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CORRIGENDUM TO "REFLEXIVE PROPERTY ON IDEMPOTENTS" [BULL. KOREAN MATH. SOC. 50 (2013), NO. 6, 1957–1972]

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In [2], Theorem 4.2 is incorrect, and so we here provide a correct theorem and proof.

Theorem 4.2. *If R is a non-Abelian RIP ring of minimal order, then R is of order* 16 *and is isomorphic to* $\mathrm{Mat}_2(\mathbb{Z}_2)$ *or the ring*

$$S = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{Mat}_2\left(\frac{\mathbb{Z}_2[x]}{x^2\mathbb{Z}_2[x]}\right) \mid a, d \in \mathbb{Z}_2 \textit{ and } b, c \in (x + x^2\mathbb{Z}_2[x])\frac{\mathbb{Z}_2[x]}{x^2\mathbb{Z}_2[x]} \right\}.$$

Proof. Let R be a non-Abelian RIP ring of minimal order. Then it is true that R cannot be local since local rings are Abelian. By the Wedderburn-Artin theorem, $R/J(R) \cong \sum_{i=1}^{n} \mathrm{Mat}_{k_i}(D_i)$ for some k_i's and fields D_i's. Here assume that $k_i = 1$ for all i. Then we have there cases of $|J(R)| = 2$, $|J(R)| = 4$, and $|J(R)| = 8$.

If $|J(R)| = 8$, then $R/J(R) \cong \mathbb{Z}_2$ and so R is local, a contradiction. Thus $|J(R)| = 2$ or $|J(R)| = 4$.

Let $|J(R)| = 4$. Then $R/J(R) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Since $J(R)$ is nilpotent, there exist orthogonal nonzero idempotents e_1, e_2 with $e_1 + e_2 = 1$ (i.e., $e_2 = 1 - e_1$) by [3, Proposition 3.7.2], and moreover we have

$$R = \{x + y \mid x \in Id(R), y \in J(R)\},$$

where $Id(R) = \{0, 1, e_1, e_2\}$. Then R is an RIP ring and moreover $R \cong S$ by the proof of [1, Proposition 2.7(1) and Theorem 2.11].

Let $|J(R)| = 2$. Then $R/J(R) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Since $J(R)$ is nilpotent, there exist orthogonal nonzero idempotents e_1, e_2, e_3 with $e_1 + e_2 + e_3 = 1$ by [3, Proposition 3.7.2], and moreover we have

$$R = \{x + y \mid x \in Id(R), y \in J(R)\},$$

where $Id(R) = \{0, 1, e_1, e_2, e_3, 1 - e_1, 1 - e_2, 1 - e_3\}$. For every $r = x + y \in R$ with $x \in Id(R)$ and $y \in J(R)$, we have $e_i r e_j = e_i y e_j$ for $i \neq j$ since $e_i x e_j = 0$. If $e_i y e_j \neq 0$ then $J(R) = \{0, e_i y e_j\}$. Thus $e_j J(R) e_i = 0$, and so $e_j R e_i = 0$

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since $e_j Id(R) e_i = 0$. But since R is RIP, we get $e_i R e_j = 0$ and this yields $e_i y e_j = 0$, a contradiction. Therefore we can conclude that

$$e_i R e_j = 0 \text{ for all } i, j \text{ with } i \neq j.$$

Now suppose that $eRf = 0$ for $e, f \in Id(R)$. Then e and f are orthogonal each other, say $e = e_1$ and $f = e_2$. But since R is RIP, $e_2 R e_1 = 0$ and so we get $e_2 y e_1 = 0$ since $e_2 x e_1 = 0$. This entails

$$r = (e_1 + e_2 + e_3) r (e_1 + e_2 + e_3) = e_1 r e_1 + e_2 r e_2 + e_3 r e_3.$$

Since R is non-Abelian, e_k is non-central for some $k \in \{1, 2, 3\}$. If e_1 is non-central then there exists $s \in R$ such that $e_1 s - s e_1 \neq 0$. Note $e_1 s - s e_1 \in J(R)$. Then we have

$$\begin{aligned} e_1 s - s e_1 &= (e_1 + e_2 + e_3)(e_1 s - s e_1)(e_1 + e_2 + e_3) \\ &= e_1(e_1 s - s e_1)e_1 + e_2(e_1 s - s e_1)e_2 + e_3(e_1 s - s e_1)e_3 = 0, \end{aligned}$$

a contradiction. Each case of (e_2 is non-central) and (e_3 is non-central) also induces a contradiction through a similar computation. The computations for other cases of e and f are also similar, inducing contradictions.

Summarizing, we have two cases of $J(R) = 0$ and $|J(R)| = 4$, and R is isomorphic to either $\mathrm{Mat}_2(\mathbb{Z}_2)$ (when $J(R) = 0$) or the ring S (when $|J(R)| = 4$). □

In [2], Corollary 4.3 is incorrect, and so we here provide a correct expression.

Corollary 4.3. *Let R be a ring with $J(R) = 0$. Then R is a non-Abelian RIP ring of minimal order if and only if R is a non-Abelian semiprime ring of minimal order if and only if R is a non-Abelian reflexive ring of minimal order if and only if R is a non-Abelian right idempotent reflexive ring of minimal order if and only if R is a non-Abelian left idempotent reflexive ring of minimal order.*

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