Maximum Likelihood SNR Estimation for QAM Signals Over Slow Flat Fading Rayleigh Channel

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Abstract

Estimation of signal-to-noise ratio (SNR) is an important problem in wireless communication systems. It has been studied for various constellation types and channels using different estimation techniques. Maximum likelihood estimation is a technique which provides efficient and in most cases unbiased estimators. In this paper, we have applied maximum likelihood estimation for systems employing square or cross QAM signals which are undergoing slow flat Rayleigh fading. The problem has been considered under various scenarios like data-aided (DA), non-data-aided (NDA) and partially data-aided (PDA) and the performance of each type of estimator has been evaluated and compared. It has been observed that the performance of DA estimator is best due to usage of pilot symbols, with the drawback of greater bandwidth consumption. However, this can be catered for by using partially data-aided estimators whose performance is better than NDA systems with some extra bandwidth requirement.

Keywords: SNR estimation, maximum likelihood, QAM, Rayleigh fading

1. Introduction

In most wireless communication systems, the knowledge of signal strength and consequently Signal-to-Noise Ratio (SNR) is required. This knowledge helps in many wireless applications and is generally unknown at the receiver end. Since wireless communication is of vital importance in the present communication industry, therefore SNR estimation is a widely researched area. From signal detection to cooperative networks, mobile communication to Adaptive Modulation and Coding (AMC) or link adaptation, and from cognitive radios to turbo decoding, applications of SNR estimation can be found in all. Let us consider in detail mobile communication systems, where handoffs are a very important phenomenon required when the user is mobile and the base station connected with the user cannot provide sufficient signal strength to maintain uninterrupted service. In such a case, a threshold is assigned to the signal strength which is continuously monitored and if the signal strength is in danger of falling below the threshold, the user is handed off to a different base station to avoid disconnection. This is a crucial task which needs to be performed at high speed to avoid interruption in the cellular service provided to the user, emphasizing the requirement of rapid and accurate estimation of the SNR.

Many techniques exist for estimation, among which some of the most well-known techniques include Maximum Likelihood (ML) estimation, Method of Moments (MM) and Decision-Directed (DD) method. Each of the techniques has its own merits, but the maximum likelihood approach has been selected in this paper due to its ability to achieve approximately optimal performance. ML estimation of Frequency Shift Keying (FSK) signals has been studied extensively for Rayleigh fading environment in [1-4] and for Rician channel in [5]. For the AWGN channel, a comparison of various SNR estimation schemes applied on Phase Shift Keying (PSK) signals has been presented in [6-7]. SNR estimation for PSK signals using maximum likelihood estimation has been presented for AWGN channel [8-10] and fading environments [11-12]. However, for Quadrature Amplitude Modulation (QAM), SNR estimation has been done mostly using method of moments for AWGN and fading channels [13-16]. Using ML estimation for QAM signals, SNR with additive interference, called the Signal-to-Interference Noise Ratio (SINR) has been estimated [17-18].

Since maximum likelihood estimation results in an efficient estimator for most practical cases [22], we have applied this estimation technique in this research. Communication environment consists of QAM signals transmitted through a flat fading Rayleigh environment. Estimation has been done for various scenarios depending on the type of signal packet used, namely Data-Aided (DA) or pilot-aided, Non-Data-Aided (NDA) or non-pilot-aided and Partially Data-Aided (PDA) or partially pilot-aided cases, which differ from each other in terms of the signal packet used for estimation, causing varied bandwidth consumption. The DA estimation uses a signal packet consisting only of pilot symbols, NDA scenario uses only data symbols for estimation, while PDA uses a combination of both pilot and data symbols to compromise between performance and bandwidth efficiency. Estimator performance has been evaluated using a commonly used bound on the variance of an estimator, called the Cramer-Rao Lower Bound (CRLB) [19-20]. The CRLB being a lower bound, states that the variance of any practical estimator cannot be lower than this bound, thus providing a useful means to assess the performance of designed estimators.

Paper organization has been done so that system model is described in Section 2, while Section 3 details the ML estimation procedure for DA, NDA and PDA cases. Section 4 deals with the derivation of CRLB, and performance evaluation has been discussed in Section 5. Section 6 concludes the paper.

2. System Model

As described in Section 1, we have considered a wireless communication system [23] which employs QAM signals that are transmitted through a flat fading Rayleigh environment. Noise in the form of AWGN is added to the signal and SNR is estimated at the end of the receiver's detection module, using the signal obtained at the output of the matched filter. The generalized baseband equivalent system model is shown in **Fig. 1**. When frequency flat Rayliegh fading is considered, the fading can be modeled in time domain in the form of a multiplicative gain. Here, we are considering that the fading co-efficient is varying slowly in time such that it can be considered to be constant for the range of a signal packet, which is known as slow flat fading and the signal is corrupted by complex noise in the form of AWGN. As discussed in [7], the discrete received signal can be given as:

$$r_i = \sqrt{\alpha} m_i + \sqrt{N} n_i \tag{1}$$

where r_i is the i^{th} received signal sample, m_i and n_i are the i^{th} samples of message signal and complex AWGN respectively. The coefficient of Rayleigh fading, which is considered to be same for an entire packet length due to slow fading, is given by α which is modeled by a complex Gaussian random variable. The noise has been modeled as zero mean Gaussian random variables with an overall variance of N. The power of transmitted signal is considered to be close to unity and embedded in the fading coefficient α. The fading coefficient is interacting with the transmitted signal in a multiplicative way, so it will induce a phase offset in the received signal. It has been assumed that this phase offset is already compensated by the receiver before estimating the SNR. The SNR can be defined for this case as the ratio of fading power to noise power, as:

$$\gamma = \frac{\alpha}{N} \tag{2}$$

where γ is used to represent the SNR, which is to be estimated.

The estimators have been designed for three types of received signal packets, we have considered the total packet length to be k symbols so that the packet may be described as $\boldsymbol{r} = [r_1 \ r_2 \ r_3 \ldots \ r_k]$ and the symbols may be pilot or data symbols, depending on the bandwidth requirement of the communication system.

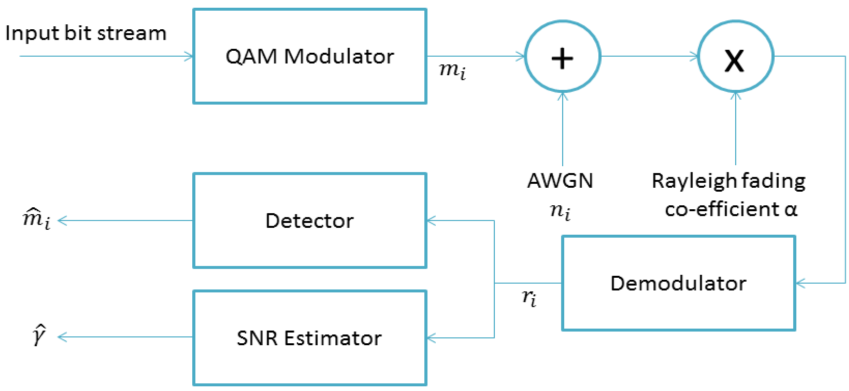

Fig. 1. Baseband equivalent block diagram of a communication system with slow-flat Rayleigh fading

3. Maximum Likelihood Estimation

In this section we have presented details of the data-aided (DA), non-data-aided (NDA) and partially data-aided (PDA) ML estimators. An important property of ML estimation is applied here, which allows the SNR to be estimated using the estimates of the individual parameters that define the SNR. So, rather than trying to estimate the SNR as a whole, we will find the ML estimates of the fading coefficient α and the noise variance N, which gives the ML estimate of SNR as:

$$\hat{\gamma}_{ML} = \frac{\hat{\alpha}_{ML}}{\hat{N}_{ML}} \tag{3}$$

where $\hat{\gamma}_{ML}$, $\hat{\alpha}_{ML}$ and $\hat{N}_{ML}$ are the ML estimates of SNR γ, Rayleigh fading coefficient α and the noise variance N, respectively.

The received signal and all of its components are complex valued, so for ease we may write the in-phase and quadrature components of the i^{th} received sample as:

$$r_{i_I} = \sqrt{\alpha} m_{i_I} + \sqrt{N} n_{i_I} \tag{4}$$

$$r_{i_Q} = \sqrt{\alpha} m_{i_Q} + \sqrt{N} n_{i_Q} \tag{5}$$

where I and Q subscripts represent in-phase and quadrature respectively.

The first thing required in order to find ML estimators is the knowledge of probability density function (PDF). The PDF of noise is Gaussian which can be represented for i^{th} sample as:

$$p(n_i) = p\left(n_{i_I}, n_{i_Q}\right) = \frac{1}{\pi N} exp\left(\frac{-({n_{i_I}}^2 + {n_{i_Q}}^2)}{N}\right) \tag{6}$$

where $p(.)$ represents the probability function and $\exp(.)$ is the exponential function.

The PDF of in-phase and quadrature components of noise, which have been defined in Eqs. (4) and (5) respectively, can therefore be written as:

$$p(r_{i_I}) = \frac{1}{\sqrt{\pi N}} \exp\left(-\frac{\left(r_{i_I} - \sqrt{\alpha} m_{i_I}\right)^2}{N}\right) \tag{7}$$

$$p\left(r_{i_Q}\right) = \frac{1}{\sqrt{\pi N}} \exp\left(-\frac{\left(r_{i_Q} - \sqrt{\alpha} m_{i_Q}\right)^2}{N}\right) \tag{8}$$

The joint PDF of in-phase and quadrature components of r_i can be written as:

$$p(r_i) = p\left(r_{i_I}, r_{i_Q}\right) = \frac{1}{\pi N} \exp\left(-\frac{\left(r_{i_I} - \sqrt{\alpha} m_{i_I}\right)^2 + \left(r_{i_Q} - \sqrt{\alpha} m_{i_Q}\right)^2}{N}\right) \tag{9}$$

Using the PDF described above, we will derive the DA, NDA and PDA estimators for SNR in separate subsections.

3.1 Data-Aided Estimator

The data-aided estimation uses only pilot symbols for the estimation procedure, which provides the most efficient results possible using ML estimation technique, with the only drawback being higher bandwidth consumption. However, since many communication systems involve some pilot symbol transmission for other purposes, we can use the pilot symbol transmission to our aid to get better estimation of SNR. For the data-aided case, we have set the total packet length described in Section 2 to be $k = g$ symbols in order to avoid confusion with other estimation procedures. We can modify Eq. (9) to get the joint PDF for complete received signal packet of length g as:

$$p(\boldsymbol{r_I}, \boldsymbol{r_Q}) = \prod_{i=1}^{g} p\left(r_{i_I}, r_{i_Q}\right) \tag{10}$$

$$= \frac{1}{(\pi N)^g} \exp\left[\frac{-1}{N}\left(\sum_{i=1}^{g}\left(r_{i_I} - \sqrt{\alpha} m_{i_I}\right)^2 + \sum_{i=1}^{g}\left(r_{i_Q} - \sqrt{\alpha} m_{i_Q}\right)^2\right)\right]$$

where $\boldsymbol{r_I}$ and $\boldsymbol{r_Q}$ are the in-phase and quadrature components of the received signal packet $\boldsymbol{r}$.

The next step in finding an ML estimator is to find the log-likelihood function, which is the log of the complete signal PDF, given in Eq. (10). It can be written as:

$$\Lambda_{DA} = \ln p(\boldsymbol{r_I}, \boldsymbol{r_Q}) = -g \ln(\pi N) - \frac{1}{N}\left[\sum_{i=1}^{g}\left(r_{i_I} - \sqrt{\alpha} m_{i_I}\right)^2 + \sum_{i=1}^{g}\left(r_{i_Q} - \sqrt{\alpha} m_{i_Q}\right)^2\right] \tag{11}$$

where Λ_{DA} is the log-likelihood function for the data-aided case.

The maximum likelihood estimates of the unknown parameters α and N can be found by partial differentiation of the log-likelihood function with respect to the unknown parameter and maximizing the result. Thus we get the ML estimates as:

$$\hat{\alpha}_{DA} = \left[\frac{\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right)}{\sum_{i=1}^{g}\left({m_{i_I}}^2 + {m_{i_Q}}^2\right)}\right]^2 \tag{12}$$

$$\hat{N}_{DA} = \frac{1}{g}\sum_{i=1}^{g}\left({r_{i_I}}^2 + {r_{i_Q}}^2\right) - \frac{\left[\frac{1}{g}\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right)\right]^2}{\frac{1}{g}\sum_{i=1}^{g}\left({m_{i_I}}^2 + {m_{i_Q}}^2\right)} \tag{13}$$

where $\hat{\alpha}_{DA}$ and $\hat{N}_{DA}$ are the ML estimates of α and N respectively for the DA case.

We can use the average energy concepts for square and cross QAM signals discussed in [21] to simplify the above expressions as:

$$\hat{\alpha}_{DA} = \frac{\left[\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right)\right]^2}{\varepsilon_{av}^2} \tag{14}$$

$$\hat{N}_{DA} = \frac{1}{g}\sum_{i=1}^{g}|r_i|^2 - \frac{1}{\varepsilon_{av}}\left[\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right)\right]^2 \tag{15}$$

Using the results obtained in Eqs. (14) and (15) in Eq. (3) gives us the final form of the DA estimator as:

$$\hat{\gamma}_{DA} = \frac{\hat{\alpha}_{DA}}{\hat{N}_{DA}} = \frac{\left[\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right)\right]^2}{\frac{\varepsilon_{av}^2}{g}\sum_{i=1}^{g}|r_i|^2 - \varepsilon_{av}\left[\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right)\right]^2} \tag{16}$$

3.2 Non-Data-Aided Estimator

The non-data-aided estimator or NDA estimator has been designed for low bandwidth systems or systems with a data rate constraint. In this case, we use only the data symbols to estimate the SNR without any prior knowledge of the transmitted data. When required, we use the estimated version of tranmitted data found by the receiver's detection module. Due of the inevitable detection errors at the receiver end, the performance of this type of estimation is bound to be less accurate than the DA estimation which uses exact information. For this case, we denote the packet length by $k = l$ symbols, and the estimated message signal samples are given by $\widehat{m}_i$. The PDF of received signal packet is given by:

$$p(\boldsymbol{r_I}, \boldsymbol{r_Q}) = \prod_{i=1}^{l} p\left(r_{i_I}, r_{i_Q}\right)$$
$$= \frac{1}{(\pi N)^l} \exp\left[\frac{-1}{N}\left(\sum_{i=1}^{l}\left(r_{i_I} - \sqrt{\alpha}\widehat{m}_{i_I}\right)^2 + \sum_{i=1}^{l}\left(r_{i_Q} - \sqrt{\alpha}\widehat{m}_{i_Q}\right)^2\right)\right] \tag{17}$$

where $\widehat{m}_{i_I}$ and $\widehat{m}_{i_Q}$ are the in-phase and quadrature components of the detected data.

The log-likelihood function can be found to be:

$$\Lambda_{NDA} = \ln p(\boldsymbol{r_I}, \boldsymbol{r_Q}) = -l\ln(\pi N) - \frac{1}{N}\left[\sum_{i=1}^{l}\left(r_{i_I} - \sqrt{\alpha}\widehat{m}_{i_I}\right)^2 + \sum_{i=1}^{l}\left(r_{i_Q} - \sqrt{\alpha}\widehat{m}_{i_Q}\right)^2\right] \tag{18}$$

The maximization of partial differentials of Eq. (18) with respect to the unknown parameters gives us the ML estimates as:

$$\hat{\alpha}_{NDA} = \left[\frac{\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)}{\sum_{i=1}^{l}\left(\widehat{m}_{i_I}{}^2 + \widehat{m}_{i_Q}{}^2\right)}\right]^2 \tag{19}$$

$$\widehat{N}_{NDA} = \frac{1}{l}\sum_{i=1}^{l}\left(r_{i_I}{}^2 + r_{i_Q}{}^2\right) - \frac{\left[\frac{1}{l}\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)\right]^2}{\frac{1}{l}\sum_{i=1}^{l}\left(\widehat{m}_{i_I}{}^2 + \widehat{m}_{i_Q}{}^2\right)} \tag{20}$$

Using the concept of average energy as done for the DA case, we get the final form of NDA estimator using Eq. (3) as:

$$\hat{\gamma}_{NDA} = \frac{\hat{\alpha}_{NDA}}{\widehat{N}_{NDA}} = \frac{\left[\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)\right]^2}{\frac{\varepsilon_{av}^2}{l}\sum_{i=1}^{l}|r_i|^2 - \varepsilon_{av}\left[\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)\right]^2} \tag{21}$$

3.3 Partially Data-Aided Estimator

We can combine the two types of estimators designed previously to get a hybrid form which is named partially data-aided or PDA estimator. This estimator combines the characteristics of DA and NDA estimators such that the signal packet used for estimation contains both pilot and data symbols. It is advantageous to do so as the bandwidth constraint of a system can be checked by using data symbols and the use of pilot symbols improves the estimation. The received signal packet is constructed from a combination of g pilot symbols and l data symbols such that the total packet length is $k = g + l$. The received signal packet is given as:

$$\boldsymbol{r} = [r_1 \quad r_2 \quad \dots \quad r_g \quad r_{g+1} \quad r_{g+2} \quad \dots \quad r_{g+l}] \tag{22}$$

Since the PDA estimator is a combination of DA and NDA estimators, the PDF of the received signal for PDA estimator can be found from the PDFs given in Eqs. (10) and (17) as:

$$p(\boldsymbol{r_I}, \boldsymbol{r_Q}) = \prod_{i=1}^{g} p\left(r_{i_I}, r_{i_Q}\right) \prod_{i=1}^{l} p\left(r_{i_I}, r_{i_Q}\right)$$

$$= \frac{1}{(\pi N)^{g+l}} \exp\left[\frac{-1}{N}\left(\sum_{i=1}^{g}\left(r_{i_I} - \sqrt{\alpha} m_{i_I}\right)^2 + \sum_{i=1}^{g}\left(r_{i_Q} - \sqrt{\alpha} m_{i_Q}\right)^2 + \sum_{i=1}^{l}\left(r_{i_I} - \sqrt{\alpha}\widehat{m}_{i_I}\right)^2 + \sum_{i=1}^{l}\left(r_{i_Q} - \sqrt{\alpha}\widehat{m}_{i_Q}\right)^2\right)\right] \quad (23)$$

The log-likelihood function can be found from the above equation as:

$$\Lambda_{PDA} = \ln p(\boldsymbol{r_I}, \boldsymbol{r_Q}) = -g \ln(\pi N) - l \ln(\pi N) - \frac{1}{N}\left[\sum_{i=1}^{g}\left(r_{i_I} - \sqrt{\alpha} m_{i_I}\right)^2 + \sum_{i=1}^{g}\left(r_{i_Q} - \sqrt{\alpha} m_{i_Q}\right)^2 + \sum_{i=1}^{l}\left(r_{i_I} - \sqrt{\alpha}\widehat{m}_{i_I}\right)^2 + \sum_{i=1}^{l}\left(r_{i_Q} - \sqrt{\alpha}\widehat{m}_{i_Q}\right)^2\right] \quad (24)$$

By differentiating with respect to unknown parameter and maximizing, the estimates of α and N can be found as:

$$\hat{\alpha}_{PDA} = \left[\frac{\frac{1}{g}\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right) + \frac{1}{l}\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)}{\frac{1}{g}\sum_{i=1}^{g}\left({m_{i_I}}^2 + {m_{i_Q}}^2\right) + \frac{1}{l}\sum_{i=1}^{l}\left({\widehat{m}_{i_I}}^2 + {\widehat{m}_{i_Q}}^2\right)}\right]^2 \quad (25)$$

$$\widehat{N}_{PDA} = \frac{1}{g}\sum_{i=1}^{g}\left({r_{i_I}}^2 + {r_{i_Q}}^2\right) + \frac{1}{l}\sum_{i=1}^{l}\left({r_{i_I}}^2 + {r_{i_Q}}^2\right) - \frac{1}{\varepsilon_{av}^2}\left[\frac{1}{g}\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right) + \frac{1}{l}\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)\right]^2 \quad (26)$$

Using the concept of average energy and Eqs. (25) and (26) we can write the final form of PDA ML estimator for SNR as:

$$\hat{\gamma}_{PDA} = \frac{\hat{\alpha}_{PDA}}{\widehat{N}_{PDA}} = \frac{\left[\frac{1}{g}\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right) + \frac{1}{l}\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)\right]^2}{\varepsilon_{av}^2\left[\frac{1}{g}\sum_{i=1}^{g}|r_i|^2 + \frac{1}{l}\sum_{i=1}^{l}|r_i|^2\right] - \varepsilon_{av}\left[\frac{1}{g}\sum_{i=1}^{g}\left(r_{i_I} m_{i_I} + r_{i_Q} m_{i_Q}\right) + \frac{1}{l}\sum_{i=1}^{l}\left(r_{i_I}\widehat{m}_{i_I} + r_{i_Q}\widehat{m}_{i_Q}\right)\right]^2} \quad (27)$$

4. Cramer-Rao Lower Bound

In order to verify the performance of an estimator, its estimation error or estimation variance needs to be compared with some bound. Cramer-Rao Lower Bound (CRLB) is one such bound on the variance of an unbiased estimator. When an estimator does not have any bias, the lowest possible variance it can achieve is given by the CRLB. We have evaluated the performance of

DA, NDA and PDA estimators by plotting the normalized mean square error (NMSE), so in this section we will find CRLB in terms of NMSE. CRLB has been derived for the DA case, which can be used as a measure for the NDA and PDA cases also. The bound on the variance of an estimator is given in [22] as:

$$CRLB = var(\theta) \geq \frac{\partial f(\theta)}{\partial \theta} I^{-1}(\theta) \frac{\partial f(\theta)}{\partial \theta}^{T} \tag{28}$$

where $var(.)$ represents the variace, θ represents the unknown parameter which is to be estimated, in our case $\theta = \gamma$ and $I(\theta)$ is the Fisher Information matrix. The function $f(\theta)$ represents the transformation function used to estimate the unknown parameter, which in our case is given by:

$$f(\theta) = f(\gamma) = \frac{\alpha}{N} \tag{29}$$

The partial derivative given in Eq. (28) has been found to be:

$$\frac{\partial f(\gamma)}{\partial \alpha} \frac{\partial f(\gamma)}{\partial N} = \begin{bmatrix} \frac{1}{N} & \frac{-\alpha}{N^2} \end{bmatrix} \tag{30}$$

The Fisher Information matrix is given by:

$$I(\gamma) = \begin{bmatrix} -E\left[\frac{\partial^2 \Lambda}{\partial \alpha^2}\right] & -E\left[\frac{\partial^2 \Lambda}{\partial \alpha \partial N}\right] \\ -E\left[\frac{\partial^2 \Lambda}{\partial \alpha \partial N}\right] & -E\left[\frac{\partial^2 \Lambda}{\partial N^2}\right] \end{bmatrix} \tag{31}$$

where Λ is the log-likelihood function and $E[.]$ denotes the expectation operation.

In order to find the Fisher Information matrix, we find the partial derivatives in Eq. (31) by using the log-likelihood function given in Eq. (11) and then the expected values are found as:

$$E\left[\frac{\partial^2 \Lambda}{\partial \alpha^2}\right] = \frac{-k\varepsilon_{av}}{2N\alpha} \tag{32}$$

$$E\left[\frac{\partial^2 \Lambda}{\partial \alpha \partial N}\right] = 0 \tag{33}$$

$$E\left[\frac{\partial^2 \Lambda}{\partial N^2}\right] = \frac{-k}{N^2} \tag{34}$$

The matrix is then given as:

$$I(\gamma) = \begin{bmatrix} \frac{k\varepsilon_{av}}{2N\alpha} & 0 \\ 0 & \frac{k}{N^2} \end{bmatrix} \tag{35}$$

Finding the inverse of the Fisher Information matrix and putting values in Eq. (28) gives us the CRLB in terms of variance as:

$$var\{\hat{\gamma}\} \geq \frac{2\gamma}{k\varepsilon_{av}} + \frac{\gamma^2}{k} \tag{36}$$

For an unbiased estimator, variance is equivalent to mean square error (MSE). Assuming prior to confirmation that the DA estimator is unbiased, we can get the bound for NMSE by normalizing the bound given in Eq. (36) as:

$$\frac{CRLB}{\gamma^2} = NMSE\{\hat{\gamma}\} \geq \frac{2}{k\gamma\varepsilon_{av}} + \frac{1}{k} \tag{37}$$

5. Performance Evaluation

In this section the performance of the DA, NDA and PDA ML SNR estimators has been analyzed for QAM signals undergoing Rayleigh fading. Two forms of performance measures have been used in this paper, namely the normalized mean square error (NMSE) and the normalized sample bias. The NMSE has been calculated as:

$$NMSE\ \{\hat{\gamma}\} = E[(\hat{\gamma} - \gamma)^2] \tag{38}$$

where $\hat{\gamma}$ is the estimated value of SNR and γ is the true value. The normalized sample bias is given as:

$$\frac{Bias\{\hat{\gamma}\}}{\gamma} = \mathrm{E}\left[\frac{\hat{\gamma} - \gamma}{\gamma}\right] \tag{39}$$

In computer simulations, the expectation operator has been treated as sample mean. We have calculated sample mean by performing $10{,}000$ trials of estimation while the total packet length k has been fixed to $1{,}000$. For the NMSE, the effect of varying constellation size, i.e. M has been considered. The bias of the estimators has been plotted for various constellation sizes. A comparison of the performance of the three types of estimators has also been carried out. The performance of each type of estimator has been discussed in the following subsections.

5.1 DA Estimator Performance

The performance of DA estimator has been discussed in detail in this subsection. As discussed previously, this type of estimator has complete knowledge of the transmitted signal, which means this is not a bandwidth efficient method. However, the accuracy of estimation is greatly enhanced due to use of pilot symbols. The packet length has been fixed to $g = 1{,}000$ symbols for observing the NMSE and bias for various constellation sizes.

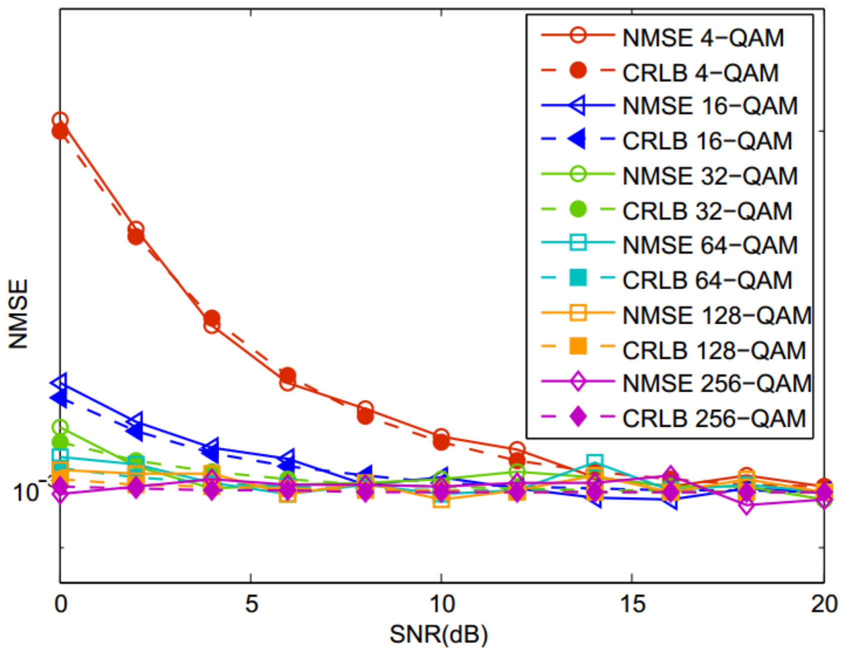

Fig. 2. NMSE and CRLB of DA estimator for square and cross M-QAM

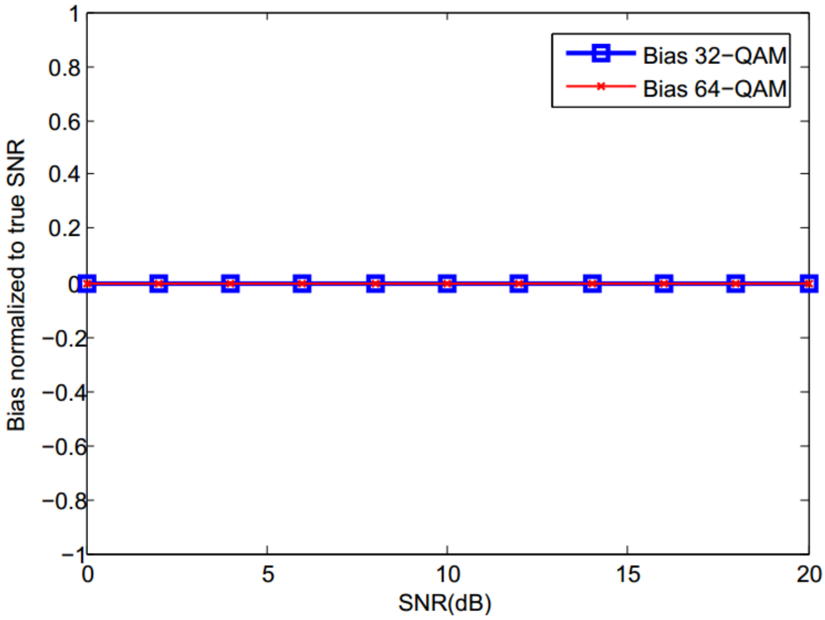

Fig. 3. Bias of DA estimator for square and cross QAM

Fig. 2 shows the NMSE and CRLB for various square and cross M-QAM. We have plotted the results for square QAM using $M = 4, 16, 64$ and 256 and for cross QAM using $M = 32$ and 128. It can be seen that all constellations approach the bound eventually as SNR is increased. However due to presence of fading the estimation error curve is not very smooth, specially for higher order QAM. The bias of DA estimator has been shown in **Fig. 3** for 32-QAM and 64-QAM. It can be seen that the DA estimator is unbiased as there is zero normalized bias for the entire range of SNR. The bias for DA estimators was seen to be zero for all values of constellation size M considered in this paper.

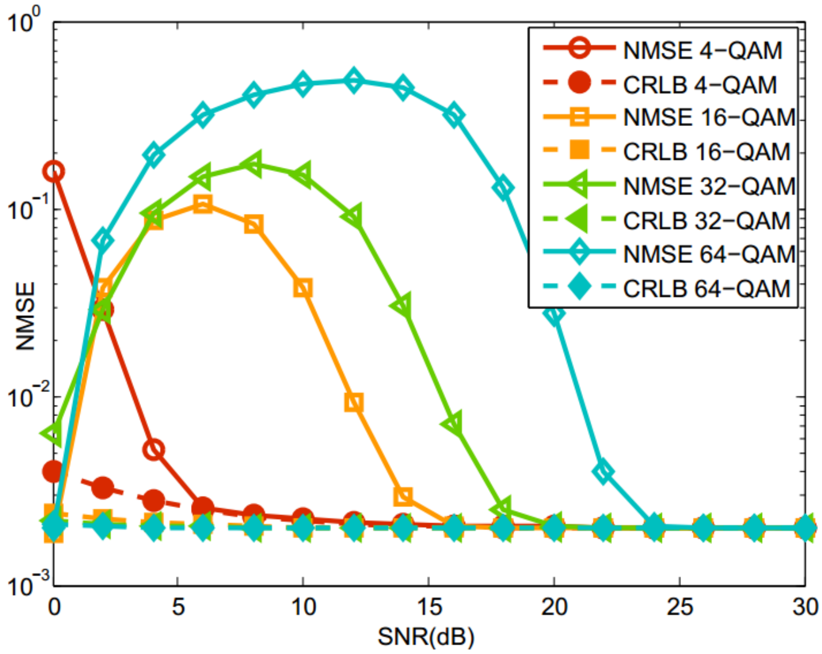

Fig. 4. NMSE and CRLB of NDA estimator for M-QAM

5.2 NDA Estimator Performance

The NDA estimator is designed to reduce the bandwidth consumption of the system. It uses only data symbols and estimated transmitted symbols (which include errors) for SNR estimation. Due to this, the performance of this estimator is drastically different from the DA estimator as fading causes more errors in estimation. Like the DA case, we have observed the effect of varying constellation size as well as the bias of NDA estimator for various M-QAM. The packet length l has been set to 1000 symbols for all cases.

The effect of varying constellation size for square and cross M-QAM has been observed using NMSE and CRLB in **Fig. 4** for QAM of order 4, 16, 32 and 64. The performance of higher order QAM has not been shown as the results were not as significant due to presence of fading and estimation errors. It can be seen from the figure that due to use of only data symbols, the performance of estimators becomes poor as the constellation size is increased because error probability increases. The bias for NDA estimator can be observed from **Fig. 5** and it can be seen that due to data symbols, the bias increases with increase in constellation size due to increased error probability.

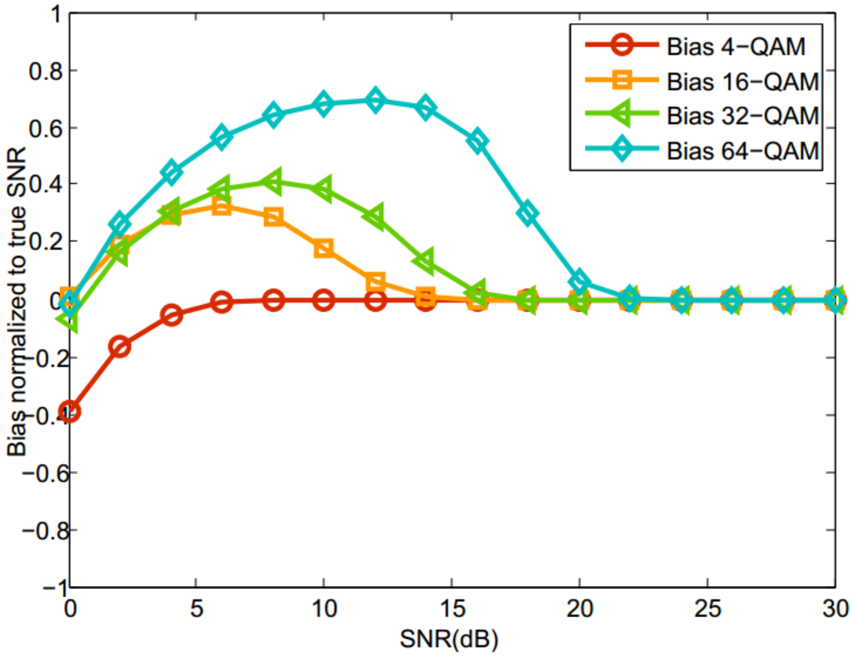

Fig. 5. Bias of NDA estimator for square and cross QAM

5.3 PDA Estimator Performance

The PDA estimator is a hybrid of the DA and NDA estimators. In this method, a percentage of the received signal packet used for estimation consists of pilot symbols while remaining are data symbols. This is done to improve the performance of the system while keeping it bandwidth efficient. We have fixed the total packet length k to 1000 symbols and started computer simulations using 10% data symbols and gradually increasing the percentage to 50%. The effect has been shown for 64-QAM using NMSE and CRLB in **Fig. 6** and bias has been plotted in **Fig. 7**. It can be seen that the performance deteriorates as the number of data symbols is increased. Both the figures show that the characteristics of PDA estimator approach those of DA estimator as SNR is increased. Typically around 6-8 dBs, the performance of PDA estimator approaches that of DA estimator for 64-QAM.

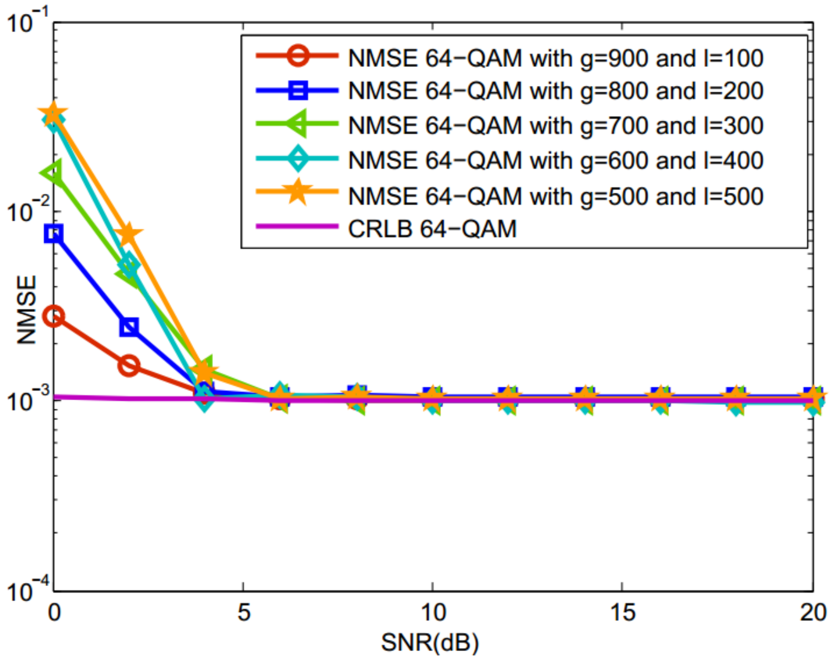

Fig. 6. NMSE and CRLB of PDA estimator for 64-QAM

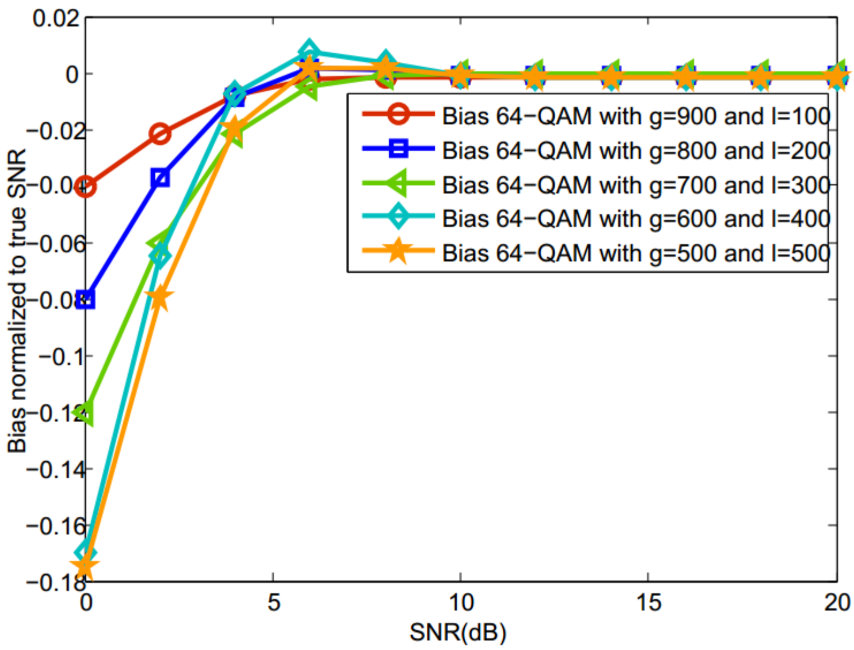

Fig. 7. Bias of PDA estimator for 64-QAM

5.4 Comparison of Estimator Performances

In order to ascertain the pros and cons of using the different types of estimators, we have compared the performances of the three estimators designed in this paper. The NMSE of the three estimators has been plotted for 64-QAM signals in **Fig. 8** along with the CRLB. It can be seen from these curves that the NMSE for DA estimator is lowest whereas NDA estimator has poor performance in the low SNR region due to fading and estimation errors. The bias of the three types of estimators has been plotted in **Fig. 9**. This figure shows that the bias is increased when number of data symbols is increased in the received packet, with the bias being maximized for all data symbols, i.e. NDA case. A comparison of various properties of the three estimator types has been given in **Table 1** for 64-QAM signals in Rayleigh channel.

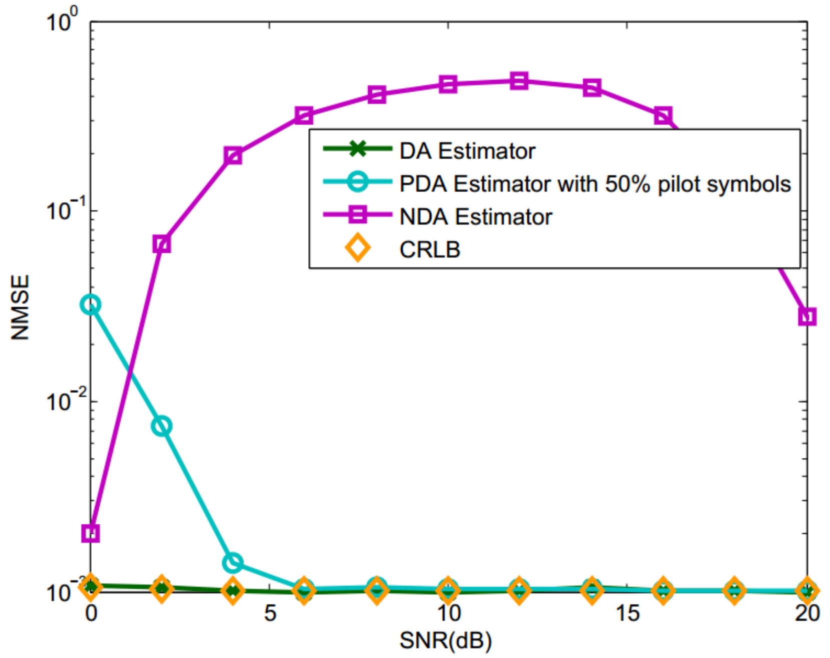

Fig. 8. Comparison of NMSE of DA, NDA and PDA estimators for 64-QAM

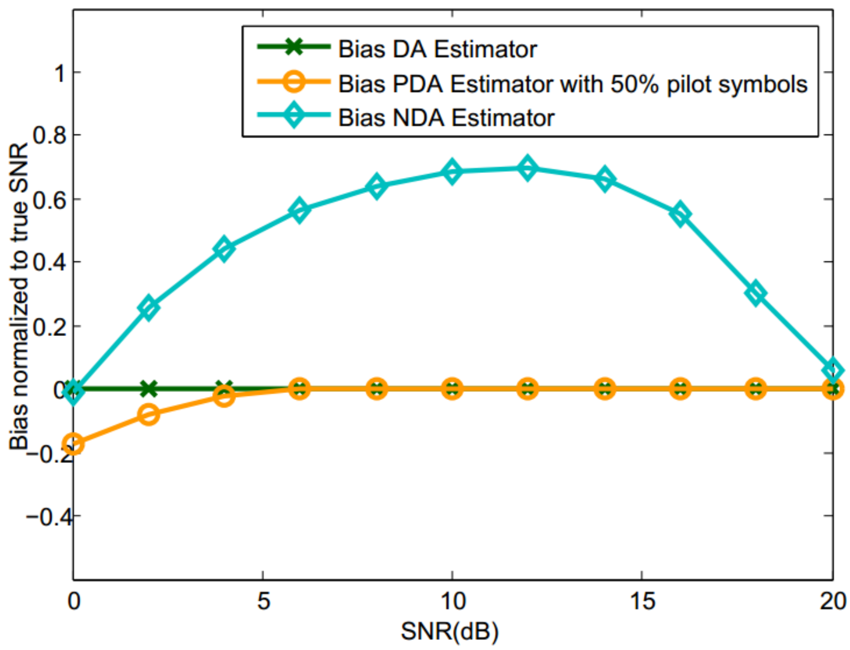

Fig. 9. Comparison of bias of DA, NDA and PDA estimators for 64-QAM

Table 1. Comparison of DA, NDA and PDA estimators for Rayleigh channel

Estimator Type	Type of received symbols used	Bias	Extra Bandwidth Requirement	Value of SNR for which estimator achieves CRLB
DA	Pilot symbols	Unbiased	Yes, same as packet length	0 dB
NDA	Data symbols	Around 0.6 for low to medium SNR range	No	25 dB
PDA	Pilot and data symbols	Bias around -0.2 for low SNR	Yes, depending on number of pilot symbols	6 dB

5. Conclusion

In this paper, we have considered the problem of SNR estimation for square and cross QAM signals in slow flat-fading Rayleigh environment. We have designed estimators using maximum likelihood estimation technique for data-aided, non-data-aided and partially data-aided scenarios. It has been observed that the estimators perform best in data-aided scenario, while PDA and NDA estimators show greater error and achieve CRLB for larger values of SNR because of the presence of fading and detection errors. However, the bandwidth consumption is highest for DA estimator and lowest for the NDA estimator. The bias of estimators has also been observed showing that DA estimator is unbiased while NDA and PDA estimators indicate presence of bias especially in regions of lower SNR, which is another cause of performance deterioration as compared to the DA estimator.

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