

Efficiency Increase and Input Power Decrease of Converted Prototype Pump Performance

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Abstract

The performance of a prototype pump converted from that of its model pump shows an increase in efficiency brought about by a decrease in friction loss. As the friction force working on impeller blades causes partial peripheral motion on the outlet flow from the impeller, the increase in the prototype's efficiency causes also a decrease in its input power. This paper discusses results of analyses on the behavior of the theoretical head or input power of a prototype pump. The equation of friction-drag coefficient for a flat plate was applied for the analysis of hydraulic loss in impeller blade passages. It was revealed that the friction-drag of a flat plate could be, to a certain degree, substituted for the friction drag of impeller blades, i.e. as a means for analyzing the relationship between a prototype pump's efficiency increase and input power decrease.

Keywords: Performance conversion, Model pump, Efficiency increase, Input power decrease, Scale effect

1. Introduction

The efficiency of a prototype pump shows a higher value than that of a model pump. This is the result of a decrease in the fluid friction coefficient. The friction force working on impeller blades drags the water in impeller blade passages towards the rotational direction of the impeller, thus increasing the peripheral component of the impeller outlet flow, as well as the impeller input power. Compared with the friction coefficient of model impeller blades, that of prototype impeller blades shows a decrease owing to a decrease in relative roughness. Correspondingly, there is a decrease in the ratio of the peripheral component of the impeller outlet flow versus the peripheral velocity at the impeller outlet, and accordingly a decrease in the input power coefficient of the prototype impeller [1], which is not well known among engineers handling hydraulic machinery.

IEC 60193 [2] specifies that the efficiency increase in the prototype shall be attributed all to the decrease in the power coefficient both for water-turbine and pump operations. On the other hand, JIS B 8327:2002 [3] stipulating performance conversion from a model to a prototype pump specifies that the efficiency increase in the prototype shall be attributed wholly to the increase in the head coefficient increase. Besides, IEC 62097 [4] published in 2009 specifies that the efficiency increase in the prototype shall be attributed to the head coefficient increase for the pump operation of reversible water turbine. However, JIS B 8327:2013 [5] published recently specifies that the efficiency increase is attributed half to the head increase and half to the input power decrease, based on the results reported by the author [1].

As for the issue on whether an efficiency increase is attributable to a head or input power coefficient, Kurokawa [6] states that an increase in an impeller's Reynolds Number increases the slip factor of blades and accordingly decreases in turn the theoretical head or input power. He proceeds to say that a decrease in input power cancels the expected head increase in the impeller due to a decrease in the friction coefficient. He concludes that the head coefficient of a prototype increases only in proportion to the value of decrease in loss head in the pump component, not including the impeller. Kurosawa's view, however, was not reflected in the relevant JIS standard until 2013.

Hutton and Fay [7] reported test results on a centrifugal pump, where Reynolds number was changed from 4×10^5 to 6×10^6 , ranging from hydraulically smooth to rough impeller passage surface. The results clarify that the shaft power decreases and the delivery head once increases and then decreases when Reynolds number increases within the hydraulically smooth range. However, when Reynolds number changes within the hydraulically rough surface range, both the shaft power and delivery head remain the same. They didn't attempt a detailed analysis on the behaviour of the shaft power and delivery head.

Recently, Ida [8] studied changes between the theoretical heads of a model and a prototype on the basis of the relationship

between the boundary layer thickness on impeller blades and the theoretical head. A change in the outlet flow angle due to a decrease in the boundary layer thickness on prototype impeller blades is investigated there, the condition being a shock-free inlet flow on a flat blade cascade. It was revealed that even under a shock-free inlet a slight rotation occurred in the outlet flow from the impeller owing to a drag or friction caused by the blades. Ida states that the change in the direction of relative outlet flow is less than 0.1° at most, and that the change in performance due to manufacturing tolerance is rather larger than that in the theoretical head due to the Reynolds Number.

His study was restricted to the change in outlet flow angle, based on the behaviour of the boundary layer only on blades and not on the shroud and disk. The shroud and disk surfaces conjointly work to deliver water to the outlet, and accordingly the friction force on these surfaces also cause rotational motion in the flow from the impeller, which was not taken into consideration in his study.

Pelz and Stonjek [9] proposed a new method for turbomachinery performance prediction, but this is related solely to hydraulic efficiency, not to the change in head and input power. Bellary and Samad [10] reported the results of numerical analysis on performance of a centrifugal pump for four different viscous liquids. The analysis was made on velocity vectors and pressure contours, and not on the relationship between efficiency step-up, head increase and input power decrease.

The author [1] investigated previously the relationship between the efficiency increase from the model to the prototype and the power coefficient decrease on the basis of the relationship between the loss head versus the blade drag force and the theoretical head.

This paper analyses how much efficiency increase from the model to the prototype is brought about by the head coefficient increase and by the power coefficient decrease, based directly on friction loss analysis of impeller blade passages, where not only blade surfaces but also shroud and disk surfaces are taken into consideration.

Efficiency conversion equation specified in JIS B 8327 [3] [5] is derived based on the friction-drag coefficient equation on a single flat plate, and the results of the conversion provides reasonable results within a certain degree of accuracy, and therefore, the friction analysis is carried out in this paper on the basis of the same equation on a single flat plate.

2. Change in Blade Drag and Input Power

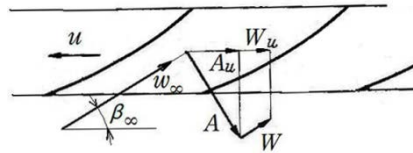


Fig. 1 Forces working on impeller blades

When a flow around axial flow impeller blades, as shown in Fig. 1, is considered, the momentum difference of water given by an impeller between inlet and outlet for inlet flow with no pre-rotation equals to the sum of peripheral components of lift and drag forces working on blades and blade passages, and is shown as follows.

$$\rho v_{a\infty} z b t v_{u2} = z (W \cos \beta_{\infty} + A \sin \beta_{\infty}) \quad (1)$$

where the drag W is comprised of the profile drag and friction drag, and the former is caused only by a blade and the latter by all blade passage surfaces.

The lift force A on a blade and the drag force W on a blade passage are expressed by using the lift and drag coefficients, C_A , C_D and C_F , and relevant surface areas, S_A and S_F , where S_A denotes the projected area of a blade and S_F the surface area composing a blade passage, see eq. (8). Then, eq. (1) can be written as follows.

$$\rho v_{a\infty} b t v_{u2} = (\rho/2) w_{\infty}^2 \{ (C_D S_A + C_F S_F) \cos \beta_{\infty} + C_A S_A \sin \beta_{\infty} \} \quad (2)$$

When expressed in dimensionless equation, eq. (2) can be rewritten as follows.

$$v_{u2}/u_2 = (1/2) \{ w_{\infty}^2 / (v_{a\infty} u_2) \} \times \{ (C_D S_A + C_F S_F) \cos \beta_{\infty} + C_A S_A \sin \beta_{\infty} \} / b t \quad (3)$$

The displacement thickness of the boundary layer on blade passage surfaces for the prototype is thinner than that for the model, and, therefore, strictly speaking, there is no similarity between their main flows through blade passages. Nevertheless, their lift and profile drag coefficients, C_A and C_D , are taken to be the same, the only difference being their friction coefficient. The difference in peripheral force of the prototype and model is caused by the difference in the friction drag coefficient, $(C_{FP} - C_{FM})$. As such, the difference in the peripheral velocity component of the prototype and model is expressed as follows.

$$v_{u2P}/u_{2mP} - v_{u2M}/u_{2mM} = (1/2) (w_{\infty}^2 / v_{a\infty} u_2) (C_{FP} - C_{FM}) (S_F / b t) \cos \beta_{\infty} \quad (4)$$

where the subscripts such as P and M are omitted from those dimensionless terms which are the same for both prototype and model such as $w_{\infty}^2 / v_{a\infty} u_2$ and $S_F / b t$. The same shall apply hereinafter.

Based on the above relation, head coefficient difference $\Delta\psi_{th}$ for the centrifugal prototype and model pumps is obtained in the following. Here it is assumed that:

- (1) the representative velocity of blade flow w_{∞} is the vector mean of the relative velocities at the blade inlet and outlet,
- (2) the representative impeller diameter D_{∞} is the arithmetic mean diameter of the inlet and outlet of central blade section dividing the blade passage equally, on reference to a result that the friction loss coefficient for the mixed flow impeller is expressed reasonably by the arithmetic mean diameter of the inlet and outlet [11],
- (3) the representative blade pitch t , width b and meridian velocity $v_{a\infty}$ are those at the diameter D_{∞} , and
- (4) the angle of the representative flow β_{∞} is the same both for the model and prototype.

Based on the following relations,

$$\frac{w_{\infty}^2}{v_{a\infty} u_{2m}} = \frac{(v_{a\infty} / \sin \beta_{\infty})^2}{v_{a\infty}} \frac{v_{a2}}{v_{a2}} \frac{1}{u_{2m}} = \frac{v_{a\infty}}{v_{a2}} \frac{\varphi}{\sin^2 \beta_{\infty}} \quad (5)$$

$$\psi_{th} = \frac{H_{th}}{u_{2m}^2/g} = \frac{u_{2m}v_{u2}/g}{u_{2m}^2/g} = \frac{v_{u2}}{u_{2m}} \quad (6)$$

Equation (4) can be rewritten as follows,

$$\begin{aligned} \Delta\psi_{th} &= \psi_{thP} - \psi_{thM} \\ &= \frac{1}{2} (C_{FP} - C_{FM}) \frac{S_F}{bt} \frac{v_{a\infty}}{v_{a2}} \varphi \frac{\cos \beta_\infty}{\sin^2 \beta_\infty} \\ &= (C_{FP} - C_{FM}) \frac{l}{t} \left(1 + \frac{t \sin \beta_\infty}{b}\right) \frac{v_{a\infty}}{v_{a2}} \varphi \frac{\cos \beta_\infty}{\sin^2 \beta_\infty} \end{aligned} \quad (7)$$

where the friction drag working area S_F is replaced by

$$S_F = 2l(b + t \sin \beta_\infty) \quad (8)$$

The velocity causing friction loss at impeller tip periphery is the relative velocity w_∞ for closed impellers and the absolute velocity v_∞ for open impellers, and the former is ordinarily larger than the latter, especially for high specific speed impellers. Accordingly the second term in eq. (8) is adjusted for the case of open type impellers in this paper, as referred to in Section 4.

Equation (7) gives the difference in the theoretical head, and thus the difference in the input power coefficient, between the model and prototype, as the theoretical head denotes input power per unit weight flow. Correspondingly, the value of $\Delta\psi_{th} / \psi_{thM}$ gives the ratio of change in input power coefficient versus the coefficient for the model.

The skin friction coefficient of a flat plate [12] as shown below is used in this paper for the coefficient C_F in eq. (7) as in the paper by Ida [8]

$$C_F = (1.89 + 1.62 \log \frac{l}{k_s})^{-2.5} \quad (9)$$

3. Hydraulic Loss in Impeller

In the former section the difference in the impeller work, i.e. the theoretical head, of the model and prototype was examined using the friction coefficient of a flat plate for estimating the friction loss in the impeller. The effectiveness of the estimation is discussed here.

Work done by the drag force on blades is equal to the energy lost in the impeller [13]. Accordingly,

$$\rho g b z t v_{a\infty} H_{LI} = \frac{1}{2} z \rho w_\infty^3 (C_D b l + C_F S_F) \quad (10)$$

The dimensionless head loss in the impeller ψ_{LI} can be expressed as follows.

$$\psi_{LI} = \frac{g H_{LI}}{u_{2m}^2} = \frac{1}{2} \left(\frac{C_D b l + C_F S_F}{bt} \right) \left(\frac{w_\infty}{u_{2m}} \right)^3 \frac{u_{2m} v_{a2}}{v_{a2} v_{a\infty}} \quad (11)$$

In eq. (11), blade width b , meridian velocity $v_{a\infty}$ and average relative velocity w_∞ are taken at the mean diameter between the inlet and outlet as representative values of blades, for the purpose of applying the equation to centrifugal impellers as well. Therefore, the difference in impeller head loss between the prototype and model is written as follows,

$$\begin{aligned} \Delta\psi_{LI} &= \frac{1}{2} (C_{FP} - C_{FM}) \frac{S_F}{bt} \left(\frac{w_\infty}{u_{2m}} \right)^3 \frac{1}{\varphi} \frac{v_{a2}}{v_{a\infty}} \\ &= (C_{FP} - C_{FM}) \frac{l}{t} \left(1 + \frac{t \sin \beta_\infty}{b}\right) \left(\frac{w_\infty}{u_{2m}} \right)^3 \frac{1}{\varphi} \frac{v_{a2}}{v_{a\infty}} \end{aligned} \quad (12)$$

where the profile drag coefficient C_D is assumed to be the identical for both the prototype and model.

The efficiency conversion equation specified in JIS B 8327 [3] gives reasonable results with a certain degree of uncertainty, where the efficiency converted in JIS B 8327 [3] is in fact the hydraulic efficiency. The difference of impeller head loss between the prototype and model can be obtained also by the hydraulic efficiency increase, calculated by the conversion equation in JIS B 8327 [3], based on the following assumptions:

- (1) Hydraulic loss in the impeller is half of the total hydraulic loss in the pump; accordingly, the friction loss decrease in the impeller is also half of the total friction loss decrease in the pump, where the results of tests carried out on axial flow pumps by Toyokura [14] is referred to.
- (2) The ratio of friction loss versus the total hydraulic loss and the friction loss decreasing ratio due to Reynolds Number are the same for both the impeller and fixed passages.

In the previous paper [1] the efficiency conversion formula given in JIS B 8327 [3] was referred to, and therefore the same formula is referred to in this paper for the purpose of convenience in the comparison of calculated results. The hydraulic efficiency of the prototype pump is given as follows,

$$\eta_{hP} = \eta_{hM} [1 + \delta_{EM} \{1 - 1.08(D_P/D_M)^{-0.18}\}] \quad (13)$$

$$\text{where } \delta_{EM} = (1 - \eta_{hM}) \{1.4Ns^{-0.1} - 0.07\} \quad (14)$$

The term $(1 - \eta_{hM})$ is replaced by 0.1 in JIS B 8327 [3] by assuming η_{hM} to be 0.9, but in this paper the term is given by the exact expression, as there are cases of lower hydraulic efficiency.

Hydraulic head loss coefficient ψ_L is given as follows,

$$\psi_L = \psi_{th}(1 - \eta_h) \quad (15)$$

$$\text{where } 1 - \eta_h = \frac{H_{th} - H}{H_{th}} = \frac{\psi_L}{\psi_{th}}$$

Hydraulic loss in the impeller $\Delta\psi_{LIC}$ is assumed to be half of the loss in the pump and then,

$$\Delta\psi_{LIC} = \psi_{LIP} - \psi_{LIM} = \psi_{thP}(1 - \eta_{hP})/2 - \psi_{thM}(1 - \eta_{hM})/2$$

Here ψ_{thP} is assumed to be equal to ψ_{thM} as the difference between them is small compared with $(1 - \eta_{hP})$, and then

$$\Delta\psi_{LIC} = \psi_{thM}(\eta_{hM} - \eta_{hP})/2 \quad (16)$$

Validity of applying the friction loss coefficient of a flat plate to the estimation of hydraulic loss in the impeller can be verified by comparing the results of eqs. (12) and (16).

4. Verification of the Blade Drag Effect

4.1 Axial Flow Impeller

Differences in the theoretical head coefficient, being equal to the input power coefficient, and those of the impeller hydraulic loss coefficient between the model and prototype are obtained by referring to test results of three axial flow pumps [15][16][17]. The scale ratio of prototype pumps is assumed to be 8, and the roughness of the prototype's blade surface to be twice as rough as that of the model, as eq. (13) is given on the assumption of the roughness ratio of two.

Table 1 shows dimensions, dimensionless performance and chord-spacing ratio of the pumps examined.

Table 2 shows the friction-drag coefficient of impeller blades C_{FM} and C_{FP} [eq. (9)], the theoretical head difference between the prototype and model [eq. (7)] and its ratio against the theoretical head of model, calculated based on the flat plate friction drag coefficient. As the impellers are of open type and have no shroud, it is unnecessary to take the friction loss on impeller tip periphery into account. Accordingly, the area subjected to friction drag, eq. (8), was calculated with the second term in the parentheses multiplied by 1/4. Friction loss caused on the casing inner surface around the impeller tip is generated by absolute velocity, which is much lower than the relative velocity, and thus the loss there is neglected here. As shown in Table 2, the decrease in theoretical head, i.e., the decrease in input power, between the model and prototype is almost 1% in all cases.

Table 3 shows the measured hydraulic efficiency η_{hM} , hydraulic efficiency increase between the model and prototype, the decrease in impeller head loss calculated by flat plate friction, $\Delta\psi_{LI}$ [eq. (12)], and calculated by efficiency conversion equation, $\Delta\psi_{LIC}$ [eq. (16)], and the ratio between them. As shown in Table 3, the ratio $\Delta\psi_{LI}/\Delta\psi_{LIC}$ is 1.02~1.12, i.e., almost 1.0, and this supports the idea that the behavior of friction in an axial impeller can be explained by the flat plate friction drag coefficient.

Table 1 Dimensions of axial flow pumps studied

No.	N_s	D_o	D_i	D_m	ϕ	ψ	β_∞	$(l/t)_\infty$
1	1550	296	130	228.6	0.38	0.23	23.66	0.75
2	1550	356	172	279.6	0.32	0.2	19.92	0.77
3	1550	370	185	292.5	0.316	0.196	19.61	0.78

Table 2 Friction-drag coefficients and differences between the theoretical head of the model and prototype, calculated based on flat plate friction drag coefficient

No.	l (m)	k_{sM} (μm)	C_{FM}	C_{FP}	$\Delta\psi_{th}$	$\Delta\psi_{th}/\psi_{thM}$
1	0.135	12	0.00482	0.00367	-0.00227	-0.0086
2	0.169	12	0.00460	0.00351	-0.00260	-0.0113
3	0.179	12	0.00454	0.00348	-0.00265	-0.0117

Table 3 Comparison between the hydraulic loss coefficients of the model and prototype, calculated applying the efficiency conversion and flat plate friction decrease

No.	η_{hM}	δ_E	η_{hP}	$\Delta\eta_h$	$\Delta\eta_h/\eta_{hM}$	$\Delta\psi_{LI}$	$\Delta\psi_{LIC}$	$\Delta\psi_{LI}/\Delta\psi_{LIC}$
1	0.870	0.078	0.8875	0.0175	0.0201	-0.00235	-0.00231	1.016
2	0.850	0.090	0.8697	0.0197	0.0232	-0.00259	-0.00232	1.115
3	0.840	0.096	0.8608	0.0208	0.0248	-0.00262	-0.00243	1.081

4.2 Centrifugal and Mixed-Flow Impellers

The same calculation as done in the previous sub-section was performed on centrifugal and mixed-flow impellers. Here also the scale ratio between the prototype and model is assumed to be 8, while the ratio of the blade roughness surface between them is assumed also to be two, as in the case of the axial flow pumps.

Table 4 shows head coefficient ψ , flow coefficient ϕ , impeller dimensions, number of blades z , blade angle at the mean diameter β_∞° and inlet velocity v_{a1} . Design constants ψ and ϕ and inlet-outlet diameter ratio were determined based on the design chart by Stepanoff [18] with blade outlet angle assumed to be 25° .

Table 5 shows the blade surface roughness of the model k_{sM} , blade width at the mean diameter b_∞ , friction-drag coefficients C_{FM} and C_{FP} , head coefficient difference $\Delta\psi_{th}$ and its ratio against head coefficient of the model $\Delta\psi_{th}/\psi_{thM}$. The blade width at the mean diameter was determined with the tip inlet angle assumed to be 17° .

Table 4 Dimensions of centrifugal and mixed flow pumps studied

No.	N_s	ψ	φ	D_{1o} (mm)	D_{1o}/D_2	D_{1i}/D_{1o}	D_{∞} (mm)	z	β_{∞}°	$(l/t)_{\infty}$	v_{a1}
1	200	0.51	0.12	180	0.44	0.7	282.2	6	19.7	2.03	5
2	280	0.495	0.13	200	0.5	0.6	282.5	6	21.1	1.88	5
3	400	0.45	0.16	220	0.57	0.5	279.9	5	23.4	1.65	5
4	560	0.39	0.2	240	0.77	0.4	247.2	5	26.7	1.50	4.5
5	800	0.31	0.26	260	0.92	0.35	238.7	5	29.6	1.40	4.5
6	1120	0.25	0.295	280	1.1	0.35	232.2	4	30.1	1.10	4.2

Table 5 Friction-drag coefficients and differences between the theoretical head of the model and prototype calculated based on the flat plate friction drag coefficient

No.	N_s	k_{sM} (μm)	b_{∞} (mm)	C_{FM}	C_{FP}	$\Delta\psi_{th}$	$\Delta\psi_{th}/\psi_{thM}$
1	200	12	28.8	0.00410	0.00317	-0.0063	-0.0106
2	280	12	36.4	0.00416	0.00321	-0.0049	-0.0087
3	400	12	42.8	0.00413	0.00319	-0.0041	-0.0080
4	560	12	59.8	0.00431	0.00332	-0.0030	-0.0068
5	800	12	69.9	0.00440	0.00338	-0.0020	-0.0055
6	1120	12	85.5	0.00444	0.00341	-0.0016	-0.0054

Table 6 Comparison between the hydraulic loss coefficients of the model and prototype calculated applying efficiency conversion and flat plate friction decrease

No.	N_s	η_{hM}	δ_E	η_{hP}	$\Delta\eta_h$	$\Delta\eta_h/\eta_{hM}$	$\Delta\psi_{LI}$	$\Delta\psi_{LIC}$	$\Delta\psi_{LI}/\Delta\psi_{LIC}$
1	200	0.855	0.109	0.8790	0.0240	0.0281	-0.00253	-0.00717	0.353
2	280	0.872	0.093	0.8929	0.0209	0.0239	-0.00207	-0.00592	0.350
3	400	0.879	0.085	0.8981	0.0191	0.0218	-0.00189	-0.00489	0.386
4	560	0.879	0.081	0.8974	0.0184	0.0210	-0.00162	-0.00409	0.397
5	800	0.843	0.102	0.8650	0.0220	0.0261	-0.00127	-0.00405	0.312
6	1120	0.843	0.098	0.8642	0.0212	0.0252	-0.00117	-0.00315	0.370

Table 6 shows the assumed model hydraulic efficiency η_{hM} , friction loss ratio δ_E , converted prototype hydraulic efficiency η_{hP} , hydraulic efficiency difference $\Delta\eta_h$, its ratio $\Delta\eta_h/\eta_{hM}$, hydraulic loss difference between model and prototype impellers $\Delta\psi_{LI}$, hydraulic loss difference calculated by converted efficiency $\Delta\psi_{LIC}$ and its ratio $\Delta\psi_{LI}/\Delta\psi_{LIC}$.

Friction-drag working area used for calculating differences in head coefficients and impeller head losses was obtained by multiplying 1/2 or 1/3 to the second term in eq. (8) for the cases of pumps of the specific speed of 800 and 1120, with consideration given on the difference between the diameters of the tip and hub.

As seen in Table 5, the decreasing ratio of theoretical head $\Delta\psi_{th}/\psi_{thM}$ was -1.1% for low specific speed impellers and -0.6% for high specific speed impellers. The ratio $\Delta\psi_{LI}/\Delta\psi_{LIC}$ in Table 6 is 0.312~0.397 in contrast to approximately 1.0 for axial impellers in Table 3, which will be discussed in the following section.

5. Discussions

5.1 Difference in Hydraulic Impeller Loss of the Model and Prototype

Examination is made to verify whether the flat plate friction loss equation, eq. (9), is able to be duly substituted for the friction loss equation for estimating impeller blade passage losses. This is done by comparing the results calculated based on the flat plate friction drag, eq. (12), with those calculated based on hydraulic efficiency conversion, eq. (16).

As seen in Table 6 for centrifugal and mixed flow impellers, the values of $\Delta\psi_{LI}/\Delta\psi_{LIC}$ are below 0.4 for all cases, although the values are between 1.0 and 1.1 for the cases of axial flow impellers (see Table 3). This indicates that the friction loss calculated by the flat plate friction equation is small, almost 1/3 of the actual friction loss.

Shirakura [19] carried out a test for the purpose of clarifying the effect of the impeller blade surface roughness on the hydraulic efficiency of the pump. He states that the difference in impeller head caused by different blade surface roughnesses is in average about 3.3 times as large as the hydraulic loss difference calculated by applying flat plate turbulent boundary layer equation on four surfaces composing impeller passages. The hydraulic loss caused by curvature and divergence in the blade passages is assumed in his calculation to be independent of the surface roughness.

Furthermore, Ida [11] reports in his paper on the characteristics of mixed flow pumps that the friction loss coefficient of impeller passages is about 2.8 times as large as the friction loss coefficient of a straight pipe. This is based on the analysis on hydraulic loss at shock-free flow, where the loss comprised of only friction loss (Normally the hydraulic loss includes losses in bends and divergence, but in his study the losses are classified into shock and friction losses; therefore, in a shock-free flow the loss is all friction loss.).

Thus, the values of $\Delta\psi_{LI}/\Delta\psi_{LIC}$ presented in Table 6 show about the same results as is given by Shirakura [19] and Ida [11], and can be claimed to be reasonable.

In the case of axial flow impellers, Table 3, however, the values of $\Delta\psi_{LI}/\Delta\psi_{LIC}$ are 1.0~1.1. In axial impellers, the blades compose straight cascades, the normal spacing between blades $t \sin\beta_\infty$ is nearly equal to blade width b which is the same from inlet to outlet, and accordingly the cross sections of flow passages remain almost unchanged. In contrast, blades compose circular cascades in centrifugal and mixed flow impellers; therefore, the peripheral spacing between blades expands remarkably from inlet to outlet, although the expansion ratio is about the same as that of the axial flow impellers. That is, the passage cross-sections change from a nearly square section at the inlet to a slender rectangular section at the outlet. This is considered to be the main reason for the difference in the values of $\Delta\psi_{LI}/\Delta\psi_{LIC}$ for centrifugal and mixed flow impellers versus that for axial impellers.

As a conclusion, the flat plate friction equation can be applied for estimating friction loss in axial flow impellers which have no serious problems. As for its application for centrifugal and mixed flow impellers, due consideration must be given to that the equation does not offer a complete solution.

Table 7 Converted Performance of Prototype pump for a case of $D_P/D_M = 8$ and $\beta_\infty = 20^\circ$, reported previously by the Author [1]

Ns	η_M	η_{hM}	$\Delta\psi_{th}/\psi_{thM}$	η_{hP}	ψ_P/ψ_M	$\Delta\eta_h/\eta_{hM}$
200	0.80	0.855	-0.016	0.872	1.004	0.020
280	0.82	0.872	-0.014	0.889	1.005	0.020
400	0.84	0.879	-0.012	0.896	1.006	0.019
560	0.84	0.879	-0.011	0.895	1.007	0.018
800	0.83	0.843	-0.010	0.858	1.008	0.018
1120	0.83	0.843	-0.009	0.857	1.008	0.018
1600	0.82	0.837	-0.008	0.851	1.009	0.017

5.2 Difference in the Theoretical Head Coefficient

Difference in the ratio of theoretical head coefficient of the model and prototype, calculated by eq. (7), versus the theoretical head coefficient of the model, $\Delta\psi_{th}/\psi_{thM}$, gives the change in input power coefficient of the prototype. As far as axial impellers are concerned (Table 2), the values of $\Delta\psi_{th}/\psi_{thM}$ are $-0.009 \sim -0.012$, which correspond to $-45 \sim -48\%$ of hydraulic efficiency increase $\Delta\eta_h/\eta_{hM}$ ($= 0.02 \sim 0.025$), i.e. the decrease in the input power coefficient.

The results of hydraulic efficiency conversion reported in the previous paper[1] are shown in Table 7. For the case of $Ns = 1600$, the value of $\Delta\psi_{th}/\psi_{thM}$ is -0.008 and about a half of hydraulic efficiency increase $\Delta\eta_h/\eta_{hM}$ ($= 0.017$) in negative value.

In contrast, for the cases of centrifugal and mixed flow impellers ($Ns = 200 \sim 1120$), Table 5, the values of $\Delta\psi_{th}/\psi_{thM}$ are $-0.0106 \sim -0.0054$, which correspond to about $-38 \sim -21\%$ of the values of hydraulic efficiency increase $\Delta\eta_h/\eta_{hM}$ ($= 0.0281 \sim 0.0252$), i.e. the decrease in the input power coefficient. However, the results of the previous paper [1] give the values of $\Delta\psi_{th}/\psi_{thM}$ of $-0.016 \sim -0.009$, corresponding to about $80 \sim 50\%$ of hydraulic efficiency increase $\Delta\eta_h/\eta_{hM}$ ($= 0.02 \sim 0.018$) in negative value.

As a conclusion, the flat plate friction equation can be applied for estimating a decrease in the input power coefficient of prototype pump for axial flow impellers without any serious problem. The equation can also be used for centrifugal and mixed flow impellers; however, it is noted that the amount of decrease is about a half of that calculated by the efficiency conversion formula.

5.3 Working Area of Friction-Drag

In the analysis of the measured hydraulic loss, the impeller passage is substituted by a channel comprising 4 flat surfaces [19] or a pipe with an equivalent hydraulic mean depth [11]. When the impeller work is discussed based on the forces working on blades, not only the friction forces on both upper and lower blade surfaces but also the forces on both disk and shroud surfaces should be taken into account (see eq. (2)). This is because the power is transmitted to water through the impeller passage comprising 4 surfaces. The impeller work cannot be carried out only by the blade surfaces. Without disk and shroud surfaces power cannot be transmitted to the water discharged from the impeller.

When the slip factor of blades is examined, with consideration given on change in friction-drag related with scale effect, it is not reasonable to study the effect of friction-drag referring to the friction forces only on blade surfaces. The effect of disk and shroud surface friction in the study of scale effect becomes larger with decreasing value of Ns , as the blade height becomes lower compared with impeller outlet diameter.

6. Conclusions

The difference in theoretical head coefficient (equal to input power coefficient) of a model and a prototype with a scale ratio of 8 and surface roughness ratio of 2 was obtained, where the flat plate friction coefficient equation was applied for estimating the friction-drag on impeller blades. In addition, the difference in the hydraulic loss coefficient of the model and prototype impellers was obtained based on the efficiency step-up calculated by the conversion formula. The validity of substituting a flat plate friction-drag equation for estimating the friction loss in an impeller was investigated based on a comparison of these differences.

The results are summarized as follows.

- (1) The hydraulic efficiency of the prototype pumps shows higher values than that of the models by $0.0240 \sim 0.018$ depending on Ns of $200 \sim 1550$.
- (2) The theoretical head coefficient, equal to input power coefficient, of the prototype shows a decrease of about a half of hydraulic efficiency increase between the model and prototype for axial flow impellers, but a decrease of about $40 \sim 23\%$ of the hydraulic efficiency increase for centrifugal and mixed flow impellers.

- (3) The difference in impeller hydraulic loss between the model and prototype, calculated using the flat plate friction-drag equation, agrees with that derived by the hydraulic efficiency conversion for axial impellers, but is about 1/3 for centrifugal and mixed flow impellers.
- (4) The difference shown above among axial flow, centrifugal and mixed flow impellers is considered to be caused by the difference between linear and circular cascades.
- (5) The flat plate friction-drag equation can be duly applied for discussion of the relation between hydraulic efficiency increase and input power coefficient decrease of the prototype pump, provided that the hydraulic friction loss calculated gives underestimated values for a circular cascade.
- (6) When examining impeller work with hydraulic friction considered, not only the blade surfaces but also the disk and shroud surfaces should be taken into account, as these 4 surfaces work conjointly to deliver water through the impeller.

Nomenclature

A	Lift force working on a blade		Subscripts
b	Passage width		
C	Coefficient		
D	Impeller diameter	A	Lift force
g	Acceleration due to gravity	a	Meridian component
H	Head	C	Calculated by efficiency conversion equation
k_s	Surface roughness	D	Profile drag
l	Blade length	F	Friction drag
N_s	Specific speed	h	hydraulic
Q	Flow rate	I	Impeller
S	Area	i	Hub or disk
t	Peripheral spacing of blades	L	Hydraulic loss
u	Peripheral velocity	M	Model
v	Absolute flow velocity	m	Square mean of values at disk and shroud
w	Relative flow velocity	o	Tip or shroud
W	Drag force working on a blade passage	P	Prototype
z	Number of blades	th	Theoretical
β	Blade angle or angle of relative flow	u	Peripheral component
δ_E	Friction loss ratio (total friction loss head / pump total head)	v	Volumetric
η	Efficiency	1	Impeller inlet
ρ	Fluid density	2	Impeller outlet
φ	Flow coefficient = v_{a2} / u_{2m}	∞	Representative value (value at arithmetic mean diameter of inlet and outlet of central blade section dividing blade passage equally)
ψ	Head coefficient = gH / u_{2m}^2		

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