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INFLUENCE OF SLIP CONDITION ON RADIATIVE MHD FLOW OF A VISCOUS FLUID IN A PARALLEL POROUS PLATE CHANNEL IN PRESENCE OF HEAT ABSORPTION AND CHEMICAL REACTION.

M. VENKATESWARLU[1,†], D. VENKATA LAKSHMI[2], AND G. DARMAIAH[3]

[1]DEPARTMENT OF MATHEMATICS, V. R. SIDDHARTHA ENGINEERING COLLEGE, KANUR, KRISHNA (DIST), ANDHRA PRADESH, INDIA-520007
E-mail address: mvsr2010@gmail.com

[2]DEPARTMENT OF MATHEMATICS, BAPATLA WOMEN'S ENGINEERING COLLEGE, BAPATLA, GUNTUR (DIST), ANDHRA PRADESH, INDIA-522102

[3]DEPARTMENT OF MATHEMATICS, NARASARAOPETA ENGINEERING COLLEGE, NARASARAOPETA, GUNTUR, (DIST), ANDHRA PRADESH, INDIA-522601

ABSTRACT. The present investigation deals, heat and mass transfer characteristics with the effect of slip on the hydromagnetic pulsatile flow through a parallel plate channel filled with saturated porous medium. Based on the pulsatile flow nature, exact solution of the governing equations for the fluid velocity, temperature and concentration are obtained by using two term perturbation technique subject to physically appropriate boundary conditions. The expressions of skin friction, Nusselt number and Sherwood number are also derived. The numerical values of the fluid velocity, temperature and concentration are displayed graphically whereas those of shear stress, rate of heat transfer and rate of mass transfer at the plate are presented in tabular form for various values of pertinent flow parameters. By increasing the slip parameter at the cold wall the velocity increases whereas the effect is totally reversed in the case of shear stress at the cold wall.

NOMENCLATURE

a	distance between two parallel plates	a^*	mean absorption coefficient
C	species concentration	C_f	skin-friction coefficient
C_0	species concentration at the cold wall	c_p	specific heat at constant pressure
B_0	uniform magnetic field	C_1	species concentration at the heated wall
Da	Darcy parameter	D_m	chemical molecular diffusivity

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† Corresponding author.

Gm	Solutal Grashof number
g	acceleration due to gravity
j_w	mass flux
K_r^*	dimensional chemical reaction parameter
K_T	thermal conductivity of the fluid
N	radiation parameter
n	frequency of oscillation
Q_0	dimensional heat absorption parameter
q_w	heat flux
Sh	Sherwood number
T_1	fluid temperature at the heated wall
t	dimensional time
u	fluid velocity in x- direction
Gr	thermal Grashof number
H	non-dimensional heat absorption term
K	permeability of porous medium
Kr	non-dimensional chemical reaction
M	magnetic parameter
Nu	Nusselt number
$\Pr$	Prandtl number
q_r	radiating flux vector
Sc	Schmidt number
T	fluid temperature
T_0	fluid temperature at the cold wall
U	A scaled velocity
v	fluid velocity in y- direction

GREEK SYMBOLS

β_c	coefficient of concentration expansion
ν	kinematic coefficient of viscosity
ϕ	A scaled concentration
ϕ_2	dimensional heated wall slip parameter
σ_e	electrical conductivity
τ_w	shear stress
η	A scaled coordinate
γ	non-dimensional cold wall slip
σ^*	Stefan-Boltzmann constant
β_T	coefficient of thermal expansion
ω	A scaled frequency
ϕ_1	dimensional cold wall slip parameter
ρ	fluid density
τ	non dimensional time
ψ	A scaled coordinate
θ	A scaled temperature
σ	non-dimensional heated wall slip

1. Introduction

The heat transfer enhancement is one of the most important technical aims for engineering systems due to its wide applications in electronics, cooling systems, fire and combustion modeling, development of metal waste from spent nuclear fuel, next-generation solar film collectors, heat exchangers technology, applications in the field of nuclear energy and various thermal systems. Sparrow and Cess [1] were one of the initial investigators to consider temperature dependent heat absorption on steady stagnation point flow and heat transfer. Chamkha [2] investigated on the unsteady MHD convective heat and mass transfer past a semi infinite vertical permeable moving plate with heat absorption. Analytical solutions for hydromagnetic free convection of a particulate suspension from an inclined plate with heat absorption were presented by Ramadan and Chamkha [3]. Ishak [4] worked mixed convection boundary layer flow over a horizontal plate with thermal radiation.

In recent years, the flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their application in many branches of science and technology, viz., in the field of agriculture engineering to study the underground water resources, seepage of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. The convection problem in porous medium has also important applications in geothermal reservoirs and geothermal energy extractions. Convection in porous media was documented by Nield and Bejan [5]. Prasad and Reddy [6] presented the radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation. Venkateswarlu et al. [7] discussed the radiation effects on MHD boundary layer flow of liquid metal over a porous stretching surface in porous medium with heat generation.

The study of MHD flows have stimulated considerable interest due to its important physical applications in solar physics, meteorology, power generating systems, aeronautics and missile aerodynamics, cosmic fluid dynamics and in the motion of Earth's core. Magnetofluid dynamics for engineers and applied physicists was documented by Cramer and Pai [8]. In a broader sense, MHD has applications in three different subject areas, such as astrophysical, geophysical and engineering problems. In light of these applications, free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike, has been studied by Cheng and Minkowycz [9]. Raptis and Kafoussias [10] presented Magnetohydrodynamic free convection flow and mass transfer through porous medium bounded by an infinite vertical porous plate with constant heat flux, due to the importance of mass transfer and that of applied magnetic field in the study of star and planets. Recently Turkyimazoglu and Pop [11] investigated analytically Soret and heat source effects on the unsteady radiative MHD free convection flow from an impulsively started infinite vertical plate. Venkateswarlu et al. [12] presented the thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion.

In most of the chemical engineering processes, chemical reaction occurs between a foreign mass and the fluid. Chemical reactions can be classified as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in numerous industrial applications viz. polymer production, drying evaporation at the surface of a water body, energy transfer in a wet cooling tower, generating electric power, manufacturing of ceramics or glassware, food processing etc. Chamkha [13] investigated MHD flow over a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Afify [14] studied the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Ibrahim et al. [15] analyzed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Bakr [16] discussed the effects of chemical reaction on MHD free convection and mass transfer flow of a micro polar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Recently, Venkateswarlu and Padma [17] analyzed the unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction. Manjula et al. [18] presented the influence of thermal radiation and chemical reaction on MHD flow, heat and mass transfer over a stretching surface.

Studies related to the oscillatory fluid flow are increasingly important in recent times due to its numerous applications in many real life problems. Some of these include, Makinde and Mhone [19] studied the combined effects of radiative heat transfer and MHD on oscillatory flow in a channel filled with porous medium. Mahmood and Ali [20] investigated the effect of Navier slip imposed on the lower wall on the unsteady hydromagnetic oscillatory flow of an incompressible viscous fluid in a planer channel filled with porous medium. In addition, Abdul-Hakeem and Sathiyanathan [21] presented analytical solution for two-dimensional oscillatory flow of an incompressible viscous fluid, through a highly porous medium bounded by an infinite vertical plate. While Umavathi et al. [22] studied the unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel.

Nanofluids enhance thermal conductivity of the base fluid enormously, which are also very stable and have no additional problems, such as sedimentation, erosion, additional pressure drop and non-Newtonian behavior, due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement. These suspended nanoparticles can change the transport and thermal properties of the base fluid. The boundary layer in Newtonian and porous media filled by nanofluids over fixed and moving boundaries have been recently considered by Fahad et al [23], Kuznetsov and Nield [24], Abbasi [25], Khan and Pop [26], Bachok et al. [27, 28] and Hayat et al. [29].

The objective of the present study is to investigate the influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of heat absorption and chemical reaction. Therefore, in the present work, the physical problem as described in Adesanya and Makinde [30] is considered. We should in prior emphasize that our intention is not to reproduce the results of Adesanya and Makinde [30]. In fact, the model that we consider differs considerably from that of Adesanya and Makinde [30] in that we use a better approach in the formulation, use a proper radiation term, introduce a heat absorption parameter and chemical reaction parameter. Analytical closed form solutions are presented for the momentum, energy and concentration equations using some proper change of variables. The following strategy is pursued in the rest of the paper. Section two presents the formation of the problem. The analytical solutions are presented in section three. Results are discussed in section four and finally section five provides a conclusion of the paper.

2. Formation of the Problem

We consider the unsteady laminar slip flow of an incompressible, viscous and electrically conducting fluid through a channel with non-uniform wall temperature bounded by two parallel plates separated by a distance a. The channel is assumed to be filled with a saturated porous medium. A uniform magnetic field of strength B_0 is applied perpendicular to the plates. The above plate is heated at constant temperature and the thermal radiation effect is also taken in to account. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate K_r^* between the diffusing species and the fluid. Initially i.e. at time $t \leq 0$, both the fluid and plate are at rest and at uniform temperature T_0. Also species concentration within the fluid is maintained at uniform concentration C_0. Geometry of the problem is presented in Fig. 1. Here, u- fluid velocity in x- direction, v- fluid velocity along y- direction, p-

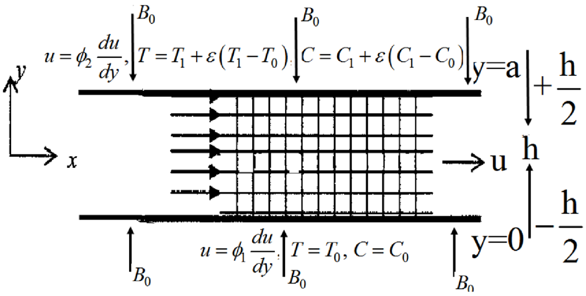

FIGURE 1. Geometry of the problem

fluid pressure, g- acceleration due to gravity, ρ- fluid density, β_T- coefficient of thermal expansion, β_C- coefficient of concentration volume expansion, t- time, K- permeability of porous medium, B_0- magnetic induction, T- fluid temperature, T_0- temperature at the cold wall, K_T- thermal diffusivity of the fluid, Q_0- dimensional heat absorption parameter, q_r- radiative heat flux, C- species concentration in the fluid, C_0- concentration at the cold wall, σ_e- fluid electrical conductivity, c_p- specific heat at constant pressure, D_m- chemical molecular diffusivity, ν- kinematic viscosity of the fluid and K_r^*- dimensional chemical reaction parameter respectively.

We choose a Cartesian coordinate system $(x,\ y)$ where x- lies along the centre of the channel, y- is the distance measured in the normal section such that $y = a$ is the channel's half width as shown in the figure below. Under the assumptions made by Adesanya and Makinde [30], as well as of the usual Boussinesq's approximation, the equations of conservation of mass, momentum, energy and concentration governing the free convective nonlinear boundary layer flow over a laminar porous plate in porous medium can be expressed as:

Continuity equation:

$$\frac{\partial \nu}{\partial y} = 0 \tag{2.1}$$

Momentum equation:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T\left(T - T_0\right) + g\beta_C\left(C - C_0\right) - \frac{\sigma_e B_0^2}{\rho}u - \frac{\nu}{K}u \tag{2.2}$$

Energy equation:

$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho c_p}\left(T - T_0\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{2.3}$$

Diffusion equation;

$$\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} - K_r^* \left(C - C_0 \right) \tag{2.4}$$

We should in prior warn the reader that our model is not the same as that Adesanya and Makinde [30] in which the heat absorption and chemical reaction effects were not taken into account.

Assuming that slipping occurs between the plate and fluid, the corresponding initial and boundary conditions of the system of partial differential equations for the fluid flow problem are given below

$$\left.\begin{array}{l} u = \phi_1 \dfrac{du}{dy},\ T = T_0,\ C = C_0 \quad \text{at} \quad y = 0 \\ u = \phi_2 \dfrac{du}{dy},\ T = T_1 + \epsilon (T_1 - T_0) \exp(\text{int}),\ C = C_1 + \epsilon (C_1 - C_0) \exp(\text{int}) \text{ at } y = a \end{array}\right\} \tag{2.5}$$

where T_1- fluid temperature at the heated plate, C_1- species concentration at the heated plate, ϕ_1- cold wall dimensional slip parameter, ϕ_2- heated wall dimensional slip parameter, n- frequency of oscillation and $\epsilon \ll 1$ is a very small positive constant.

Following Rapits [31], by using the Rosseland approximation, the radiative flux vector q_r can be written as:

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma^* \left(T_0^4 - T^4 \right) \tag{2.6}$$

where σ^* and a^* are the Stefan–Boltzmann constant and the mean absorption coefficient respectively. We assume that the difference between fluid temperature T and cold wall temperature T_0 within the flow is sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in Taylor series T^4 about the cold wall temperature T_0 and neglecting the second and higher order terms, we have

$$T^4 \cong 4T_0^3 T - 3T_0^4 \tag{2.7}$$

Using equations (2.6) and (2.7) in equation (2.3). We obtain

$$\frac{\partial T}{\partial t} = \frac{K_T}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{16 a^* \sigma^* T_0^3}{\rho c_p} \left(T - T_0 \right) - \frac{Q_0}{\rho c_p} \left(T - T_0 \right) \tag{2.8}$$

We introduce the following non-dimensional variables

$$\left.\begin{array}{l} \psi = \dfrac{x}{h},\ \eta = \dfrac{y}{h},\ U = \dfrac{h}{\nu} u,\ P = \dfrac{h^2}{\rho \nu^2} p,\ \gamma = \dfrac{\phi_1}{h},\ \sigma = \dfrac{\phi_2}{h}, \\ \omega = \dfrac{h^2}{\nu} n,\ \tau = \dfrac{\nu}{h^2} t,\ \theta = \dfrac{T - T_0}{T_1 - T_0},\ \phi = \dfrac{C - C_0}{C_1 - C_0} \end{array}\right\} \tag{2.9}$$

Equations (2.2), (2.4) and (2.8) reduce to the following non-dimensional form

$$\frac{\partial U}{\partial \tau} = -\frac{dP}{d\psi} + \frac{\partial^2 U}{\partial \eta^2} + Gr\theta + Gm\phi - \left[M + \frac{1}{Da} \right] U \tag{2.10}$$

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2} - (N+H)\,\theta \tag{2.11}$$

$$\frac{\partial\phi}{\partial\tau} = \frac{1}{Sc}\frac{\partial^2\phi}{\partial\eta^2} - Kr\phi \tag{2.12}$$

Here $Gr = \frac{g\beta_T(T_1-T_0)h^3}{\nu^2}$ is the thermal buoyancy force, $Gm = \frac{g\beta_C(C_1-C_0)h^3}{v^2}$ is the concentration buoyancy force, $M = \frac{\sigma_e B_0^2 h^2}{\rho v}$ is the magnetic parameter, $Da = \frac{K}{h^2}$ is the Darcy parameter, $\Pr = \frac{\rho c_p \nu}{K_T}$ is the Prandtl number, $N = \frac{16a^*\sigma^* h^2 T_0^3}{\rho c_p \nu}$ is the thermal radiation parameter, $H = \frac{Q_0 h^2}{\rho c_p \nu}$ is the heat source parameter, $Sc = \frac{\nu}{D_m}$ is the Schmidt number and $Kr = \frac{h^2}{\nu}K_r^*$ is the chemical reaction parameter respectively.

Initial and boundary conditions, presented by equation (2.5), in non-dimensional form, are given by

$$\left.\begin{aligned} &U = \gamma\frac{dU}{d\eta}, \quad \theta = 0, \quad \phi = 0 \ \text{ at } \ \eta = 0 \\ &U = \sigma\frac{dU}{d\eta}, \quad \theta = 1 + \epsilon\exp\left(i\omega\tau\right), \quad \phi = 1 + \epsilon\exp\left(i\omega\tau\right) \quad \text{at} \quad \eta = 1 \end{aligned}\right\} \tag{2.13}$$

Following Adesanya and Makinde [30], for purely an oscillatory flow we take the pressure gradient of the form

$$\lambda = -\frac{dP}{d\psi} = \lambda_0 + \epsilon\exp(i\omega t)\lambda_1 \tag{2.14}$$

where λ_0- and λ_1- are constants and ω is the frequency of oscillation.

It is now important to calculate physical quantities of primary interest, which are the local wall shear stress or skin friction coefficient, the local surface heat flux and the local surface mass flux. Given the velocity, temperature and concentration fields in the boundary layer, the shear stress τ_w, the heat flux q_w and mass flux j_w are obtained by

$$\tau_w = \mu\left[\frac{\partial u}{\partial y}\right] \tag{2.15}$$

$$q_w = -K_T\left[\frac{\partial T}{\partial y}\right] \tag{2.16}$$

$$j_w = -D_m\left[\frac{\partial C}{\partial y}\right] \tag{2.17}$$

In non-dimensional form the skin-friction coefficient Cf, heat transfer coefficient Nu and mass transfer coefficient Sh are defined as

$$Cf = \frac{\tau_w}{\rho\left(\nu/h\right)^2} \tag{2.18}$$

$$Nu = \frac{hq_w}{K_T\left(T_1 - T_0\right)} \tag{2.19}$$

$$Sh = \frac{hj_w}{D_m\left(C_1 - C_0\right)} \tag{2.20}$$

Using non-dimensional variables in equation (2.9) and equations (2.15) to (2.17) into equations (2.18) to (2.20), we obtain the physical parameters

$$Cf = \left[\frac{\partial U}{\partial \eta}\right] \tag{2.21}$$

$$Nu = -\left[\frac{\partial \theta}{\partial \eta}\right] \tag{2.22}$$

$$Sh = -\left[\frac{\partial \phi}{\partial \eta}\right] \tag{2.23}$$

3. Solution of the problem

Equations (2.10) to (2.12) are coupled non-linear partial differential equations and these cannot be solved in closed form. So, we reduce these non-linear partial differential equations into a set of ordinary differential equations, which can be solved analytically. This can be done by assuming the trial solutions for the velocity, temperature and concentration of the fluid as (see, Singh *et al.* [32] and Siva Kumar *et al* [33])

$$U(\eta,\tau) = U_0(\eta) + \epsilon\exp(i\omega\tau)U_1(\eta) + o(\epsilon^2) \tag{3.1}$$

$$\theta(\eta,\tau) = \theta_0(\eta) + \epsilon\exp(i\omega\tau)\theta_1(\eta) + o(\epsilon^2) \tag{3.2}$$

$$\phi(\eta,\tau) = \phi_0(\eta) + \epsilon\exp(i\omega\tau)\phi_1(\eta) + o(\epsilon^2) \tag{3.3}$$

Substituting equations (3.1) to (3.3) into equations (2.10) to (2.12), then equating the harmonic and non–harmonic terms and neglecting the higher order terms of $o\left(\epsilon^2\right)$, we obtain

$$U_0'' - \left[M + \frac{1}{Da}\right]U_0 = -\left[Gr\theta_0 + Gm\phi_0 + \lambda_0\right] \tag{3.4}$$

$$U_1'' - \left[M + \frac{1}{Da} + i\omega\right]U_1 = -\left[Gr\theta_1 + Gm\phi_1 + \lambda_1\right] \tag{3.5}$$

$$\theta_0'' - \Pr\left(N + H\right)\theta_0 = 0 \tag{3.6}$$

$$\theta_1'' - \Pr\left(N + H + i\omega\right)\theta_1 = 0 \tag{3.7}$$

$$\phi_0'' - ScKr\phi_0 = 0 \tag{3.8}$$

$$\phi_1'' - Sc\left(Kr + i\omega\right)\phi_1 = 0 \tag{3.9}$$

Initial and boundary conditions, presented by equation (2.13), can be written as

$$\left.\begin{array}{l} U_0 = \gamma\dfrac{dU_0}{d\eta},\ U_1 = \gamma\dfrac{dU_1}{d\eta},\ \theta_0 = 0,\ \theta_1 = 0,\ \phi_0 = 0,\ \phi_1 = 0 \quad \text{at} \quad \eta = 0 \\ U_0 = \sigma\dfrac{dU_0}{d\eta},\ U_1 = \sigma\dfrac{dU_1}{d\eta},\ \theta_0 = 1,\ \theta_1 = 1,\ \phi_0 = 1,\ \phi_1 = 1 \quad \text{at} \quad \eta = 1 \end{array}\right\} \tag{3.10}$$

where the prime denotes the ordinary differentiation with respect to η.

The analytical solutions of equations (3.4) to (3.9) with the boundary conditions in equation (3.10) are given by

$$U_0 = A_{17}\exp(-m_5\eta) + A_{16}\exp(m_5\eta) + A_3 + \frac{A_1\sinh(m_1\eta)}{\sinh(m_1)} - \frac{A_2\sinh(m_3\eta)}{\sinh(m_3)} \tag{3.11}$$

$$U_1 = A_{34}\exp(-m_6\eta) + A_{33}\exp(m_6\eta) + A_{20} + \frac{A_{18}\sinh(m_2\eta)}{\sinh(m_2)} - \frac{A_{19}\sinh(m_4\eta)}{\sinh(m_4)} \tag{3.12}$$

$$\theta_0 = \frac{\sinh(m_1\eta)}{\sinh(m_1)} \tag{3.13}$$

$$\theta_1 = \frac{\sinh(m_2\eta)}{\sinh(m_2)} \tag{3.14}$$

$$\phi_0 = \frac{\sinh(m_3\eta)}{\sinh(m_3)} \tag{3.15}$$

$$\phi_1 = \frac{\sinh(m_4\eta)}{\sinh(m_4)} \tag{3.16}$$

By substituting equations (3.11) to (3.16) into equations (3.1) to (3.3) we obtained solutions for the fluid velocity, temperature and concentration and are presented in the following form

$$\begin{aligned} U(\eta,\tau) = &\left[A_{17}\exp(-m_5\eta) + A_{16}\exp(m_5\eta) + A_3 + \frac{A_1\sinh(m_1\eta)}{\sinh(m_1)} - \frac{A_2\sinh(m_3\eta)}{\sinh(m_3)}\right] \\ &+ \epsilon\exp(i\omega\tau)\Bigg[A_{34}\exp(-m_6\eta) + A_{33}\exp(m_6\eta) \\ &+ A_{20} + \frac{A_{18}\sinh(m_2\eta)}{\sinh(m_2)} - \frac{A_{19}\sinh(m_4\eta)}{\sinh(m_4)}\Bigg] \end{aligned} \tag{3.17}$$

$$\theta(\eta,\tau) = \left[\frac{\sinh(m_1\eta)}{\sinh(m_1)}\right] + \epsilon\exp(i\omega\tau)\left[\frac{\sinh(m_2\eta)}{\sinh(m_2)}\right] \tag{3.18}$$

$$\phi(\eta,\tau) = \left[\frac{\sinh(m_3\eta)}{\sinh(m_3)}\right] + \epsilon\exp(i\omega\tau)\left[\frac{\sinh(m_4\eta)}{\sinh(m_4)}\right] \tag{3.19}$$

3.1. **Skin friction.** From the velocity field, the skin friction at the plate can be obtained, which in non dimensional form is given by

$$\begin{aligned} Cf = &\left[A_{16}m_5e^{m_5\eta} - A_{17}m_5e^{-m_5\eta} + \frac{A_1m_1\cosh(m_1\eta)}{\sinh(m_1)} - \frac{A_2m_3\cosh(m_3\eta)}{\sinh(m_3)}\right] \\ &+\epsilon\exp(i\omega\tau)\Bigg[A_{33}m_6e^{m_6\eta} - A_{34}m_6e^{-m_6\eta} \\ &+\frac{A_{18}m_2\cosh(m_2\eta)}{\sinh(m_2)} - \frac{A_{19}m_4\cosh(m_4\eta)}{\sinh(m_4)}\Bigg] \end{aligned} \tag{3.20}$$

3.2. **Nusselt number.** From temperature field, we obtained heat transfer coefficient which is given in non-dimensional form as

$$Nu = -\left[\frac{m_1\cosh(m_1\eta)}{\sinh(m_1)}\right] - \epsilon\exp(i\omega\tau)\left[\frac{m_2\cosh(m_2\eta)}{\sinh(m_2)}\right] \tag{3.21}$$

3.3. **Sherwood number.** From concentration field, we obtained mass transfer coefficient which is given in non-dimensional form as

$$Sh = -\left[\frac{A_3 m_3 \cosh(m_3\eta)}{\sinh(m_3)}\right] - \epsilon \exp(i\omega\tau)\left[\frac{A_5 m_4 \cosh(m_4\eta)}{\sinh(m_4)}\right] \tag{3.22}$$

4. Results and discussion

In order to investigate the influence of various physical parameters such as thermal Grashof number Gr, solutal Grashof number Gm, Darcy parameter Da, pressure gradient λ, magnetic parameter M, cold wall slip parameter γ, heated wall slip parameter σ, heat absorption parameter H, Prandtl number Pr, radiation parameter N, chemical reaction parameter Kr and mass diffusion parameter Sc on the flow-field, fluid velocity U, temperature θ and concentration ϕ have been studied analytically and computed results of the analytical solutions from equations (3.17) to (3.19) are displayed graphically from Figs. 2 to 18 for various values of these physical parameters. The numerical values of skin friction, Nusselt number and Sherwood number computed from analytical solutions, presented by equations (3.20) to (3.22) are presented in tabular form in Tables 1 to 4 for various values of different physical parameters. In the present study following default parameter values are adopted for computations: $Gr = 2,\ Gm = 2,\ M = 1,\ Da = 0.5,\ \tau = 0,\ \lambda = 1,\ \Pr = 0.71,\ N = 1,\ H = 0.5,\ Sc = 0.30,\ Kr = 0.5,\ \gamma = 0.1,\ \sigma = 0.1,\ \omega = 1,$ and $\epsilon = 0.001$. Therefore all the graphs and tables are corresponding to these values unless specifically indicated on the appropriate graph or table.

Figs. 2 and 3 shows the fluid velocity profile variations with the cold wall slip parameter γ and the heated wall slip parameter σ. It is observed that, the fluid velocity U increases on

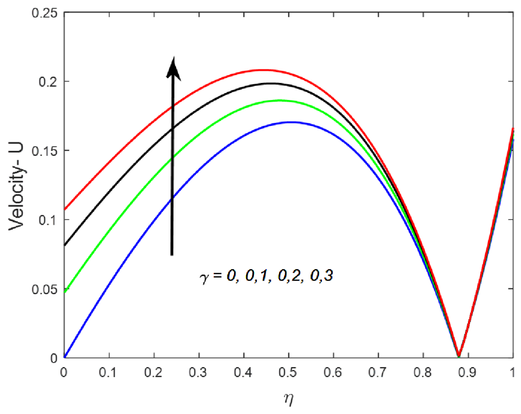

FIGURE 2. Influence of cold wall slip parameter on velocity profiles.

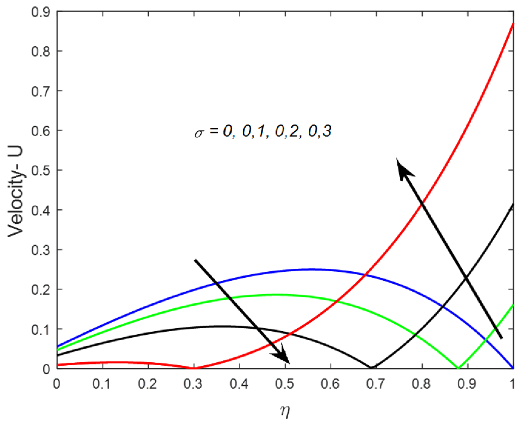

FIGURE 3. Influence of heated wall slip parameter on velocity profiles.

increasing the cold wall slip parameter γ thus enhancing the fluid flow. The cold wall slip parameter did not cause any appreciable effect on the heated wall. An increase in the heated wall slip parameter σ decreases the fluid velocity minimally at the cold wall and increasing the heated wall slip parameter causes a flow reversal towards the heated wall. It is observed that $\sigma = 0$ corresponds to the pulsatile case with no slip condition at the heated wall in Fig 3.

The nature of fluid velocity and concentration in presence of foreign species such as Hydrogen $(Sc = 0.22)$, Helium $(Sc = 0.30)$, Water vapour $(Sc = 0.60)$, Ammonia $(Sc = 0.78)$ is shown in Figs. 4 and 5. Physically, Schmidt number signifies the relative strength of viscosity

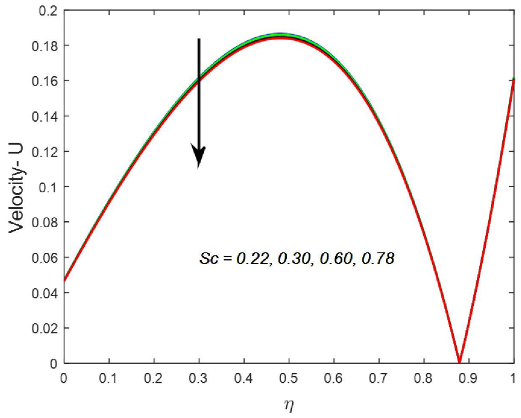

FIGURE 4. Influence of Schmidt number on velocity profiles.

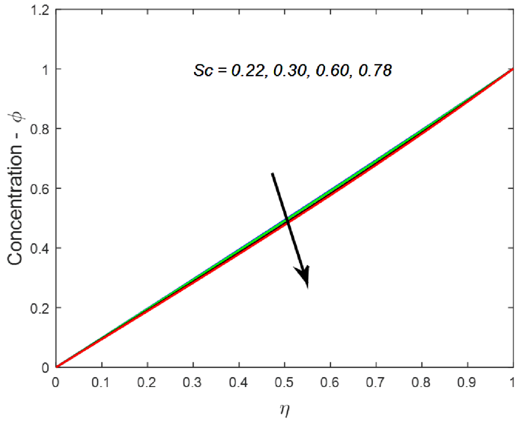

FIGURE 5. Influence of Schmidt number on concentration profiles.

to chemical molecular diffusivity. Therefore the Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic and concentration boundary layers. This causes the concentration buoyancy effects to decrease the fluid velocity. It is observed that velocity and concentration decreases on increasing Schmidt number in Figs. 4 and 5.

Figs. 6 and 7 demonstrate the influence of chemical reaction parameter Kr on the velocity and species concentration. It is observed that, both velocity U and species concentration ϕ decreases on increasing the chemical reaction parameter Kr. This implies that, chemical reaction tends to reduce the fluid velocity and species concentration. It is observed that, from Figs. 8 and 9 both the fluid velocity U and temperature θ decreases on increasing the radiation parameter N.

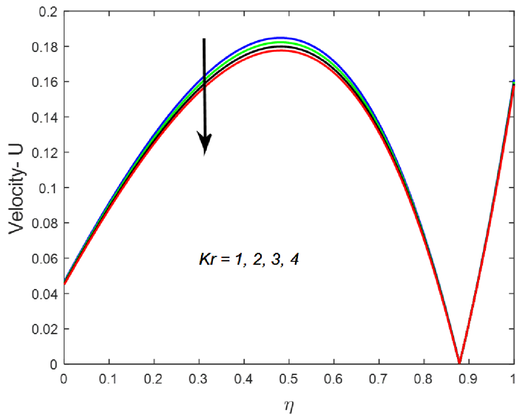

FIGURE 6. Influence of chemical reaction parameter on velocity profiles.

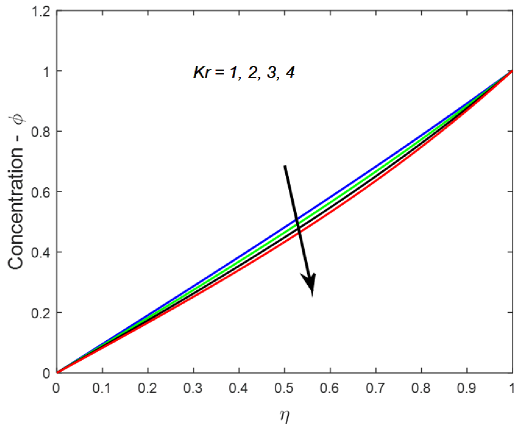

FIGURE 7. Influence of chemical reaction parameter on concentration profiles.

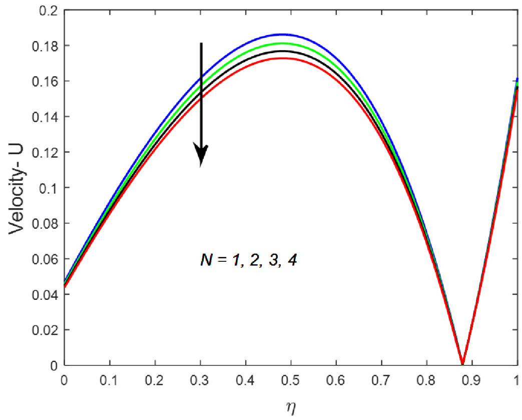

FIGURE 8. Influence of radiation parameter on velocity profiles.

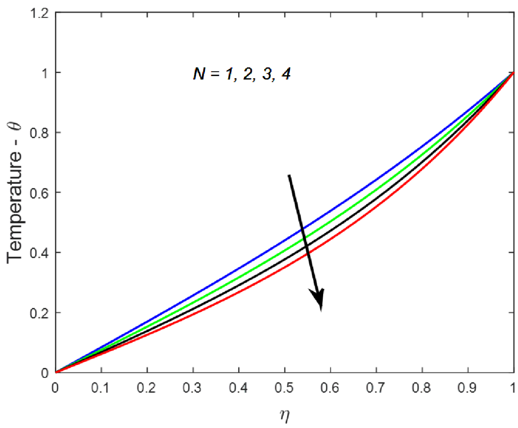

FIGURE 9. Influence of radiation parameter on temperature profiles.

Figs. 10 and 11, demonstrate the plot of fluid velocity U and temperature θ for a variety of heat absorption parameter H. It is seen in figures that, the fluid velocity and temperature decrease on increasing the heat absorption parameter. This implies that heat absorption tend to retard the fluid velocity and temperature. This is because radiation and heat absorption have tendency to reduce fluid temperature.

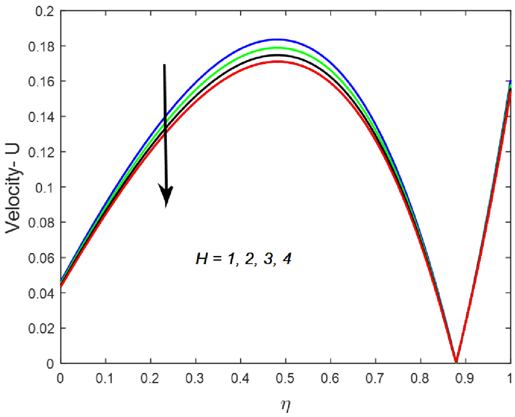

FIGURE 10. Influence of heat absorption parameter on velocity profiles.

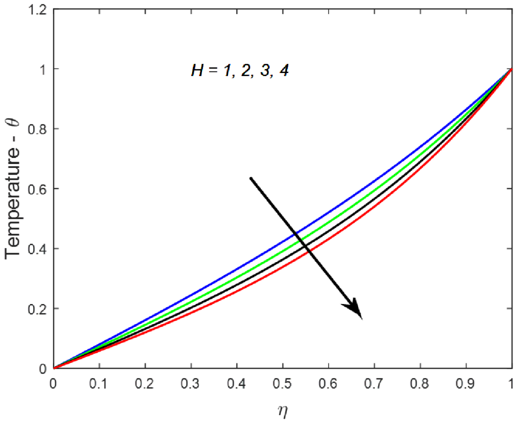

FIGURE 11. Influence of heat absorption parameter on temperature profiles.

Figs. 12 and 13, shows the plot of fluid velocity U and temperature θ of the flow field against different values of Prandtl number Pr taking other parameters are constant. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The values of the Prandtl number are chosen for air $(\mathrm{Pr} = 0.71)$, electrolytic solution $(\mathrm{Pr} = 1.00)$, water $(\mathrm{Pr} = 7.00)$ and water at $4°C$ $(\mathrm{Pr} = 11.40)$. It is evident from Figs. 12 and 13, velocity U and temperature θ decreases on increasing Prandtl number Pr. Thus higher prandtl number leads to faster cooling of the plate.

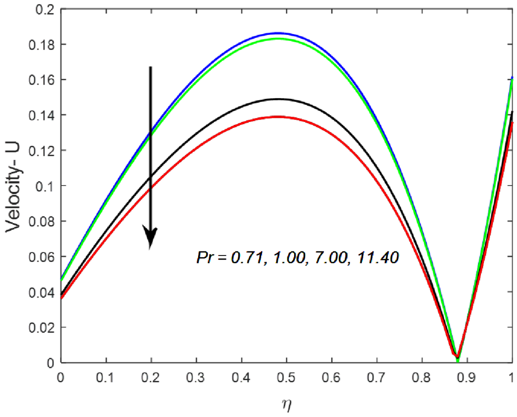

FIGURE 12. Influence of Prandtl number on velocity profiles.

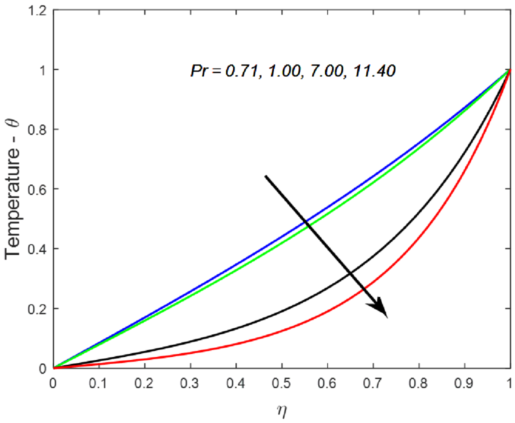

FIGURE 13. Influence of Prandtl number on temperature profiles.

Fig.14 shows the variation of fluid velocity U with the Darcy parameter Da. The graph shows that an increase in the Darcy parameter increases the fluid flow except at the flow reversal point at the heated wall.

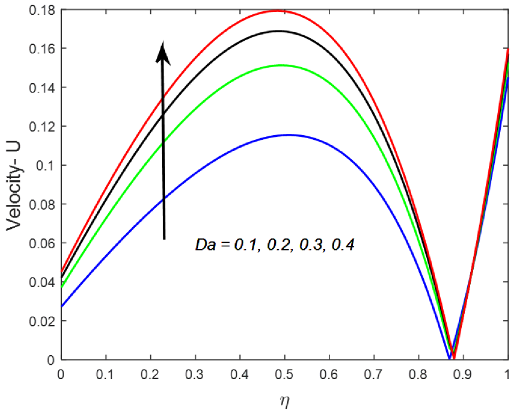

FIGURE 14. Influence of Darcy parameter on velocity profiles.

Fig.15 demonstrates the influence of pressure gradient λ on the fluid velocity U. It is observed that, the fluid velocity U increases on increasing the pressure gradient λ.

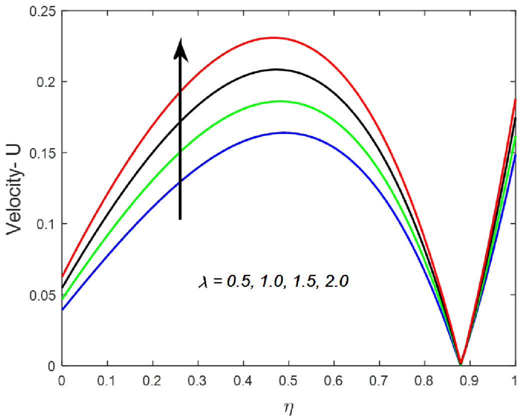

FIGURE 15. Influence of pressure gradient on velocity profiles.

Fig.16 depicts the influence of magnetic field intensity on the variation of fluid velocity. It is noticed that, an increase in the magnetic parameter M decreases the fluid velocity U due to the resistive action of the Lorenz forces except at the heated wall where the reversed flow induced by wall slip caused an increase in the fluid velocity. This implies that magnetic field tends to decelerate fluid flow.

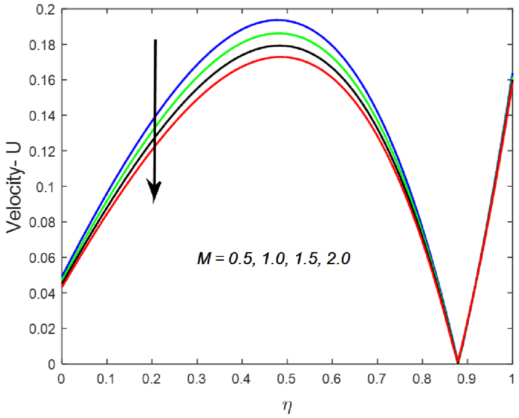

FIGURE 16. Influence of magnetic parameter on velocity profiles.

The effects of thermal Grashof number Gr and solutal Grashof number Gm on the velocity U of the flow field are presented in Figs. 17 and 18. Physically, thermal Grashof number Gr signifies the relative strength of thermal buoyancy force to viscous hydrodynamic force in the boundary layer. Solutal Grashof number Gm signifies the relative strength of species buoyancy force to viscous hydrodynamic force in the boundary layer. A study of the curves shows that thermal Grashof number Gr and solutal Grashof number Gm accelerates the velocity of the flow field at all points. This is due to the reason that there is an enhancement in thermal buoyancy force and concentration buoyancy force.

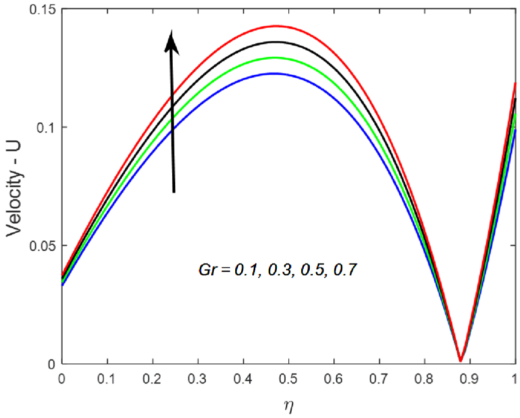

FIGURE 17. Influence of Grashof number on velocity profiles.

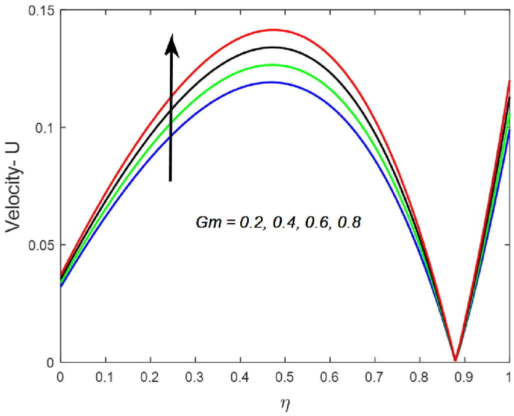

FIGURE 18. Influence of Solutal Grashof number on velocity profiles.

From Tables 1 and 2, it is clear that the skin friction Cf increases on increasing thermal Grashof number Gr, solutal Grashof number Gm, Darcy parameter Da and pressure gradient λ whereas it decreases on increasing magnetic parameter M, Prandtl number $\Pr$, radiation parameter N, heat absorption parameter H, Schmidt number Sc and chemical reaction parameter Kr at both cold and heated walls. The skin friction coefficient decreases at the cold wall and it increases at the heated wall on increasing the cold wall slip parameter γ and heated wall slip parameter σ.

TABLE 1. Effect of $Gr,\ Gm,\ M,\ Da,\ \gamma$, and σ on skin friction coefficient when $\Pr = 0.71,\ N = 1,\ H = 0.5,\ Sc = 0.30,\ Sr = 1,\ Kr = 0.5,\ \tau = 0,\ \omega = 1,\ \epsilon = 0.001$.

Gr	Gm	M	Da	γ	σ	Skin friction Cf	
						Cold wall	Heated wall
0.1	2.0	1.0	0.5	0.1	0.1	0.3302	0.9901
0.3	2.0	1.0	0.5	0.1	0.1	0.3450	1.0562
0.5	2.0	1.0	0.5	0.1	0.1	0.3597	1.1223
0.7	2.0	1.0	0.5	0.1	0.1	0.3745	1.1884
2.0	0.2	1.0	0.5	0.1	0.1	0.3212	0.9905
2.0	0.4	1.0	0.5	0.1	0.1	0.3378	1.0602
2.0	0.6	1.0	0.5	0.1	0.1	0.3544	1.1299
2.0	0.8	1.0	0.5	0.1	0.1	0.3710	1.1996
2.0	2.0	0.5	0.5	0.1	0.1	0.4921	1.6383
2.0	2.0	1.0	0.5	0.1	0.1	0.4706	1.6179
2.0	2.0	1.5	0.5	0.1	0.1	0.4508	1.5993
2.0	2.0	2.0	0.5	0.1	0.1	0.4325	1.5822
2.0	2.0	1.0	0.1	0.1	0.1	0.2717	1.4510
2.0	2.0	1.0	0.2	0.1	0.1	0.3713	1.5273
2.0	2.0	1.0	0.3	0.1	0.1	0.4210	1.5717
2.0	2.0	1.0	0.4	0.1	0.1	0.4508	1.5993
2.0	2.0	1.0	0.5	0.0	0.1	0.5597	1.5814
2.0	2.0	1.0	0.5	0.1	0.1	0.4706	1.6179
2.0	2.0	1.0	0.5	0.2	0.1	0.4060	1.6444
2.0	2.0	1.0	0.5	0.3	0.1	0.3570	1.6644
2.0	2.0	1.0	0.5	0.1	0.0	0.5570	1.3250
2.0	2.0	1.0	0.5	0.1	0.1	0.4706	1.6179
2.0	2.0	1.0	0.5	0.1	0.2	0.3351	2.0769
2.0	2.0	1.0	0.5	0.1	0.3	0.0924	2.8997

TABLE 2. Effect of $\Pr,\ N,\ H,\ Sc,\ Kr$ and λ on skin friction coefficient when $Gm = 2,\ Gr = 2,\ \tau = 0,\ M = 1,\ Da = 0.5,\ \gamma = 0.1,\ \sigma = 0.1,\ \omega = 1,\ \epsilon = 0.001$.

Pr	N	H	Sc	Kr	λ	Skin friction Cf	
						Cold wall	Heated wall
0.71	1.0	0.5	0.30	0.5	1.0	0.4706	1.6179
1.00	1.0	0.5	0.30	0.5	1.0	0.4630	1.6025
7.00	1.0	0.5	0.30	0.5	1.0	0.3832	1.4220
11.40	1.0	0.5	0.30	0.5	1.0	0.3613	1.3593
0.71	1.0	0.5	0.30	0.5	1.0	0.4706	1.6179
0.71	2.0	0.5	0.30	0.5	1.0	0.4586	1.5933
0.71	3.0	0.5	0.30	0.5	1.0	0.4480	1.5714
0.71	4.0	0.5	0.30	0.5	1.0	0.4387	1.5517
0.71	1.0	1.0	0.30	0.5	1.0	0.4644	1.6053
0.71	1.0	2.0	0.30	0.5	1.0	0.4531	1.5821
0.71	1.0	3.0	0.30	0.5	1.0	0.4432	1.5613
0.71	1.0	4.0	0.30	0.5	1.0	0.4345	1.5425
0.71	1.0	0.5	0.22	0.5	1.0	0.4715	1.6196
0.71	1.0	0.5	0.30	0.5	1.0	0.4706	1.6179
0.71	1.0	0.5	0.60	0.5	1.0	0.4674	1.6115
0.71	1.0	0.5	0.78	0.5	1.0	0.4655	1.6078
0.71	1.0	0.5	0.30	1.0	1.0	0.4674	1.6115
0.71	1.0	0.5	0.30	2.0	1.0	0.4612	1.5993
0.71	1.0	0.5	0.30	3.0	1.0	0.4554	1.5877
0.71	1.0	0.5	0.30	4.0	1.0	0.4500	1.5767
0.71	1.0	0.5	0.30	0.5	0.5	0.3922	1.4879
0.71	1.0	0.5	0.30	0.5	1.0	0.4706	1.6179
0.71	1.0	0.5	0.30	0.5	1.5	0.5490	1.7478
0.71	1.0	0.5	0.30	0.5	2.0	0.6274	1.8778

From Table 3, it is clear that the heat transfer coefficient Nu increases at the cold wall and it decreases at the heated wall on increasing the Prandtl number $\Pr$, radiation parameter N and heat absorption parameter H.

TABLE 3. Effect of Pr, N and H on heat transfer coefficient when $\tau = 0,\ \omega = 1,\ \epsilon = 0.001$.

Pr	N	H	Nusselt number Nu	
			Cold wall	Heated wall
0.71	1.0	0.5	−0.8432	−1.3334
1.00	1.0	0.5	−0.7885	−1.4577
7.00	1.0	0.5	−0.2543	−3.2537
11.40	1.0	0.5	−0.1324	−4.1417
0.71	1.0	0.5	−0.8432	−1.3334
0.71	2.0	0.5	−0.7565	−1.5332
0.71	3.0	0.5	−0.6815	−1.7189
0.71	4.0	0.5	−0.6162	−1.8924
0.71	1.0	1.0	−0.7982	−1.4353
0.71	1.0	2.0	−0.7177	−1.6277
0.71	1.0	3.0	−0.6478	−1.8070
0.71	1.0	4.0	−0.5867	−1.9752

From Table 4, it is clear that the mass transfer coefficient Sh increases at the cold wall and it decreases at the heated wall on increasing the Schmidt number Sc and chemical reaction parameter Kr.

TABLE 4. Effect of Sc and Kr on the mass transfer coefficient $\tau = 0,\ \omega = 1,$ and $\epsilon = 0.001$.

Sc	Kr	Sherwood number Sh	
		Cold wall	Heated wall
0.22	0.5	−0.9829	−1.0374
0.30	0.5	−0.9764	−1.0506
0.60	0.5	−0.9526	−1.0992
0.78	0.5	−0.9388	−1.1279
0.30	1.0	−0.9526	−1.0992
0.30	2.0	−0.9075	−1.1936
0.30	3.0	−0.8652	−1.2847
0.30	4.0	−0.8257	−1.3726

5. Conclusions

In this paper we have studied analytically the influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of heat absorption and chemical reaction.From the present investigation the following conclusions can be drawn:

- Cold wall slip parameter, Darcy parameter, pressure gradient, Thermal Grashof number and solutal Grashof number are tend to accelerate the fluid velocity whereas Schmidt number, chemical reaction parameter, radiation parameter, heat absorption parameter, magnetic parameter and Prandtl number and have reverse effect on it.
- Radiation parameter, heat absorption parameter and Prandtl number are tendency to retard the fluid temperature.
- Schmidt number and chemical reaction parameter have tendency to decelerate the species concentration.
- Thermal Grashof number, Solutal Grashof number, Darcy parameter and pressure gradient are tend to accelerate the skin friction coefficient whereas magnetic parameter, Prandtl number, radiation parameter, heat absorption parameter, Schmidt number and chemical reaction parameter have a reverse effect on the skin friction at both cold and heated walls. Skin friction coefficient decreases on increasing the cold wall slip parameter and heated wall slip parameter at the cold wall whereas it has a reverse effect at the heated wall.
- Radiation parameter, heat absorption parameter and Prandtl number have tendency to increase the heat transfer coefficient at the cold wall whereas they have tendency to retard the heat transfer coefficient at the heated wall.
- Schmidt number and chemical reaction parameter have tendency to accelerate the mass transfer coefficient at the cold wall whereas they have tendency to retard the mass transfer coefficient at the heated wall.

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Appendix

$$m_1 = \sqrt{\Pr(N+H)}, \qquad m_2 = \sqrt{\Pr(N+H+i\omega)}, \qquad m_3 = \sqrt{ScKr},$$

$$m_4 = \sqrt{Sc(Kr+i\omega)}, \qquad m_5 = \sqrt{M+\frac{1}{Da}}, \qquad m_6 = \sqrt{M+\frac{1}{Da}+i\omega},$$

$$A_1 = \frac{Gr}{m_5^2 - m_1^2}, \qquad A_2 = \frac{Gm}{m_3^2 - m_5^2}, \qquad A_3 = \frac{\lambda_0}{m_5^2},$$

$$A_4 = \frac{\gamma m_1 A_1}{\sinh(m_1)}, \qquad A_5 = \frac{\gamma m_3 A_2}{\sinh(m_3)}, \qquad A_6 = A_4 - (A_3 + A_5)$$
$$A_7 = A_1 \left[\sigma m_1 \coth(m_1) - 1\right], \quad A_8 = A_2 \left[\sigma m_3 \coth(m_3) - 1\right],$$
$$A_9 = A_7 - (A_3 + A_8), \qquad A_{10} = 1 + \gamma m_5, \qquad A_{11} = 1 - \gamma m_5, \qquad A_{12} = 1 + \sigma m_5,$$
$$A_{13} = 1 - \sigma m_5, \qquad A_{14} = A_{11} A_{12} \exp(-m_5) - A_{10} A_{13} \exp(m_5),$$
$$A_{15} = A_6 A_{12} \exp(-m_5) - A_9 A_{10}, \qquad A_{16} = \frac{A_{15}}{A_{14}}, \qquad A_{17} = \frac{A_6 - A_{11} A_{16}}{A_{10}},$$
$$A_{18} = \frac{Gr}{m_6^2 - m_2^2}, \qquad A_{19} = \frac{Gm}{m_4^2 - m_6^2}, \qquad A_{20} = \frac{\lambda_1}{m_6^2}, \qquad A_{21} = \frac{\gamma m_2 A_{18}}{\sinh(m_2)},$$
$$A_{22} = \frac{\gamma m_4 A_{19}}{\sinh(m_4)}, \qquad A_{23} = A_{21} - (A_{20} + A_{22}), \qquad A_{24} = A_{18} \left[\sigma m_2 \coth(m_2) - 1\right],$$
$$A_{25} = A_{19} \left[\sigma m_4 \cot(m_4) - 1\right], \qquad A_{26} = A_{24} - (A_{20} + A_{25}), \qquad A_{27} = 1 + \gamma m_6,$$
$$A_{28} = 1 - \gamma m_6, \qquad A_{29} = 1 + \sigma m_6, \qquad A_{30} = 1 - \sigma m_6,$$
$$A_{31} = A_{28} A_{29} \exp(-m_6) - A_{27} A_{30} \exp(m_6), \qquad A_{32} = A_{23} A_{29} \exp(-m_6) - A_{26} A_{27},$$
$$A_{33} = \frac{A_{32}}{A_{31}}, \qquad A_{34} = \frac{A_{23} - A_{28} A_{33}}{A_{27}}$$

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