Bandwidth-Efficient Selective Retransmission for MIMO-OFDM Systems

Muhammad Zia, Tamoor Kiani, Nazar A. Saqib, Tariq Shah, and Hasan Mahmood

In this work, we propose an efficient selective retransmission method for multiple-input and multiple-output (MIMO) wireless systems under orthogonal frequency-division multiplexing (OFDM) signaling. A typical received OFDM frame may have some symbols in error, which results in a retransmission of the entire frame. Such a retransmission is often unnecessary, and to avoid this, we propose a method to selectively retransmit symbols that correspond to poor-quality subcarriers. We use the condition numbers of the subcarrier channel matrices of the MIMO-OFDM system as a quality measure. The proposed scheme is embedded in the modulation layer and is independent of conventional hybrid automatic repeat request (HARQ) methods. The receiver integrates the original OFDM and the punctured retransmitted OFDM signals for more reliable detection. The targeted retransmission results in fewer negative acknowledgements from conventional HARQ algorithms, which results in increasing bandwidth and power efficiency. We investigate the efficacy of the proposed method for optimal and suboptimal receivers. The simulation results demonstrate the efficacy of the proposed method on throughput for MIMO-OFDM systems.

Keywords: HARQ, FEC, log-likelihood ratio, MIMO-OFDM, selective retransmission.

Manuscript received Jan. 22, 2014; revised July 27, 2014; accepted Sept. 11, 2014.

Muhammad Zia (corresponding author, mzia@ucdavis.edu), Tamoor Kiani (ktamoor@gmail.com), and Hasan Mahmood (hasan@qau.edu.pk) are with the Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan.

Tariq Shah (stariqshah@gmail.com) is with the Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan.

Nazar A. Saqib (nazar.abbas@seecs.edu.pk) is with the College of Electronics & Mathematics, NUST, Islamabad, Pakistan.

I. Introduction

Over the past decade, multiple-input and multiple-output (MIMO) communication systems have been the focus of the research community due to their potential ability to achieve system capacity and channel diversity [1]–[3]. Orthogonal frequency-division multiplexing (OFDM) and orthogonal frequency-division multiple access (OFDMA) technologies allow low-complexity receiver design with frequency diversity that is suitable for resource allocation. Consequently, MIMO-OFDM and OFDMA modulations are embedded in contemporary wireless communication standards, such as Long-Term Evolution (LTE) [4]. The complexity of an optimal receiver for MIMO-OFDM systems grows exponentially with the number of transmitting antennas and constellation size. Thus, suboptimal receiver designs, such as that of a zero-forcing (ZF) equalizer, provide a trade-off between complexity and performance.

In spite of the capacity and diversity gains offered by MIMO-OFDM communication systems, wireless signals inevitably encounter channel distortions and fading, which results in packet errors. The automatic repeat request (ARQ) is an effective method to achieve error-free transmission by retransmitting bad packets. Forward error correction (FEC) codes substantially enhance the reliability of the decoded bit stream to reduce packet loss. HARQ methods are powerful means of combating channel distortion to ensure an error-free and high-throughput communication link by integrating observations of multiple transmissions corresponding to the same packet. Thus, HARQ retransmission methods are a key feature of recent wireless communications [4]. Conventional HARQ methods overcome the drawback of disjoint ARQ and FEC codes design [5] by exploiting observations from multiple

http://dx.doi.org/10.4218/etrij.15.0114.0050

transmissions to perform joint decoding. In Chase combining HARQ (CC-HARQ), also known as HARQ Type-I, when decoding errors occur, the receiver saves observations corresponding to the bad packet in a buffer and requests the retransmission of the same information by sending a NACK signal. Due to independent channel realizations, joint decoding at the receiver achieves diversity gain, which results in a low bit error rate (BER). However, in incremental redundancy HARQ (IR-HARQ), also known as Type-II HARQ Type-II, if the cyclic redundancy check fails, then the receiver sends a NACK signal to the transmitter. In response to the NACK signal, it transmits additional parity bits. In Type-III HARQ, in response to the NACK signal, the transmitter retransmits information bits of the code word (similar to Chase combining) and additional parity bits [4]. Thus, Type-III HARQ is a combination of CC-HARQ and IR-HARQ. There are multiple rounds of HARQ until an error-free packet is received or the maximum number of retransmission rounds is met. Note that FEC decoders have high complexity; therefore, fewer HARQ retransmission rounds result in lower computational costs.

Most of the research on HARQ involves ARQ and FEC [6]–[10]. Very little research is conducted on diversity combining techniques [11]–[13] at the modulation layer to enhance power and bandwidth efficiency. The methods in [11]–[13] retransmit partial information in a predetermined fashion without exploiting channel state information (CSI). Furthermore, these methods have high joint decoding complexity for frequency-selective channels. Recent work in [14] proposes selective retransmission at the modulation layer for single-input–single-output (SISO) systems for OFDM signaling. The selective retransmission approach in [14] targets information symbols corresponding to low-gain subcarriers for retransmission.

In this paper, we extend the work in [14] for MIMO-OFDM systems. In SISO-OFDM systems, the subcarrier channel norm is a measure of channel quality. However, for MIMO-OFDM systems, the norm of the channel matrix, $H(\ell)$, of the ℓth subcarrier is not a good measure of channel quality [15]–[16]. The condition number of the channel matrix of a MIMO-OFDM subcarrier provides a good measure of the quality of the subcarrier, especially for suboptimal receivers at high signal-to-noise ratio (SNR) [17]. Several reports discuss the impact of the condition number on channel quality [18]–[20]. The impact of condition number on the received SNR is presented in [18]–[19]. In the proposed selective retransmission approach for MIMO-OFDM systems, we use condition number $\kappa(\ell)$ as a measure of channel quality to select the subcarriers for retransmission. The motivation for selective retransmission stems from the fact that in a typical failed packet there are fewer corrupt bits. The full retransmission of the packet is unnecessary, and the receiver can recover from error using only a partial retransmitting of the packet. The MIMO-OFDM signaling allows retransmission of susceptible symbols corresponding to those subcarriers that have a large condition number. In [17], the condition number of the MIMO channel is used to choose a modulation scheme. In the proposed method, we use the condition number $\kappa(\ell)$ of the channel matrix corresponding to the ℓth subcarrier as an indicator of channel quality for selective retransmission. We select poor-quality (large condition number) subcarriers for retransmission. The proposed receiver computes the condition numbers of N_s subcarriers and the log-likelihood ratio (LLR) from the observations corresponding to good-quality subcarriers. The observations corresponding to the poor-quality subcarriers are preserved, and this is then followed by the selective retransmission request to the transmitter at the modulation level. The proposed selective retransmission method is independent of conventional ARQ and HARQ protocols. We investigate the throughput gain of the selective retransmission method under MIMO-OFDM signaling over conventional ARQ and HARQ methods. We also provide throughput analysis of the proposed approach for optimal and suboptimal decoding methods. Note that the selective retransmission at the modulation level is independent of conventional HARQ and is effective to lower the average number of retransmissions. As a result of the low average number of retransmissions, the conventional HARQ methods achieve higher throughput with lower complexity.

Note that for SISO-OFDM systems, receiver complexity is much lower as compared to MIMO-OFDM systems, especially for maximum-likelihood (ML) decoders. In MIMO-OFDM systems, the complexity of ML decoding increases exponentially with the number of transmitting antennas and cardinality of the constellation set $\mathcal{A}$. Thus, the complexity of ML receivers for MIMO-OFDM systems is not tractable for large numbers of transmitting antennas or the constellation set $\mathcal{A}$. Suboptimal decoding methods, such as ZF and minimum mean square error (MMSE), offer viable alternative at the expense of performance loss. In this work, we investigate the efficacy of the proposed selective retransmission on conventional HARQ methods for optimal and suboptimal decoders.

We organize this manuscript as follows. First, we present the system model and problem formulation of selective HARQ in Section II. In Section III-1, we discuss the optimal and suboptimal receivers to compute LLR. The selective retransmission method and throughput analysis are provided in Section III-2. The performance of the proposed method is presented in Section IV. Finally, we present the conclusion in Section V.

II. System Model and Problem Formulation

1. System Model

We consider a MIMO communication system under OFDM signaling over a frequency-selective channel with bit-interleaved coded modulation (BICM) [21]–[22]. The transmitter and receiver are equipped with n_t and n_r antennas, respectively. The information bit vector, **u**, is encoded by an FEC encoder to form a codeword, $\mathcal{C}$. The codeword $\mathcal{C}$ is then interleaved by an interleaver, π, into a bit vector, $\tilde{\mathcal{C}}$. The interleaved bit vector $\tilde{\mathcal{C}}$ is mapped onto the symbol set $\mathcal{A}$ and then circularly demultiplexed into n_t symbol streams. Let $\mathbf{s}_k = [s_k(1) \ldots s_k(N_s)]^T$ be the information symbols vector to be modulated into an OFDM signal $\mathbf{x}_k = [x_k(1) \ldots x_k(N_s)]^T$ of N_s subcarriers for transmission by the kth antenna, as shown in Fig. 1. We assume that the frequency-selective channel $\mathbf{h}_{i,j}$ from the jth transmit antenna to the ith receive antenna is of order L. Thus, each OFDM signal vector $\mathbf{x}_k$ is transmitted over the frequency-selective channel. There are $n_r \times n_t$ frequency-selective channels between transmitter and receiver. OFDM signaling converts each frequency-selective channel vector $\mathbf{h}_{i,j}$ into N_s parallel subcarriers. MIMO-OFDM signaling with N_s subcarriers converts frequency-selective channels into N_s parallel MIMO flat-fading channel matrices $H(\ell) \in \boldsymbol{C}^{n_r \times n_t}$, where $\ell = 1, 2, \ldots, N_S$ is the index of a subcarrier [23]. Let $\mathbf{b}(\ell) = \left[s_1(\ell) \ldots s_{n_t}(\ell)\right]^T \in \boldsymbol{C}^{n_t \times 1}$ be the transmitted symbol vector from n_t transmit antennas over flat-fading channel matrix $H(\ell)$. Then, we have

$$\mathbf{z}(\ell) = H(\ell)\mathbf{b}(\ell) + \mathbf{w}(\ell), \qquad (1)$$

where element $H_{i,j}(\ell) \in \boldsymbol{C}$ of flat-fading channel matrix $H(\ell)$ is a complex gain of the ℓth subcarrier between the jth transmit and the ith receive antenna. There are N_s many $H(\ell)$ matrices of complex gains of order $n_r \times n_t$ obtained by applying an FFT matrix to each frequency-selective channel vector $\mathbf{h}_{i,j}$, where $i = 1, \ldots, n_r$ and $j = 1, \ldots, n_t$. There are n_r observation vectors $\mathbf{y}_1, \ldots, \mathbf{y}_{n_r}$ corresponding to n_r receive antennas. An FFT transformation is applied to each observation vector after removing the cyclic prefix (CP). Elements of each transformed vector that correspond to the ℓth subcarrier are stacked to construct vector $\mathbf{z}(\ell)$, as given in (1). The receiver then computes the LLR metric vector

$$\Lambda(:,\ell) = \begin{bmatrix} \lambda_1(\ell) \\ \vdots \\ \lambda_{n_t \log_2 M}(\ell) \end{bmatrix} \in R^{n_t \log_2 M \times 1} \qquad (2)$$

that corresponds to the ℓth subcarrier of the MIMO-OFDM system, shown in Fig. 1. Note that $\Lambda(:,\ell)$ is the ℓth column of the LLR metric Λ in (3). In (2), M is the cardinality of the constellation set $\mathcal{A}$. Thus, the LLR metric for all N_s subcarriers can be written in matrix form as follows:

$$\Lambda = \left[\Lambda(:,1) \ldots \Lambda(:,N_s)\right] \in R^{n_t \log_2 M \times N_s}. \qquad (3)$$

The LLR matrix Λ is multiplexed circularly column-wise into the single vector $\tilde{\mathcal{L}}$ of the LLR metric. That is, $\tilde{\mathcal{L}} = \mathrm{vec}\{\Lambda\}$, where the operator $\mathrm{vec}\{\Lambda\}$ generates a vector by stacking the columns of matrix Λ. The multiplexed LLR stream $\tilde{\mathcal{L}}$ is then de-interleaved by π^{-1} into LLR vector $\mathcal{L}$, which is used by the FEC decoder to decode the information bit vector. The output of FEC, $\hat{\mathbf{u}}$, is an estimate of the transmitted information bit vector (**u**).

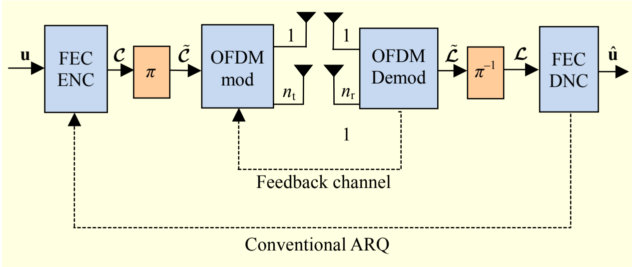


Fig. 1. System model for selective retransmission under MIMO-OFDM signaling.

We propose the selective retransmission of symbol vectors that are transmitted on poor-quality subcarriers at the modulation level for the aforementioned model presented in Fig. 1. The proposed selective retransmission enhances the throughput of conventional ARQ, CC-HARQ, and IR-HARQ methods. The targeted retransmission of information symbols from the OFDM frame is based on the condition number $\kappa(\ell)$ of the channel matrix of the ℓth subcarrier.

2. Condition Number

The condition number is a measure of the singularity of a matrix. The large condition number of channel matrix $H(\ell)$ of the ℓth subcarrier is an indicator that the channel matrix is nearly singular. Note that for subcarriers of large condition number, small changes in observation (measurement error due to noise) result in large changes in the estimate of the information. Furthermore, small variations in the coefficients of ill-conditioned channel matrices, due to channel estimation error, significantly impairs the performance of the receiver. The condition number $\kappa(\ell)$ of a channel matrix of a subcarrier can be written as

$$\kappa(\ell) = \frac{\rho_{\max}}{\rho_{\min}}, \qquad (4)$$

http://dx.doi.org/10.4218/etrij.15.0114.0050

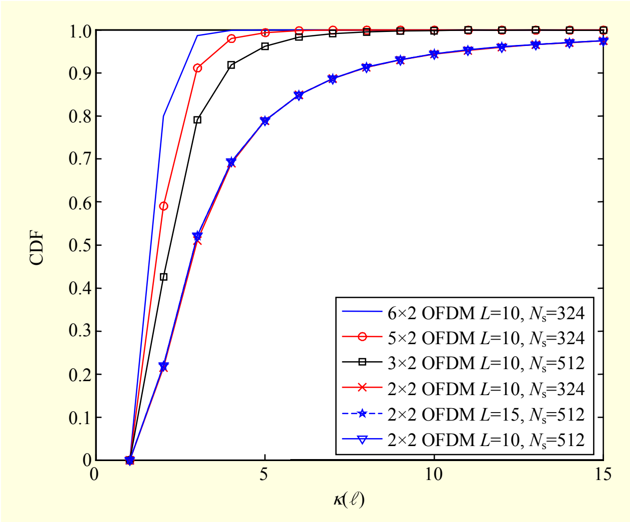


Fig. 2. Empirical CDF of condition number $\kappa(\ell)$ of MIMO-OFDM subcarriers as a function of receive antennas and channel length.

where $\rho_{\max}$ and $\rho_{\min}$ are the largest and smallest singular values of channel matrix $H(\ell)$ of the ℓth subcarrier, respectively [17]. It can be seen from Fig. 2 that the cumulative distribution function (CDF) of the condition number of MIMO-OFDM channels is independent of both the number of subcarriers and the channel order of frequency-selective channels. However, the condition numbers of the channel matrices of MIMO-OFDM subcarriers improves by increasing the number of receiver antennas.

3. Problem Formulation

In this work, we exploit both the frequency diversity of OFDM modulation and the condition numbers of the channel matrices of subcarriers for selective retransmission of the information symbols in MIMO wireless communication systems. The spatial multiplexing of information symbols (V-BLAST) over channel matrix $H(\ell)$ is affected by the condition number $\kappa(\ell)$. Note that the condition number $\kappa(\ell)$ of a channel matrix is a measure of how much correlation exists among paths, and it affects receiver performance, particularly for those suboptimal receivers in the high SNR regime.

In the proposed method, we omit the retransmission of information symbols corresponding to good-quality subcarriers to avoid unnecessary retransmission of reliable subcarriers. The proposed method requests retransmission of the information symbols corresponding to those subcarriers with channel matrices of large condition number $\kappa(\ell) \geq \tau_c$ prior to computing LLR. The receiver preserves observation of those channel matrices with high condition number selected for retransmission to compute LLR jointly. Note that targeted retransmission and joint detection for low-quality subcarriers at the modulation level is independent of conventional ARQ and HARQ (CC-HARQ and IR-HARQ) methods. Consequently, selective retransmission at the modulation layer enhances the channel quality, which results in a lower average number of conventional ARQ and HARQ frames.

Let $H_p(\ell)$ be the channel matrix corresponding to the retransmitted information symbol vector $\mathbf{b}(\ell)$ of the MIMO-OFDM system. Due to large latency, we assume that $H(\ell)$ and $H_p(\ell)$ are independent. Remember that $H(\ell)$ is the channel matrix of the ℓth subcarrier of the MIMO-OFDM system during first transmission. The observation vector $\mathbf{z}_p(\ell)$ of the retransmitted symbol vector is

$$\mathbf{z}_p(\ell) = H_p(\ell)\mathbf{b}(\ell) + \mathbf{w}_p(\ell), \tag{5}$$

where $\mathbf{w}_p$ is a noise vector corresponding to the large condition number $\kappa(\ell)$ subcarrier from the first transmission. Thus, the joint system model for selective retransmission is

$$\tilde{\mathbf{z}}(\ell) = \begin{bmatrix} \mathbf{z}(\ell) \\ \mathbf{z}_p(l) \end{bmatrix} = \begin{bmatrix} H(\ell) \\ H_p(\ell) \end{bmatrix} \mathbf{b}(\ell) + \begin{bmatrix} \mathbf{w}(\ell) \\ \mathbf{w}_p(\ell) \end{bmatrix} \tag{6}$$

$$= \mathcal{H}(\ell)\mathbf{b}(\ell) + \mathbf{v}(\ell), \tag{7}$$

where $\mathbf{v}(\ell) = \left[\mathbf{w}^{\mathrm{T}}(\ell)\ \mathbf{w}_p^{\mathrm{T}}(\ell)\right]^{\mathrm{T}}$. The objective of selective retransmission is to omit resending high-quality subcarriers, which results in increased power and bandwidth efficiency. The probability of bit error in high-quality subcarriers is very low, and their retransmission contributes to both bandwidth and energy overhead. For MIMO-OFDM systems, the joint channel matrix $\mathcal{H}(\ell)$ corresponding to the subcarrier with large condition number during the first transmission is a block matrix of order $2n_r \times n_t$. Since there is large latency between the first transmission of the OFDM frame and subsequent selective retransmission of poor subcarriers at the modulation level, the two channel realizations are assumed to be independent. Due to the independent channel realizations of the multiple transmissions, joint detection of the retransmitted information symbols can achieve higher channel diversity, which results in a lower BER. The threshold τ_c on the condition number $\kappa(\ell)$ depends upon the strength of FEC.

The proposed additional retransmission at the modulation layer requires partial channel feedback, along with a NACK signal, for selective retransmission. The partial channel state information is fed back to the transmitter in terms of the bit-map vector of N_s bits. Each bit in the bit-map vector is an indicator whether to retransmit information corresponding to its subcarrier. The transmitter can append retransmitted information

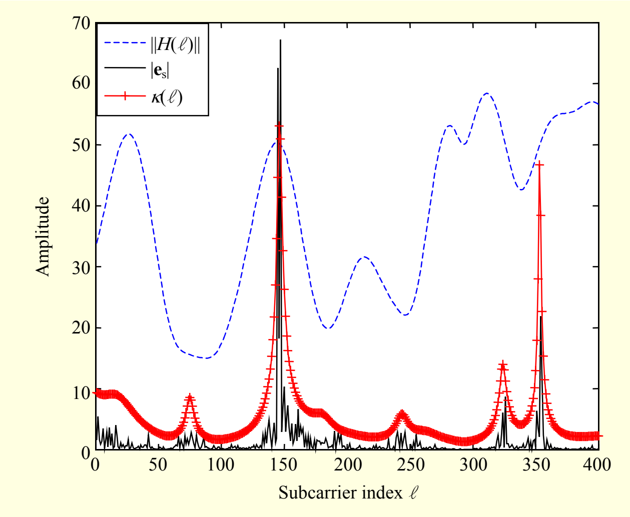


Fig. 3. Channel condition number $\kappa(\ell)$, channel gain $\|H(\ell)\|$, and slicer error $|\mathbf{e}_s|$ for MIMO-OFDM signaling over Rayleigh fading channel.

symbols to the new MIMO-OFDM frame at a predetermined location. For example, a retransmitted symbol vector can be appended to the beginning of an OFDM symbol. Furthermore, it can be observed from the condition number of a realization of the MIMO-OFDM channel in Fig. 3 that feedback information can be reduced by forming clusters of condition numbers of subcarriers. The receiver assigns one feedback bit to each cluster, resulting in less feedback.

The clusters of high condition numbers cause burst error and eventually affect the error-correcting capability of the FEC. To mitigate the effect of such clustering, we use BICM [21] to disperse bit errors. Note that for MIMO-OFDM systems, the condition number $\kappa(\ell)$ is a measure of the channel quality for selective retransmission. Figure 3 also reveals the impact of the condition number of the channel matrices of subcarriers on the receiver performance of MIMO-OFDM systems. In Fig. 3, we consider the slicer error, $\mathbf{e}_s = \mathbf{b}(\ell) - \mathrm{dec}\{\hat{\mathbf{b}}(\ell)\}$, as a quality measure of a MIMO-OFDM subcarrier. The norm of the slicer error, $\|\mathbf{e}_s\|$, is higher for subcarriers that have a large condition number; this is in spite of the higher channel norm $\|\mathcal{H}(\ell)\|^2$ for Rayleigh-fading channels. Therefore, retransmission of subcarriers with large condition numbers is more effective when trying to enhance the channel quality.

The forward-error decoding algorithms process soft information (log-likelihood) to decode information bits. The complexity of the ML receiver to evaluate LLR increases exponentially by increasing the number of transmitting antennas, n_t. However, the computational cost of the LLR for joint detection from two transmissions of a low-quality subcarrier is very close to that of single transmission. The ML detection of a MIMO receiver with single transmission is itself prohibitively complex. Therefore, suboptimal receivers, such as MMSE- or ZF-based demodulation of MIMO BICH [24], offer a trade-off between complexity and performance. In this work, we consider both optimal and suboptimal receivers for selective retransmission over a MIMO-OFDM channel. In our simulation analysis, we compare the performances of the optimal and suboptimal receivers to observe the efficacy of the proposed method. Next, we provide an overview of the BICM-OFDM receiver, with a particular focus on selective retransmission at the physical-layer level.

III. BICM-OFDM Receiver and Throughput

In this section, we discuss optimal and suboptimal LLR computation and throughput of selective retransmission for MIMO-OFDM systems.

1. BICM-OFDM Receiver

We discuss both an optimal and a suboptimal LLR computation of a MIMO system employing OFDM signaling. In Section II, we discuss a MIMO-OFDM model that implements the selective retransmission scheme. An FEC decoder processes LLR vector $\mathcal{L}$ to estimate information bits. The low-complexity suboptimal approaches to compute LLR [25]–[26] at the expense of performance. It is clear from (7) that the selective retransmission does not cause a significant increase in the complexity of the ML method of LLR computation as compared to that caused by a single transmission. Now, we discuss the LLR computation of ML, ZF, and MMSE receivers.

Consider the MIMO-OFDM system with BICM over a frequency-selective channel, shown in Fig. 1, with n_t transmitting antennas and n_r receiving antennas. There are N_s information symbol streams $\mathbf{b}(\ell)$, $\ell = 1, 2, \ldots, N_s$, each of which is of length n_t. Each symbol stream $\mathbf{b}(\ell)$ carries $\log_2 MN_s$ interleaved bits of codeword $\mathcal{C}$, where M is the cardinality of the constellation set $\mathcal{A}$. The exact LLR of the ith bit of the ℓth symbol stream is

$$\lambda_i(\ell) = \log\left(\frac{\sum_{\mathbf{b}(\ell)\in\chi_1^i} \exp\left(\frac{-1}{\sigma^2} \|\tilde{\mathbf{z}}(\ell) - \mathcal{H}(\ell)\mathbf{b}(\ell)\|^2 \right)}{\sum_{\mathbf{b}(\ell)\in\chi_0^i} \exp\left(\frac{-1}{\sigma^2} \|\tilde{\mathbf{z}}(\ell) - \mathcal{H}(\ell)\mathbf{b}(\ell)\|^2 \right)} \right), \quad (8)$$

where $\chi_0^i \subset \mathcal{A}\times\mathcal{A}$, $\chi_1^i \subset \mathcal{A}\times\mathcal{A}$, and $i = 1, \ldots, \log_2|\mathcal{A}\times\mathcal{A}|$. Consider an example of a MIMO-OFDM system with $n_t = 2$ and bits mapped to a 4-QAM constellation.

Table 1. Super symbol mapping for ML receiver.

S. No.	$\mathbf{b}(\ell)$		$\lambda_1(\ell)$	$\lambda_2(\ell)$	$\lambda_3(\ell)$	$\lambda_4(\ell)$
1	S_1	S_1	0	0	0	0
2	S_1	S_2	0	0	0	1
3	S_1	S_3	0	0	1	1
4	S_1	S_4	0	0	1	0
5	S_2	S_1	0	1	0	0
6	S_2	S_2	0	1	0	1
7	S_2	S_3	0	1	1	1
8	S_2	S_4	0	1	1	0
9	S_3	S_1	1	1	0	0
10	S_3	S_2	1	1	0	1
11	S_3	S_3	1	1	1	1
12	S_3	S_4	1	1	1	0
13	S_4	S_1	1	0	0	0
14	S_4	S_2	1	0	0	1
15	S_4	S_3	1	0	1	1
16	S_4	S_4	1	0	1	0

The super constellation set $\mathcal{A}\times\mathcal{A}$ has 16 elements, and each point in the set $\mathcal{A}\times\mathcal{A}$ carries four information bits. That is, each symbol vector $\mathbf{b}(\ell)$ has two elements from set $\mathcal{A}$ and carries four information bits. It is obvious from Table 1 that we can divide the super set $\mathcal{A}\times\mathcal{A}$ into two subsets denoted by χ_0^i and χ_1^i to evaluate LLR $(\lambda_i(\ell))$.

The complexity of the ML bit metric $\Lambda(:,\ell)$ of a MIMO-BICM system with n_t transmit antennas is $|\mathcal{A}\times\mathcal{A}|^{n_t}$. Since

$$\log\left(\sum_{\mathbf{b}(\ell)\in\chi_1^i}\exp\left(\frac{-1}{\sigma^2}\left\|\tilde{\mathbf{z}}(\ell)-\mathcal{H}(\ell)\mathbf{b}(\ell)\right\|^2\right)\right) \approx -\min_{\mathbf{b}(\ell)\in\chi_1^i}\left(\frac{\left\|\tilde{\mathbf{z}}(\ell)-\mathcal{H}(\ell)\mathbf{b}(\ell)\right\|^2}{\sigma^2}\right), \tag{9}$$

the approximate LLR [25] of the ith bit of the ℓth OFDM stream is

$$\lambda_i(\ell)=\frac{1}{\sigma^2}\min_{\mathbf{b}(\ell)\in\chi_0^i}\left\|\tilde{\mathbf{z}}(\ell)-\mathcal{H}(\ell)\mathbf{b}(\ell)\right\|^2 -\frac{1}{\sigma^2}\min_{\mathbf{b}(\ell)\in\chi_1^i}\left\|\tilde{\mathbf{z}}(\ell)-\mathcal{H}(\ell)\mathbf{b}(\ell)\right\|^2. \tag{10}$$

The linear LLR metric calculation methods such as ZF and MMSE [25]–[26] provide variable alternative, which have a complexity equal to $O(n_t^3)$ for MIMO quadrature amplitude modulation (QAM). These linear methods compute LLR in two steps. First, an estimate of the symbol vector $\mathbf{b}(\ell)$ is obtained. Then this symbol estimate is used to separately compute LLR for the bits of each symbol [25]–[26].

Let $\mathbf{b}_Z(\ell)$ be the ZF estimate of the transmitted symbol vector over the OFDM subcarrier channel matrix $\mathcal{H}(\ell)$, then

$$\mathbf{b}_Z(\ell)=\mathcal{H}(\ell)^{\dagger}\tilde{\mathbf{z}}(\ell)=\mathbf{b}(\ell)+\tilde{\mathbf{w}}(\ell), \tag{11}$$

where $\tilde{\mathbf{w}}(\ell)=\mathcal{H}(\ell)^{\dagger}\mathbf{v}(\ell)$ and $\mathcal{H}(\ell)^{\dagger}=\left(\mathcal{H}(\ell)^{\mathrm{H}}\mathcal{H}(\ell)\right)^{-1}\times\mathcal{H}^{\mathrm{H}}(\ell)$. The covariance matrix of $\tilde{\mathbf{w}}(\ell)$ is

$$R_{\tilde{\mathrm{w}}}=\sigma_{\mathrm{w}}^2\left(\mathcal{H}(\ell)^{\mathrm{H}}\mathcal{H}(\ell)\right)^{-1}. \tag{12}$$

Note that $\mathbf{b}_Z(\ell)=\left[\hat{s}_1(\ell)\ \dots\ \hat{s}_{n_t}(\ell)\right]$. The ZF LLR metric $\lambda_{Z,i}^k(\ell)$ is

$$\lambda_{Z,i}^k(\ell)=\frac{1}{\sigma_{\tilde{\mathrm{w}},k}}(\min_{s\in\chi_0^i}|\hat{s}_k(\ell)-s|^2-\min_{s\in\chi_1^i}|\hat{s}_k(\ell)-s|^2), \tag{13}$$

where $k=1,\dots,n_t$. For the ZF receiver in (13), $\chi_0^i\subset\mathcal{A}$ and $\chi_1^i\subset\mathcal{A}$ with $i=1,\dots,\log_2 M$. Thus, there are $n_t\log_2 M$ LLRs corresponding to the vector $\mathbf{b}(\ell)$ of the ℓth subcarrier. Similarly, the MMSE estimate of the transmitted symbol vector of n_t elements corresponding to the ℓth subcarrier is

$$\mathbf{b}_M(\ell)=\left(\mathcal{H}(\ell)^{\mathrm{H}}\mathcal{H}(\ell)+\sigma_{\mathrm{w}}^2\mathbf{I}\right)^{-1}\mathcal{H}(\ell)\tilde{\mathbf{z}}(\ell). \tag{14}$$

Thus, the LLR metric for the MMSE receiver is

$$\lambda_{M,i}^k(\ell)=\frac{W_{k,k}}{1-W_{k,k}}\left(\min_{s\in\chi_0^i}\left|\frac{\hat{s}_k}{W_{k,k}}-s\right|^2-\min_{s\in\chi_1^i}\left|\frac{\hat{s}_k}{W_{k,k}}-s\right|^2\right), \tag{15}$$

where $W_{k,k}$ are the diagonal elements of $\mathbf{W}=(\mathbf{I}-\mathbf{R}_{\tilde{\mathrm{w}}})^{-1}$ and vector $\mathbf{b}_M(\ell)=\left[\hat{s}_1(\ell)\dots\hat{s}_{n_t}(\ell)\right]$ is the MMSE estimate of n_t symbols of the ℓth subcarrier. The complexity of LLR computation is linear in terms of the number of transmit antennas. The throughput results of optimal and suboptimal receivers are provided in Section IV.

2. Throughput Under Selective Retransmission

In this section, we discuss the throughput of conventional ARQ and HARQ methods under selective retransmission for MIMO-OFDM systems. In this investigation, we consider only one selective retransmission of poor subcarriers of each OFDM frame. That is, each subcarrier is evaluated for retransmission based on the condition number of its channel matrix. Even after the selective retransmission of the poor subcarriers, there is a chance that the condition number of the newly stacked, taller channel matrix, $\mathcal{H}(\ell)$, is greater than τ_c, as shown in Fig. 2. An extension to multiple selective retransmissions to further improve the condition number is straight forward. After the single retransmission of poor-quality subcarriers of an OFDM frame, the demodulator jointly computes the LLR of the received bits from the observation vector $\tilde{\mathbf{z}}(\ell) \in \mathcal{C}^{2n_r \times 1}$. Note that for good-quality subcarriers, the receiver computes the LLR of the received bits from the vector $\mathbf{z}(\ell) \in \mathcal{C}^{n_r \times 1}$. The LLR vector $\mathcal{L}$ is an input vector to the FEC decoder and can be computed using the optimal (ML) and suboptimal (ZF and MMSE) methods discussed in Section III-1. The selective retransmission at the modulation layer enhances the channel quality for conventional ARQ and HARQ methods, which results in fewer retransmission requests.

We denote the ARQ method with selective retransmission at the modulation layer enabled, by S-ARQ. The CC-HARQ and IR-HARQ methods when selective retransmission at the physical layer is active are denoted by SCC-HARQ and SIR-HARQ, respectively. We also denote CC without FEC as CC-ARQ.

The throughput of ARQ and HARQ methods can be evaluated from the BER of the receiver. Let P_{c_1} and P_{e_1} be the probabilities of receiving an error-free packet and an erroneous packet, each of length L_P, during the first transmission, respectively, with a BER probability of P_{b_1}, under the assumption that the probability of a bit to be erroneous is independent of the other bits in the frame. Similarly, P_{c_2} and P_{e_2} are the probabilities of receiving an error-free packet and an erroneous packet, each of length L_P, respectively, for a retransmission having a BER probability of P_{b_2}. Note that for joint detection, $P_{b_1} > P_{b_2}$. Then, the average number of retransmissions to deliver an error-free packet is calculated to be [14]

$$T_a = \frac{2P_{c_1} - P_{c_1}^2 - 3P_{c_1}P_{c_2} + 2P_{c_2} + P_{c_1}^2 P_{c_2}}{(P_{c_1} + P_{c_2} - P_{c_1}P_{c_2})^2}. \tag{16}$$

Note that for simple ARQ, $P_{c_1} = P_{c_2}$, then $T_a = 1/P_{c_1}$, and the throughput is then $\eta = 1/T_a = P_{c_1}$ [27]. When selective retransmission is enabled at the modulation level, we denote the overhead due to selective retransmission by m. The targeted retransmission improves the BER of the receiver. The throughput of a receiver, η, is the ratio of error-free received bits, k, to the total number of bits transmitted, $\tilde{n}$, to deliver k bits. Under selective retransmission at the modulation level with overhead m, we have

$$\tilde{n} = k(m+1)T_a. \tag{17}$$

Note that (17) provides the throughput of the S-ARQ method. For $m = 0$, S-ARQ becomes the conventional ARQ method. To compute T_a from (16) for the ARQ and S-ARQ methods, $P_{c_1} = P_{c_2}$. In SCC-HARQ, selective retransmission is enabled to improve the channel quality for CC-HARQ. Let R be the code rate of FEC for the CC-HARQ method. With m as overhead due to selective retransmission, we have

$$\tilde{n} = \frac{k}{R}(m+1)T_a. \tag{18}$$

Let $\tilde{T} = \lfloor T_a \rfloor$ and $\Delta T = T_a - \lfloor T_a \rfloor$ be the integer and fractional parts of the average number of transmissions to receiver error-free k bits, respectively. For IR-HARQ with rate R_1 and R_2 under selective retransmission (SIR-HARQ), we have

$$\tilde{n} = k(m+1)\left(\left\lfloor \frac{\tilde{T}}{2} \right\rfloor \frac{1}{R_2} + \frac{1}{R_1}\Delta T\right), \ \tilde{T} \text{ is even}, \tag{19}$$

and

$$\tilde{n} = k(m+1)\left(\left\lfloor \frac{\tilde{T}}{2} \right\rfloor \frac{1}{R_2} + \frac{1}{R_1} + (\frac{1}{R_2} - \frac{1}{R_1})\Delta T\right), \ \tilde{T} \text{ is odd}. \tag{20}$$

The throughput of the receiver is $\eta = k/\tilde{n}$, where we compute $\tilde{n}$ using (17), (18), (19), or (20) depending upon the retransmission method. For the throughput of a receiver without selective retransmission, $m = 0$.

For selective retransmission, the receiver evaluates the condition numbers for N_s matrices of order $n_r \times n_t$, where n_t and n_r are the number of transmit and receive antennas, respectively. The complexity of computing singular values is $\mathrm{O}(n_r n_t^2)$. Thus, the complexity of N_s many condition numbers is $\mathrm{O}(N_s n_r n_t^2)$ [28]. The proposed selective retransmission computes N_s many condition numbers once during each coherent time T_c. Furthermore, the complexity of the LLR computation for a single transmission and for joint detection from multiple transmissions of the same vector is similar [3]. Thus, the computation of N_s condition numbers once during each coherent time T_c is the computational overhead from selective retransmission.

IV. Simulations

Now, we are ready to present the performance of our proposed selective retransmission method embedded at the modulation layer. We present the efficacy of the selective retransmission method with respect to BER for a given MIMO-OFDM system and throughput gain as compared to Chase-combining without FEC (CC-ARQ). The throughput gain of a conventional ARQ with selective retransmission enabled (S-ARQ) over simple ARQ is presented in this section. We also provide a comparison of the throughput between SIR-HARQ and conventional IR-HARQ. Note that SIR-HARQ is

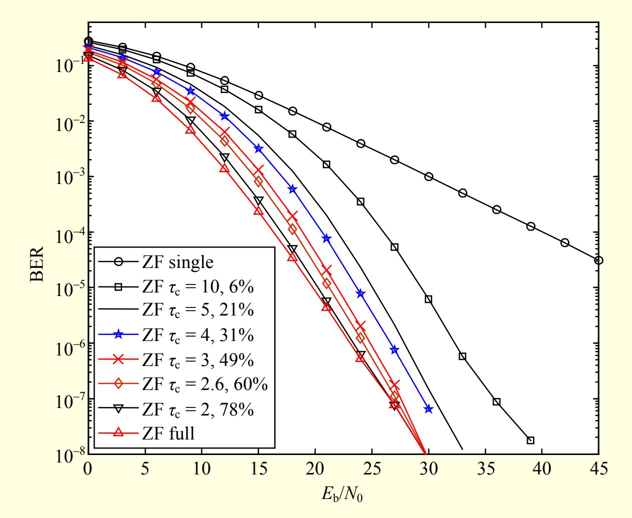


Fig. 4. BER of ZF receiver of 2 × 2 MIMO-OFDM system for different threshold τ_c on condition number $\kappa(\ell)$ without FEC.

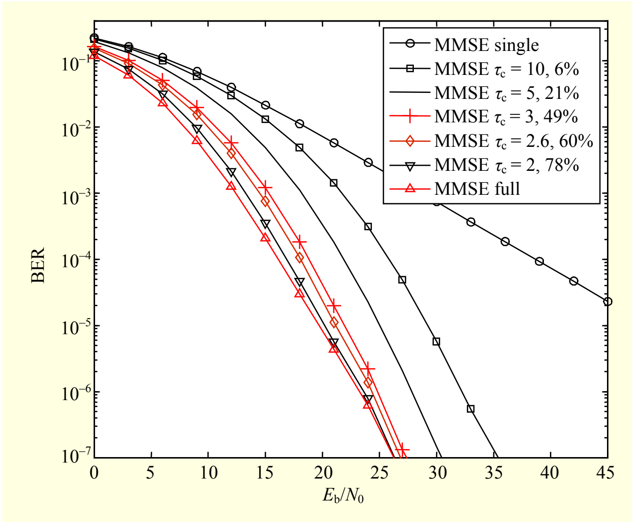


Fig. 5. BER of MMSE receiver of 2 × 2 MIMO-OFDM system for different threshold τ_c on condition number $\kappa(\ell)$ without FEC.

actually IR-HARQ with selective retransmission enabled at the modulation layer. In the simulation setup, we consider the block-fading frequency-selective Rayleigh fading channel. The path gains of the MIMO channel are independent and identically distributed with zero mean and unit variance. We consider MIMO-OFDM modulation with $N_s = 648$ subcarriers. Note that the conventional retransmission methods, such as CC-HARQ and IR-HARQ, are unaware of the proposed selective retransmission at the modulation layer. We encode information bits with LDPC code at rates $R_1 = 3/4$ and $R_2 = 1/2$ for IR-HARQ. The code rate of the first transmission is $R_2 =$ 1/2. In the case of NACK, the transmitter sends more parity information, which results in a low code rate of $R_2 = 1/2$. We use the sum–product algorithm with 50 iterations to decode information bits for coded selective HARQ. We compare the BER of the selective retransmission (S-ARQ) with the full retransmission (CC-ARQ) to study the efficacy of the proposed method for optimal (ML) and suboptimal (ZF and MMSE) detection with and without FEC codes. The objective of this investigation is to present low-complexity power and bandwidth-efficient transceiver design.

First, we present a BER performance comparison of the selective retransmission and CC without FEC with optimal (ML) and suboptimal receivers. This comparison reveals that a full retransmission is unnecessary to recover packet errors. Targeted retransmission achieves a BER performance equivalent to that of a CC. This is due to the fact that for a given noise strength, the probability of error in the decoded bits corresponding to the subcarriers of channel matrices with large corresponding condition number is higher. The proposed retransmission approach targets information corresponding to such subcarriers for retransmission. The performance of joint detection under selective retransmission is bounded by the BER of a full retransmission (CC). Therefore, we compare the BER of the selective retransmission with CC for different threshold values on condition number $\kappa(\ell)$. Figure 4 compares the BER performances of a selective retransmission with CC for a ZF receiver. The condition number of the MIMO channel has a significant impact on the performance of suboptimal receivers as compared to optimal detection [16]. The simulation results reveal that by retransmitting 49% of the information bits, which correspond to the information symbols retransmitted over subcarriers of corresponding condition number $\kappa(\ell) \geq \tau_c = 3$, the performance gap between full retransmission and selective retransmission is marginal. This performance gap diminishes in high SNR regimes. A similar behavior can be observed for an MMSE receiver, as shown in Fig. 5.

Figure 6 compares the throughput of the proposed method (S-ARQ) for suboptimal receivers (ZF and MMSE) with CC-ARQ without FEC. Note that we decrease the information in the retransmitted frame as E_b/N_0 increases. It can be observed that S-ARQ provides significant throughput gain over CC-ARQ. The throughput of the conventional ARQ method is much lower than that of S-ARQ and CC-ARQ. This is due to the fact that the ARQ method does not save observations from the first transmission; hence, it does not perform joint detection. The throughput of S-ARQ is better than CC-ARQ for E_b/N_0 in the range of 15 dB to 35 dB. Figure 6 also shows that the throughput of the ZF and MMSE suboptimal receivers is similar.

To observe the impact of feedback information on the throughput of the proposed method for selective retransmission,

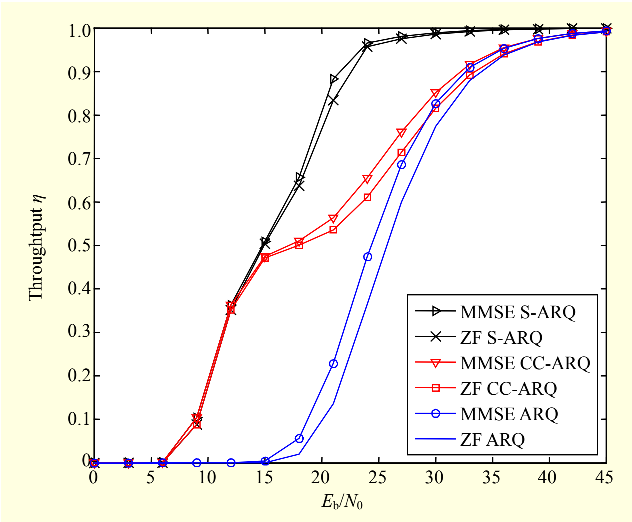


Fig. 6. Throughput comparison of S-ARQ, CC-ARQ, and simple ARQ with ZF and MMSE, without FEC.

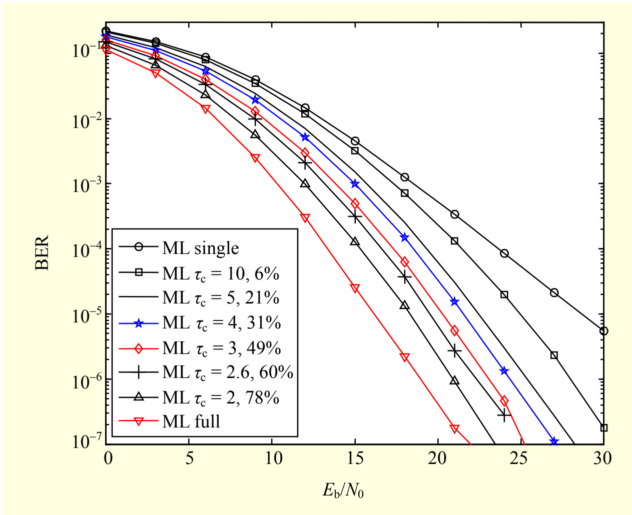


Fig. 8. BER of ML receiver of a 2 × 2 MIMO-OFDM system for different thresholds τ_c on condition number $\kappa(\ell)$ without FEC.

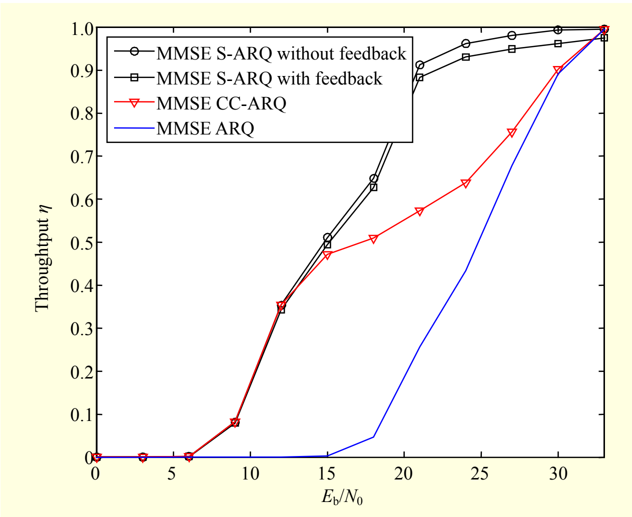


Fig. 7. Impact of channel feedback on the throughput of the proposed selective retransmission for a Doppler shift of 30 Hz.

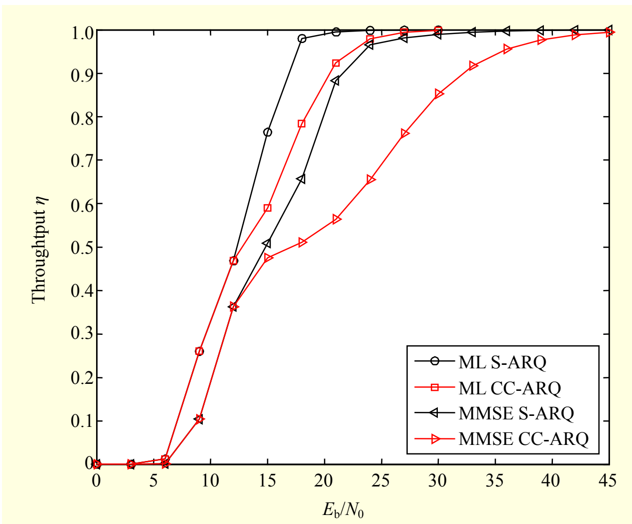


Fig. 9. Throughput comparison of ML and MMSE for S-ARQ, CC-ARQ, and ARQ.

we consider a 7-tap MIMO channel model for LTE [29] with 10 MHz bandwidth and 256 subcarriers. Note that the condition number of the MIMO-OFDM subcarrier channel matrix is sensitive to agitation. In the simulation setup, we update the condition numbers after every 15 OFDM symbols for a Doppler frequency of 30 Hz. That is, 256-bit channel feedback information is sent to the transmitter after every 15 symbols for selective retransmission. For 4-QAM constellation, for one bit transmitted to the receiver, there is an overhead of 0.03 bits in the feedback path. Figure 7 shows the impact of channel feedback information on the throughput for the MMSE receiver under a Doppler frequency of 30 Hz. Simulation results reveal that the impact of this feedback information on the throughput is marginal. The channel feedback has more impact on the throughput at higher E_b/N_0. We also observed a similar trend for the ZF receiver. Next, we discuss the BER performance and throughput of the ML receiver for MIMO-OFDM systems.

Figure 8 shows the BER performance of the ML receiver for a single transmission, S-ARQ, and CC. The BER improvement in Fig. 8 shows that the BER gap between full retransmission and selective retransmission with an optimal receiver is larger as compared to suboptimal receivers (ZF and MMSE), as shown in Figs. 4 and 5. The throughput of the CC-ARQ and S-ARQ methods with an ML receiver is compared in Fig. 9. It can be observed from Fig. 9 that the S-ARQ

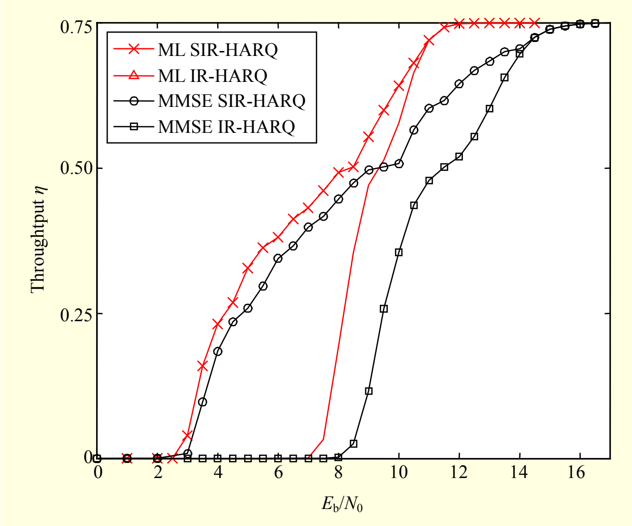


Fig. 10. Throughput comparison of SIR-HARQ and IR-HARQ with ML and MMSE receivers under an LDPC code of rate R=3/4 and R=1/2.

method with MMSE receiver achieves an equivalent throughput of CC-ARQ with ML receiver. It can be noticed that retransmission using the condition number is more effective for low-complexity suboptimal receivers.

The throughput comparison of conventional IR-HARQ and SIR-HARQ is presented in Fig. 10. Note that SIR-HARQ is conventional IR-HARQ with selective retransmission enabled at the modulation level. In SIR-HARQ, first, one round of selective retransmission is completed at the modulation layer using the condition number as the retransmission criterion. Then, LLR is computed for the FEC decoder. The selective retransmission significantly improves LLR and provides good channel quality to the conventional HARQ methods. A throughput comparison between IR-HARQ and SIR-HARQ is provided in Fig. 9. SIR-HARQ with ML receiver also provides a throughput gain over the conventional IR-HARQ. Figure 10 also compares the throughput gap between the ML and MMSE receivers. The throughput of the ML receiver is higher than that of the MMSE receiver for both IR-HARQ and SIR-HARQ.

V. Conclusion

This work presented a selective retransmission method for MIMO-OFDM systems using the condition numbers of the channel matrices of subcarriers as its retransmission criterion. The targeted retransmission at the modulation layer is independent of conventional ARQ and HARQ methods and provides enhanced channel quality for conventional ARQ and HARQ schemes. We present a BER and throughput analysis for optimal and suboptimal receivers and compare the performance of selective ARQ and selective HARQ methods with conventional ARQ and HARQ methods. The simulation results demonstrate a significant throughput improvement in conventional ARQ and HARQ methods when selective retransmission is enabled. The proposed method can be customized to improve the throughput of the link for an application based on the BER requirement and strength of FEC.

References

[1] L. Zheng and D. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, May 2003, pp. 1073–1096.

[2] S. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, Oct. 1998, pp. 1451–1458.

[3] E.G. Larsson and P. Stoica, "*Space-Time Block Coding for Wireless Communication*," Cambridge, England: Cambridge University Press, 2003.

[4] 3GPP Release 10, *IEEE Standard for Long Term Evolution of the 3GPP Radio Technology*. http:/www.3gpp.org

[5] S. Lin and P.S. Yu, "A Hybrid ARQ Scheme with Parity Retransmission for Error Control of Satellite Channels," *IEEE Trans. Commun.*, vol. 30, no. 7, July 1982, pp. 1701–1719.

[6] S. Park and D. Love, "Hybrid ARQ Protocol for Multi-antenna Multicasting Using a Common Feedback Channel," *IEEE Trans. Commun.*, vol. 59, no. 6, June 2011, pp. 1530–1542.

[7] C. Shen and M.P. Fitz, "Hybrid ARQ in Multiple-Antenna Slow Fading Channels: Performance Limits and Optimal Linear Dispersion Code Design," *IEEE Trans. Inf. Theory*, vol. 57, no. 9, Sept. 2011, pp. 5863–5883.

[8] B. Mikki and T. Eriksson, "On the Average Rate of HARQ-Based Quasi-Static Spectrum Sharing Networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, Jan. 2012, pp. 65–77.

[9] J.-F. Cheng, "Coding Performance of Hybrid ARQ Schemes," *IEEE Trans. Commun.*, vol. 54, no. 6, June 2006, pp. 1017–1029.

[10] P. Wu and N. Jindal, "Performance of Hybrid-ARQ in Block-Fading Channels: A Fixed Outage Probability Analysis," *IEEE Trans. Commun.*, vol. 58, no. 4, Apr. 2010, pp. 1129–1141.

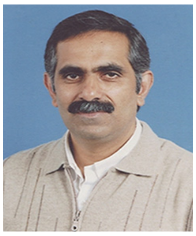
[11] H. Samra, Z. Ding, and P.M. Hahn, "Optimal Symbol Mapping Diversity for Multiple Packet Transmissions," *IEEE Int. Conf. Acoust., Speech Signal Process.*, Hong Kong, China, vol. 4, Apr. 6–10, 2003, pp. 181–184.

[12] M. Zia and Z. Ding, "Joint ARQ Receiver Design for Bandwidth Efficient MIMO Systems," *IEEE Global Telecommun. Conf.*, New Orleans, LO, USA, Nov. 30–Dec. 4, 2008, pp. 1–5.

[13] X. Liang, C.-M. Zhao, and Z. Ding, "Piggyback Retransmissions over Wireless MIMO Channels: Shared Hybrid-ARQ (SHARQ) for Bandwidth Efficiency," *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, Aug. 2013, pp. 3770–3782.

[14] M. Zia et al., "Selective HARQ Transceiver Design for OFDM

System," *IEEE Commun. Lett.*, vol. 17, no. 12, Oct. 2013, pp. 2229–2232.
[15] J. Maurer, G. Matz, and D. Seethaler, "Low-Complexity and Full-Diversity MIMO Detection Based on Condition Number Thresholding," *IEEE Int. Conf. Acoust., Speech Signal Process.*, Honolulu, HI, USA, vol. 3, Apr. 15–20, 2007, pp. 61–64.
[16] D. Tse and P. Viswanath, "*Fundamentals of Wireless Communication*," Cambridge, England: Cambridge University Press, 2005.
[17] A. Forenza et al., "Adaptive MIMO Transmission Scheme: Exploiting the Spatial Selectivity of Wireless Channels," *IEEE Veh. Technol. Conf.*, Stockholm, Sweden, vol. 5, May 30–June 1, 2005, pp. 3188–3192.
[18] J. Zik, *Maximizing LTE Performance through MIMO Optimization*, PCTEL Inc., Germantown, PA, USA, Apr. 2011.
[19] "MIMO Performance and Condition Number in LTE Test," Application Note, Agilent Technologies.
[20] N. Kita et al., "Measurement of Demmel Condition Number for 2×2 MIMO-OFDM Broadband Channels," *IEEE Veh. Technol. Conf.*, Los Angeles, CA, USA, vol. 1, May 17–19, 2004, pp. 294–298.
[21] G. Caire, G. Taricco, and E. Biglieri, "Bit-Interleaved Coded Modulation," *IEEE Trans. Inf. Theory*, vol. 44, no. 3, May 1998, pp. 927–946.
[22] S.H. Muller, "Coding Approaches for Multiple Antenna Transmission in Fast Fading OFDM," *IEEE Trans. Signal Process.*, vol. 50, no. 10, Oct. 2002, pp. 2442–2450.
[23] E.G. Larsson, P. Stoica, and J. Li, "Orthogonal Space-Time Block Codes: Maximum Likelihood Detection for Unknown Channels and Unstructured Interferences," *IEEE Trans. Signal Process.*, vol. 51, no. 2, Feb. 2003, pp. 362–372.
[24] D. Seethaler, G. Matz, and F. Hlawatsch, "An Efficient MMSE-Based Demodulator for MIMO Bit-Interleaved Coded Modulation," *IEEE Global Telecommun. Conf.*, Dallas, TX, USA, vol. 4, Nov. 29–Dec. 3, 2004, pp. 2455–2459.
[25] M. Butler and I. Collings, "A Zero-Forcing Approximate Log-Likelihood Receiver for MIMO Bit-Interleaved Coded Modulation," *IEEE Commun. Lett.*, vol. 8, no. 2, Feb. 2004, pp. 105–107.
[26] R. Ghaffar and R. Knopp, "Reduced Complexity Soft Detection for BICM MIMO OFDM System," *Int. Conf. Comput., Contr. Commun.*, Karachi, Pakistan, vol. 4, Feb. 17–18, 2009, pp. 1–5.
[27] S. Lin, D.J. Costello, and M.J. Miller, "Automatic-Repeat-Request Error-Control Schemes," *IEEE Commun. Mag.*, vol. 22, no. 12, Dec. 1984, pp. 5–17.
[28] G.H. Golub and C.F. Van Loan, "*Matrix Computation*," Baltimore, USA: JHU Press, 2013.
[29] N. Arsalane et al., "3GPP Channel Model Emulation with Analysis of MIMO-LTE Performances in Reverberation Chamber," *Int. J. Antennas Propag.*, Jan. 2012.

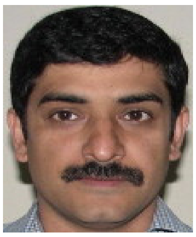


Muhammad Zia received his MS degree in electronics in 1991 and his MPhil in 1999, both from the Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan. He received his PhD degree in electrical engineering from the Department of Electrical and Computer Engineering, University of California, Davis, CA, USA, in 2010. He is currently with the Department of Electronics, Quaid-i-Azam University, as an assistant professor.

Tamoor Kiani received his BS degree in electronics engineering from Mohammad Ali Jinnah University in 2010 and his MPhil degree in electronics from the Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan. He is an active member of the Pakistan Engineering Counsel. His research interests are in the area of wireless communications and signal processing.

Nazar A. Saqib received his MS and MPhil degrees in electronics from Quaid-i-Azam University, Islamabad, Pakistan, in 1993. He received his PhD degree in electrical engineering from the Center for Research and Advanced Studies, National Polytechnic Institute, Mexico. He is currently working as an associate professor at the Department of Computer Engineering, National University of Sciences and Technology, College of Electrical and Mechanical Engineering, Islamabad, Pakistan.

Tariq Shah is working as a tenured associate professor at the Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan. He received his MS and MPhil degrees in mathematics from the Quaid-i-Azam University, Islamabad, Pakistan, in 1989 and 1991, respectively. He received his PhD degree in mathematics from the Department of Algebra, Faculty of Mathematics, University of Bucharest, Romania, in 2000.



Hasan Mahmood received his MS degree in electronics from Quaid-i-Azam University, Islamabad, Pakistan, in 1991 and his MS degree in electrical engineering from the University of Ulm, Germany, in 2002. From 1994 to 2000, he was with the Department of Electronics, Quaid-i-Azam University, as a faculty member. He received his PhD degree in electrical engineering from the Stevens Institute of Technology, Hoboken, NJ, USA, in 2007. He is currently with the Department of Electronics, Quaid-i-Azam University, as an assistant professor.