Energy-Efficient Power Allocation for Cognitive Radio Networks with Joint Overlay and Underlay Spectrum Access Mechanism

Jiakuo Zuo, Li Zhao, Yongqiang Bao, and Cairong Zou

Traditional designs of cognitive radio (CR) focus on maximizing system throughput. In this paper, we study the joint overlay and underlay power allocation problem for orthogonal frequency-division multiple access–based CR. Instead of maximizing system throughput, we aim to maximize system energy efficiency (EE), measured by a "bit per Joule" metric, while maintaining the minimal rate requirement of a given CR system, under the total power constraint of a secondary user and interference constraints of primary users. The formulated energy-efficient power allocation (EEPA) problem is nonconvex; to make it solvable, we first transform the original problem into a convex optimization problem via fractional programming, and then the Lagrange dual decomposition method is used to solve the equivalent convex optimization problem. Finally, an optimal EEPA allocation scheme is proposed. Numerical results show that the proposed method can achieve better EE performance.

Keywords: Energy efficiency, power allocation, OFDMA, cognitive radio, fractional programming.

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I. Introduction

Due to the advantages of spectrum utilization, cognitive radio (CR) has garnered considerable attention [1]–[2]. According to the known ways of utilizing the licensed primary users' (PUs') spectrums, CR techniques can be classified into underlay CR and overlay CR. In overlay systems, unlicensed secondary users (SUs) are allowed to utilize idle spectrum bands. In underlay systems, PUs and SUs are allowed to coexist in the same spectral band but under tolerable transmit power constraints [3]–[4].

There have been many researches on overlay and underlay CR systems. In this paper, we focus on the researches of orthogonal frequency-division multiple access (OFDMA)–based CR networks. For overlay OFDMA–based CR networks, in [4], a fast barrier method was first proposed to obtain the optimal resource allocation strategy with reasonable complexity, and then a simple heuristic resource allocation method with lower complexity, which can approximate the optimal solution, was also presented. Reference [5] studied the power allocation problem for OFDMA-based CR systems with statistical interference constraints. Reference [6] presented a new design formulation for joint subcarrier assignment and power allocation in OFDMA ad hoc CR networks. For underlay OFDMA–based CR networks, [7] proposed an iterative partitioned water-filling algorithm and a recursive power allocation algorithm to obtain the optimal power allocation, respectively. Reference [8] analyzed the achievable capacity of a secondary service over a fading environment based on a primary network. However, the above works study the resource allocation problem for either overlay or underlay OFDMA–based CR networks. Recently, a hybrid spectrum sharing model has been proposed that considers both overlay and underlay spectrum access mechanisms. In [9]–[10], the resource allocation for OFDMA-based CR with a joint overlay and underlay spectrum access mechanism was studied; the optimal and suboptimal algorithms to solve the resource allocation problem were also presented. In [11], by creating a soft decision CR to exploit the unused and underused spectral regions, a novel hybrid waveform to combine both the underlay waveform and the overlay waveform was proposed.

We find that all the above researches focus on improving the throughput of the respective CR networks. However, energy efficiency (EE) is also an important issue in CR networks. Because of the exponential traffic growth with the popularity of smartphones and the limited energy supply with higher prices, energy-efficient wireless communications have drawn increasing attention recently [12]–[13]. In [14], a water-filling factor–aided search method was proposed to solve the energy-efficient power allocation (EEPA) problem for OFDMA-based CR networks. In [15], the EEPA problem was addressed via parametric programming, and then an iterative algorithm was presented. Reference [16] studied the EEPA in OFDMA-based CR with cooperative relay and proposed a barrier method to solve the power allocation problem. Considering the minimal throughput requirements and proportional fairness of CR users, [17] proposed a bisection-based algorithm to solve the EEPA problem. Reference [18] studied both the subcarrier allocation problem and the power allocation problem for OFDMA-based CR networks from an EE perspective and developed an efficient fast barrier method to find the optimal solution of the resource allocation problem. Reference [19] studied the energy-efficient opportunistic spectrum access strategies for OFDMA-based CR networks with multiple SUs and developed optimal and suboptimal methods to solve the above problem. In [20], the throughput and EE optimization under quality-of-service constraints for MIMO-based CR systems are studied. In [21], a promising framework of spectrum sharing strategy selection based on EE was proposed for MIMO-based CR interference channels.

In this paper, we study the EEPA problem for OFDMA-based CR networks with a joint overlay and underlay spectrum access mechanism. Our aim is to maximize the CR system's EE, measured by "bit per Joule" metric, while maintaining the minimal rate requirements of the system, under the total power constraint of an SU and interference constraints of PUs. However, the above problem is non-convex. To make it solvable, first, an equivalent convex problem is derived based on fractional programming (FP) [22]. Then, the Lagrange dual decomposition method [23] is used to find the optimal solution of the equivalent convex problem. At last, an efficient iterative algorithm is proposed to solve the joint overlay and underlay EEPA problem for OFDMA-based CR networks.

The rest of this paper is organized as follows. Section II provides both the system model and the optimization problem. In Section III, the new optimal EEPA scheme is presented. Simulation results are presented in Section IV, and conclusions are drawn in Section V.

II. Signal Model and Problem Statement

Consider an OFDMA-based CR network where an SU coexists with a group of PUs in side-by-side bands. Each SU refers to a secondary user transmitter (SUT) and a secondary user receiver (SUR) link. Similarly, each PU refers to a primary user transmitter (PUT) and a primary user receiver (PUR) link. The system model is shown in Fig. 1. The whole available bandwidth, W, is divided into N subchannels, each with bandwidth $\Delta f = W/N$, and the SU is allowed to use the whole spectrum. Without loss of generality, assume that there are L subchannels that are underutilized by PUs (that is, *underlay* subchannels) and K subchannels that are unused by PUs (that is, *overlay* subchannels), where $L + K = N$. As shown in Fig. 2, the K overlay subchannels and L underlay subchannels are distributed side by side. The total number of available subchannels for the SU is N.

Let p_n and h_n denote the transmit power of the SU and the channel fading gains between SUT and SUR on the nth subchannel, respectively; the transmission rate of the SU on the nth subchannel can be expressed as

$$r_n = \Delta f \log_2\left(1 + \frac{|h_n|^2 p_n}{\sigma_0^2 + \varUpsilon_n}\right), \tag{1}$$

where σ_0^2 is the additive white Gaussian noise variance and $\varUpsilon_n$ is the interference power introduced by all PUTs to the

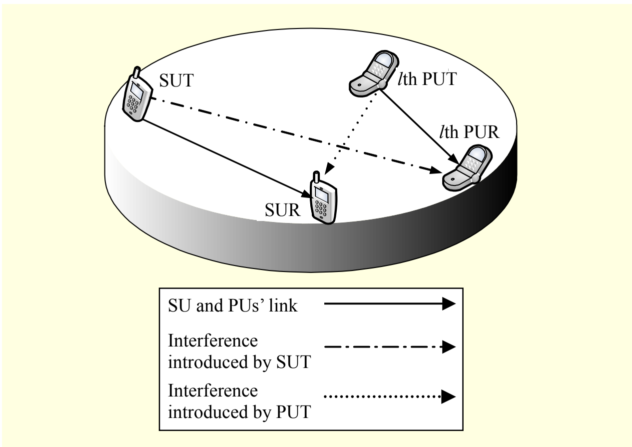

Fig. 1. System model.

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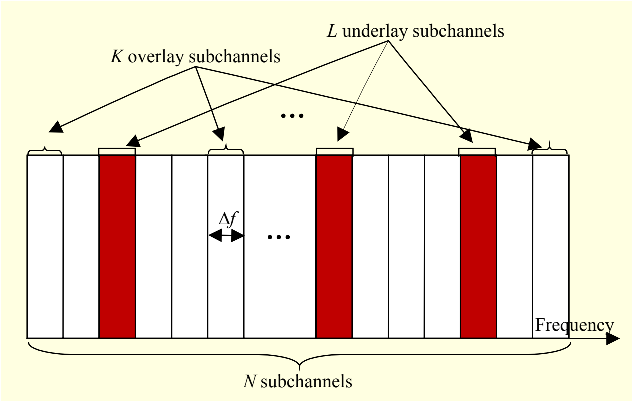

Fig. 2. Distribution of underlay and overlay subchannels in considered CR system.

SUR on the nth subchannel, which is defined as follows [24]–[25]:

$$\Upsilon_n = \sum_{l=1}^{L} \left| h_{l,n}^{\mathrm{PS}} \right|^2 \int_{d_{n,l}-\Delta f/2}^{d_{n,l}+\Delta f/2} E_{\mathrm{pds}}(w)\, \mathrm{d}w, \tag{2}$$

where $h_{l,n}^{\mathrm{PS}}$ denotes the channel fading gains from the lth PUT to the SUR on the nth subchannel, $d_{n,l}$ denotes the spectral distance between the nth SU subchannel and the lth PU subchannel, $E_{\mathrm{pds}}(w)$ is the power density spectrum of the PU signal, and w is the normalized frequency. Then, the total transmission rate of the SU is defined as follows:

$$R(\mathbf{p}) = \Delta f \sum_{n=1}^{N} \log_2\left(1 + \frac{|h_n|^2 p_n}{\sigma_0^2 + \Upsilon_n}\right), \tag{3}$$

where $\mathbf{p} = [p_1, p_2, \ldots, p_N]$.

The total interference to the lth PU introduced by SU is [9]–[10]

$$I_l = \sum_{n=1}^{N} p_n \left| h_{l,n}^{\mathrm{SP}} \right|^2 \vartheta_{l,n}, \tag{4}$$

where $\vartheta_{n,l} = T_{\mathrm{S}} \int_{d_{n,l}-\Delta f/2}^{d_{n,l}+\Delta f/2} \left(\frac{\sin(\pi f T_{\mathrm{S}})}{\pi f T_{\mathrm{S}}} \right) \mathrm{d}f$, $h_{l,n}^{\mathrm{SP}}$ denotes the channel fading gains from the SUT to the lth PUR on the nth subchannel, f denotes frequency, and T_{s} is the symbol duration.

The total power consumption of SUT is modeled as

$$P^{\mathrm{total}}(\mathbf{p}) = \tau \sum_{n=1}^{N} p_n + P^{\mathrm{c}}, \tag{5}$$

where τ denotes the power amplifier efficiency and P^{c} denotes the power consumption of circuits and base station facilities.

The EE of the SU can be defined as

$$\zeta^{\mathrm{EE}}(\mathbf{p}) = \frac{\Delta f \sum_{n=1}^{N} \log_2\left(1 + \frac{|h_n|^2 p_n}{\sigma_0^2 + \Upsilon_n}\right)}{\tau \sum_{n=1}^{N} p_n + P^{\mathrm{c}}}. \tag{6}$$

In this paper, the power allocation problem is to maximize the EE of the SU. Therefore, the EEPA problem is formulated as

$$\begin{aligned} \mathrm{OP1}:\ & \max_{\mathbf{p}\geq 0} \frac{\Delta f \sum_{n=1}^{N} \log_2\left(1 + \frac{|h_n|^2 p_n}{\sigma_0^2 + \Upsilon_n}\right)}{\tau \sum_{n=1}^{N} p_n + P^{\mathrm{c}}}. \\ \text{s.t.}\ & \mathrm{C}_1: \sum_{n=1}^{N} p_n \leq P_{\max}, \\ & \mathrm{C}_2: \sum_{n=1}^{N} p_n \left| h_{l,n}^{\mathrm{SP}} \right|^2 \vartheta_{l,n} \leq I_l^{\mathrm{th}}, \forall l \in \mathfrak{L}, \\ & \mathrm{C}_3: \Delta f \sum_{n=1}^{N} \log_2\left(1 + \frac{|h_n|^2 p_n}{\sigma_0^2 + \Upsilon_n}\right) \geq R_{\min}, \end{aligned} \tag{7}$$

where $P_{\max}$ is the total power budget of SU; I_l^{th} is the interference threshold of the lth PU; $\mathfrak{L}$ is the set of underlay subchannels; $R_{\min}$ is the minimal rate requirement of the SU; C_1 is the transmission power constraint; C_2 is the interference constraints of the PUs; and C_3 guarantees the target rate requirement of the SU.

Since OP1 is not convex, it is difficult to solve it directly. To make it solvable, OP1 is first transformed into a convex problem, and then an iterative EEPA method is proposed.

III. Optimal EEPA Algorithm

In this section, we first use FP [22] to transform OP1 into a convex optimization problem, and then we provide the equivalent conditions. Subsequently, an optimal iterative algorithm is proposed.

Now, we define a new objective function to be

$$\phi(\mathbf{p}, \gamma) = R(\mathbf{p}) - \gamma P^{\mathrm{total}}(\mathbf{p}), \tag{8}$$

where γ is a positive parameter.

We introduce another optimization problem as follows:

$$\begin{aligned} \mathrm{OP2}:\ & \max_{\mathbf{p}\in\Theta} \phi(\mathbf{p}, \gamma) \\ \text{s.t.}\ & \mathrm{C}_1: \sum_{n=1}^{N} p_n \leq P_{\max}, \\ & \mathrm{C}_2: \sum_{n=1}^{N} p_n \left| h_{l,n}^{\mathrm{SP}} \right|^2 \vartheta_{l,n} \leq I_l^{\mathrm{th}}, \forall l \in \mathfrak{L}, \\ & \mathrm{C}_3: \Delta f \sum_{n=1}^{N} \log_2\left(1 + \frac{|h_n|^2 p_n}{\sigma_0^2 + \Upsilon_n}\right) \geq R_{\min}. \end{aligned} \tag{9}$$

Let $\varphi(\gamma)=\max_{\mathbf{p}\in\Theta}\phi(\mathbf{p},\gamma)$, where Θ denotes the feasible region of OP1 or OP2. Now, let us assume $\mathbf{p}^*=\arg\max_{\mathbf{p}\in\Theta}\frac{R(\mathbf{p})}{P^{\text{total}}(\mathbf{p})}$; thus, we can define the optimal value of OP1 as

$$\gamma^*=\frac{R(\mathbf{p}^*)}{P^{\text{total}}(\mathbf{p}^*)}=\max_{\mathbf{p}\in\Theta}\frac{R(\mathbf{p})}{P^{\text{total}}(\mathbf{p})}. \tag{10}$$

To relate OP1 and OP2, the following theorem is introduced [22]:

Theorem. The optimal solution $\mathbf{p}^*$ achieves the optimal value γ^* of OP1, if and only if

$$\begin{aligned}\varphi(\gamma^*)&=\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma^*P^{\text{total}}(\mathbf{p})\right\}\\&=R(\mathbf{p}^*)-\gamma^*P^{\text{total}}(\mathbf{p}^*)\\&=0,\end{aligned} \tag{11}$$

where $R(\mathbf{p})\geq 0,\ P^{\text{total}}(\mathbf{p})>0.$

Proof. See Appendix.

From the theorem, we can see that OP1 can be transformed into an equivalent problem OP2 with γ^*, which has the same optimal solution. Therefore, to solve OP1 is to find the optimal solution of OP2 for a given parameter γ and then update γ until the theorem is fulfilled. Thus, we can focus on solving the equivalent problem OP2.

In the following, we use the Lagrange dual decomposition method [23] to solve OP2 for a given γ. The Lagrange dual function of the primal problem OP2 can be written as

$$D\left(\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right)=\max_{\mathbf{p}\geq 0}\text{La}\left(\mathbf{p},\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right), \tag{12}$$

where $\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}$ are dual variables and $\text{La}\left(\mathbf{p},\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right)$ is the Lagrange function, which is defined as

$$\begin{aligned}&\text{La}\left(\mathbf{p},\{\theta_i\}_{i=1}^{5},\{\alpha_n\}_{n=1}^{N}\right)\\&\quad=\sum_{n=1}^{N}\Delta f\log_2\left(1+\frac{|h_n|^2p_n}{\sigma_0^2+\Upsilon_n}\right)-\gamma\left(\tau\sum_{n=1}^{N}p_n+P^{\text{c}}\right)\\&\quad-\alpha_0\left(\sum_{n=1}^{N}p_n-P_{\max}\right)-\sum_{l=1}^{L}\theta_l\left(\sum_{n=1}^{N}p_n\left|h_{l,n}^{\text{SP}}\right|^2\vartheta_{l,n}-I_l^{\text{th}}\right)\\&\quad+\beta_0\left(R-R_{\min}\right).\end{aligned} \tag{13}$$

Accordingly, the dual problem of OP2 can be expressed as

$$\min_{\alpha_0,\beta_0,\theta_l\geq 0}D\left(\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right). \tag{14}$$

To solve the dual problem, (14), we need to find the solution of (12). We observe that (12) can be rewritten as

$$\begin{aligned}&D\left(\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right)\\&\quad=\sum_{n=1}^{N}L_n\left(\mathbf{p},\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right)-\gamma P^{\text{c}}+\alpha_0P_{\max}\\&\quad+\sum_{l=1}^{L}\theta_lI_l^{\text{th}}-\beta_0R_{\min},\end{aligned} \tag{15}$$

where $L_n\left(\mathbf{p},\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right)$ is defined as

$$\begin{aligned}&L_n\left(\mathbf{p},\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}\right)\\&\quad=\max_{\mathbf{p}\geq 0}\left(\left(1+\beta_0\right)r_n-\left(\gamma\tau+\alpha_0+\sum_{l=1}^{L}\theta_l\left|h_{l,n}^{\text{SP}}\right|^2\vartheta_{n,l}\right)p_n\right).\end{aligned} \tag{16}$$

According to the Karush–Kuhn–Tucker conditions, the optimal solutions of (16) are given by

$$p_n=\left(\frac{\left(1+\beta_0\right)\Delta f}{\ln 2\left(\gamma\tau+\alpha_0+\sum_{l=1}^{L}\theta_l\left|h_{l,n}^{\text{SP}}\right|^2\vartheta_{n,l}\right)}-\frac{\sigma_0^2+\Upsilon_n}{|h_n|^2}\right)^+. \tag{17}$$

Substituting (17) into (13) and then the result back into (12), we obtain the optimal dual function for the given values of the dual variables. The optimal dual variables can be obtained from the dual problem (14) using the subgradient method [26]. The dual variables can be updated as

$$\begin{aligned}\alpha_0&=\left(\alpha_0+\varpi_0\left(P_{\max}-\sum_{n=1}^{N}p_n\right)\right)^+,\\\theta_l&=\left(\theta_l+\varsigma_l\left(I_l^{\text{th}}-\sum_{n=1}^{N}p_n\left|h_{l,n}^{\text{SP}}\right|^2\vartheta_{l,n}\right)\right)^+,\\\alpha_0&=\left(\alpha_0+\eta_0\left(R-R_{\min}\right)\right)^+,\end{aligned} \tag{18}$$

where ϖ_0, $\{\varsigma_l\}_{l=1}^{L}$, and η_0 are step lengths.

According to the aforementioned analysis, we propose a new optimal EEPA algorithm, which is termed as OEEPA and tabulated as follows:

Algorithm. OEEPA.

1: **initialization**: initial γ, $\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}$ and maximum tolerance δ.

2: **repeat (out loop)**

3: **repeat (inner loop)**

4: update p_n via (17), $n=1,2,\ldots,N$

5: update dual variables $\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}$ via (18)

6: **until** $\alpha_0,\beta_0,\{\theta_l\}_{l=1}^{L}$ converge

7: Update $\gamma=R(\mathbf{p})/P^{\text{total}}(\mathbf{p})$

8: **until** $|\phi(\mathbf{p},\gamma)|\leq\delta$

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Remark: the proposed iterative algorithm consists of two nested loops. In the inner loop (step 4), computing p_n for all n requires a complexity of $O(N)$ (multiplications and additions). Thus, the complexity of solving OP2 is $O(t_I N)$, where t_I is the number of inner iterations required in a subgradient search. If the number of outer loop iterations is t_O, then the total computational complexity of the proposed algorithm is $O(t_O t_I N)$. In the inner loop, a subgradient algorithm is used to solve OP2. Research in [26] shows that a subgradient algorithm with constant step length can converge to the optimal solution of convex optimization problems within a small range. Therefore, the inner loop converges to the proofs solution of OP2 within a small range. The detailed proofs of the convergence of the outer loop; that is, FP can be found in [22].

IV. Performance Simulations

We present some numerical experiments to evaluate the performance of our proposed scheme. Assume the bandwidth for each subchannel Δf is set to 0.3125 MHz. The magnitudes of all channel fading coefficients follow a Rayleigh distribution and are independent. Without lose of generality, let $\tau = 1$, $P^{\mathrm{c}}=10^{-2}$W, $\sigma_0^2 = 10^{-6}$ W, and $I_l^{\mathrm{th}} = I_{\mathrm{th}}$. The step lengths ϖ_0, $\{\varsigma_l\}_{l=1}^{L}$, and η_0 are set to be 10^{-3}. All the results have been averaged over 1,000 iterations.

To emphasize the advantages of the proposed scheme, we introduce two power allocation problems. The first one is an EEPA problem for an OFDMA-based CR system with an overlay spectrum access mechanism; its optimization problem is as follows:

$$
\begin{aligned}
\text{OP3}: \ & \max_{\mathbf{p}\geq 0} \frac{\Delta f \sum_{n=1}^{K} \log_2\left(1+\frac{|h_n|^2 p_n}{\sigma_0^2+\Upsilon_n}\right)}{\tau \sum_{n=1}^{K} p_n + P^{\mathrm{c}}}. \\
\text{s.t.} \ & \mathrm{C}_1: \sum_{n=1}^{K} p_n \leq P_{\max}, \\
& \mathrm{C}_2: \sum_{n=1}^{K} p_n \left|h_{l,n}^{\mathrm{SP}}\right|^2 \vartheta_{l,n} \leq I_l^{\mathrm{th}}, \forall l \in \mathfrak{L}, \\
& \mathrm{C}_3: \Delta f \sum_{n=1}^{K} \log_2\left(1+\frac{|h_n|^2 p_n}{\sigma_0^2+\Upsilon_n}\right) \geq R_{\min}.
\end{aligned} \tag{19}
$$

Note that the only difference between OP1 and OP3 is that only overlay subchannels are available for the SU in OP3. Reference [18] studied energy-efficient design for multi-user OFDMA-based CR systems with overlay spectrum access mechanisms. If we set the number of SUs in [18] to be one, then OP3 is the same as formula (33) in [18]. The fast barrier method proposed in [18] can also be used to solve OP3. We name the above method to solve OP3 as OSAM. The second power allocation problem is that of throughput maximization, whose associated optimization problem is as follows:

$$
\begin{aligned}
\text{OP4}: \ & \max_{\mathbf{p}\geq 0} \Delta f \sum_{n=1}^{N} \log_2\left(1+\frac{|h_n|^2 p_n}{\sigma_0^2+\Upsilon_n}\right). \\
\text{s.t.} \ & \mathrm{C}_1: \sum_{n=1}^{N} p_n \leq P_{\max}, \\
& \mathrm{C}_2: \sum_{n=1}^{N} p_n \left|h_{l,n}^{\mathrm{SP}}\right|^2 \vartheta_{l,n} \leq I_l^{\mathrm{th}}, \forall l \in \mathfrak{L}, \\
& \mathrm{C}_3: \Delta f \sum_{n=1}^{N} \log_2\left(1+\frac{|h_n|^2 p_n}{\sigma_0^2+\Upsilon_n}\right) \geq R_{\min}.
\end{aligned} \tag{20}
$$

Note that the only difference between OP1 and OP3 is that OP4 is intended to maximize the system throughput. If we add the minimum rate requirement constraint C_3 to formula (8) in [9], then OP4 is the same as formula (8) in [9] (although the constraints in [9] are written in vector form, they are the same as the constraints in OP4). Both OP4 and (8) with the minimum rate requirement constraint in [9] are convex; the optimal solutions of them can be obtained via standard convex optimization techniques [23]. We name the scheme to solve OP3 problem as TMPA.

We first investigate the convergence of our proposed algorithm. Figure 3 illustrates the evolution of the proposed iterative algorithm for different total power budgets. The interference threshold is $I_{\mathrm{th}} = 10^{-6}$ W, the minimum rate requirement is $R_{\min} = 1$ Mbit/s, and the number of underlay subchannels and overlay subchannels is set to $L = 2$ and $K = 10$, respectively. We investigate three cases: $P_{\max} = 0.5\times10^{-4}$ W, $P_{\max} = 2\times10^{-4}$ W and $P_{\max} = 10\times10^{-4}$ W. Since the proposed OEEPA consists of two loops, we only consider the effect of the number of outer loop iterations t_O and set the number of inner iterations t_I large enough to guarantee that the inner loop can find the optimal solution of OP2. It can be observed in Fig. 3 that OEEPA converges to the optimal value within seven iterations for all considered values of the total power budget. The maximum EE can be improved when there is a greater total power budget, since this will lower the probability of system outage.

The EE versus total power budget under interference threshold $I_{\mathrm{th}} = 10^{-6}$ W is depicted in Fig. 4. The minimum rate requirement is set to $R_{\min} = 1$ Mbit/s, and the number of underlay subchannels is set at $L = 2$. We consider two cases of different numbers of overlay subchannels, $K = 5$ and $K = 10$. As can be seen in Fig. 4, for both cases, the EE of the three algorithms increases with the increasing of the total power budget at the beginning, because the CR network outage can

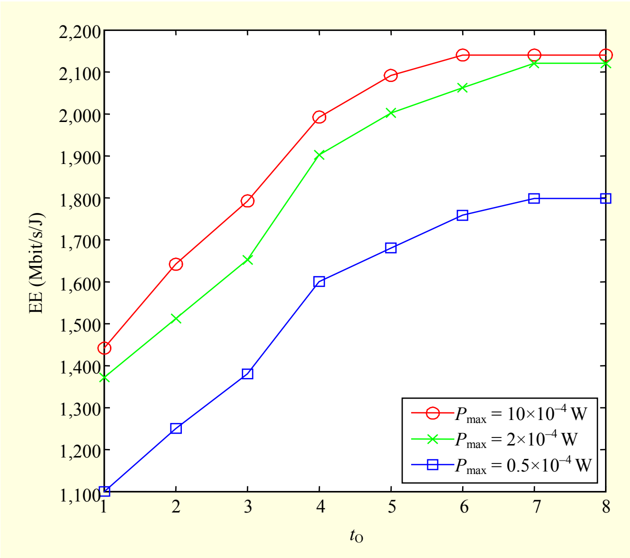

Fig. 3. EE vs. outer iterations for different total power budgets.

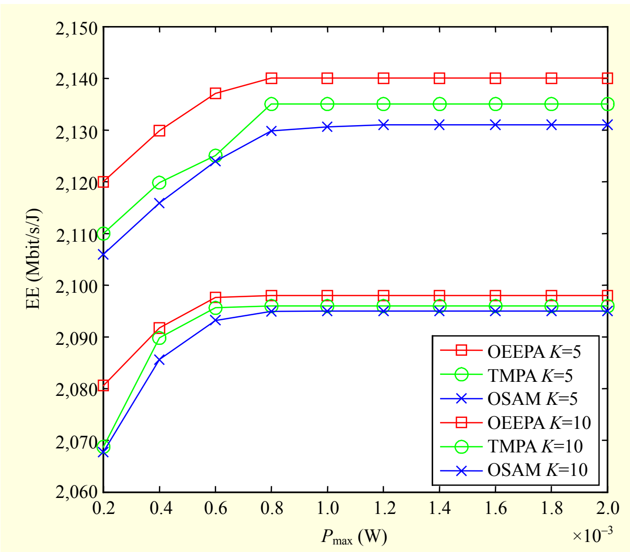

Fig. 4. EE vs. total power budget for different numbers of overlay subchannels.

be reduced with an increasing of the total power budget. When the total power budget is sufficient enough, the minimum rate requirement of the SU can be always satisfied, and then the EE of the CR network remains almost unchanged. Additionally, the EE of the three algorithms can be improved when there are more overlay subchannels that can be used. As shown in Fig. 4, OEEPA has a higher EE than that of the TMPA scheme for both the cases of K = 5 and K = 10. The reason is that the proposed OEEPA scheme aims to maximize the EE of the CR network, whereas the TMPA scheme aims to maximize the rate of the CR network. However, since all the overlay and underlay subchannels are available to the SU in OEEPA and TMPA, only overlay subchannels can be used by the SU in

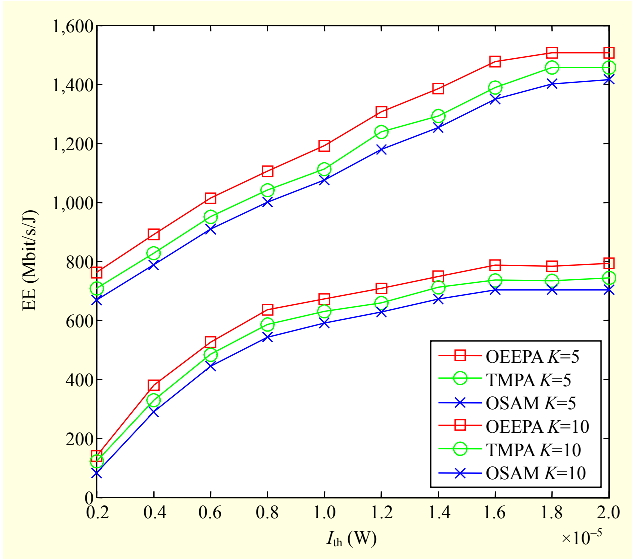

Fig. 5. EE vs. interference threshold for different numbers of overlay subchannels.

OSAM. Therefore, OEEPA and TMPA achieve higher EE than OSAM. This is because the underlay subchannels may have better channel quality; thus, the CR network with a joint overlay and underlay spectrum access mechanism can use these underlay subchannels to achieve a higher transmission rate and save transmit power [10].

Figure 5 depicts the EE versus the interference threshold I_{th} for different numbers of overlay subchannels. In Fig. 5, the total power budget is set to $P_{max} = 10^{-4}$ W, the minimum rate requirement is set to R_{min} = 1 Mbit/s, and the number of underlay subchannels is set to L = 2. There are two cases of different numbers of overlay subchannels, K = 5 and K = 10. It is shown in Fig. 5 that the EE of the three algorithms grows with the growth of the interference threshold. This is because the lower the interference threshold is, the more the CR network suffers outage. We also have that the proposed OEEPA algorithm has better performance than TMPA, and OEEPA and TMPA achieve higher EE than OSAM. The reasons are the same as those given for Fig. 4.

In Fig. 6, we evaluate the EE of OEEPA algorithm versus the interference threshold I_{th} for different numbers of underlay subchannels. The total power budget is set to $P_{max} = 10^{-4}$ W, and the number of overlay subchannels is set to K = 10. We investigate three cases: L = 2, L = 4, and L = 6. As can be seen in Fig. 6, when there are more underlay subchannels, the EE can be improved, but the improvement is limited. This is because the power allocated to each underlay subchannel is limited by the interference threshold constrains of PUs; more power allocated to each underlay subchannel will result in a higher probability of system outage.

Finally, we investigate the EE as a function of the minimal

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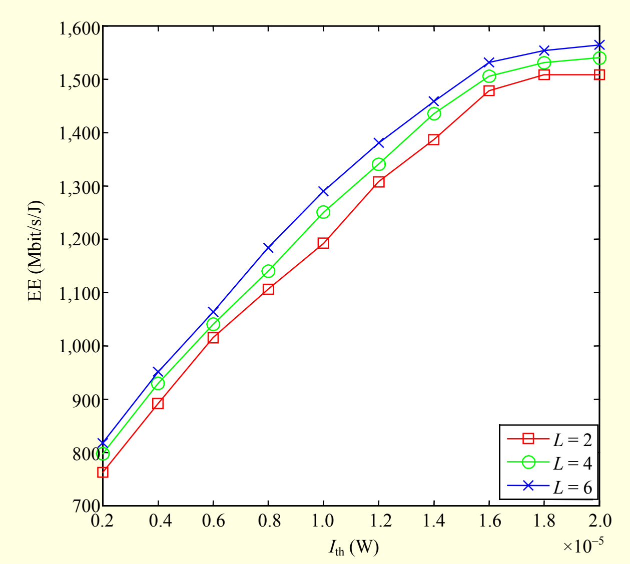


Fig. 6. EE vs. interference threshold for different numbers of underlay subchannels.

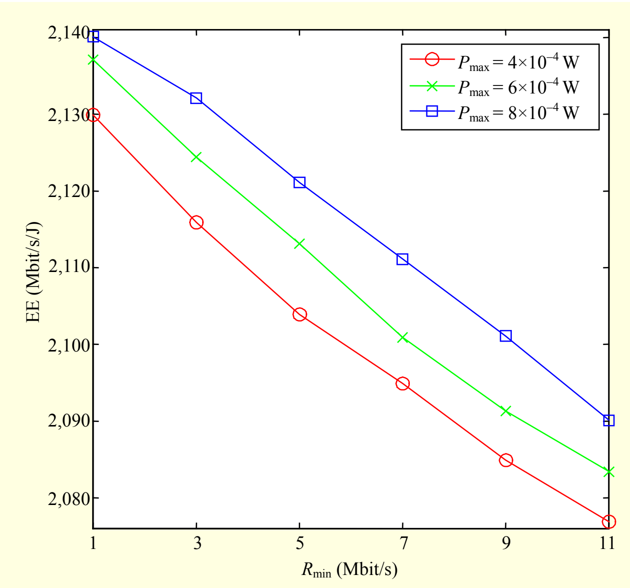


Fig. 7. EE vs. minimal rate requirement for different total power budgets.

rate requirement. The interference threshold is set to I_{th} = 10^{-6} W, and the minimum rate requirement is set to R_{min} = 1 Mbit/s. The number of underlay and overlay subchannels is set to L=2 and K = 10, respectively. We investigate three cases: P_{max} = 4 × 10^{-4} W, P_{max} = 6 × 10^{-4} W, and P_{max} = 8 × 10^{-4} W. As can be seen in Fig. 7, EE decreases with the growth of the rate requirement. This is because when the rate requirement is high, to meet the rate requirement, much power is allocated to the SU (the growth of rate requirements will result in an exponential increase of power consumption). At the same time, the growth of the rate requirements also results in more frequent system outages.

V. Conclusion

In this paper, we investigated the power allocation for a joint overlay and underlay OFDMA-based cognitive radio network. Different from traditional throughput maximization methods, we solve the power allocation problem from the perspective of energy efficiency (EE). The new energy-efficient power allocation (EEPA) problem is difficult to solve directly, since the new formulated optimization problem is nonconvex. To solve it efficiently, the new problem is firstly transformed into an equivalent convex problem via a fractional programming algorithm, and then the equivalent problem is solved via the Lagrange dual decomposition method. Finally, a new iterative EEPA scheme is proposed. The computational complexity and convergence of the proposed algorithm are analyzed at the end. To show the improvement in EE, we compared the proposed scheme with the traditional throughput maximization scheme and the overlay spectrum access mechanism scheme. Simulation results show that the proposed scheme can improve the EE.

Appendix

Before proving the theorem, we first analyze the characteristics of the function $\varphi(\gamma)$.

Lemma. The function $\varphi(\gamma)$ is strictly monotonic decreasing and convex over γ. The function $\varphi(\gamma)=0$ has a unique solution, γ^*.

Proof. Let $\mathbf{p}^*_{\gamma_i} = \arg\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma_i P^{\text{total}}(\mathbf{p})\right\}$, where i = 1, 2 and $\gamma_1 > \gamma_2 \ge 0$. Here, $\mathbf{p}^*_{\gamma_1}$ and $\mathbf{p}^*_{\gamma_2}$ are distinct optimal solutions. Therefore, the optimal solution of $\max_{\mathbf{p}\in\Theta}\{R(\mathbf{p})-\gamma_2 P^{\text{total}}(\mathbf{p})\}$ is $\mathbf{p}^*_{\gamma_2}$, and its maximum value is $\varphi(\gamma_2)=R(\mathbf{p}^*_{\gamma_2})-\gamma_2 P^{\text{total}}(\mathbf{p}^*_{\gamma_2})$. Since $\mathbf{p}^*_{\gamma_1}$ is not the optimal solution of $\max_{\mathbf{p}\in\Theta}\{R(\mathbf{p})-\gamma_2 P^{\text{total}}(\mathbf{p})\}$, $\{R(\mathbf{p}^*_{\gamma_1})-\gamma_2 P^{\text{total}}(\mathbf{p}^*_{\gamma_1})\}$ cannot achieve the maximum value of $R(\mathbf{p}^*_{\gamma_2})-\gamma_2 P^{\text{total}}(\mathbf{p}^*_{\gamma_2})$. Thus, we have

$$\begin{aligned}\varphi(\gamma_2) &= \max_{\mathbf{p}\in\Theta}\{R(\mathbf{p})-\gamma_2 P^{\text{total}}(\mathbf{p})\}\\ &= R(\mathbf{p}^*_{\gamma_2})-\gamma_2 P^{\text{total}}(\mathbf{p}^*_{\gamma_2})\\ &> R(\mathbf{p}^*_{\gamma_1})-\gamma_2 P^{\text{total}}(\mathbf{p}^*_{\gamma_1}).\end{aligned} \tag{21}$$

Since $\gamma_1 > \gamma_2 \ge 0$, we have

$$R\left(\mathbf{p}_{\gamma_1}^*\right)-\gamma_2 P^{\text{total}}\left(\mathbf{p}_{\gamma_1}^*\right)>R\left(\mathbf{p}_{\gamma_1}^*\right)-\gamma_1 P^{\text{total}}\left(\mathbf{p}_{\gamma_1}^*\right)$$
$$=\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma_1 P^{\text{total}}(\mathbf{p})\right\} \tag{22}$$
$$=\varphi(\gamma_1).$$

Combining with (21) and (22), we have $\varphi(\gamma_2)>\varphi(\gamma_1)$, since $\gamma_1>\gamma_2$; therefore, $\varphi(\gamma)$ is a strictly monotonic decreasing function.

For $\forall\gamma_1,\gamma_2\geq 0$, $\gamma_1\neq\gamma_2$, and $0\leq t\leq 1$, let $\gamma_3=t\gamma_1+(1-t)\gamma_2$ and $\mathbf{p}_{\gamma_3}^*=\arg\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma_3 P^{\text{total}}(\mathbf{p})\right\}$. Then, we have

$$\begin{aligned}
&\varphi(t\gamma_1+(1-t)\gamma_2)=\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma_3 P^{\text{total}}(\mathbf{p})\right\}\\
&=R\left(\mathbf{p}_{\gamma_3}^*\right)-\gamma_3 P^{\text{total}}\left(\mathbf{p}_{\gamma_3}^*\right)\\
&=R\left(\mathbf{p}_{\gamma_3}^*\right)-\left[t\gamma_1+(1-t)\gamma_2\right]P^{\text{total}}\left(\mathbf{p}_{\gamma_3}^*\right)\\
&=t\left[R\left(\mathbf{p}_{\gamma_3}^*\right)-\gamma_1 P^{\text{total}}\left(\mathbf{p}_{\gamma_3}^*\right)\right]+(1-t)\left[R\left(\mathbf{p}_{\gamma_3}^*\right)-\gamma_2 P^{\text{total}}\left(\mathbf{p}_{\gamma_3}^*\right)\right]\\
&\leq t\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma_1 P^{\text{total}}(\mathbf{p})\right\}+(1-t)\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma_2 P^{\text{total}}(\mathbf{p})\right\}\\
&=t\varphi(\gamma_1)+(1-t)\varphi(\gamma_2).
\end{aligned} \tag{23}$$

Therefore, $\varphi(\gamma)$ is convex over γ.

Since $R(\mathbf{p})\geq 0$ and $P^{\text{total}}(\mathbf{p})>0$, $\lim_{\gamma\to+\infty}\varphi(\gamma)=-\infty$ and $\lim_{\gamma\to-\infty}\varphi(\gamma)=+\infty$; thus, together with the strictly monotonic decreasing property of $\varphi(\gamma)$, we conclude that $\varphi(\gamma^*)=0$ has a unique solution γ^*. ■

Proof of theorem.

(A) Assume that the optimal solution and optimal value of OP1 are $\mathbf{p}^*$ and γ^*, respectively, which means

$$\gamma^*=\frac{R\left(\mathbf{p}^*\right)}{P^{\text{total}}\left(\mathbf{p}^*\right)}=\max_{\mathbf{p}\in\Theta}\frac{R(\mathbf{p})}{P^{\text{total}}(\mathbf{p})}\geq\frac{R(\mathbf{p})}{P^{\text{total}}(\mathbf{p})}. \tag{24}$$

Then, we have

$$R(\mathbf{p})-\gamma^* P^{\text{total}}(\mathbf{p})\leq 0\,, \tag{25}$$

$$R\left(\mathbf{p}^*\right)-\gamma^* P^{\text{total}}\left(\mathbf{p}^*\right)=0\,. \tag{26}$$

From (25) and (26), we have the maximum value of $\varphi(\gamma^*)$ is 0, which means that $\varphi(\gamma^*)=\max_{\mathbf{p}\in\Theta}\left\{R(\mathbf{p})-\gamma^* P^{\text{total}}(\mathbf{p})\right\}=0$ and that this then has an optimal solution $\mathbf{p}^*$.

(B) Assume that the optimal solution of OP2 with γ^* is $\mathbf{p}_{\gamma^*}^*$, such that $\varphi(\gamma^*)=R\left(\mathbf{p}_{\gamma^*}^*\right)-\gamma^* P^{\text{total}}\left(\mathbf{p}_{\gamma^*}^*\right)=0$. Then, we have

$$R(\mathbf{p})-\gamma^* P^{\text{total}}(\mathbf{p})\leq R\left(\mathbf{p}_{\gamma^*}^*\right)-\gamma^* P^{\text{total}}\left(\mathbf{p}_{\gamma^*}^*\right)=0\,. \tag{27}$$

Further, we have

$$\frac{R(\mathbf{p})}{P^{\text{total}}(\mathbf{p})}\leq\gamma^* \quad\text{and}\quad \frac{R\left(\mathbf{p}_{\gamma^*}^*\right)}{P^{\text{total}}\left(\mathbf{p}_{\gamma^*}^*\right)}=\gamma^*. \tag{28}$$

Equation (28) implies that the optimal solution $\mathbf{p}_{\gamma^*}^*$ for the equivalent problem OP2 with γ^* is also the optimal solution of OP1.

Both (A) and (B) complete the proof of theorem. ■

References

[1] C.-X. Wang et al., "Cellular Architecture and Key Technologies for 5G Wireless Communication Networks," *IEEE Commun. Mag*., vol. 52, no. 2, Feb. 2014, pp. 122–130.

[2] E.Z. Tragos et al., "Spectrum Assignment in Cognitive Radio Networks: A Comprehensive Survey," *IEEE Commun. Surveys Tutorials*, vol. 15, no. 3, Jan. 2013, pp. 1108–1135.

[3] J. Oh and W. Choi, "A Hybrid Cognitive Radio System: A Combination of Underlay and Overlay Approaches," *IEEE Veh. Technol. Conf*., Ottawa, Canada, Sept. 6–9, 2010, pp. 1–5.

[4] M. Ge and S. Wang, "Fast Optimal Resource Allocation is Possible for Multiuser OFDM-Based Cognitive Radio Networks with Heterogeneous Services," *IEEE Trans. Wireless Commun*., vol. 11, no. 4, Apr. 2012, pp. 1500–1509.

[5] G. Bansal, M.J. Hossain, and V.K. Bhargava, "Adaptive Power Loading for OFDM-Based Cognitive Radio Systems with Statistical Interference Constraint," *IEEE Trans. Wireless Commun*., vol. 10, no. 9, Sept. 2011, pp. 2786–2791.

[6] T.N. Duy and L.N. Tho, "Distributed Resource Allocation for Cognitive Radio Networks with Spectrum Sharing Constrains," *IEEE Trans. Veh. Technol*., vol. 60, no. 7, Sept. 2011, pp. 3436–3449.

[7] P. Wang et al., "Power Allocation in OFDM-Based Cognitive Radio Systems," *IEEE Global Telecommun. Conf*., Washington, DC, USA, Nov. 26–30, 2007, pp. 4061–4065.

[8] M.G. Khoshkholgh, K. Navaie, and H. Yanikomeroglu, "On the Impact of the Primary Network Activity on the Achievable Capacity of Spectrum Sharing over Fading Channels," *IEEE Trans. Wireless Commun*., vol. 8, no. 4, Apr. 2009, pp. 2100–2111.

[9] G. Bansal, O. Duval, and F. Gagnon, "Joint Overlay and Underlay Power Allocation Scheme for OFDM-Based Cognitive Radio Systems," *IEEE Veh. Technol. Conf*., Taipei, Taiwan, May 16–19, 2010, pp. 1–5.

[10] G. Bansal et al., "Subcarrier and Power Allocation for OFDMA-Based Cognitive Radio Systems with Joint Overlay and Underlay Spectrum Access Mechanism," *IEEE Trans. Veh. Technol*., vol. 62, no. 3, Mar. 2013, pp. 1111–1122.

[11] V. Chakravarthy et al., "A Novel Hybrid Overlay/Underlay Cognitive Radio Waveform in Frequency Selective Fading Channels," *Int. Conf. Cognitive Radio Oriented Wireless Netw.*

Commun., Hannover, Germany, June 22–24, 2009, pp. 1–6.

[12] D. Feng et al., "A Survey of Energy-Efficient Wireless Communications," *IEEE Commun. Surveys Tutorials*, vol. 15, no. 1, Feb. 2013, pp. 167–178.

[13] S. Kim, B.G. Lee, and D. Park, "Energy-Per-Bit Minimized Radio Resource Allocation in Heterogeneous Networks," *IEEE Trans. Wireless Commun*., vol. 13, no. 4, Apr. 2014, pp. 1862–1873.

[14] J. Mao et al., "Energy Efficiency Optimization for OFDM-Based Cognitive Radio Systems: A Water-Filling Factor Aided Search Method," *IEEE Trans. Wireless Commun*., vol. 12, no. 5, May 2013, pp. 2366–2375.

[15] Y. Wang et al., "Optimal Energy-Efficient Power Allocation for OFDM-Based Cognitive Radio Networks," *IEEE Commun. Lett*., vol. 16, no. 9, Sept. 2012, pp. 1420–1423.

[16] M. Ge and S. Wang, "Energy-Efficient Power Allocation for Cooperative Relay Cognitive Radio Networks," *IEEE Wireless Commun. Netw. Conf*., Shanghai, China, Apr. 7–10, 2013, pp. 691–696.

[17] W.J. Shi and S. Wang, "Energy-Efficient Resource Allocation in Cognitive Radio Systems," *IEEE Wireless Commun. Netw. Conf*., Shanghai, China, Apr. 7–10, 2013, pp. 4618–4623.

[18] S. Wang, M. Ge, and W. Zhao, "Energy-Efficient Resource Allocation for OFDM-Based Cognitive Radio Networks," *IEEE Trans. Commun*., vol. 61, no. 8, Aug. 2013, pp. 3181–3191.

[19] X. Cong, L. Lu, and G.Y. Li, "Energy-Efficient Spectrum Access in Cognitive Radios," *IEEE J. Sel. Areas Commun*., vol. 32, no. 3, Mar. 2014, pp. 550–562.

[20] S. Akin and M.C. Gursoy, "On the Throughput and Energy Efficiency of Cognitive MIMO Transmissions," *IEEE Trans. Veh. Technol*., vol. 62, no. 7, Mar. 2013, pp. 3245–3260.

[21] W. Zhong and J. Wang, "Energy Efficient Spectrum Sharing Strategy Selection for Cognitive MIMO Interference Channels," *IEEE Trans. Signal Process*., vol. 61, no. 14, July 2013, pp. 3705–3717.

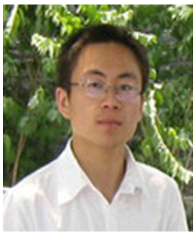
[22] W. Dinkelbach, "On Nonlinear Fractional Programming," *Manag. Sci*., vol. 13, no. 7, Mar. 1967, pp. 492–498.

[23] S. Boyd and L. Vandenberghe, "Interior-Point Methods," in *Convex Optimization*, London, UK: Cambridge University Press, 2004, pp. 562–622.

[24] G. Bansal, M.J. Hossain, and V.K. Bhargava, "Optimal and Suboptimal Power Allocation Schemes for OFDM-Based Cognitive Radio Systems," *IEEE Trans. Wireless Commun*., vol. 7, no. 11, Nov. 2008, pp. 4710–4718.

[25] T. Weiss et al., "Mutual Interference in OFDM-Based Spectrum Pooling Systems," *IEEE Veh. Technol. Conf*., vol. 4, May 17–19, 2004, pp. 1873–1877.

[26] S. Boyd, L. Xiao, and A. Mutapcic, *Subgradient Methods*, Stanford University, Oct. 1, 2003. Accessed June 18, 2014. https://web.stanford.edu/class/ee392o/subgrad_method.pdf



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