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Research Paper

Magnetization of a Modified Magnetic Quantum Dot

Dae-Han Park and Nammee Kim*
Department of Physics, Soongsil University, Seoul 06978, Korea

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Abstract The energy dispersion and magnetization of a modified magnetic dot are investigated numerically. The effects of additional electrostatic potential, magnetic field non-uniformity, and Zeeman spin splitting are studied. The modified magnetic quantum dot is a magnetically formed quantum structure that has different magnetic fields inside and outside of the dot. The additional electrostatic potential prohibits the ground-state angular momentum transition in the energy dispersion as a function of the magnetic field inside the dot, and provides oscillation of the magnetization as a function of the chemical potential energy. The magnetic field non-uniformity broadens the shape of the magnetization. The Zeeman spin splitting produces additional peaks on the magnetization.

Keywords: Magnetization, Magnetic quantum dot, Hybrid quantum structure, Spin effect

I. Introduction

The pioneer work of Kummel et al. [1] showed that de Haas-van Alphen oscillation [2] was not shown in realistic semiconductor quantum wells or wires, because Landau levels can no longer be used to describe the quantized orbit energy, due to the coupling of electric and magnetic confinements. The coupling of electric and magnetic confinements changes the pattern of magnetization. Moreover, magnetization as a function of chemical potential, i.e., the number of electrons in quantum structures, exhibits very pronounced oscillations. Even though many works have been conducted on quantum wells or wires in various geometries of magnetic fields [3], the detailed properties of these quantum oscillations as a function of chemical potentials have not attracted much attention some time. However, many researchers have recently regained interest in the magnetic properties of semiconductor quantum structures due to their spintronic applications [4].

Magnetically confined quantum structures, such as the magnetic quantum dot, magnetic antidot, magnetic quantum ring, and magnetic superlattice, have attracted much attention in the last few decades. These quantum structures are formed by spatially inhomogeneous magnetic fields in a two-dimensional electron gas (2DEG) [5,6]. Recently, the study of 2DEG-based magnetic quantum structures has expanded to 2D-plane-based quantum structures [7-9]. Energy dispersions and wave functions of a modified magnetic quantum dot have been studied by the author's group [10]. Including the modified quantum dot, the energy spectra of electrons and transport properties in the quasi 1-dimensional (1D) wire having these quantum structures in the conductor region, have been actively investigated for other magnetic quantum structures. However, the magnetization of magnetic quantum structures has not been studied to date, due to the lack of application need, and the numerical calculation complexity.

In this paper, we present a numerical study on the magnetization of the modified magnetic quantum dot. We investigate the effects of the additional electrostatic potential energy and Zeeman spin splitting on the quantum oscillations of magnetization as a function of chemical potentials. The modified magnetic quantum dot is formed by spatially in-homogeneous distributions of magnetic fields. Electrons are magnetically confined to the plane where magnetic fields inside and outside of the dot are different, i.e., $(0, 0, B_1)$ for $r < r_0$, and $(0, 0, B_2)$ for $r > r_0$. Here, r_0 is the radius of the dot.

II. Theory: Wavefunctions and Magnetization with the Zeeman Effect

The spin degenerated energy dispersions and wave functions of the modified magnetic quantum dot have previously been studied. To study the effects of spin on the magnetization, we add the Zeeman effects $g^* \mu_B \vec{B} \cdot \vec{\sigma}$ to the Hamiltonian as follows,

$$\left\{ \frac{1}{2m^*}(P + e\vec{A})^2 + V(r) + g^* \mu_B \vec{B} \cdot \vec{\sigma} \right\} \psi(\vec{r}) = E \psi(\vec{r}) \qquad (1)$$

Here, g^* is the electron gyromagnetic factor, μ_B is the Bohr magneton, $\vec{\sigma}$ is the spin operator, and V(r) is the additional electrostatic potential. After solving this

*Corresponding author
E-mail: nammee@ssu.ac.kr

Hamiltonian, the wavefunctions become [10]

$$R_{nm}(r) = C_1(\sqrt[4]{\gamma_1^2+\alpha^2}r)^{\sqrt{m^2+\delta s_0^2}} e^{-\frac{1}{2}\sqrt{\gamma_1^2+\alpha^2}r^2}$$

$$M(a,B;\sqrt{\gamma_1^2+\alpha^2}r^2)(r>r_0)$$

$$a_1 = -\frac{1}{2}\left(\frac{\varepsilon_{nm}-\frac{g^*m^*}{2m_0}}{\gamma_1^2+\alpha^2}-\frac{\gamma m}{\sqrt{\gamma_1^2+\alpha^2}}-\sqrt{m^2+\delta s_0^2}-1\right)$$

$$b_1 = \sqrt{m^2+\delta s_0^2}+1 \qquad (2)$$

$$R_{nm}(r) = C_1(\sqrt[4]{\gamma_2^2+\alpha^2}r)^{\sqrt{m_{eff}^2+\delta s_0^2}} e^{-\frac{1}{2}\sqrt{\gamma_1^2+\alpha^2}r^2}$$

$$U(c,d;\sqrt{\gamma_1^2+\alpha^2}r^2)(r<r_0)$$

$$c = -\frac{1}{2}\left(\frac{\varepsilon_{nm}-\frac{g^*m^*}{2m_0}}{\sqrt{\gamma_2^2+\alpha^2}}-\frac{\gamma m}{\sqrt{\gamma_2^2+\alpha^2}}-\sqrt{m_{eff}^2+\delta s_0^2}-1\right)$$

$$d = \sqrt{m_{eff}^2+\delta s_0^2}+1 \qquad (3)$$

where, $m_{eff} = m+(\gamma_1-\gamma_2)s_0$. All quantities are expressed in dimensionless units by letting $\hbar\omega_0\left(=\frac{\hbar eB_0}{2m^*}\right)$, the inverse length $\beta(=\sqrt{m^*\omega_0/\hbar})$ be 1 and $r_0^2 \to S_0$. While the cyclotron frequency for the external magnetic field B_0 is $\omega_c = \frac{eB_0}{m^*}$, we define the standard frequency $\omega_0 = \frac{eB_0}{2m^*}$, for an easy expression of electrostatic potentials in the familiar way. In these units, $\hbar^2/m^* = \hbar\omega_0/\beta^2 \to 1$, where ω_0 is the standard frequency in units of energy.

The magnetization of an electron system is a thermodynamic state function defined as $M = \left.\frac{\partial F}{\partial B}\right|_{T,N_s}$, where $F = U - TS$ is the free energy of the system, N_s is the electron density, U is the ground state energy, and S is the entropy [11,12]. At T=0 K, the ground state energy U is calculated by summing up the energies of all occupied states, and the magnetization is equal to the average magnetic moment ,$\langle M\rangle = \sum f_\alpha^0\left[-\frac{\partial E_\alpha}{\partial B}\right]$, where f_α^0 is the Fermi distribution function, and E_α are energy eigenvalues of the single electron state characterized by the set of quantum number α [1,3]. For the numerical calculation, we use the parameters of InAs : $m^*/m_0 = 0.0239$, $g^* = -15$.

III. Results and Discussion

1. Effects of electrostatic potentials on energy dispersion

We study the effects of electrostatic potentials on the energy dispersion by including $V(r) = \frac{\delta}{2r^2}m^*\omega_0^2 r_0^4 + \frac{1}{2}m^*\alpha^2\omega_0^2 r^2$ in Eq. (1). Here, δ and α are parameters, and determine the strength of the antidot and the parabolic potential, respectively. The standard cyclotron frequency is $\omega_0 = \frac{eB_0}{2m^*}$.

Figure 1 shows the effects of electrostatic potentials on the energy dispersion of the 2DEG system with various combinations of α, δ as a function of $\gamma = B/B_0$. Figure 1(a) is the case of having parabolic potential only

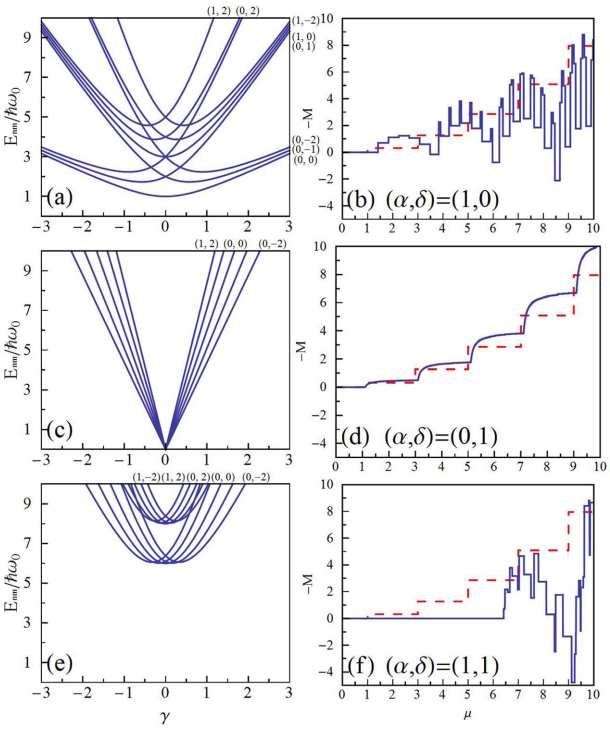

Figure 1. (a), (c), and (e) Eigenenergies for the states (n, m): n =0, 1, and m=–2, –1, 0, 1, 2 of a 2EDG system as a function of γ, and (b), (d), and (f) the corresponding electron magnetization with γ=1, for various combinations of α and δ in dimensionless units.

$(\alpha = 1, \delta = 0)$. Figure 1(c) is the case of having the antidot potential only $(\alpha = 0, \delta = 1)$, while Figure 1(e) is the case of having both parabolic potential and antidot potentials $(\alpha = 1, \delta = 1)$. Figures 1(b), (d), and (f) show the magnetization of electrons in a 2DEG system with various combinations of α, δ as a function of the chemical potential, corresponding to Figures 1(a), (c), and (e), respectively. The dashed lines indicate the magnetization of the conventional 2DEG system. When only the antidot potential is applied (in Figure 1(d)), the magnetization curve is smoothened along the step-like magnetization. However, when the parabolic potential is applied (Figures 1(b) and (f)), oscillations as a function of the chemical potential are produced. Here, we do not show the results of de Haas-van Alphen oscillation, because we know that this oscillation is broken in realistic semiconductor quantum wells or wires, because the quantized orbit energy can no longer be described by Landau levels, due to the coupling of electric and magnetic confinements, as shown in Figure 1. The results show that we can produce negative magnetization at a certain region of the chemical potential by applying the parabolic confinement to electrons in the spin-degenerated 2DEG system.

2. Effects of magnetic field non-uniformity on magnetization

In order to know the effects of magnetic field non-uniformity, we do not include additional electrostatic potentials in the numerical calculation in this section. In

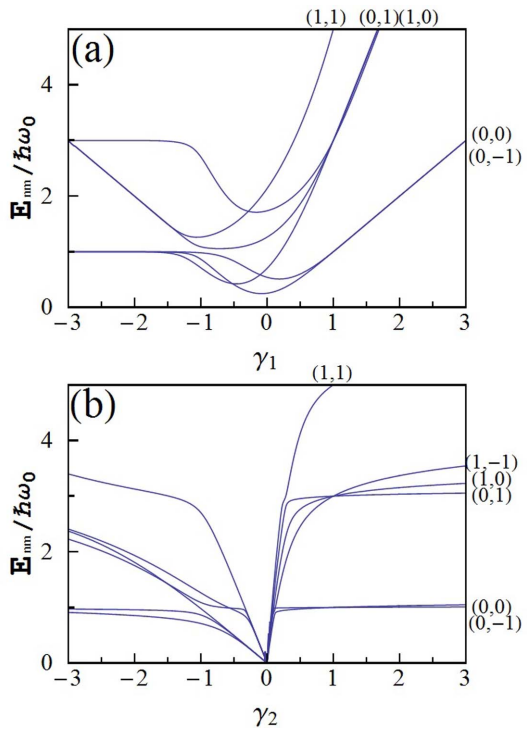

Figure 2. Eigenenergies for the states (n, m): n=0, 1, and m=−1, 0, 1, of a modified magnetic quantum dot as a function of (a) γ_1 (at γ_2=1) and (b) γ_2 (at γ_1=1) with $r_0 = \sqrt{5}$.

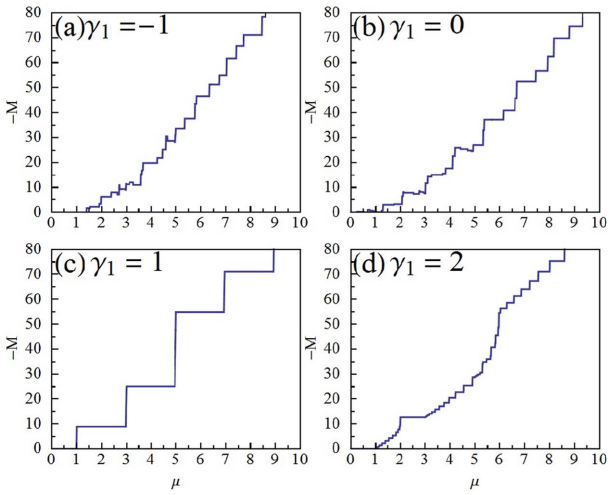

Figure 3. The electron magnetization of a modified magnetic quantum dot with $r_0 = \sqrt{5}$ and μ=4 for various γ_1 in dimensionless units. No additional electrostatic potentials are considered.

Figure 2, we show the effect of magnetic field non-uniformity on the energy dispersion of a modified quantum dot. We plot the low level of states (n, m) to see the trend of the energy spectra. Figure 2(a) shows a fixed the magnetic field outside of the dot $(\gamma_2 = 1)$, changing magnetic field inside of the dot γ_1. The energy dispersion exhibits quite different features depending on the direction of the magnetic fields inside and outside the dot. When $\gamma_1(\gamma_1 > 0)$ increases, (n, m<0) states approach the Landau level of $(2n+1)|\gamma_1|\hbar\omega_0$, while when $|\gamma|(\gamma_1 < 0)$ increases, (n, m<0) states approach the Landau level of $(2n+1)|\gamma_2|\hbar\omega_0$. These results show that as $\gamma_1(\gamma_1 > 0)$ increases, (n, m<0) states are located in the deeper region of the dot and as $|\gamma_1|(\gamma_1 < 0)$ increases, (n, m<0) states are located far away from the dot. Figure 2(b) shows a fixed magnetic field inside of the dot and changing magnetic field outside of the dot γ_2. When both $\gamma_2(\gamma_2 > 0)$ and $|\gamma_2|(\gamma_2 < 0)$ increase, (n, m<0) states are located deeper inside the dot, and approach the Landau level of $(2n+1)|\gamma_1|\hbar\omega_0$. This means that the increase of the magnetic field of outside the dot has no effects on the electrons in ground states.

Figure 3 shows the effect of magnetic field non-uniformity on the electron magnetization of the modified quantum dot. Figure 3(a) shows the case of opposite directions of magnetic fields inside and outside of the magnetic dot. When we calculate, we choose $\gamma_2 = 1$. Figure 3(b) shows the case of the conventional quantum dot. Figure 3(c) shows the case of the conventional 2DEG system. Figure 3(d) shows the case of the quasi-anti magnetic dot that has stronger magnetic field inside of the dot, than outside of the dot. These figures show that the change of magnetic field non-uniformity produces little change in the shape of the magnetization oscillation; however, the magnetic field non-uniformity itself (Figures 3(a), (b), and (c)) has broadened the step-like magnetization of the 2DEG system in Figure 3(c).

3. The Zeeman effects on energy dispersion and magnetization

Figure 4 shows the effects of the Zeeman spin splitting on energy dispersion. The solid lines show the energy dispersion of a modified magnetic dot with parabolic potential with spin degeneracy. This figure appears symmetric for positive and negative γ_1 with low order of states (n, m), but they could demonstrate asymmetric features if high order of states (n, m) are included. However in comparison with Fig. 2(a), it is clear that it shows a less asymmetric energy spectrum. The (0, 0) state

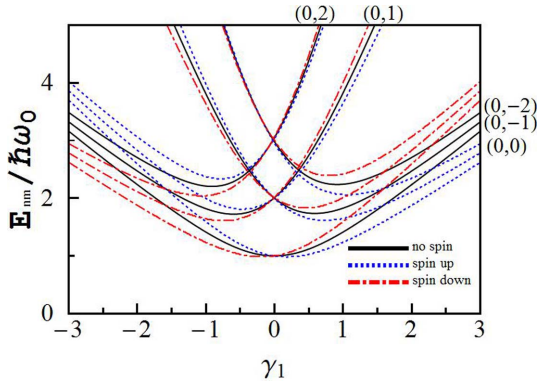

Figure 4. Eigenenergies for the states (*n, m*): *n*=0, and *m*=−2, −1, 0, 1, 2 of a modified magnetic quantum dot as a function of γ_1 with γ_2=1, $r_0 = \sqrt{5}$ and (α,δ)=(1,0) in dimensionless units. Solid lines indicate eigenenergies with no spin, dotted-lines indicate spin-up, and dash-dotted-lines indicate spin-down eigenenergies. For the Zeeman effect, the parameters of InAs of $\frac{m^*}{m_0}$ = 0.0239, g^* = −15 are used.

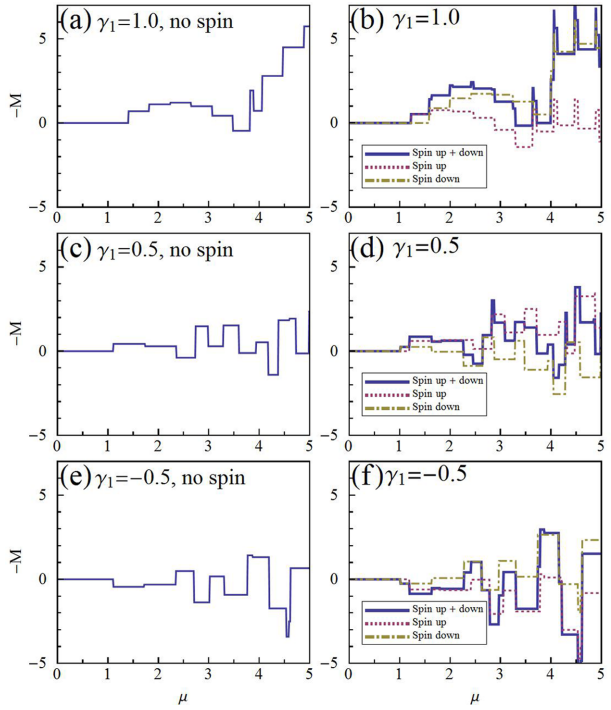

Figure 5. The electron magnetization of a modified magnetic quantum dot with $r_0 = \sqrt{5}$, μ=4 and (α,δ)=(1,0) in dimensionless units. For (a), (c), and (e), there are no spin effects. For (b), (d), and (f), the Zeeman spin splitting is considered.

always becomes the ground state by applying parabolic potential, and electrons have higher energy than in Fig. 2(a), because they gain energy from the additional parabolic potential. The dotted-lines show the spin-up states, while the dash-dotted-lines indicate the spin-down states. Because we use the parameters of InAs, $m^*/m_0 = 0.0239$, $g^* = -15$, and the size of energy splitting is $g^* m^*/m_0 \gamma_1 = 0.3585\gamma_1 \hbar\omega_0$.

Figure 5 shows the electron magnetization of the modified magnetic quantum dot with $r_0 = \sqrt{5}, \mu = 4$, and $(\alpha, \delta) = (1, 0)$ in dimensionless units. Figures 5(a), (c), and (e) show the oscillation as a function of the chemical potential due to the additional parabolic potential. These also show that we can produce negative magnetization at certain regions of the chemical potential without spin effects. With introducing the Zeeman spin splitting, we can see the extra peaks in Figures 5(b), (d), and (f), because each energy state splits to two spin states, and the magnetization is the sum of $\frac{\partial E_\alpha}{\partial B}$ of the spin-up and spin-down energy dispersions. The extra peaks occur at the specific chemical potential values where spin-up and spin-down magnetizations have a large difference.

IV. Conclusion

The energy dispersion and magnetization of a modified magnetic dot with considering additional electrostatic potential and Zeeman spin splitting have been investigated numerically. Negative magnetization at a certain region of the chemical potential can be produced, and applying an additional parabolic electrostatic potential to electrons in the spin-degenerated 2DEG system produces oscillation as a function of the chemical potential. The magnetic field non-uniformity produces little change in the shape of the oscillated magnetization, besides broadening the step-like magnetization shape of the 2DEG system. Introducing Zeeman spin splitting into the calculation, extra peaks are produced in the magnetization of electrons in the modified magnetic quantum dot system.

Acknowledgements

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