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영상 복원을 위한 통합 베이즈 티코노프 정규화 방법

류재홍*

A Unified Bayesian Tikhonov Regularization Method for Image Restoration

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요 약

본 논문은 영상 복원 문제에 대한 정규화 모수를 찾는 새로운 방법을 제시한다. 사전 정보가 없으면 티코노프(Tikhonov) 정규화 모수를 선택하기 위한 일반화된 교차 검증법이나 L자형 곡선 검정 등의 별도의 최적화 함수가 필요하다. 본 논문에서는 티코노프 정규화에 대한 통합된 베이즈 해석을 소개하고 영상 복원 문제에 적용한다. 티코노프 정규화 모수와 베이즈 하이퍼 모수들의 관계를 정립하고 최대 사후 확률과 근거 프레임워크를 사용한 정규화 모수를 구하는 공식을 제시한다. 실험결과는 제안하는 방법의 효능을 보여준다.

ABSTRACT

This paper suggests a new method of finding regularization parameter for image restoration problems. If the prior information is not available, separate optimization functions for Tikhonov regularization parameter are suggested in the literature such as generalized cross validation and L-curve criterion. In this paper, unified Bayesian interpretation of Tikhonov regularization is introduced and applied to the image restoration problems. The relationship between Tikhonov regularization parameter and Bayesian hyper-parameters is established. Update formular for the regularization parameter using both maximum a posteriori(: MAP) and evidence frameworks is suggested. Experimental results show the effectiveness of the proposed method.

키워드

Bayesian Interpretation, Evidence Framework, Tikhonov Regularization, Image Restoration
베이즈 해석, 근거 프레임워크, 티코노프 정규화, 영상 복원

Ⅰ. Introduction

Image restoration is the process to recover an original image from distorted one by using an appropriate degradation model[1]. Image restoration is an example of inversion problems in the ill-posed systems[2,3]. In the linear degradation model, we assume that a given input image f is blurred by a Point Spread Function(: PSF) h and further distorted by a Gaussian noise η. This can be written in the form

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y) \tag{1}$$

where symbol ★ denotes convolution.
In the Fourier transform, we have

$$G(u,v) = H(u,v)F(u,v) + N(u,v). \tag{2}$$

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In the inverse filtering, we have

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}. \quad (3)$$

This formula shows that if $H(u,v)$ is zero or very small in the high frequency region and $N(u,v)$ is still not vanished in the corresponding region, the second term $N(u,v)/H(u,v)$ is amplified. Thus we need a remedy solving ill-posed inverse problem.

Wiener filter requires priori information such as power spectrums of original image and noise. If the noise level is known a priori, Morozov's Discrepancy Principle(: MDP) can be applied for the determination of regularization parameter in the constrained least squares image restoration. Otherwise separate optimization functions such as Generalized Cross Validation(: GCV) function and L-curve criterion are suggested as alternative methods in the literature[4-7].

In this paper, unified Bayesian interpretation of Tikhonov regularization is suggested and applied to the image restoration problems. In section II, square residual and smoothing term in the frequency domain are introduced to solve for Tikhonov regularization parameter. In section III and IV, Bayesian update of Tikhonov regularization parameter is introduced and applied to the image restoration problems. The relationship between Tikhonov regularization parameter and Bayesian hyperparameters is established. Update formular for the regularization parameter using both Maximum A Posteriori(: MAP) and evidence frameworks is suggested. In section V, experimental results show the effectiveness of the proposed method followed by the conclusion and reference sections.

II. Regularization Parameter Selection in the Frequency Domain

In the Tikhonov regularization for an ill-posed problem $\boldsymbol{Hf} = \boldsymbol{g}$, we seek a limiting vector $\hat{\boldsymbol{f}}_\lambda$ to fit data $\boldsymbol{g}$ in least squares sense with penalty term for large normed solution in the cost function as

$$J(\hat{f}) = \frac{1}{2}(\parallel \boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}} \parallel^2 + \lambda \parallel \boldsymbol{C}\hat{\boldsymbol{f}} \parallel^2). \quad (4)$$

Regularized estimation vector is defined as

$$\hat{\boldsymbol{f}}_\lambda = (\boldsymbol{H}^T\boldsymbol{H} + \lambda \boldsymbol{C}^T\boldsymbol{C})^{-1}\boldsymbol{H}^T\boldsymbol{g}. \quad (5)$$

Here, $\boldsymbol{H}$ and $\boldsymbol{C}$ are the block circulant matrices of a PSF h and a smoothing operator p respectively. Smoothing functional usually given by the Laplacian operator. λ is the Tikhonov regularization parameter. $\boldsymbol{g}$ and $\hat{\boldsymbol{f}}_\lambda$ are the column vectors stacking columns of a degraded image g and a recovered image $\hat{f}_\lambda$ respectively. Both $\boldsymbol{H}$ and $\boldsymbol{C}$ matrices have the dimension, MN by MN. That is, $m = n = MN$ with image dimension M by N.

In the frequency domain, block circulant matrices $\boldsymbol{H}$ and $\boldsymbol{C}$ are diagonalized. We denote them $\boldsymbol{S}$ and $\boldsymbol{T}$ respectively. Let $\boldsymbol{z}$ and $\boldsymbol{b}$ be the column vectors of the Fourier transform of $\boldsymbol{f}$ and $\boldsymbol{g}$ respectively.

Regularized estimation vector is defined as

$$\hat{z}_\lambda = (\boldsymbol{S}^H\boldsymbol{S} + \lambda \boldsymbol{T}^H\boldsymbol{T})^{-1}\boldsymbol{S}^H\boldsymbol{b}, \quad (6)$$

where superscript H denotes the Hermitian or conjugate transpose.

Square residual is defined as

$$\parallel \xi_\lambda \parallel^2 = \sum_{i=1}^{n} \frac{\lambda^2 |t_i|^4}{(|s_i|^2 + \lambda |t_i|^2)^2} |b_i|^2. \quad (7)$$

Here, s_i and t_i are the diagonal elements of the matrices $\boldsymbol{S}$ and $\boldsymbol{T}$ respectively. b_i is the element of column vector $\boldsymbol{b}$ stacking columns of 2D Fourier transform of the degraded image g.

Smoothing term is defined as

$$\| \boldsymbol{C}\hat{\boldsymbol{f}}_\lambda \|^2 = \sum_{i=1}^{n} \frac{|s_i|^2 |t_i|^2}{(|s_i|^2 + \lambda |t_i|^2)^2} |b_i|^2. \tag{8}$$

Ⅲ. Unified Bayesian Interpretation of Tikhonov Regularization

MAP estimator maximizes the posterior pdf $p(\boldsymbol{f}|\boldsymbol{g})$ which can be expressed using Bayes′ law[8] as following

$$p(\boldsymbol{f}|\boldsymbol{g}) = \frac{p(\boldsymbol{g}|\boldsymbol{f})\, p(\boldsymbol{f})}{p(\boldsymbol{g})}, \tag{9}$$

$$Posterior = \frac{Likelihood \times Prior}{Evidence}. \tag{10}$$

Assume that both error and original image are Gaussian random vectors, we have

$$p(\boldsymbol{g}|\boldsymbol{f}) = \frac{1}{(2\pi\sigma_\xi^2)^{m/2}} exp\left[- \frac{\| \boldsymbol{Hf} - \boldsymbol{g} \|^2}{2\sigma_\xi^2} \right], \tag{11}$$

$$p(\boldsymbol{f}) = \frac{1}{(2\pi\sigma_{Cf}^2)^{n/2}} exp\left[- \frac{\| \boldsymbol{Cf} \|^2}{2\sigma_{Cf}^2} \right]. \tag{12}$$

By taking the negative log for the Bayes′ law, we have the MAP interpretation of Tikhonov regularization as

$$\lambda = \frac{\sigma_\xi^2}{\sigma_{Cf}^2}. \tag{13}$$

We can interpret the parameter λ as a global scalar proportionality measure analogous to the parametric Wiener filter with setting $\boldsymbol{C}$ to the identity matrix.

In the evidence framework[9], we have fixed-point iteration known as the MacKay update

$$\chi_F^2 \equiv 2\alpha E_F = \gamma, \tag{14}$$

$$\chi_D^2 \equiv 2\beta E_D = m - \gamma \tag{15}$$

or

$$\alpha = \gamma / 2E_F, \tag{16}$$

$$\beta = (m - \gamma)/2E_D. \tag{17}$$

In these equations, α and β are unknown hyper-parameters

$$\alpha = \frac{1}{\sigma_{Cf}^2},\ \beta = \frac{1}{\sigma_\xi^2}. \tag{18}$$

E_F and E_D are regularization and cost terms respectively

$$E_F = \frac{1}{2} \| \boldsymbol{Cf} \|^2, \tag{19}$$

$$E_D = \frac{1}{2} \| \boldsymbol{\xi} \|^2 \tag{20}$$

and γ is the number of effective parameters

$$\gamma = \sum_{i=1}^{n} \frac{\beta\mu_i}{\beta\mu_i + \alpha\nu_i}. \tag{21}$$

Here, μ_i and ν_i denote singular value of $\boldsymbol{H}^T\boldsymbol{H}$ and $\boldsymbol{C}^T\boldsymbol{C}$ respectively.

Then we combine the MAP and the evidence frameworks into the unified Bayesian update formular for the Tikhonov regularization parameter as a fixed point iteration method.

$$\lambda = \frac{\alpha}{\beta} = \frac{\gamma}{m-\gamma}\frac{E_D}{E_F} \tag{22}$$

with

$$\gamma = \sum_{i=1}^{n} \frac{\mu_i}{\mu_i + \lambda\nu_i}. \tag{23}$$

Now we have the unified Bayesian interpretation of Tikhonov regularization.

Ⅳ. A New Image Restoration Method using Unified Bayesian Regularization

We have the Bayesian update formular for the Tikhonov regularization parameter in the frequency domain using equation (7) and (8) as

$$\lambda = \frac{\gamma}{m-\gamma}\frac{\| \xi \|^2}{\| \boldsymbol{Cf} \|^2}, \tag{24}$$

$$\lambda = \frac{\gamma}{m-\gamma}\frac{\sum_{i=1}^{n}\frac{\lambda^2|t_i|^4}{(|s_i|^2+\lambda|t_i|^2)^2}|b_i|^2}{\sum_{i=1}^{n}\frac{|s_i|^2|t_i|^2}{(|s_i|^2+\lambda|t_i|^2)^2}|b_i|^2} \tag{25}$$

with

$$\gamma = \sum_{i=1}^{n} \frac{\mu_i}{\mu_i + \lambda\nu_i} = \sum_{i=1}^{n}\frac{|s_i|^2}{|s_i|^2+\lambda|t_i|^2}. \tag{26}$$

Here, we see that the number of effective parameters is equal to the sum of filter factors[10].

After calculating λ using the fixed point iteration, we obtain $\hat{z}$ as following

$$\hat{z} = (\boldsymbol{S}^H\boldsymbol{S} + \lambda\boldsymbol{T}^H\boldsymbol{T})^{-1}\boldsymbol{S}^H\boldsymbol{b} \tag{27}$$

where superscript H denotes the Hermitian or conjugate transpose.

Reshaping the column vector $\hat{z}$ into 2D matrix to obtain $\hat{F}(u,v)$, we finally get the estimation of original image by taking inverse 2D FFT.

Ⅴ. Experimental Results

We report the experiments with the new image restoration method proposed in the previous two sections. Figure 1 shows the satellite image data from the USAF Phillips Laboratory, Laser and Imaging Directorate, Kirtland AFB, NM[11].

We compare the unified Bayesian(: UB) method with the conventional techniques MDP, GCV, L-curve and Wiener filter. Results are depicted in the figures 2 , 3 and 4. First row shows the restored image data having negative pixel components and the second row further processed by using projection with non-negativity constraint. Visual inspection shows that MDP and L-curve results in an over smoothed estimation with Laplacian smoother and GCV depicts under smoothing with identity matrix.

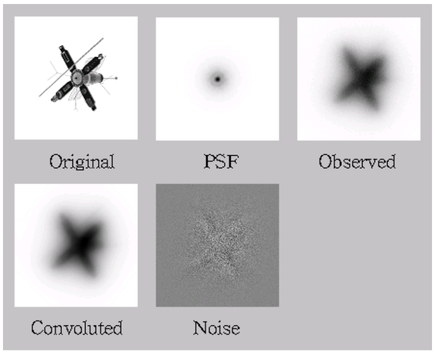

Fig. 1 Satellite image data.

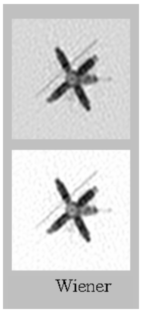

Fig. 2 Restored images with Wiener filter as benchmark.

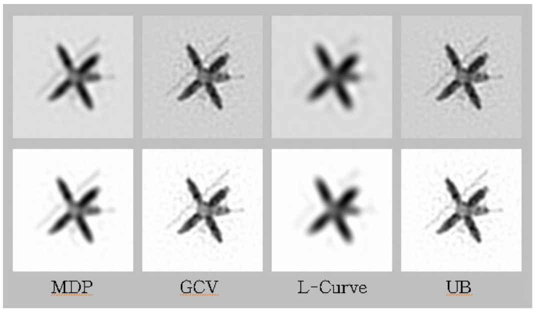

Fig. 3 Restored images with Laplacian smoother.

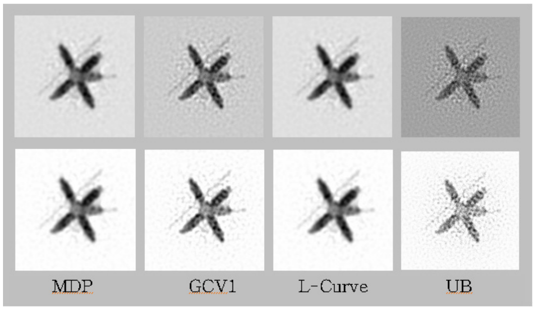

Fig. 4 Restored images with identity smoother.

The new UB method shows comparable results with conventional methods using smoothing operator. However, the UB filter depicts under smoothing with identity matrix that is more severe compared to the GCV method.

Image restoration performance is measured by the figure-of-merit functions such as relative error (RE), signal to noise ratio (SNR), peak SNR (PSNR) and improvement of SNR (ISNR)[7,12-14].

Table 1 and 2 show the image restoration performance with remarking value of regularization parameter λ. Results of Wiener filter is included for comparison benchmark. Here, we report only the restored image data projected with non-negativity constraint. We have the equivalent results as visual inspection on the restored images.

Table 1. Performance Results using Laplacian smoother with Wiener filter as benchmark.

Measure / Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
MDP	0.3803	8.397	22.03	5.356	8.29e-2
GCV	0.3449	9.247	22.87	6.206	4.45e-3
L-curve	0.4509	6.917	20.55	3.876	1.00e-0
UB	0.3489	9.146	22.77	6.106	7.91e-3
Wiener	0.3243	9.781	23.41	6.740	N/A

Table 2. Performance Results with identity smoothing operator.

Measure / Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
MDP	0.3648	8.758	22.39	5.717	4.75e-4
GCV	0.3575	8.935	22.56	5.895	9.80e-5
L-curve	0.3623	8.818	22.45	5.777	4.38e-4
UB	0.5694	4.891	18.52	1.850	1.72e-5

Ⅵ. Conclusions

In this paper, unified Bayesian interpretation of Tikhonov regularization is suggested and applied to the image restoration problems. The relationship between Tikhonov regularization parameter and Bayesian hyper-parameters is established. Update formular for the regularization parameter using both maximum a posteriori and evidence frameworks is suggested. Fixed point iteration can be applied for the regularization parameter. Experimental results show the comparable performance of the unified Bayesian method.

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