through a hydrophobic inner layer to an outer hydrophilic layer and then evaporate it to the atmosphere. A variety of wicking tests have been used to measure wicking distance. Wicking...
distance has been measured visually by human eye or digital video camera with aid of image analysis technique (Memariyan & Ekhtiyari, 2010; Su, Fang, & Chen, 2007). To acquire better image, dyes might be added in water. However, these methods have some problems in the applications since added dye used for tracking the movement of water should not affect wicking behavior (no interaction between dye and fiber). When the images are analyzed by the digital camera system, the threshold value should be set up prior to converting gray images into pure black and pure white images. The binary converted images represent completely wet and dry area without informing on partially wet (gray) area. This technique is also not suitable for differently colored fabrics, thick or multilayered composite fabrics because grayscale and brightness of the image, affecting the determination of the threshold value, appear differently depending on the material color, the background light, and also the surface image of the material cannot detect water content in the inner part of material. An alternative method is to directly weigh the water absorbed fabric using the electronic balance in the equilibrium state and then the measured data are introduced into the model equations to describe the dynamic wicking behavior. The aim of this work is therefore to measure the wicking parameters using an electronic balance and then to analyze wicking behaviors based on the model equations with the measured parameters.

The equations to model absorbent behavior have been made differently according to absorption flow mechanisms, i.e., capillary flow mode or diffusion flow mode, depending on pore morphology and surface properties of constituent materials (Chatterjee & Gupta, 2002; van Brakel & Heertjes, 1977). In the capillary flow mode, it has been assumed that the porous space consists of a bundle of straight capillary tubes of same diameter, in which absorption proceeds completely saturating fabric without showing the concentration gradient in the flow direction (Figure 1(a)). The mathematically modeling equation for capillary flow mode was made by replacing the hydraulic pressure term in Hagen–Poiseuille’s equation by capillary pressure defined by Laplace’s equation (Lucas, 1918; Washburn, 1921).
On the other hand, the diffusion flow mode occurs mostly when porous space has significantly irregular pore sizes and the constituent material is highly hygroscopic and swell-able like a pulp material, in which a wide concentration gradient appears in the wicking direction and fluid has a tendency to spontaneously move from high concentration to low one (Figure 1(b)). Fick’s equation was used to mathematically model the diffusion behaviors (Gillespie, 1958; Richard, 1931; Rudd, 1960). The diffusion coefficient in the Fick’s equation was given as a function of the permeability coefficient defined in Darcy law and concentration gradient along wicking distance (Richard, 1931).

The determination of flow mechanisms, either capillary or diffusion mode, is needed before developing the absorbency equations since the equation is selected suitable for the flow mechanism. The flow mechanisms could be known from the capillary sorption curve which is obtained through the measurement of concentration gradient of absorbed material (water) along capillary pressure head in the equilibrium state (van Brakel & Heertjes, 1977). The wicking behaviors in most of the textile yarns and fabrics are, however, highly complicated due to the nature of diverse pore shapes and distributions, which are also governed by the fiber surface properties, yarn twist, and many factors. The authors have studied the wicking behaviors of cotton knitted fabrics. At different twist levels of cotton yarn, vertical wicking strip and siphoning tests were implemented to determine capillary pressure gradient and permeability coefficient which are key parameters in the model equations. The simulation analysis with the semi-empirical equations and data processing technique were conducted to objectively evaluate the absorbent performance.

II. Theory

Absorbency modeling equation has been determined differently according to flow mechanisms. Flow mechanism can be characterized from capillary sorption curve derived experimentally from step by step measurements of fluid concentrations at different pressure head \( H_s \) in the equilibrium state (Figure 1). Capillary flow mode occurs mostly in the porous material consisting of uniform pore size completely saturating the material as shown in Figure 1(a) while the diffusion flow mode occurs in highly hygroscopic porous material of irregular pore sizes partially saturating material, which results in a wide concentration gradient as shown in Figure 1(b).

1. Capillary flow

The simplest structural equation to model capillary absorption flow is attributed to Lucass (1918) and Washburn (1921). They considered that porous material consists of a bundle of cylindrical tubes of same diameter. Substituting the capillary pressure \( P_s \) into Poiseuille’s equation (1), the wicking rate in vertical direction was given by:

\[
\frac{dx}{dt} = \left( \frac{r^2}{8\eta x} \right) \Delta P = \left( \frac{r^2}{8\eta x} \right) \left( P_s - \rho g x \right) \tag{1}
\]

Here \( x \) is the wicking distance at time \( t \), \( \Delta P \) is the hydraulic pressure, \( r \) is the pore radius, \( \eta \) is the liquid viscosity, \( \rho \) is the liquid
density, and \( g \) is the gravitational acceleration.

Capillary pressure is defined at the equilibrium height in the vertical wicking test:

\[
P_c = \frac{2 \gamma \cos \theta}{r} = \rho g H_e
\]  

(2)

Here \( \gamma \) is the liquid surface tension, and \( \theta \) is the contact angle, and \( H_e \) is the capillary pressure head in the equilibrium state as shown in Figure 1(a).

Integration of the equation (1) gives the equation which characterizes the vertical wicking in a capillary flow mode:

\[
\ln \left(1 - \frac{x}{H_e}\right)^{-1} - \frac{x}{H_e} = \frac{r^2 \rho g t}{8 \eta H_e}
\]

(3)

At low value of time (\( t \)) the wicking height (\( x \)) is low as compared to the equilibrium height (\( H_e \)). Then the equation (3) reduces to the following equation by the Taylor’s approximation:

\[
\left(\frac{x}{H_e}\right)^2 = \frac{r^2 \rho g t}{8 \eta H_e}
\]

(4)

Substituting \( H_e \) obtained from the relationship (2) into the equation (4), well known Washburn’s equation for capillary absorption is derived as follows:

\[
x = \sqrt{\frac{2 \gamma \cos \theta}{\eta}} t^{1/2}
\]

(5)

For horizontal flow, the gravity term in the equation (1) is zero and therefore, the result is the same as that given by the Washburn equation (5).

Since it is not easy to find the mean value of capillary radius (\( r \)) of textile material for the application of Washburn’s equation (5), Poiseuille’s equation (1) has been replaced by Darcy’s equation (6), i.e., semi-empirical equations (Cho, Choi, & Hong, 2015).

\[
\frac{dx}{dt} = \left(\frac{k_s}{\eta x}\right) \Delta P
\]

(6)

Here \( k_s \) is the permeability coefficient measured experimentally in the saturated condition.

Comparing equations (1) and (6), the relationship between permeability coefficient and capillary radius can be found as follows.

\[
k_s = \frac{r^2}{8}
\]

(7)

Substituting the equation (7) into the equations (3) and (4), the wicking equations for capillary flow in the vertical direction and the horizontal direction can be derived respectively as follows.

\[
\ln \left(1 - \frac{x}{H_e}\right)^{-1} - \frac{x}{H_e} = \frac{k_s \rho g t}{\eta H_e}
\]

(8)

\[
x = \sqrt{\frac{k_s \rho g H_e}{\eta}} t^{1/2}
\]

(9)

To simulate the wicking distance as function of time based on the equations, the permeability constant (\( k_s \)) and capillary pressure head (\( H_e \)) should be known.

2. Diffusion Flow

In diffusion flow, fluid moves spontaneously from a region of higher concentration to one of...
lower concentration in porous structure. Fick's second law can be introduced to determine the concentration as a function of distance and time.

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( D(C) \frac{\partial C}{\partial x} \right) \quad (10)
\]

Here \( C \) is the concentration, \( t \) is the time, \( x \) is the wicking distance, and \( D(C) \) is the diffusion coefficient.

In the simplest and fairly common case where the diffusion coefficient is constant, the equation (10) becomes

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad (11)
\]

Using boundary conditions of \( C(x,0)=0 \) and \( C(0,t)=1 \), the solution to equation (11) becomes (Chatterjee & Gupta, 2002):

\[
C(x,t) = \text{erfc} \left( \frac{x}{2 \sqrt{D t}} \right) \quad (12)
\]

Here \( \text{erfc} \) is the complementary error function.

To describe concentration profile as a function of distance and time with the equation (12), the value of the diffusion coefficient should be known. Diffusion coefficient was determined in terms of permeability and capillary pressure as follows (Richard, 1931).

Under a given pressure gradient, flow through pores in unsaturated condition follows Darcy's law.

\[
v = \frac{Q}{A} = \frac{k_u(C)}{\eta} \frac{\partial P}{\partial x} = \rho g k_u(C) \frac{\partial (H-x)}{\partial C} \frac{\partial C}{\partial x} \quad (13)
\]

Here \( v \) is the velocity, \( Q \) is the flow rate of the fluid volume, \( A \) is the cross-sectional area perpendicular to the flow direction, \( k_u(C) \) is the relative permeability coefficient at a degree of concentration \( C \) in the unsaturated flow condition, \( \eta \) is the viscosity, and \( P(=\rho g H - \rho g x) \) is the hydraulic pressure in which \( \rho g H \) represents the capillary pressure for diffusion flow and \( \rho g x \) is the gravitational force retarding the wicking rate in the vertical wicking.

In porous network structures with different pore sizes, several authors have confirmed cubic relationship between relative permeability and concentration degree experimentally (Wyckoff & Botset, 1936) or theoretically (Fatt, 1956; Lin & Slattery, 1982).

\[
k_u(C) = k_s C^3 \quad (14)
\]

Here \( k_s \) is the permeability under the condition of complete saturation. The equation of continuity for one-dimensional flow with velocity of \( v \), i.e., the equation of mass conservation, is given as:

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\rho g k_u(C)}{\eta} C^3 \frac{\partial (H-x)}{\partial C} \frac{\partial C}{\partial x} \right) \quad (15)
\]

Substituting equation (13) and (14) into equation (15), and then applying a chain rule to the resultant, the equation for diffusion flow can be derived as follows:

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\rho g k_u(C)}{\eta} C^3 \frac{\partial (H-x)}{\partial C} \frac{\partial C}{\partial x} \right) \quad (16)
\]
To solve the non-linear partial differential equation (16) for vertical wicking, numerical method should be introduced. In the work, the diffusion flow in the horizontal direction with mean value of diffusion coefficient was considered, in which the gravitational term on the right hand side of the equation (16) is negligible.

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\rho g k_x}{\eta} C^3 \frac{\partial H}{\partial x} \right) \]  

(17)

Comparing the equations (17) to the equation (10), the diffusion coefficient can be defined as follows.

\[ D(C) = \frac{\rho g k_x}{\eta} C^3 \frac{\partial H}{\partial C} \]  

(18)

The equation (17) becomes the same as the equation (11) when the mean value of diffusion coefficient is calculated from the equation (18) as follows:

\[ D = \int_0^1 D(C) dC = \int_0^1 \frac{\rho g k_x}{\eta} C^3 \frac{\partial H}{\partial C} dC \]  

(19)

In order to obtain the mean value of the diffusion coefficient given in equation (19), the permeability coefficient \( k_p \), and the pressure gradient against concentration \( (\partial H/\partial C) \) (or concentration gradient \( (\partial C/\partial H) \)) should be measured experimentally. If the mean value of the diffusion coefficient is determined, the concentration profiles as function of wicking distance and time can be calculated from the equation (12).

III. Experimental

1. Materials

For the experiment, cotton single pique knitted fabrics were prepared since the pique structure is more porous and breathable, and less deformable than the plain structure so that the pique is widely used for polo shirts, blouses, pants or household items such as quilts or bedding knitted. Cotton yarns were spun at two different twist levels. Fabric thickness was measured on FAST system. Specimens used are listed in Table 1.

2. Permeability

Siphoning device was used to measure the permeability (Figure 2). A sample was cut to 5cm (course direction)\(\times\)25cm (wale direction) dimensions. It was soaked with water and then hung over two reservoirs containing water at the different levels. Water was forced to flow through fabric by siphoning action. Fluid amount \( (Q) \) passed through was measured after regular time intervals. Permeability coefficient \( (k_p) \) in

<table>
<thead>
<tr>
<th>Table 1. Materials for Experiment</th>
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<tbody>
<tr>
<td><strong>Type</strong></td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>S16</td>
</tr>
<tr>
<td>S30</td>
</tr>
</tbody>
</table>
Darcy’s equation (6) was calculated as follows:

\[ k_s = \frac{\eta}{\rho g} \frac{Q}{A} \]  \hspace{1cm} (20)

3. Capillary sorption curve

When the end of the sample was immersed vertically into the fluid reservoir, liquid started to rise along the length (Figure 1). After 30 min, the advance ceased and the front reached the maximum height with concentration gradient in the wicking direction. Specimen was then pulled out of the reservoir and cut into strips of 2 cm length along the fabric starting from the bottom. The weights of the cut pieces were measured on the microbalance to derive capillary sorption curve in the equilibrium state. Capillary pressure \( P_c(C) \) corresponding to a given fabric height \( H(C) \) is given by:

\[ P_c(C) = \rho g H(C) \]  \hspace{1cm} (21)

Here \( \rho \) is the water density. Concentration degree was measured by the difference between the weights of wet and dry samples. Figure 3 shows the capillary sorption curves derived experimentally. From Figure 3, the mean value of the capillary pressure \( \rho g H_c \) in the equation (9) for the capillary flow mode (completely saturated flow) is defined as follows:

\[ P_c = \int_0^1 P_c(C) dC = \rho g \int_0^1 H(C) dC = \rho g H_c \]  \hspace{1cm} (22)

Figure 4 shows the procedure to numerically derive the mean value of diffusion coefficient \( D \) and capillary pressure \( P_c \) from the capillary sorption curve. To reduce the errors in numerical calculation of diffusion coefficient \( D \) and capillary pressure \( P_c \), the data were selected at the middle point of each interval in capillary sorption curve and calculated as follows.

\[ D = \frac{k_s \rho g}{\eta} \int_0^1 C^3 dH = \frac{k_s \rho g}{\eta} \sum_{i=0}^{n-1} C_{i+1/2} \Delta H_i \]  \hspace{1cm} (23)

Figure 2. Schematic Diagram of Equipment for Fabric In-planar Permeability Measurement
Cheol-Jae Hong · Byung-Jick Kim / Model-Based Simulation Analysis of Wicking Behavior in Hygroscopic Cotton Fabric

\[ C_{i+1/2} = \frac{C_{i+1} + C_i}{2} \]

\[ P_c = \rho g \int_{0}^{1} H(C) dC = \rho g \sum_{i=0}^{n-1} H_{i+1/2} \Delta C_i \]  

(24)

\[ H_{i+1/2} = \frac{H_{i+1} + H_i}{2} \]

Here \( n \) is the total number of cut strips.

IV. Results and Discussion

The permeability \( (k) \), capillary pressure \( (P_c) \), and diffusion coefficient \( (D) \) measured experimentally are presented in Table 2.

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**Figure 3.** Concentration Gradient Curves and Mean Capillary Pressure for 100% Saturation Flow

**Figure 4.** Calculations of Diffusion Coefficient \( (D) \) and Capillary pressure \( (P_c) \) from the Capillary Sorption Curve
Table 2. Experimental Data for Wicking Simulation

<table>
<thead>
<tr>
<th>Type</th>
<th>$k_a$ (10^{-2} \text{cm}^2)</th>
<th>$P_c (\mu g H_c)$ (10^3 \text{dyne/cm}^2)</th>
<th>$D$ (10^{-2} \text{cm}^2/\text{sec})</th>
</tr>
</thead>
<tbody>
<tr>
<td>S16</td>
<td>3.54</td>
<td>7.066</td>
<td>7.339</td>
</tr>
<tr>
<td>S30</td>
<td>4.02</td>
<td>5.325</td>
<td>6.275</td>
</tr>
</tbody>
</table>

Figure 5 shows capillary sorption curves from which the mean values of capillary pressure and diffusion coefficient were calculated. The lower twist level of yarn showed the lower value of the permeability coefficient, but higher values of the capillary pressure and diffusion coefficient (Table 2). Insertion of a twist into yarn makes the fibers lie closer together and yarn become compacted, which leads to the decrease in yarn diameter, leaving high interstitial spaces (pore size) between yarns in the fabric. From the equations (7) and (2), it could be known that the increase of pore size gave the increase of permeability and the decrease of capillary pressure. Diffusion coefficient is given by product of permeability and capillary pressure as known in the equation (23). In cotton fabrics with low twist yarns, even though they have a low fluid permeability, they have much higher capillary potential energy (capillary pressure).

Wicking behavior in diffusion flow mode could be estimated from the equation (12) with the measured value of diffusion coefficient ($D$). Figure 6 shows the results of concentration against wicking distance at different time intervals. Conventional wicking test has been usually conducted by the visual measurement of the wicking front as function of time. In case of diffusion flow, however, the concentration where human could discern the wicking front might appear differently depending on the human eyesight (or threshold value when digital camera is used), fabric color, background light. In the work, we assumed that human eye could detect the wicking front when it was saturated with 30 percent of concentration. Figure 7(b) shows the wicking distances (the front with 30% concentration) as a function of time. The distance...
Figure 6. Concentration Profiles Simulated from the Diffusion Model Equation (the Wicking Distance Calculated from the Capillary Model Equation Agreed with the Distance at the 6.6% Concentration Calculated from the Concentration Profile)

from the diffusion model (Figure 7(b)) was compared to the distance from capillary model equation (9) (Figure 7(a)). Wicking rate of the high twist level of yarn was lower than that of the low twist level because of the decrease of capillary pressure even with the increase of permeability.

The distances calculated from capillary model equation (9) matched the distance of the diffusion flow chart (Figure 6) and found to be same with distance at 6.6% concentration. However, 6.6% concentration is too low for human eye to detect the front. The result indicates that the wicking rate calculated from capillary model is faster than the wicking rate from the diffusion model in which the wicking front is detectable at 30% concentration. In the capillary model, pore is assumed to be cylindrical tube through which the flow goes in straight direction. However, since the absorption is driven by the capillary action (pressure) at the common interfacial surface of solid–liquid(wet)–air(dry) as defined by the equation (2), fluid tends to move along the fiber orientation, i.e., tortuous path of yarn in knitted fabric, which could result in the decrease in the rate. Additionally, in hygroscopic cotton fabric with porous network structure of different pore radii, fluid tends to diffuse from the large size of pore (low capillary pressure) into the small size (high capillary pressure) or further into the amorphous region of cotton fiber at the given wicking distance, which could retard the capillary wicking rate due to the loss of energy for capillary flow into diffusion energy. When correction factor ($\tau$) which is defined as the wicking length calculated from the capillary model to that from the diffusion model, the simple equation (9) which has been widely used in textile industries can be modified to describe the diffusion front as follows.

$$x = \frac{1}{\tau} \sqrt{\frac{k_1 \rho g H_c}{\eta} t^{1/2}} \quad (25)$$

Figure 8 shows the comparison of wicking distances calculated from capillary model and diffusion model. When the human eyesight is
Figure 7. Wicking Distance Against Time: (a) the Distance Calculated from the Capillary Flow Model, (b) the Distance Calculated from the Diffusion Model, with the Assumption that the Human Eye Detects the Wicking Front at a 30% Concentration

Figure 8. Relationship Between the Wicking Distances Calculated from the Capillary Model and from the Diffusion Model, with the Assumption that the Wicking Front was Measured at 30% Concentration

assumed to detect the wicking front at the concentration of 30%, the correction factor ($\tau$) was found to be 1.78.

V. Conclusion

The work was undertaken to study the
wicking behaviors in cotton knitted fabrics. To characterize the wicking mechanisms in the fabrics, capillary sorption curves were derived experimentally. Highly hygroscopic cotton fabrics absorbed water with significantly wide concentration gradient in the flow direction, indicating the wicking mechanism was governed by a diffusion flow mode. Semi-empirical diffusion model was derived to simulate the wicking behaviors being compared to classical capillary model. The simulation results showed that the wicking rate from the capillary model in which the fluid was assumed to move in the straight direction completely saturating the wicking front was faster than that from the diffusion model. In capillary flow, tortuous flow path and diffusion between different pore sizes retards the wicking rate in reality. Capillary flow model equation could be modified with introducing the tortuous correction factor. Wicking test in textile industries has been conducted by a visual measurement. However, it could involve many difficulties in precisely detecting the wicking front, especially in a diffusion flow mode. To avoid any problems in visual measurement, the models based wicking analysis was necessary, in which the key parameters were measured using an electronic balance. The experimental techniques with the balance can be applied especially well for highly hydrophilic porous textile materials since the siphoning action used to measure the permeability occurs through pores and the equilibrium wicking height to derive the capillary pressure gradient curve is risen enough to be measurable by highly hydrophilic surface property of constituent material. The models and experimental methods in the work can provide tool to objectively evaluate textile wicking performances.

References


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