

# Real-Time Orbit Determination for Future Korean Regional Navigation Satellite System

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This paper presents an algorithm for Real-Time Orbit Determination (RTOD) of navigation satellites for the Korean Regional Navigation Satellite System (KRNSS), when the navigation satellites generate ephemeris by themselves in abnormal situations. The KRNSS is an independent Regional Navigation Satellite System (RNSS) that is currently within the basic/preliminary research phase, which is intended to provide a satellite navigation service for South Korea and neighboring countries. Its candidate constellation comprises three geostationary and four elliptical inclined geosynchronous orbit satellites. Relative distance ranging between the KRNSS satellites based on Inter-Satellite Ranging (ISR) is adopted as the observation model. The extended Kalman filter is used for real-time estimation, which includes fine-tuning the covariance, measurement noise, and process noise matrices. Simulation results show that ISR precision of 0.3–0.7 m, ranging capability of 65,000 km, and observation intervals of less than 20 min are required to accomplish RTOD accuracy to within 1 m. Furthermore, close correlation is confirmed between the dilution of precision and RTOD accuracy.

**Keywords:** extended Kalman filter, inter-satellite ranging, Korean regional navigation satellite system, real-time orbit determination

## 1. INTRODUCTION

Satellite Navigation Systems (SNSs) are used widely for a variety of navigation services. While Global Navigation Satellite Systems (GNSSs) provide free commercial-level services around the world, some countries are developing their own Regional Navigation Satellite Systems (RNSSs), e.g., India (Indian Regional Navigation Satellite System, IRNSS), Japan (Japanese Regional Advanced Navigation Satellites, JRANS), and China (Beidou/COMPASS) (United Nations 2010). Such RNSSs usually cover non-global/limited areas and they are intended to provide an accurate local navigation service based on both the global and the regional systems under normal conditions, but also to provide a stand-alone/compensatory navigation service in abnormal situations when the GNSSs might be unavailable.

The Korean Regional Navigation Satellite System (KRNSS) is current at the basic/preliminary research stage. Comprising three Geostationary Orbit (GEO) and four

Elliptical Inclined Geosynchronous Orbit (EIGSO) satellites as a candidate constellation, the KRNSS is intended to provide local navigation services for the region around the Korean Peninsula (Choi et al. 2013). Successful operation of an SNS requires precise Orbit Determination (OD). Thus, SNSs usually collect long-term observational data of navigation satellites at ground stations and perform post/batch process OD to determine the state vectors (i.e., positions and velocities). The state vectors are then converted into ephemeris data for the SNS, which are transmitted periodically to the navigation satellites from the ground stations. A study of the OD of the KRNSS under normal conditions has been undertaken by Choi (2014). However, in an abnormal situation, when ground stations cannot uplink/upload the ephemeris data to the navigation satellites, the satellites must generate their own ephemeris data to provide a continuous navigation solution. In such circumstances, because of high memory requirements and computational loads, the post/batch process OD is unsuitable, but Real-

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Time Orbit Determination (RTOD) is appropriate for the sequential processing of the observational data.

The main objective of the current paper is to develop an RTOD algorithm for the navigation satellites of the future KRNSS in abnormal situations. The well-known Extended Kalman Filter (EKF) is used for real-time estimation, which includes fine-tuning the initial covariance matrix, measurement noise matrix, and process noise matrix. In an effort to maintain operation without connection to ground stations, relative distance ranging between the KRNSS satellites based on Inter-Satellite Ranging (ISR) is adopted as the observation model (Wolf 2000). Multi-faceted parametric studies show that OD precision is affected by important parameters such as the ISR precision, ranging distance capability, and observational time interval.

The remainder of this paper is organized as follows. The KRNSS is introduced briefly in Section 2. Section 3 deals with the observational model, dynamic model, and real-time estimation theory for configuring the RTOD. Section 4 presents the RTOD simulation program and the associated results. Section 5 concludes the discussion.

## 2. KOREAN REGIONAL NAVIGATION SATELLITE SYSTEM (KRNSS)

The KRNSS currently considers three GEO and four EIGSO satellites as the candidate constellation. Fig. 1 shows the ground tracks of the KRNSS satellites in the Earth-Centered-Earth-Fixed coordinate system.

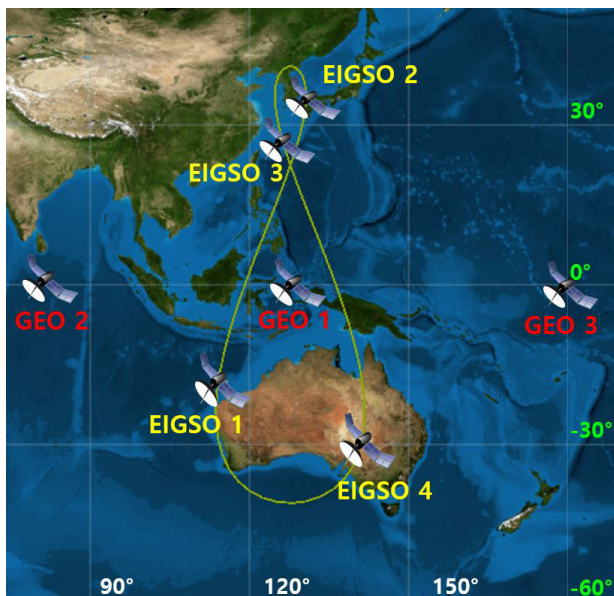


Fig. 1. Ground tracks of the Korean Regional Navigation Satellite System (KRNSS).

The orbits of the three GEO satellites are located at 80° E, 127° E, and 180° E in consideration of geometrical visibility (Choi 2014). The orbits of the four EIGSO satellites are inclined at 41° to provide coverage of the Korean Peninsula for a large proportion of their periods (Choi et al. 2013). This constellation was designed to receive at least four navigation signals in South Korea at any one time. Table 1 shows the orbital characteristics of the KRNSS satellites.

## 3. REAL-TIME ORBIT DETERMINATION (RTOD) ALGORITHM

Typical OD algorithms require three principal components: measurement model, dynamic model, and estimation.

As the KRNSS is designed currently to operate only with domestic ground stations, it is subject to visibility limitations. In an effort to mitigate this geometrical constraint, relative distance ranging between satellites using ISR is employed (Wolf 2000; Choi 2014). In relative distance ranging between satellites, it is critical to measure the signal travel time, the accuracy of which is affected by a variety of sources of error (Wolf 2000). In this paper, the total error in the observational data (ranging precision) includes Gaussian random noise with 0.1-100 m magnitude to describe the satellite clock offsets, ionospheric delay, tropospheric delay, and multipath error (Wolf 2000):

$$\text{Distance} = (\text{Time}_{\text{sat}(RX)} - \text{Time}_{\text{sat}(TX)}) \times (\text{Speed of light}) \quad (1)$$

where  $\text{Time}_{\text{sat}(RX)}$  and  $\text{Time}_{\text{sat}(TX)}$  represent the reception and the transmission times of the signal, respectively.

In consideration of the altitude of the KRNSS satellites, their dynamical equations of motion incorporate the asymmetric gravitational field of the Earth, the third-body perturbation of the Sun and the Moon, and the Solar Radiation Pressure (SRP):

$$\ddot{\vec{r}} = -\frac{\mu_E}{r^3} \vec{r} + \vec{a}_{geo} + \vec{a}_{3rd} + \vec{a}_{SRP} \quad (2)$$

Table 1. Duration of the longitude drift from the different origins near domestic GEO satellites

Satellites	Semimajor Axis (km)	Eccentricity	Inclination (deg)	Argument of Perigee (deg)
EIGSO 1	42,164	0.075	41	270
EIGSO 2	42,164	0.075	41	270
EIGSO 3	42,164	0.075	41	270
EIGSO 4	42,164	0.075	41	270
GEO 1	42,164	0	0	undefined
GEO 2	42,164	0	0	undefined
GEO 3	42,164	0	0	undefined

where  $\vec{r}$  is the position vector,  $\mu_E$  is the gravitational constant of the Earth,  $\vec{a}_{geo}$  is the asymmetric gravitational field perturbation of the Earth,  $\vec{a}_{3rd}$  is the third-body perturbation due to the Sun and the Moon, and  $\vec{a}_{SRP}$  is the perturbation due to SRP (Vallado & McClain 2007). Table 2 displays the important parameter settings of the dynamics used in the simulations.

The state vectors are estimated using the EKF algorithm in real time (Kalman 1960), which is composed mainly of the predictor and the corrector. The state vector ( $\hat{X}_k$ ), the covariance matrix ( $\hat{P}_k$ ), and the state transition matrix ( $\Phi^T(t_{k+1}, t_k)$ ) are used to predict the state vector ( $\bar{X}_{k+1}$ ) and the covariance matrix ( $\bar{P}_{k+1}$ ) via the following equations (Brown & Hwang 1997):

$$X = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^T \quad (3)$$

$$\bar{X}_{k+1} = \Phi(t_{k+1}, t_k) \hat{X}_k \quad (4)$$

$$\bar{P}_{k+1} = \Phi(t_{k+1}, t_k) \hat{P}_k \Phi^T(t_{k+1}, t_k) + Q \quad (5)$$

where  $Q$  represents the process noise matrix.

The predicted state vector ( $\bar{X}_{k+1}$ ) and covariance matrix ( $\bar{P}_{k+1}$ ) are then corrected to the state vector ( $\hat{X}_{k+1}$ ) and the covariance matrix ( $\hat{P}_{k+1}$ ) by the following equations:

$$K_{k+1} = \bar{P}_{k+1} H_{k+1}^T [H_{k+1} \bar{P}_{k+1} H_{k+1}^T + R_{k+1}]^{-1} \quad (6)$$

$$\hat{P}_{k+1} = \bar{P}_{k+1} - K_{k+1} H_{k+1} \bar{P}_{k+1} \quad (7)$$

$$\hat{X}_{k+1} = \bar{X}_{k+1} + K_{k+1} [y_{k+1} - H_{k+1} \bar{X}_{k+1}] \quad (8)$$

where

$$H_{k+1} = \frac{\partial \rho}{\partial X} = \begin{bmatrix} \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}} & \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}} & \frac{z-z_1}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{x-x_n}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}} & \frac{y-y_n}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}} & \frac{z-z_n}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}} & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

**Table 2.** Model and parameters in the dynamic model

Model / Parameter	Description	References
ECI Frame	J2000	(Tapley et al. 2004)
Aspherical Earth Gravity	JGM3	(Tapley et al. 1996)
Planetary Ephemeris	Astronomical Almanac	(Seidelmann 2006)
Precession/Nutation	IAU 1976	(Seidelmann 2006)
Solar Radiation Pressure Reflectivity Coefficient	$C_r : 1.3$	
Satellite Spec'	Cross section : 4.81m <sup>2</sup> Mass : 1,465 kg	GPS block IIF (Los Angeles Air Force Base 2014)
Numerical Integration	Runge Kutta 45	ODE45, MATLAB

Here,  $K_{k+1}$  is the Kalman gain,  $y_{k+1}$  is the observational data, and  $R$  is the measurement noise matrix (Zarchan & Musoff 2000). Symbol  $\rho$  represents the relative distance between satellites,  $[x_n \ y_n \ z_n]$  represents the coordinates of the satellite, and subscript  $n$  is the number of observational data.  $H_{k+1}$  is the partial derivative of the measurement function with respect to the state vector, which is determined numerically.

## 4. ORBIT DETERMINATION (OD) SIMULATIONS AND RESULTS

Simulations are conducted to identify the requirements necessary to achieve meter-level accuracy of RTOD. The time span of the simulation is fixed at 25 hr in consideration of the orbital period of the EIGSO satellites. The RTOD error at each time step is calculated as the distance between the actual locations of the navigation satellites and their positions estimated by EKF. The RTOD accuracy is defined as the Root Mean Square (RMS) over the entire duration of the simulation. For comparative analysis, the true positions of the satellites are generated artificially by propagating the dynamic equations of motion from the initial states without any errors.

### 4.1 EKF Matrices Optimization

The EKF algorithm is first optimized by fine-tuning the covariance matrix, measurement noise matrix, and process noise matrix based on their physical meanings.

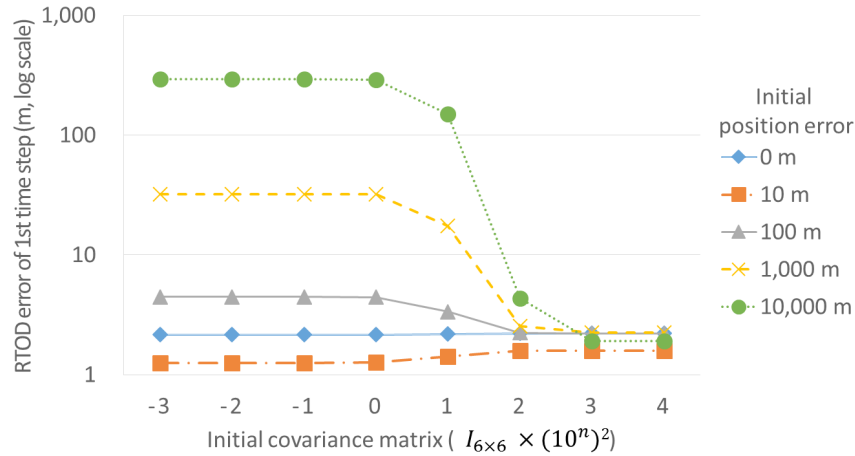


Fig. 2. Magnitude of position error at the 1<sup>st</sup> time step vs. initial covariance matrix (EIGSO 1).

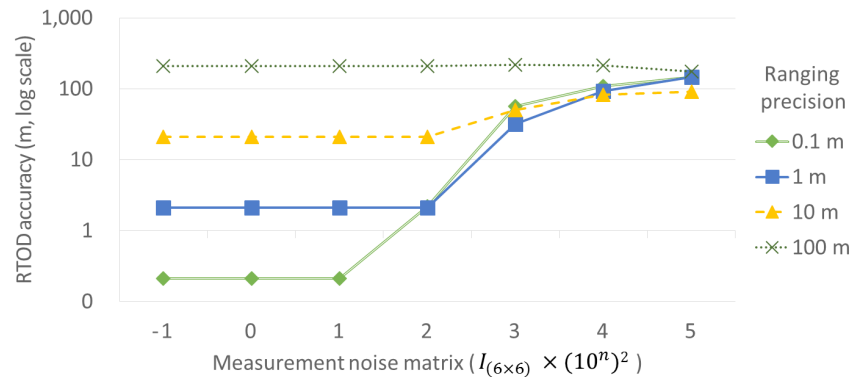


Fig. 3. RTOD accuracy vs. measurement noise matrix (EIGSO 1).

The covariance matrix in EKF physically represents the magnitude of the errors included in the state vector. It is used mainly for calculating the Kalman gain (Zarchan & Musoff 2000). A large magnitude of the covariance matrix implies large errors in the estimated state vector. When estimating the state vector at the subsequent time step, a relatively bigger covariance matrix (bigger than the ranging accuracy) yields a larger Kalman gain, and the observational data become more influential on the state vectors of the navigation satellites. While the covariance matrix is updated automatically in the EKF algorithm, its initial value (the initial covariance matrix) must be selected carefully by the users. To analyze the sensitivity, the initial covariance matrix is tuned while the ranging accuracy is fixed at 1 m. Fig. 2 shows the strong relation between the RTOD error at the initial time step and the initial covariance matrix. When the order of the initial covariance matrix component is equal to or is one degree smaller than that of the initial position error, the position converges at the first time step, which is consistent with its physical meaning. Fig. 2 also confirms that if the diagonal elements of the initial covariance matrix

are  $(10^3)^2$ , the positional error at the first time step converges to a constant value, regardless of any initial error.

The measurement noise matrix represents the magnitude of the error included in the observational data. It is used principally for computing the Kalman gain (Zarchan & Musoff 2000). The physical characteristics of the matrix are related to the weights of the observational model. In an attempt to establish an appropriate measurement noise matrix, the ranging precision is varied from 0.1–100 m, while the total observation time is fixed at 25 hr. In Fig. 3, when the order of the measurement noise matrix component is 1–2 orders higher than that of the ranging precision, the RTOD accuracy represents the best accuracy stably, which also reflects its physical meaning. As a result, the measurement noise matrix is determined using the square of ranging precision.

The process noise matrix considers the unexpected errors of the developed system and the omitted term of the equations of motion. When computing the covariance matrix, a larger process noise matrix yields a bigger covariance matrix and eventually, the Kalman gain becomes

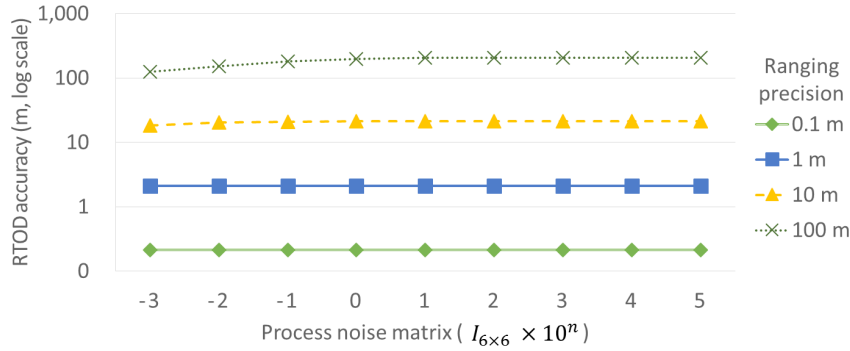


Fig. 4. RTOD accuracy vs. process noise matrix (EIGSO 1).

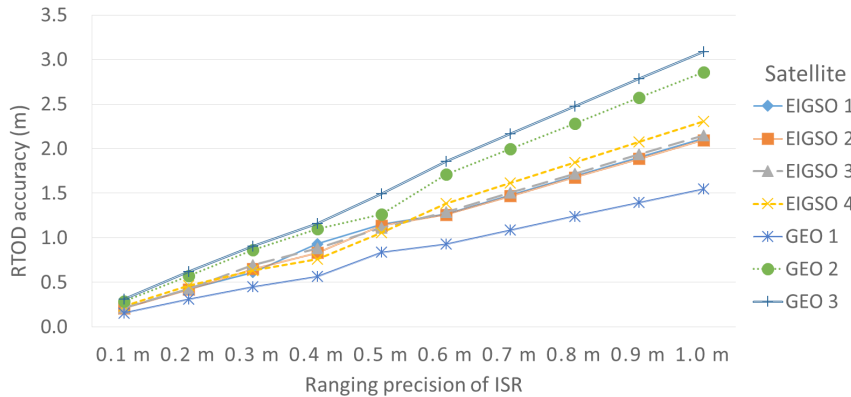


Fig. 5. RTOD accuracy vs. ranging precision.

greater (Zarchan & Musoff 2000). As this paper considers simulations using virtual data, it is not simple to determine the process noise matrix. In this work, simulations are performed with a specific range of process noise matrices that do not lead to a singularity of the EKF. In the simulation process of determining the process noise matrix, the ranging precision is varied from 0.1–100 m, while the total observation time is fixed at 25 hr. In Fig. 4, when the magnitude of the process noise matrix component is the same as or is bigger than the ranging precision, the RTOD accuracy is worst. In order to consider the worst-case scenario and to obtain realistic results, the component of process noise matrix is determined empirically using by the same magnitude of ranging precision.

#### 4.1 OD Results

##### 4.2.1 OD Accuracy with Ranging Precision

Simulations are performed to identify the requirements necessary to achieve meter-level accuracy of RTOD. Fig. 5 presents the RTOD accuracy of the KRNSS satellites versus ranging precision, which shows that ranging precision of 0.3–0.7 m is essential to achieve 1-meter-level of RTOD

accuracies. The linearity of RTOD accuracy with regard to the ranging precision confirms their strong relation. While the RTOD accuracy of the GEO 1 satellite, located at the center of the KRNSS satellite group, shows the best accuracy, the GEO 2 and 3 satellites, located in the outermost positions, show the worst; the EIGSO satellites maintain similar accuracies to each other. These results indicate there should be a strong relationship between RTOD accuracy and the geometrical configuration of the satellites.

##### 4.2.2 OD Accuracy and PDOP

In an effort to scrutinize the relationship between RTOD accuracy and the geometrical configuration of the satellites, the Dilution Of Precision (DOP), which is an indicator of the geometrical configuration, is calculated (Hofmann-Wellenhof et al. 2001). In an SNS, the numeral 6 represents good DOP. Generally, the DOP is calculated based on the positions of the navigation satellites with respect to the location of the service receiver. However, here, using ISR to perform the RTOD, the DOP of a particular satellite is calculated based on the positions of the other navigation satellites. This DOP is called the inversed GNSS DOP (Choi et al. 2014). As the clock error is not considered separately in the simulation, the positional

**Table 3.** PDOP and RTOD accuracy by ranging precision

Satellites	Ranging Precision			PDOP
	0.100 m	1.000 m	10.000 m	
	OD Accuracy	OD Accuracy	OD Accuracy	
EIGSO 1	0.211 m	2.116 m	21.153 m	2.120
EIGSO 2	0.210 m	2.096 m	20.957 m	2.122
EIGSO 3	0.214 m	2.146 m	21.455 m	2.119
EIGSO 4	0.230 m	2.304 m	23.047 m	2.127
GEO 1	0.154 m	1.548 m	15.491 m	1.543
GEO 2	0.285 m	2.857 m	28.560 m	2.834
GEO 3	0.309 m	3.091 m	30.918 m	3.100

DOP (PDOP) is computed instead of the geometrical DOP. The PDOP value of each satellite is calculated using the RMS method over the entire orbital period.

It is worth noting that the values of PDOP multiplied by the ranging precision in Table 3 are very close to the RTOD accuracies. This is consistent with the results of Choi et al. (2014), showing that the positional accuracy can be obtained by the geometrical DOP multiplied by the observational precision between the satellite and receiver in the SNS. Conversely, when the magnitude of the PDOP is <6 (at least) and it maintains nearly the same magnitude without large variation, the RTOD accuracy can be estimated approximately through the PDOP, which could be used when designing orbital phases of an SNS.

4.2.3 Initial Positional Error of KRNSS Satellites

The initial positions of satellites are often obtained by the Two-Line Element (TLE) sets provided by the Joint Space Operations Center (JSpOC). However, TLEs contain significant errors that can be up to 10-30 km for GEO satellites (Kim et al. 2010). To analyze the effects of initial positional errors, it is varied between 0-10 km, while the ranging accuracy is fixed at 1 m because this study adopts 0.45 m ranging precision of the ISR equipment of GPS IIR

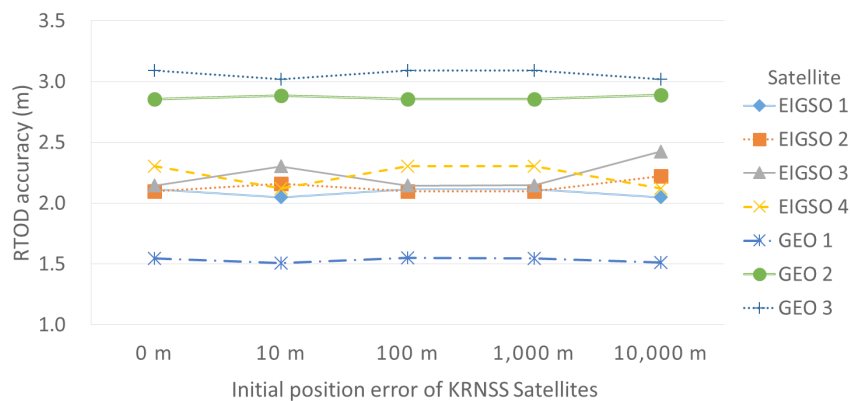
(Xu et al. 2012). Fig. 6 shows there is no significant change in RTOD accuracy regardless of the variation of initial positional errors. This is because the initial covariance matrix is selected appropriately and the ranging precision is highly accurate compared with the initial positional errors.

4.2.4 OD Accuracy and Limitation on Relative Ranging

As relative ranging between satellites is the only observational data in the current analysis, it is significant to understand the effects of the variation of the relative distance between satellites. During one orbital period, the minimum relative distance between the EIGSO 2 and 3 satellites is about 3,100 km, and the maximum relative distance between the GEO 2 and 3 satellites is about 64,600 km. Hence, the KRNSS must be designed to be capable of covering a range of about 65,000 km. According to Toyoshima (2005), the current ISR technology can measure the distance between the Earth and the Moon. If the maximum observational range is restricted, the number of observational data decreases and consequently, the RTOD accuracy reduces. Fig. 7 shows the distribution of observational data with respect to the relative distance between satellites during one orbital period. It can be seen in Fig. 7 that a ranging capability of at least 65,000 km is required to maintain the meter-level accuracy of RTOD.

4.2.5 OD Accuracy and Observation Time Interval

All the simulations have been conducted with 10 min observational time intervals. Here, the observational time interval is varied from 1-60 min in order to study its relation with RTOD accuracy. While Fig. 8 shows no consistent tendency between RTOD accuracy against observational time interval, the fluctuation of the RTOD accuracies does increase when the observational time interval is >20 min.



**Fig. 6.** RTOD accuracy vs. initial positional error.

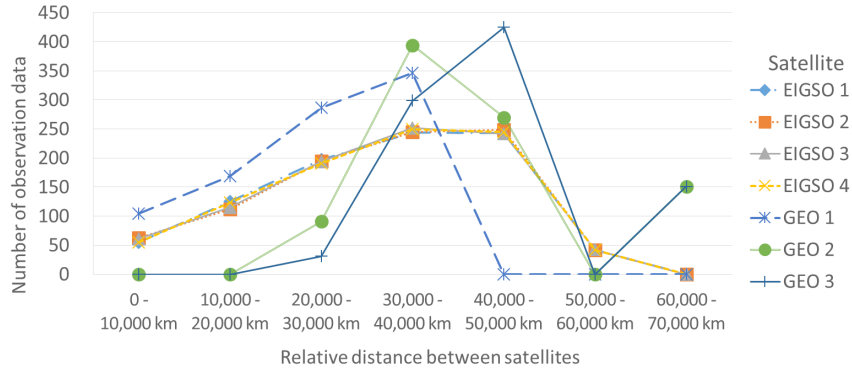


Fig. 7. Relative range distribution of observational data.

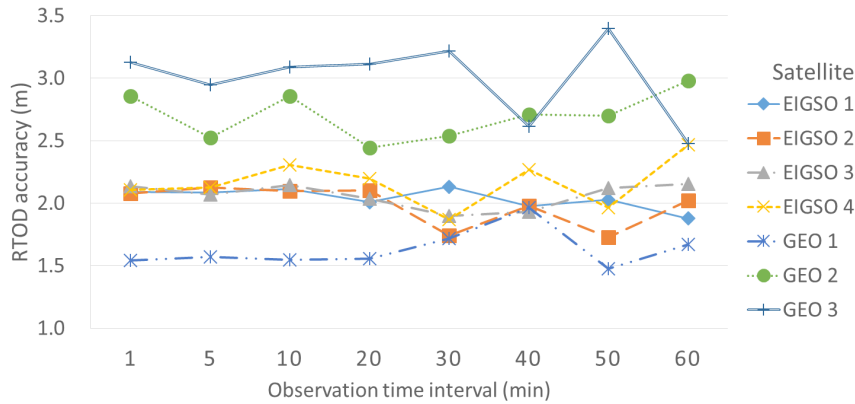


Fig. 8. RTOD accuracy vs. observational time interval.

Hence, the observational time interval of the KRNS should be <20 min for stable RTOD accuracy.

time interval of <20 min are also required to maintain the required RTOD accuracy.

### 5. CONCLUSIONS

RTOD has been conducted to produce ephemeris data for KRNS navigation satellites under the assumption of an abnormal situation in which communication between the navigation satellites and the ground station is unavailable. Relative distance ranging between the satellites using ISR was adopted, and the EKF algorithm was employed to investigate the requirements necessary to achieve meter-level accuracy of RTOD. The fine-tuning process was first performed for the key matrices of the EKF (initial covariance matrix, measurement noise matrix, and process noise matrix) based on their physical meanings. According to the simulations, ranging precision of 0.3–0.7 m is required to obtain RTOD accuracy to within 1 m. The relevance between the geometrical configuration (DOP) and RTOD accuracy makes it possible to estimate the RTOD accuracy from the DOP (without actually performing RTOD). A ranging capability of at least 65,000 km and an observational

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