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The GARCH-GPD in market risks modeling: An empirical exposition on KOSPI

Cheru Atsmegiorgis[1] · Jongtae Kim[2] · Sanghoo Yoon[3]

[1]Department of Statistics, Daegu University
[23]Department of Computer Science and Statistics

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Abstract

Risk analysis is a systematic study of uncertainties and risks we encounter in business, engineering, public policy, and many other areas. Value at Risk (VaR) is one of the most widely used risk measurements in risk management. In this paper, the Korean Composite Stock Price Index data has been utilized to model the VaR employing the classical ARMA (1,1)-GARCH (1,1) models with normal, t, generalized hyperbolic, and generalized pareto distributed errors. The aim of this paper is to compare the performance of each model in estimating the VaR. The performance of models were compared in terms of the number of VaR violations and Kupiec exceedance test. The GARCH-GPD likelihood ratio unconditional test statistic has been found to have the smallest value among the models.

Keywords: ARMA, GARCH, GARCH-GPD, KOSPI, Value at Risk.

1. Introduction

Recent researches, especially in the latter half of 20th century, dealt with the determination of an explicit trade-off between risk and returns. The specific definition of risk is very important when used as the stochastic discount factors for asset pricing, it is equally important to estimate an aggregate measure of risk in portfolio of asset for determination of risk capital. In a financial risk management, the modeling of extreme market risks and its impact are important topics. Extreme market risk is risk due to extreme changes in prices (Ruppert, 2004), e.g. stock market crashes. Although the risk occurs with small probability, it has large financial consequences. The estimation of the daily Value at Risk (VaR) and expected shortfall measures are indispensable to study and understand the risk with respect to the extreme market events.

The aim of this paper is to examine and compare the ability of GARCH (Generalized Auto Regressive Conditional Heteroscedasticity) model with normal, t, generalized hyperbolic innovations and GARCH-EVT (Extreme Value Theory) modeling that are used for modeling

[1] Ph.D. candidate, Department of statistics, Daegu University, Gyeongsan 38453.
[2] Professor, Department of computer science and statistics, Daegu University, Gyeongsan 38453.
[3] Corresponding author: Assistant professor, Department of computer science and statistics, Daegu University, Gyeongsan 38453, Korea. Email: statstar@daegu.ac.kr

the VaR. Moreover a detailed theoretical overview of both traditional VaR models and extreme value theory would be discussed.

Several authors have developed models for analyzing Korean Composite Stock Price Index (KOSPI). Kim and Park (2010) showed the usefulness of GARCH-GPD. Kwon and Lee (2014) analyzed KOSPI200 and futures using VECM-CC-GARCH model and computed a hedge ratio from the estimated conditional covariance-variance matrix. Park and Baek (2014) considered multivariate GARCH models and incorporating risk management measures. Ko and Son (2015) suggested a one-factor model for the Wiener stochastic process decomposed into a systematic and an idiosyncratic risk factor. Kim (2011) analyzed the variation of KOSPI returns using a GARCH-ARJI (auto regressive jump intensity) model. Kim and Bang (2014) employed a regime switching GJR-GARCH model to KOSPI return. Kim (2014) conducted a study on the Copula-GARCH model considering the correlation between stock and bond markets.

In this paper, models considered for producing VaR are like ARMA (Auto Regressive Moving Average) (1,1)-GARCH (1,1) and GARCH-GPD (Generalized Pareto Distribution). In ARMA-GARCH model the mean and variance equations of the returns are allowed to be time varying and these are modeled by ARMA and GARCH models respectively. The innovations are assumed to be either normally, or t and generalized hyperbolic distribution. The other class of models augments the GARCH models with GPD during the periods considered in this paper. The back-testing is a part of the model validation which verifies to what extent actual losses match expected losses. It is a tool that risk managers apply to check how well their forecasts on VaR are attuned. In this paper, dynamic back-testing is used to evaluate the performance of the models. The ugarchfit function and ugarchroll function in 'rugarch' package (Ghalanos, 2015), gpdFit function in 'fExtreme' package (Wuertz, 2013) of R program were used for analysis.

2. Methodology

2.1. GARCH Model

Time series for financial data have typical non-normal characters, such as fat tails, volatility clustering and leverage effect. To describe these features, many different models have been proposed in the econometrics literature. The standard ARCH model was developed by (Engle, 1982) describing volatility dynamics. When the lag of ARCH models became too large, (Bollerslev, 1986) proposed adopting the generalized ARCH, known as the GARCH model. GARCH models have found extraordinarily wide use since they incorporate the two main stylized facts about financial returns, volatility clustering and unconditional non-normality. The most common form of the GARCH model is the GARCH (1,1) defined as;

$$\begin{cases} \gamma_t = \mu + \epsilon_t = \mu + \sigma_t z_t \\ \delta_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_t^2, \end{cases}$$

with $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$, γ_t is the actual return, μ is the expected return, δ_t is the volatility of the return on day t. Hence the conditional volatility today depends on the yesterday innovations ($\epsilon_t = \gamma_{t-1} - \mu_{t-1}$), the yesterday conditional volatility (δ_{t-1}) and the unconditional volatility ω.

ARMA (1,1)-GARCH (1,1) conditional models are used to account for the time varying nature of the mean and the variance of returns. It consists of two equations: a conditional mean equation which specifies the behavior of the returns and a conditional variance equation which describes the dynamic behavior of the conditional variance. The conditional mean equation for a GARCH model for a return is given by:

$$\gamma_t = \mu_t + \epsilon_t,$$

where $\epsilon_t = \delta_t z_t$ and $z_t \sim N(0,1)$ or $z_t \sim$ standardized studen's t distribution or hyperbolic distribution. From this it follows that $E(r_t) = \mu_t$ and $Var(r_t) = \delta_t^2$. For example, if r_t follows ARMA (1,1), then $r_t = \alpha_0 + \alpha_1 r_{t-1} + \beta_1 \epsilon_{t-1} + \epsilon_t$. The randomness in the model comes through the stochastic variables z_t, which are the residuals or the innovations of the process. These residuals are conventionally assumed to be independently and identically distributed and to follow a normal distribution. The GARCH model with normal innovations is fitted using the pseudo maximum likelihood procedure. The original ARCH/GARCH models were based on the normal distribution of the residual terms. However, Blattberg and Gonedes (1974) are some of the researchers that have examined the distributions of return data and they suggest the use of the t distribution in estimation and forecasting of market data, as it features the observed fat tailed properties of market returns. Moreover, there are researches like (Vallena and Askvik, 2014) on GARCH models that consider the generalized hyperbolic distribution for the residual. This paper considers the GARCH model with normal, t and generalized hyperbolic innovations for the residuals.

The normal distribution is a continuous probability distribution and its density function for a normal random variable is defined by a mean and the standard deviation δ as (Walck, 1996). It is commonly denoted as $N(\mu, \delta^2)$. The probability density function of the t distribution is characterized by another parameter termed degrees of freedom (df), related to the variance of the distribution (Aczel and Sounderpandian, 2009). The degrees of freedom parameter is defined by $df = n - 1$, where n is the sample size. The variance related to the df parameter is approximately defined by;

$$Var(f(t)) = \frac{df}{(df-2)} \text{ for } df > 2.$$

The other non-normal distribution considered is generalized hyperbolic distribution. The distribution originates from the standard normal distribution but is modified to incorporate non normality in skewness and kurtosis, which are the two of the parameters to be estimated in the pdf of the distribution (Nam and Gup, 2003). The distribution function of a variable X is defined by:

$$X = A + BY_{g,h(Z)} = A + B\left(\frac{\exp(gz)-1}{g}\right)\exp\left(\frac{hz^2}{2}\right).$$

According to Nam and Gup (2003), the probability function is a two step transformation of a standard normal variable $Z \sim N(0,1)$.The first step transforms the variable, to a random variable Y, such that it is only defined by the parameters g (skewness) and h (kurtosis), defined $Y_{g,h}$ (Z). The second step involves specifying the mean and variance, A and B respectively of the distribution. The kurtosis of the distribution is defined by two parameters; the shape parameter which is a measure of peakedness and lambda which is a direct measure of the tail fatness inherent in the data (Fajardo *et al*, 2005). This sums up to a total of five parameters that need to be estimated; location, scale, skewness, shape and lambda.

2.2. Generalized Pareto distribution

The Peak Over Threshold (POT) method is employed observations that exceed a given threshold u constitute extreme events. Considering the excess distribution above, the threshold u given by:

$$F_u(y) = P\left(\frac{X - U \leq y}{x > u}\right) = \frac{F(y+u) - F(u)}{1 - F(u)}.$$

The Generalized Pareto Distribution (GPD) describes the limiting distribution for modeling excesses over a certain threshold. If X is a random variable which is generalized Pareto distributed, then its distribution function has the form;

$$G_{\gamma,\beta(x)} = \begin{cases} 1 - \left(1 - \frac{\gamma x}{\beta}\right)^{-\frac{1}{\gamma}} & \text{if } \gamma \neq 0, \\ 1 - \exp\left(-\frac{x}{\beta}\right) & \text{if } \gamma = 0. \end{cases}$$

2.3. GARCH-GPD Model

The two step procedure suggested by McNeil (1999) involves a combination of the GARCH conditional model with the extreme value model. The idea is to first filter the returns, by applying the GARCH model, to obtain approximately independent and identically distributed residuals that can then be used in extreme value. The model used in this two-step procedure is given below.

$$r_i = \mu_i + \delta_i z_i,$$

where γ_i is the return at time i (with historical data available for $i = 1, 2, \cdots, t$, and z_i are random variables distributed i.i.d., and μ_i and δ_i are the mean and standard deviation of the i^{th} return. First step: Filter the observation γ_i by fitting models for the conditional variance and conditional mean.The residual from this fit has the form

$$z_i = \frac{r_i - \mu_i}{\delta_i}.$$

It is assumed (McNeil, 1999) that the residuals z_i are approximately independent and identically distributed. Second Step: Apply extreme value theroy for these residuals. That is, z_i takes the palce of γ_t in the extreme value distribution. Briefly this can be summarized as follows;

(1) The absolute exceedance above a certain threshold are calculated;

$$\widehat{z}_i - u / \widehat{z}_i > u \text{ for } i = 1, 2, \cdots, t.$$

(2) A GPD model is fitted to these exceedances and the maximum likelihood estimates of the parameters are obtained. The estimated distribution of the residuals is then given by

$$\widehat{F}_Z(Z) \approx 1 - \frac{N_u}{n}\left(1 + \frac{\widehat{\xi}(Z - \widehat{\mu})}{\widehat{\delta}}\right)^{-\frac{1}{\xi}}.$$

(3) VaR is the $(1-\alpha)^{\text{th}}$ percentile of $\hat{F}_z$, that means the estimate is

$$\widehat{Var}_\alpha = \widehat{z}_{1-a} \approx \widehat{\mu} + \frac{\widehat{\delta}}{\widehat{\xi}}\left(\left[\frac{n}{N_u}(\alpha)\right]^{-\widehat{\xi}} - 1\right).$$

Since the variable of interest stock price index is provided on the daily basis, the two step procedure will have to be repeated each day. That is, new parameter estimates will be calculated each day. The estimates for VaR of the model are therefore dynamic. A dynamic estimate of VaR for day t+1 from the above two step procedure is

$$\widehat{VaR}_{t,\alpha} = \widehat{\mu}_t + \widehat{\delta}_t \widehat{VaR}_\alpha(\widehat{z}).$$

2.4. Adequacy of VaR measures

Christoffersen (1998) described that the problem of determining the accuracy of a VaR model can be reduced to the problem of two properties which are unconditional coverage property and independence property. The unconditional coverage property places a restriction on how often VaR violations may occur. The independence property places a strong restriction on the ways in which these violations may occur.

Kupiec (1995) suggested the unconditional coverage test based on the numbers of exceedances of the VaR estimate is proportional to the expected number of exceedances. The null hypothesis of the test is

$$H_0 : p = \widehat{p} = \frac{x}{T},$$

where p is the given failure rate corresponding to the confidence level c of the VaR model, i.e. $p = (1-c)$ and $\widehat{p}$ is equal to the observed failure rate, i.e.the number of exceedances (x) divided by the sample size (T).

Christoffersen and Pelletier (2004) defined where the hit sequence follows a first order Markov sequence with switching probability matrix

$$P = \begin{bmatrix} 1-p_{01} & p_{01} \\ 1-p_{11} & p_{11} \end{bmatrix},$$

where p_{ij} is the probability of an i on day $t-1$ being followed by a j on day t.

The test of independence is

$$H_{0,ind} : p_{01} = p_{11}.$$

Finally a test of conditional coverage is

$$H_{0,ind} : p_{01} = p_{11} = p.$$

The test is set up as a likelihood ratio test. Kupiecs unconditional coverage test statistic and Christoffersen converage test statistic are defined as

$$LR_{uc} = -2\ln\left(\frac{(1-p)^{T-x}p^x}{\left[1-\frac{x}{T}\right]^{T-x}\left(\frac{x}{T}\right)^x}\right),$$

$$LR_{cc} = -2\ln\left(\frac{(1-p_{01})^{T_{01}-x_0}p_{01}^{x_0}(1-p_{11})^{T_{11}-x_1}p_{11}^{x_1}}{\left[1-\frac{x_0}{T_{01}}\right]^{T_{01}-x_0}\left(\frac{x_0}{T_{01}}\right)^{x_0}\left[1-\frac{x_1}{T_{11}}\right]^{T_{11}-x_1}\left(\frac{x_1}{T_{11}}\right)^{x1}}\right).$$

3. Data and analysis

3.1. Data

The log returns of the KOSPI from 05-01-1998 to 04-01-2016 are considered in this paper. The Figure 3.1 shows the stock price index values over the period 05-01-1998 to 04-01-2016. The log returns are plotted in Figure 3.2. Some basic statistics pertaining to the data set are summarized in Table 3.1.

As seen from Figure 3.1, KOSPI is getting increasing with time. Thus, we need to make the distribution stationary using log return transformation on the KOPSI.

Figure 3.1 KOSPI over the period 05-01-1998 to 04-01-2016

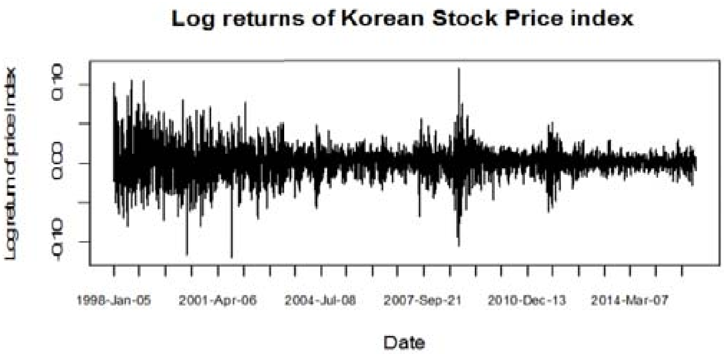

Figure 3.2 Log returns of KOSPI over the period 05-01-1998 to 04-01-2016

The Figure 3.2 depicts how log returns of KOSPI varies with time and displays an illustration of volatility clustering by transcending.

The plot of the autocorrelation function (ACF) of the log returns in Figure 3.3 shows few significant autocorrelation, providing evidence of the stationarity of the series. However, the ACF of the squared returns and the absolute returns are highly significant for all lags and decay slowly. This is a result of the volatility clustering effect and provides evidence of the existence of ARMA/GARCH effects in the time series. The log returns series is a suitable candidate for GARCH models. The Dickey-Fuller test (Table 3.1) also indicates that the series can be assumed to be stationary. First order differences of the log returns are computed, which is also a common method to achieve stationarity in a time series. Although the series is stationary, it does not follow a normal distribution, as indicated by the large excess kurtosis (4.92) and negative skewness (-0.17). The negative skewness is to be expected for an index of share prices since extreme negative returns are more likely than extreme positive returns.

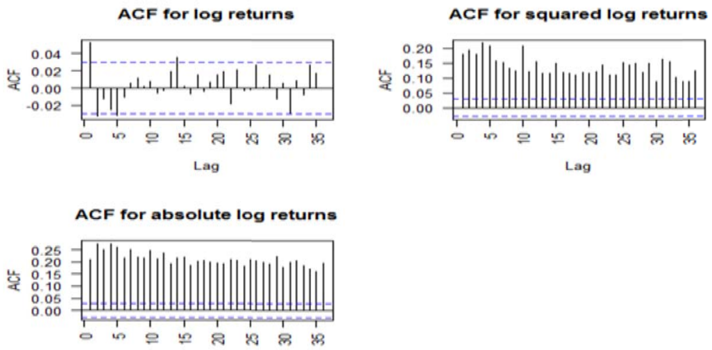

Figure 3.3 Autocorrelation Function (ACF) for log return, squared log returns and absolute log return

Table 3.1 Descriptive statistics for the log returns of KOSPI

Measures	Values of Descriptive measure
Mean	0.0004
Standard deviation	0.018
Excess Kurtosis	4.923
Skewness	-0.179
Dickey-Fuller Unit Root test	Dickey-Fuller=-15.312, p-value=0.01

3.2. The results of parameter estimation

The Table 3.2 depicted GARCH-GPD model and GARCH model fit with student t, normal and generalized hyperbolic distribution innovations for errors. For the t distribution, the shape parameter is related to the degrees of freedom parameter which determines the shape (kurtosis) of its probability density function. For the generalized hyperbolic distribution, the shape of the distribution is determined by two parameters; the shape parameter which measures the peakedness, and the GHlambda parameter which relates to the tail fatness of the probability density function (Fajardo *et al*, 2005). The parameters of the models are estimated using the maximum likelihood method.

Table 3.2 GARCH with student t, normal and generalized hyperbolic distribution for errors model fitting

parameters	GARCH-Normal		GARCH-t		GARCH-Gene. Hyp.	
	est.	s.e.	est.	s.e.	est.	s.e.
μ	0.001**	(0.000)	0.001***	(0.000)	0.001*	(0.000)
AR(1)	-0.259	(0.368)	-0.515	(0.274)	-0.622***	(0.176)
MA(1)	0.299	(0.363)	0.543*	(0.269)	0.643***	(0.173)
α_0	0.000	(0.001)	0.000	(0.000)	0.001	(0.000)
α_1	0.071***	(0.012)	0.065***	(0.007)	0.064***	(0.024)
β_1	0.928***	(0.012)	0.934***	(0.007)	0.935***	(0.024)
Shape			7.214***	(0.756)	0.250	(0.274)
Skew					-0.087***	(0.022)
Ghlambda					2.278***	(0.301)

*p<0.05, **p<0.01, ***p<0.001

The parameters for the conditional variance (β_1) and the previous days squared residuals (α_1) in Table 3.2 is highly significant for the normal, t and generalized hyperbolic distribution, indicating that including the dependence of previous days conditional variance and the previous days shock to the volatility makes the GARCH models estimation of variance more accurate. For normal innovation the inclusion of the previous days shock would make the estimation of the GARCH model more accurate. While the shape parameter, measuring kurtosis, is significant for the t distribution only and the GHlambda parameters which measures the tail fatness is significant for the generalized hyperbolic distribution. This would indicate that including a kurtosis parameter for t distribution, tail fatness parameter for generalized hyperbolic distribution has great explanatory power and makes the conditional variance estimate more precise.

Table 3.3 GARCH-GPD and GARCH with student t, normal and generalized hyperbolic distribution for errors model fitting (Threshold = 0.99)

GARCH-GPD	Estimate	Standard Deviation
shape parameter (γ)	-0.024	0.412
shape parameter (β)	0.014	0.003

3.3. Value at Risk back testing

Back-testing in VaR is a technique used to compare the predicted losses from the calculated VaR with the actual losses realized. As is generally done in forecasting practices today, we have also evaluated the results of the VaR back-testing by using the estimated models in an out-of-sample period. This is done in risk management practices in order to see how well the model performs in a sample period not used to estimate the model parameters, since we would expect the model to fit the estimation sample quite reasonably (Brooks, 2008). Thus, the in-sample period is the sample period used to estimate the models parameters and the forecast length period is the sample period held back for the forecast evaluation of the model. It is recommended to take a large proportion of sample for model parameter estimations. Therefore, the sample sizes of 3,000 and 1,513 have been used for parameter estimation and VaR forecast. The outcome of our back-testing with a rolling window is illustrated below in Fig 4. It shows the number of exceedances for each VaR model, with an alpha level set at one percent for the back-testing conducted in the specified period. All the returns are plotted and, as observed, some observations have returns lower than the VaR level.

As seen from Figure 3.4 and Table 3.4, the GARCH model with normal distribution and t distribution errors accumulate the actual exceedance of 14 and 21 respectively over the back-testing period length of 1,513. The expected exceedance for both models is 15.1 indicating a 1.4 % actual percentage for normal error and 0.9% actual percentage for t distributed error. The VaR model with generalized hyperbolic error term accumulate 8 exceedances that is a 0.5% actual percentage, while the VaR model with GARCH models augmented with GPD model generated 14 exceedances which is 0.9% actual percentage.

Thus, we have some indications that VaR models with normally distributed errors accumulates a larger number of exceedances and the VaR model with generalized hyperbolic generated the smallest number of exceedance compared to other models. However, it is depicted in the Kupiec unconditional coverage test, the generalized hyperbolic error underestimate the VaR and the exceedance are proved to be incorrect. Regarding normal distribution, there is some indication that errors for VaR do not perform well. The VaR model for symmetric GARCH-GPD model accumulate the actual exceedance of 14 exceedances over the back-testing period. The expected exceedance is 15.1 indicating a 0.9% actual percentage.

Table 3.4 shows that comparison of the VaR model for GARCH with t, normal and generalized hyperbolic distribution error and VaR model for symmetric GARCH models augmented with GPD models depicted that the normal distribution error accumulates higher number of actual exceedance or higher percentage of exceedance (1.4%) VaR to exceed the return level more frequent in terms of rapidly changing variance. Considering the number of exceedances, we found that VaR model for GARCH-GPD and GARCH-t for error term accumulates equal number of actual VaR exceedance (0.9%).

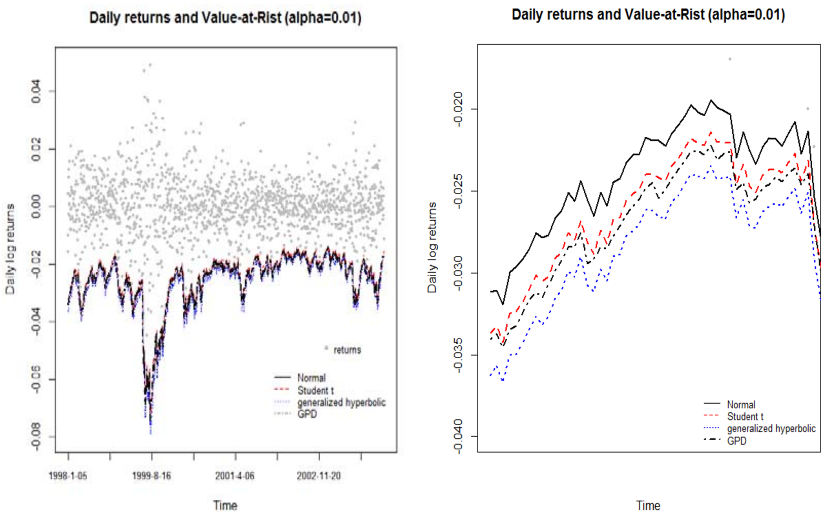

Figure 3.4 VaR back-testing in the left and the first 50 VaR back-testing in the right

Table 3.4 VaR back-testing and Kupiec test for GARCH with normal, t and generalized hyperbolic distribution for error and GARCH-GPD

Back-testingEvaluation	GARCH-Normal	GARCH-t	GARCH-Gene. Hyp.	GARCH-GPD
Length	1,513	1,513	1,513	1,513
Expected Exceed	15.1	15.1	15.1	15.1
VaR Exceed	21	14	8	14
Actual %	1.4%	0.9%	0.5%	0.9%

We can observe that all the models except the normal model matched the expected number of exceedance, whereas the normal is deviating. Table 3.5 showed that we cannot reject the normally distributed error model even though its exceedances are higher than the expected number. The unconditional coverage test depicted that the null hypothesis does not hold for generalized hyperbolic distribution. Therefore, we can conclude that GARCH-t and GARCH-GPD do better in forecasting the VaR in the specified back test length. Furthermore, the unconditional covergrage test depicts that the GARCH-GPD has smaller test statistic compared to GARCH-t and this makes it relatively better to forecast VaR.

Table 3.5 Kupiec test for GARCH with normal, t and generalized hyperbolic distribution for error and GARCH-GPD

Null Hypothesis: Correct exceedance				
Distribution	GARCH-Normal	GARCH-t	GARCH-Gene. Hyp.	GARCH-GPD
LR. uc Statistic	2.052	0.318	4.098*	0.087
P-value	0.152	0.573	0.043	0.767

4. Conclution

The aim of this paper is to compare performance of GARCH and GARCH-GPD models on the KOSPI data. In both models, the maximum likelihood estimation has been employed to estimate model parameters. The performance of GARCH and GARCH-GPD models were compared in terms of the number of VaR violations. The augmentation of the GARCH with GPD model and GARCH model with t, generalized hyperbolic innovations for error term were investigated. The comparison was in terms of the number of VaR violations. Based on the closeness of the actual number of violations to the expected number of violations, we can conclude that GARCH-t and GARCH-GPD model do better in forecasting the VaR in the specified back test length period.

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