# Joint Compensation of Transmitter and Receiver IQ Imbalance in OFDM Systems Based on Selective Coefficient Updating 

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In this paper, a selective coefficient updating (SCU) approach at each branch of the per-tone equalization (PTEQ) structure has been applied for insufficient cyclic prefix (CP) length. Because of the high number of adaptive filters and their complex adaption process in the PTEQ structure, SCU has been proposed. Using this method leads to a reduction in the computational complexity, while the performance remains almost unchanged. Moreover, the use of set-membership filtering with variable step size is proposed for a sufficient CP case to increase convergence speed and decrease the average number of calculations. Simulation results show that despite the aforementioned algorithms having similar performance in comparison with conventional algorithms, they are able to reduce the number of calculations necessary. In addition, compensation of both the channel effect and the transmitter/receiver in-phase/quadraturephase imbalances are achievable by these algorithms.

Keywords: In-phase/quadrature-phase (IQ) imbalance, per-tone equalization, set-membership filtering, dataselective updating, cyclic prefix.

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## I. Introduction

Recently, direct-conversion receivers (DCRs) and transmitters have been identified as a favorable architecture to use instead of the conventional super heterodyne configuration. DCRs have substantial excellence in cost, circuit size, and power consumption [1], but there are so many non-idealities and deficiency, such as in-phase/quadrature-phone (IQ) imbalance. In a wideband DCR, two kinds of IQ imbalance exist: frequency independent (FI) and frequency selective (FS) [2]. IQ imbalance distortion significantly degrades the received signal quality. Super heterodyne architectures can decrease these effects, but this increases the costs of the overall systems.
In [3], only FI IQ imbalance at the transmitter and receiver is considered in the presence of the carrier frequency offset, and compensation of these impairments under a sufficient CP length is developed using an adaptive least mean squares (LMS) algorithm.
In [4], the effect of receiver IQ imbalance based on OFDM systems has been studied, and in both the time domain and frequency domain, a compensation scheme using a transmitted pilot has been proposed. In addition, the proposed strategy has been extended for transmitter IQ imbalance [5]-[6]. To compensate for frequency-selective receiver IQ imbalance, a strategy has been developed based on statistical signal characteristics [7]. In [8], a compensation scheme for transmitter/receiver IQ imbalance using an improved least squares (LS) method is proposed. In this method, by applying some training data, a similar performance to the traditional LS method is obtained using time-domain channel characteristics.

In [9], a blind method based on kurtosis criteria for compensation of frequency-selective IQ imbalance is presented.

In [10], a frequency-selective IQ imbalance compensation scheme based on both the time domain and frequency domain, named as the Gaussian elimination equalizer, is proposed. In comparison with conventional LS and LMS compensation schemes, the same bit error rate (BER), but with a smaller number of training OFDM symbols, is achieved.

In [11]-[12], Tandur and Moonen considered a special case where the CP is not sufficiently long to accommodate the combined transmitter/receiver IQ imbalance and channel impulse responses. This causes an inter-symbol interference (ISI) between OFDM symbols. In these articles, a frequency domain-based per-tone equalizer has been designed to decrease the length of the equivalent filter to meet the CP length; therefore, compensations were achievable in these cases.

The main contribution of this paper is to develop two lowcomplexity compensation algorithms based on both sufficient and insufficient CP length. In addition, in our proposed algorithms, FI and FS IQ imbalances are considered at both the transmitter and the receiver. In the first proposed algorithm, a set-membership filtering technique based on a simple one-tap frequency-domain equalizer is applied with the consideration of sufficient CP length.

Our second algorithm is derived with the assumption of insufficient CP length and is based on a per-tone equalization (PTEQ) structure. In this case, the selective coefficient updating (SCU) algorithm has been extended in each branch of the PTEQ structure for complexity reduction.

The organization of this paper is as follows. We first review the development of an IQ imbalance model based on OFDM systems in Section II. Also, in this section, explanations of an IQ compensation scheme for sufficient and insufficient CP length are presented. Data-selective adaptive compensation (set-membership filtering) for sufficient CP length and adaptive compensation with SCU under insufficient CP length is derived in Section III. The computational complexity of the proposed algorithms is shown in Section IV. Our simulation results are illustrated in Section V, and finally, the conclusions will be given in Section VI.

## II. Model Description with IQ Imbalance

In this section, IQ imbalance compensation and channel impairment effects are considered, in both the transmitter and the receiver, in two cases - sufficient and insufficient CP length. In the case of sufficient CP length, there is no ISI; therefore, we can equalize the system with a simple one-tap equalizer, but in the case of insufficient CP length, it is
necessary to compensate and estimate the transmitted data.

## 1. Sufficient CP Length

Firstly, in the case of sufficient CP length, we peruse both the effects and the compensation scheme of transmitter/receiver IQ imbalance.

Let us consider $\boldsymbol{S}$ as an $N \times 1$ vector, which denotes the frequency-domain OFDM symbol, where $N$ is the number of subcarriers. Then, a baseband symbol in the time domain, denoted by $\boldsymbol{s}$, can be written as follows:

$$
\begin{equation*}
\boldsymbol{s}=\boldsymbol{P}_{\mathrm{CI}} \boldsymbol{F}_{N}^{-1} \boldsymbol{S} \tag{1}
\end{equation*}
$$

where $\boldsymbol{P}_{\mathrm{CI}}$ is the CP insertion matrix (here the length of CP is $v$ ) and $\boldsymbol{F}_{N}^{-1}$ denotes the $N \times N$ inverse discrete Fourier transform (DFT) matrix. Based on [7], an equivalent baseband symbol, $\boldsymbol{p}$, after transmission distortions can be calculated as

$$
\begin{equation*}
\boldsymbol{p}=\boldsymbol{g}_{\mathrm{ta}} \otimes \boldsymbol{s}+\boldsymbol{g}_{\mathrm{tb}} \otimes \boldsymbol{s}^{*} \tag{2}
\end{equation*}
$$

where $\otimes$ represents the linear convolution operator and $\boldsymbol{s}^{*}$ depicts the complex conjugate of $\boldsymbol{s}$. At the transmitter, jointly, FI and FS IQ imbalance can be modeled as $\boldsymbol{g}_{\mathrm{ta}}$ and $\boldsymbol{g}_{\mathrm{tb}}$ filters.

$$
\begin{align*}
& \boldsymbol{g}_{\mathrm{ta}}=\boldsymbol{F}_{N}^{-1} \boldsymbol{G}_{\mathrm{ta}}=\boldsymbol{F}_{N}^{-1}\left(\frac{\boldsymbol{H}_{\mathrm{ti}}+g_{\mathrm{t}} e^{-\mathrm{j} \phi_{\mathrm{t}}} \boldsymbol{H}_{\mathrm{tq}}}{2}\right) \\
& \boldsymbol{g}_{\mathrm{tb}}=\boldsymbol{F}_{N}^{-1} \boldsymbol{G}_{\mathrm{tb}}=\boldsymbol{F}_{N}^{-1}\left(\frac{\boldsymbol{H}_{\mathrm{ti}}-g_{\mathrm{t}} e^{\mathrm{j} \phi_{\mathrm{t}}} \boldsymbol{H}_{\mathrm{tq}}}{2}\right), \tag{3}
\end{align*}
$$

where $\boldsymbol{H}_{\mathrm{ti}}$ and $\boldsymbol{H}_{\mathrm{tq}}$ are the frequency responses of the mismatched filters in I and Q branches, respectively, and $\boldsymbol{g}_{\mathrm{t}}$ and $\phi_{t}$ are the amplitude and phase imbalance in the transmitter, respectively.
When a distorted symbol is transmitted through a quasi-static frequency-dependent channel of length $L_{\text {tap }}$, then the received baseband symbol, denoted by $\boldsymbol{r}$, can be written as

$$
\begin{align*}
\boldsymbol{r} & =\boldsymbol{c} \otimes \boldsymbol{p}+\boldsymbol{n} \\
& =\boldsymbol{c} \otimes \boldsymbol{g}_{\mathrm{ta}} \otimes \boldsymbol{s}+\boldsymbol{c} \otimes \boldsymbol{g}_{\mathrm{tb}} \otimes \boldsymbol{s}^{*}+\boldsymbol{n}  \tag{4}\\
& =\boldsymbol{c}_{\mathrm{a}} \otimes \boldsymbol{s}+\boldsymbol{c}_{\mathrm{b}} \otimes \boldsymbol{s}^{*}+\boldsymbol{n}
\end{align*}
$$

where $\boldsymbol{c}$ represents the channel in the baseband model, $\boldsymbol{c}_{\mathrm{a}}$ and $\boldsymbol{c}_{\mathrm{b}}$ can be considered as filters having a tap length of $\left(L_{\text {tap }}+L_{t}\right)-1$. They jointly reflect both the channel and transmitter IQ imbalance effects. In addition, $L_{\mathrm{t}}$ denotes the mismatched filter tap length, and $\boldsymbol{n}$ is a complex additive white Gaussian noise. Similar to (2), the received symbol $z$ can be given as

$$
\begin{equation*}
\boldsymbol{z}=\boldsymbol{g}_{\mathrm{ra}} \otimes \boldsymbol{r}+\boldsymbol{g}_{\mathrm{rb}} \otimes \boldsymbol{r}^{*} \tag{5}
\end{equation*}
$$

where $\boldsymbol{g}_{\mathrm{ra}}$ and $\boldsymbol{g}_{\mathrm{b}}$ are defined similar to $\boldsymbol{g}_{\mathrm{ta}}$ and $\boldsymbol{g}_{\mathrm{tb}}$ in (3).

By substituting (4) into (5), the following equation can be obtained:

$$
\begin{align*}
z= & \left(g_{\mathrm{ra}} \otimes \boldsymbol{c}_{\mathrm{a}}+\boldsymbol{g}_{\mathrm{rb}} \otimes \boldsymbol{c}_{\mathrm{b}}^{*}\right) \otimes \boldsymbol{s} \\
& +\left(\boldsymbol{g}_{\mathrm{ra}} \otimes \boldsymbol{c}_{\mathrm{b}}+\boldsymbol{g}_{\mathrm{rb}} \otimes \boldsymbol{c}_{\mathrm{a}}^{*}\right) \otimes \boldsymbol{s}^{*} \\
& +\boldsymbol{g}_{\mathrm{ra}} \otimes \boldsymbol{n}+\boldsymbol{g}_{\mathrm{rb}} \otimes \boldsymbol{n}^{*}  \tag{6}\\
= & \boldsymbol{d}_{\mathrm{a}} \otimes \boldsymbol{s}+\boldsymbol{d}_{\mathrm{b}} \otimes \boldsymbol{s}^{*}+\boldsymbol{n}_{\mathrm{c}} .
\end{align*}
$$

In (6), the effects of the transmitter/receiver IQ imbalances with respect to the channel impairments, are modeled by $\boldsymbol{d}_{\mathrm{a}}$ and $\boldsymbol{d}_{\mathrm{b}}$, respectively. These filters have $\left[\left(L_{\mathrm{t}}+L_{\mathrm{r}}+L_{\text {tap }}\right)-2\right]$ taps, where $L_{\mathrm{r}}$ is the length of the receiver's mismatched filter. Also, $\boldsymbol{n}_{\mathrm{c}}$ is a zero-mean improper complex noise due to the presence of receiver IQ imbalance [13].

In the case of sufficient CP, to overcome the problem of ISI, it is assumed that the CP length, $v$, is larger than the length of the filters $\boldsymbol{d}_{\mathrm{a}}$ and $\boldsymbol{d}_{\mathrm{b}}$; thus, there is no ISI between adjacent OFDM symbols. The received symbol $z$, shown in (6), can be rewritten in the frequency domain as follows:

$$
\begin{equation*}
\boldsymbol{Z}=\boldsymbol{F}_{N} \boldsymbol{P}_{\mathrm{CR}}\{\boldsymbol{z}\}=\boldsymbol{D}_{\mathrm{a}} \cdot \boldsymbol{S}+\boldsymbol{D}_{\mathrm{b}} \cdot \boldsymbol{S}_{\mathrm{m}}^{*}+\boldsymbol{N}_{\mathrm{c}}, \tag{7}
\end{equation*}
$$

where $\boldsymbol{P}_{\mathrm{CR}}$ indicates the CP removal matrix and $\boldsymbol{D}_{\mathrm{a}}, \boldsymbol{D}_{\mathrm{b}}$, and $\boldsymbol{N}_{\mathrm{c}}$ are the Fourier transforms of $\boldsymbol{d}_{\mathrm{a}}, \boldsymbol{d}_{\mathrm{b}}$, and $\boldsymbol{n}_{\mathrm{c}}$, respectively. Also, $(.)_{\mathrm{m}}$ represents the mirroring operation, which can be denoted as

$$
\boldsymbol{S}_{\mathrm{m}}[l]=\boldsymbol{S}\left[l_{\mathrm{m}}\right]
$$

where

$$
\left[l_{\mathrm{m}}\right]= \begin{cases}{[N-l+2]} & \text { for }[l]=2, \ldots, N  \tag{8}\\ {[l]} & \text { for }[l]=1\end{cases}
$$

From (7), it is clear that the transmitter and receiver IQ imbalances cause a power leak from the symbol on the mirror subcarrier, $\boldsymbol{S}_{\mathrm{m}}^{*}$, to the desired subcarrier as $\boldsymbol{S}$; this is the cause of the inter-carrier interference phenomenon.
The received symbol and its complex conjugate mirror can be summarized in matrix notation by the following equation:

The matrix $\boldsymbol{D}_{\text {tot }}[l]$ represents the joint transmitter/receiver IQ imbalance and corresponding channel impulse response impairments [14] for the received symbol denoted by $\boldsymbol{Z}_{\text {tot }}[l]$. Clearly, based on (9), we can derive a symbol estimation using a suitable linear combination of $\boldsymbol{Z}[l]$ and $\boldsymbol{Z}^{*}\left[l_{\mathrm{m}}\right]$ can be found as follows:

$$
\tilde{\boldsymbol{S}}[l]=\left[\begin{array}{ll}
\boldsymbol{W}_{\mathrm{a}}[l] & \boldsymbol{W}_{\mathrm{b}}[l]
\end{array}\right]\left[\begin{array}{c}
\boldsymbol{Z}[l]  \tag{10}\\
\boldsymbol{Z}^{*}\left[l_{\mathrm{m}}\right]
\end{array}\right],
$$

where $\boldsymbol{W}_{\mathrm{a}}[l]$ and $\boldsymbol{W}_{\mathrm{b}}[l]$ can be viewed as a two-tap frequencydomain equalizer (FEQ). In addition, $\boldsymbol{W}_{\mathrm{a}}$ and $\boldsymbol{W}_{\mathrm{b}}$ can be estimated by mean square error (MSE) criteria as follows:

$$
\min _{\boldsymbol{W}_{\mathrm{a}}[l], \boldsymbol{W}_{\mathrm{b}}[l]} \mathrm{E}\left\{\left\lvert\, \boldsymbol{S}[l]-\left[\begin{array}{ll}
\boldsymbol{W}_{\mathrm{a}}[l] & \left.\left.\boldsymbol{W}_{\mathrm{b}}[l]\right]\left.\left[\begin{array}{c}
\boldsymbol{Z}[l] \\
\boldsymbol{Z}^{*}\left[l_{\mathrm{m}}\right]
\end{array}\right]\right|^{2}\right\} . . . . ~ \tag{11}
\end{array}\right.\right.\right.
$$

After the calculation of the FEQ coefficients, they can be used to equalize the impairments that have been caused by IQ imbalances and channel effects [14].

## 2. Insufficient CP Length

Under the assumption of insufficient CP length, estimating and compensating impairments, such as IQ imbalance and channel effect, cannot be achieved by (10). According to Fig. 1, a compensation scheme containing a time-domain equalizer (TEQ) and FEQ is used. This method firstly compensates the receiver IQ imbalance and then shortens the length of the channel and transmitter/receiver IQ imbalance impulse responses effects. Consequently, the effective channel meets within the CP length in the time domain by using TEQ stages 1 and 2, respectively. Then, this method carries out a joint compensation of transmitter IQ imbalance and channel effect in the frequency domain by using a two-tap FEQ in the final stage [14]. The main drawback of this method is that it carries out the joint compensation in both the time domain and frequency domain; therefore, its complexity is high [15]. To simplify the estimation of coefficients in this structure, the TEQ method is moved to the frequency domain to obtain a unified compensation structure, which is otherwise known as a PTEQ.
A PTEQ is a unified compensation structure (see Fig. 2), where equalization is performed individually on each subcarrier after taking the DFT of the received signal $\boldsymbol{z}$. In comparison with the "TEQ + FEQ" scheme, the PTEQ scheme works at a lower sampling rate; thus, its overall implementation cost can be reduced [15]. In this structure, each branch utilizes one DFT operation and contains $L^{\prime \prime}-1$ difference terms. Finally, the transmitted symbol can be given as

$$
\tilde{\boldsymbol{S}}^{(i)}[l]=\left[\begin{array}{ll}
\boldsymbol{W}_{\mathrm{va}}^{(i)}[l] & \boldsymbol{W}_{\mathrm{vb}}^{(i)}[l]
\end{array}\right]\left[\begin{array}{c}
F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}  \tag{12}\\
\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)^{*}
\end{array}\right],
$$

where $F_{\text {ext }}[l]$ is given as

$$
F_{\mathrm{ext}}[l]=\left[\begin{array}{lll}
\boldsymbol{I}_{L^{\prime}-1} & 0_{\left(L^{\prime}-1\right) \times\left(N-L^{\prime}+1\right)} & -\boldsymbol{I}_{L^{\prime}-1}  \tag{13}\\
0_{1 \times\left(L^{\prime}-1\right)} & \boldsymbol{F}_{N}[l]
\end{array}\right],
$$



Fig. 1. TEQ with two-tap FEQ per subcarrier [14].


Fig. 2. Two-branch PTEQ OFDM in presence of IQ imbalance and insufficient CP length [15].
and $L^{\prime \prime}=\left(L^{\prime}-1\right)+L_{\mathrm{r}}$.
In the matrix in (13), the first row represents the difference terms, and the second row the DFT matrix. Now, for the $l$ th subcarrier, it is possible to obtain the PTEQ coefficients according to the following MSE minimization criteria:

According to the block diagram shown in Fig. 2, every subcarrier must be equalized based on adaptive filters. Therefore, its computational complexity is very high because there are $2 N L^{\prime \prime}$ taps that should be learnt. As an example, for
$N=64$ subcarriers and $L^{\prime \prime}=15$ for a channel with severe ISI effect, the number of taps to be learnt equals 1,920 .
In this article, to decrease the computational complexity, a new adaptive algorithm based on an SCU technique will be proposed. Also, we will apply the set-membership filtering method in the case of sufficient CP length to increase the speed of convergence and achieve a considerable reduction in the average number of computations.

## III. Proposed Method

Our proposed algorithm will consider IQ imbalance and channel impairment equalization in the following two cases:


Fig. 3. Proposed method using data-selective updating technique under sufficient CP length.
sufficient and insufficient CP length. It makes use of setmembership filtering and SCU methods based on adaptive techniques such as those found in the normalized least mean square (NLMS) algorithm.

## 1. Sufficient CP Length

Here, at first, the idea of data-selective updating (setmembership filtering) based on NLMS-type adaptive filtering with variable step size is extended to IQ and channel impairment compensation under the assumption of sufficient CP length. The proposed block diagrams are shown in Fig. 3. Set-membership filtering is used to decrease the computational complexity of the adaptive filters used and increase the rate of convergence of the adaptive algorithms. This can be achieved by controlling the step size and number of iterations used in the updating of equalizer taps in subcarriers.

The adaptation of tap weights in our proposed method, based on set-membership NLMS (SM-NLMS) with variable step size, can be realized through the following equations:

$$
\begin{gather*}
\boldsymbol{W}_{\mathrm{a}}^{(i+1)}[l]=\boldsymbol{W}_{\mathrm{a}}^{(i)}[l]+\alpha^{(i)} \cdot e^{(i)}[l] * \frac{\underline{\boldsymbol{Z}}}{} \underline{\underline{Z}}^{(i)}[l] \\
\left|\underline{\underline{Z}}^{(i)}[l]\right|  \tag{15}\\
\text { Where } \alpha^{(i)}= \begin{cases}1-\frac{\gamma}{\mid e^{(i)}[l]}[l] & \text { if } \gamma>\left|\boldsymbol{W}_{\mathrm{b}}^{(i)}[l]+\alpha^{(i)}[l]\right|, \gamma=\sqrt{5 \sigma_{n}^{2}}, \\
0 & \text { otherwise. }\end{cases}
\end{gather*}
$$

In the above, $\alpha^{(i)}$ is a variable step size, $\gamma$ is a threshold value that can be calculated based on noise variance, and $i$ denotes an instance in time. It should be noted that, according to [16], the value of $\gamma$ has to be achieved empirically. Moreover, because this method is used in an AWGN channel, the noise variance can be easily calculated in the pilot period. In the SM-NLMS algorithm, an upper bound for the $\alpha^{(i)}$ parameter is considered to limit and control the estimation error [16].
A data-selective method (set-membership filtering) can reduce the number of adaptive algorithm calculations because it doesn't update the coefficients when there is a tolerable error. Moreover, it uses a step size dependent upon noise variance to increase the rate of convergence.

## 2. Insufficient CP Length

In the case of insufficient CP length, the idea of SCU of an NLMS-type algorithm is extended to the framework of the PTEQ structure for IQ and channel-effect compensation. In this case, for a given acceptable error, only a percentage of the filter coefficients in any iteration will be updated using the SCU scheme. Figure 4 shows our proposed method based on an SCU technique in a PTEQ structure. In comparison, an adaptive algorithm based on an SCU technique leads to a considerable reduction in computations with respect to conventional counterpart adaptive methods [17].
At first, we formulate a traditional NLMS in a PTEQ structure and then an SCU-NLMS within this new structure will be developed. In a PTEQ structure, we can calculate the output of the adaptive filter as $\hat{\boldsymbol{S}}^{(i)}[l]$ at any time $i$ by using (12),


Fig. 4. Proposed two-branch PTEQOFDM in presence of IQ imbalance under insufficient CP length using SCU method.
where $\boldsymbol{W}_{\mathrm{vx}}[l]=\left[\boldsymbol{W}_{\mathrm{vx}, 0}\left[[], \boldsymbol{W}_{\mathrm{vx}, 1}[l], \ldots, \boldsymbol{W}_{\mathrm{vx}, L^{\prime \prime}-1}[l]\right]^{\mathrm{T}}(\right.$ for $x=\mathrm{a}, \mathrm{b})$ is defined as an $L^{\prime \prime} \times 1$ filter coefficient vector, and $F_{\text {ext }}^{(i)}[l] z$ and $\left(F_{\text {ext }}^{(i)}\left[l_{\mathrm{m}}\right] z\right)^{*}$ are the two $L^{\prime \prime} \times 1$ regressor vectors. The NLMS algorithm for each branch in the PTEQ structure can be derived by solving the following constrained minimization problem similar to [18]:

$$
\min _{\boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]+\boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]}\left\|\boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]+\boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}}^{(i)}[l]-\boldsymbol{W}_{\mathrm{vb}}^{(i)}[l]\right\|_{2}^{2}
$$

s.t.

$$
\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}} \boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]+\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{* \mathrm{~T}} \boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]=\boldsymbol{S}[l],
$$

where $S[l]$ is a known transmitted pilot. These equations are very complicated. So, for simplicity, we assume that in any adaptive filter, each branch can obtain the symbol $\boldsymbol{S}[l]$ after the learning procedure. Therefore, the above equations can be decomposed into two independent equations, as shown in the following equations, so as to be able to solve the problem in a simpler manner:

$$
\begin{align*}
& \qquad \min _{\boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]}\left\|\boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}}^{(i)}[l]\right\|_{2}^{2}  \tag{16}\\
& \text { s.t. } \quad\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}} \boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]=\boldsymbol{S}[l]
\end{align*}
$$

and

$$
\begin{gather*}
\min _{\boldsymbol{W}_{\mathrm{b}}^{(i+1)}[l]}\left\|\boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vb}}^{(i)}[l]\right\|_{2}^{2}  \tag{17}\\
\text { s.t. }\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{* T} \boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]=\boldsymbol{S}[l] .
\end{gather*}
$$

Based on the above decomposed equations and NLMS algorithm, the update equations to calculating the filter taps can
be written as follows:

$$
\begin{aligned}
\boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]= & \boldsymbol{W}_{\mathrm{va}}^{(i)}[l] \\
& +\frac{\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}}\left(\boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}}^{(i)}[l]\right)\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)}{\left\|\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)\right\|_{2}^{2}}
\end{aligned}
$$

and

$$
\begin{align*}
\boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]= & \boldsymbol{W}_{\mathrm{vb}}^{(i)}[l] \\
& +\frac{\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right]\right)^{*}{ }^{*}\left(\boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vb}}^{(i)}[l]\right)\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{*}}{\left\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{*}\right\|_{2}^{2}} . \tag{19}
\end{align*}
$$

In the PTEQ structure, we encounter more computation complexity because there are many branches and tap filters to learn. Therefore, a new method is needed to reduce these complexities. Hence, in this article, to decrease the computational complexity, updating a percentage of the filter coefficients at any iteration is proposed based on an SCU method to achieve an acceptable error. First, the obtained vector $\left(F_{\text {ext }}^{(i)}[l] \boldsymbol{z}\right)$ and coefficient vector $\left(\boldsymbol{W}_{\mathrm{vx}}[l]\right)$ are divided at any iteration into $P$ blocks, where all blocks are of length $L=$ $L^{\prime \prime} / P$, with $L$ being an integer. This is shown in the following:

$$
\begin{gather*}
\boldsymbol{W}_{\mathrm{vx}}[l]=\left[\boldsymbol{W}_{\mathrm{vx}, 1}^{\mathrm{T}}[l], \boldsymbol{W}_{\mathrm{vx}, 2}^{\mathrm{T}}[l], \ldots, \boldsymbol{W}_{\mathrm{vx}, P}^{\mathrm{T}}[l]\right]^{\mathrm{T}} \text { for } x=\mathrm{a}, \mathrm{~b}, \\
F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}=\left[\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)_{1}^{\mathrm{T}},\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)_{2}^{\mathrm{T}}, \ldots,\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)_{P}^{\mathrm{T}}\right]^{\mathrm{T}} . \tag{20}
\end{gather*}
$$

In the above equations, the coefficient vector sub-blocks, $\boldsymbol{W}_{\mathrm{v}, 1}[l], \boldsymbol{W}_{\mathrm{vx}, 2}[l], \ldots, \boldsymbol{W}_{\mathrm{v}, P, P}[l]$, represent the candidate subsets of
$\boldsymbol{W}_{\mathrm{vx},[ }[I]$, from which a selection is to be chosen to be updated at time instant $i$. Therefore, suppose we wish to update $B$ blocks out of $P$ blocks, then to update these blocks, a constrained minimization problem is needed that makes use of an adaptive algorithm that uses the NLMS method. Such a constrained minimization problem can be written as follows:

$$
\min _{I_{B}} \min _{\boldsymbol{W}_{\mathrm{v}, \mathrm{j}, \mathrm{l}}^{\mathrm{j} \in I_{B}}} \sum_{\mathrm{j} \in I_{B}}\left\|\boldsymbol{W}_{\mathrm{va}, \mathrm{j}}^{(i+1)}[l]+\boldsymbol{W}_{\mathrm{vb}, \mathrm{j}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}, \mathrm{j}}^{(i)}[l]-\boldsymbol{W}_{\mathrm{vb}, \mathrm{j}}^{(i)}[l]\right\|_{2}^{2}
$$

s.t.

$$
\begin{equation*}
\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}} \boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]+\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{* \mathrm{~T}} \boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]=\boldsymbol{S}[l], \tag{26}
\end{equation*}
$$

where $I_{B}=\left(\mathrm{j}_{1}, \mathrm{j}_{2}, \ldots, \mathrm{j}_{B}\right), B \leq P$. If $I_{B}$ is given and fixed, then the above equation can be solved using the Lagrange multipliers similar to [18] as follows:

$$
\begin{aligned}
J_{I_{B}}^{(i)}(l)= & \left\|\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i+1)}[l]+\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i)}[l]-\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i)}[l]\right\|_{2}^{2} \\
& +\lambda\left(\boldsymbol{S}^{(i)}[l]-\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}} \boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]\right. \\
& \left.-\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{* \mathrm{~T}} \boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]\right) .
\end{aligned}
$$

In this equation, $\lambda$ is a Lagrange multiplier. To solve the above equation, at first, we decompose it into two equations that are mathematically similar to (16) and (17). In the second step, the new cost functions using Lagrange multipliers are defined to any subset equations as the following:

$$
\begin{array}{ll}
\min _{I_{B}} & \min _{\boldsymbol{W}_{\mathrm{v}, \mathrm{j}, I_{B}}[l]} \sum_{\mathrm{j} \in I_{B}}\left\|\boldsymbol{W}_{\mathrm{va}, \mathrm{j}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}, \mathrm{j}}^{(i)}[l]\right\|_{2}^{2} \\
\text { s.t. } & \left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}} \boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]=\boldsymbol{S}[l] . \tag{27}
\end{array}
$$

The corresponding cost function using Lagrange multipliers is as follows:

$$
\begin{align*}
J_{I_{B}}^{(i)}(l)= & \left\|\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i)}[l]\right\|_{2}^{2}  \tag{22}\\
& +\lambda\left(\boldsymbol{S}^{(i)}[l]-\left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)^{\mathrm{T}} \boldsymbol{W}_{\mathrm{va}}^{(i+1)}[l]\right),
\end{align*}
$$

also,

$$
\begin{align*}
& \min _{I_{B}} \min _{\boldsymbol{W}_{\mathrm{v}, \mathrm{j},}[l]} \sum_{\mathrm{j} I_{B} \in I_{B}}\left\|\boldsymbol{W}_{\mathrm{vb}, \mathrm{j}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vb}, \mathrm{j}}^{(i)}[l]\right\|_{2}^{2}  \tag{23}\\
& \text { s.t. } \quad\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{* \mathrm{~T}} \boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]=\boldsymbol{S}[l] .
\end{align*}
$$

The corresponding cost function is as follows:

$$
\begin{align*}
J_{I_{B}}^{(i)}(l)= & \left\|\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i)}[l]\right\|_{2}^{2}  \tag{24}\\
& +\lambda\left(\boldsymbol{S}^{(i)}[l]-\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)^{* \mathrm{~T}} \boldsymbol{W}_{\mathrm{vb}}^{(i+1)}[l]\right) .
\end{align*}
$$

Now, by minimization of these cost functions, the weight vectors of the adaptive filters can be calculated as equations (25)-(26).

The $L B \times 1$ vector, denoted by $\boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i)}[l]$, is defined by

$$
\boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i)}[l]=\left[\boldsymbol{W}_{\mathrm{vx}, \mathrm{j}_{1}}^{(i) \mathrm{T}}[l], \boldsymbol{W}_{\mathrm{vx}, \mathrm{j}_{2}}^{(i) \mathrm{T}}[l], \ldots, \boldsymbol{W}_{\mathrm{vx}, \mathrm{j}_{B}}^{(i) \mathrm{T}}[l]\right]^{\mathrm{T}} \text { for } x=\mathrm{a}, \mathrm{~b},
$$

$$
\begin{aligned}
\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i+1)}[l]= & \boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i)}[l] \\
& +\frac{\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}}^{\mathrm{T}}\left(\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i)}[l]\right)\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}}}{\left\|\left(F_{\mathrm{ext}}^{(i)}[l]\right)_{I_{B}}\right\|_{2}^{2}},
\end{aligned}
$$

$$
\begin{align*}
\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i+1)}[l]= & \boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i)}[l]  \tag{25}\\
& +\frac{\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)_{I_{B}}^{* \mathrm{~T}}\left(\boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i)}[l]\right)\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right]\right)_{I_{B}}^{*}}{\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z_{I_{B}}^{*} \|_{2}^{2}\right.},
\end{align*}
$$

where $\left(F_{\text {ext }}^{(i)}[l] \boldsymbol{z}\right)_{I_{B}}$ and $\left(F_{\text {ext }}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)_{I_{B}}^{*}$ are defined by
$\left(F_{\text {ext }}^{(i)}[l] z\right)_{I_{B}}=\left[\left(F_{\text {ext }}^{(i)}[l] z\right)_{\mathrm{j}_{1}}^{\mathrm{T}}\left(F_{\text {ext }}^{(i)}[l] z\right)_{\mathrm{j}_{2}}^{\mathrm{T}} \cdots\left(F_{\text {ext }}^{(i)}[l] z\right)_{\mathrm{j}_{B}}^{\mathrm{T}}\right]^{\mathrm{T}}$,

Then, we have

$$
\begin{aligned}
& \left(F_{\mathrm{ext}}^{(i)}[l] \boldsymbol{z}\right)_{I_{B}}^{\mathrm{T}}\left(\boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i)}[l]\right) \\
& \quad=\boldsymbol{S}[l]-\left(F_{\mathrm{ext}}^{\mathrm{i}^{(i)}}[l] \boldsymbol{z}\right)_{I_{B}}^{\mathrm{T}} \boldsymbol{W}_{\mathrm{vx}, I_{B}}^{(i)}[l] \\
& \\
& \quad=e^{(i)}(l) .
\end{aligned}
$$

Therefore, using the above equation, the obtained results in (25)-(26) can be simplified to (27)-(28) below. The ultimate recursive equations for updating $B$ blocks stipulated by $I_{B}$ can be given by

$$
\begin{align*}
& \boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i+1)}[l]=\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i)}[l]+\frac{\mu\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}}}{\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}}\right\|_{2}^{2}} e^{(i)}(l), \\
& \boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i+1)}[l]=\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i)}[l]+\frac{\mu\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)_{I_{B}}^{*}}{\left\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)_{I_{B}}^{*}\right\|_{2}^{2}} e^{(i)}(l) . \tag{28}
\end{align*}
$$

In the above equations, the step size $(\mu)$ is inserted to control the convergence rate and excess MSE. In the next step, the subset $I_{B}$ must be determined. As mentioned previously, the members of this subset are chosen from a set that has $P$ entries. To select the set of $B$ blocks from $P$ blocks, a minimum squared-Euclidean-norm as a cost function is considered. Hence, the block selection problem can be formulated as

$$
\begin{align*}
I_{B} & =\underset{I_{B} \in S}{\arg \min }\left\|\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{va}, I_{B}}^{(i)}[l]\right\|_{2}^{2} \\
& =\underset{I_{B} \in S}{\arg \min }\left\|\frac{\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}} e^{(i)}(l)}{\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}}\right\|_{2}^{2}}\right\|_{2}^{2}  \tag{29}\\
& =\underset{I_{B} \in S}{\arg \max }\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{I_{B}}\right\|_{2}^{2} \\
& =\underset{I_{B} \in S}{\arg \max } \sum_{\mathrm{j} \in S}\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{\mathrm{j}}\right\|_{2}^{2},
\end{align*}
$$

where $S$ is the set of all possible subsets of size $B$. In addition, in "b" branch, the set $I_{B}$ of " b " branch can be rewritten as

$$
\begin{align*}
I_{B} & =\underset{I_{B} \in S}{\arg \min }\left\|\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i+1)}[l]-\boldsymbol{W}_{\mathrm{vb}, I_{B}}^{(i)}[l]\right\|_{2}^{2} \\
& =\underset{I_{B} \in S}{\arg \max } \sum_{\mathrm{j} \in S}\left\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] \boldsymbol{z}\right)_{\mathrm{j}}^{*}\right\|_{2}^{2} \tag{30}
\end{align*}
$$

Now, to select the optimum subset to be updated, at first, the blocks are arranged in ascending order, based on the squared Euclidean norm, using (29) and (30). Then, the subsets with $B$ number of elements that have the largest squared Euclidean norm are chosen as follows:
$\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{\mathrm{j}_{1}}\right\|_{2}^{2} \geq\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{\mathrm{j}_{2}}\right\|_{2}^{2} \geq \cdots \geq\left\|\left(F_{\mathrm{ext}}^{(i)}[l] z\right)_{\mathrm{j}_{B}}\right\|_{2}^{2}$,
$\left\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)_{\mathrm{j}_{1}}^{*}\right\|_{2}^{2} \geq\left\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)_{\mathrm{j}_{2}}^{*}\right\|_{2}^{2} \geq \cdots \geq\left\|\left(F_{\mathrm{ext}}^{(i)}\left[l_{\mathrm{m}}\right] z\right)_{\mathrm{j}_{B}}^{*}\right\|_{2}^{2}$.
In the NLMS algorithm, the step size $\mu$ should be limited by $0<\mu<2$, but in the SCU-NLMS algorithm, the step size is limited to $0<\mu<2 B / P$.

## IV. Computational Complexity

In the traditional NLMS algorithm, computational complexity arises from $L^{\prime \prime}$ multiplications in error calculation; $L^{\prime \prime}$ multiplications to update weight factors; and two multiplications and one division due to normalization. Now, considering the structure shown in Fig. 4 and applying an adaptive filter with two branches, the number of computations will be doubled in any subcarrier equalization. Considering $N$ to be the subcarrier number, the total computations in our applied system are presented in Table 1. This table shows the computational complexity of NLMS and SCU-NLMS under a PTEQ structure.
Also, in the first proposed algorithm, SM-NLMS, when updating is not carried out, only one comparison is enough for a given error threshold, and extra computations are not required. Figure 5 compares the computation complexity of Full-NLMS and SCU-NLMS based on a PTEQ structure. According to Table $1, L^{\prime \prime}=12$ and $L=1$ is assumed.

Table 1. Computational complexity of NLMS and SCU-NLMS based on PTEQ.

|  | NLMS | SCU-NLMS in proposed system |  |
| :---: | :---: | :---: | :---: |
|  |  | $P<L^{\prime \prime}$ | $P=L^{\prime \prime}$ |
| Multiplications | $2 N\left(2 L^{\prime \prime}+2\right)$ | $2 N\left(L^{\prime \prime}+B L+2\right)$ | $2 N\left(L^{\prime \prime}+B+2\right)$ |
| Divisions | $2 N$ | $2 N$ | $2 N$ |
| Comparisons | - | $2 N\left(O(P)+P \log _{2}(B)\right)$ | $2 N\left(\left\lfloor 2 \log _{2} L^{\prime \prime}\right\rfloor+2\right)$ |



Fig. 5. Computation complexity comparison of Full-NLMS and SCU-NLMS in PTEQ structure vs. number of selected blocks for updating (total number of blocks $=12$ ).

## V. Simulation Results

In this section, some simulations have been done to show the efficiency of the proposed algorithms. The size of FFT is $N=$ 64 , the CP length is $v=16$, and the applied modulation type is 64 QAM. Two kinds of channel profiles are considered: additive white Gaussian noise and multipath with $L_{\text {tap }}+1=22$ taps and exponential decaying power profile. Each path is generated as an independent complex Gaussian random variable. Each channel realization is independent of the previous one, and the BER results are depicted by averaging the BER curves over 50 independent channels.
The IQ amplitude imbalances are considered to be $\varepsilon, \varepsilon_{\mathrm{r}}=5 \%$, and the corresponding phase imbalances are assumed to be $\phi$, $\phi_{\mathrm{r}}=5^{\circ}$, at both the transmitter and the receiver, for the simulated figures (Figs. 6-10). The curves in these figures obviously illustrate that for such IQ imbalance to allow a high data rate communication, compensation is absolutely necessary. The mismatched filters must be considered in both the transmitter and the receiver when there are FS IQ imbalances. Therefore, the front-end filter mismatched impulse responses with related tap lengths $L_{\mathrm{t}}=L_{\mathrm{r}}=2$ are $\boldsymbol{h}_{\mathrm{i}}=\boldsymbol{h}_{\mathrm{i}}=[0.9,0.1]$ and $\boldsymbol{h}_{\mathrm{tq}}=\boldsymbol{h}_{\mathrm{rq}}=[0.1$, 0.9 ] for Figs. 7 and 10, respectively. In these figures, the obtained results are shown based on IQ imbalances due to both FI and FS IQ. In addition, the bound on the output error of the SM-NLMS algorithm is set to $\gamma=\sqrt{5 \sigma_{\mathrm{n}}^{2}}$, where $\sigma_{\mathrm{n}}^{2}$ is the noise variance. It should be noted that given the variances for both the signal and the noise, as well as assuming a zero-mean signal, the signal-to-noise ratio (SNR) can be defined as


Fig. 6. BER performance, 64, QAM, NLMS, and SM-NLMS based on a one-tap FEQ, only FI IQ imbalance, AWGN flat channel.


Fig. 7. BER performance, 64 QAM, NLMS, and SM-NLMS based on a one-tap FEQ, FI, and FS IQ imbalance, AWGN flat channel.


Fig. 8. BER performance, 64 QAM and NLMS in PTEQ structure, FI IQ imbalance, Rayleigh fading channel.


Fig. 9. BER performance, 64 QAM, Full NLMS, and SCUNLMS in PTEQ, only FI IQ imbalance, Rayleigh fading channel, $(\mathrm{PTEQ}$ length $=12)$.

$$
\begin{equation*}
\mathrm{SNR}=\frac{\sigma_{\text {signal }}^{2}}{\sigma_{\mathrm{n}}^{2}} \tag{31}
\end{equation*}
$$

Figure 6 represents a plot of BER versus SNR for the proposed method SM-NLMS and NLMS, based on a onetap FEQ initialization. This simulation result is obtained considering only the FI IQ imbalance. As shown in this figure, good results are obtained in both the transmitter (Tx) and the transmitter/receiver ( Rx ) IQ imbalances. In addition, a computational complexity reduction in our algorithm with respect to the conventional NLMS algorithm is achieved.

Similarly, Fig. 7 illustrates a plot of BER versus SNR for the proposed method SM-NLMS and NLMS, based on a one-tap FEQ initialization. This figure is derived from considering both FI and FS IQ imbalances. In Figs. 6 and 7, the average number of updates for all of the subcarriers is less than 50 percent, approximately. In addition, only 100 symbols are used to learn the system. As shown in Fig. 7, there is a small increase in error probability with respect to the results obtained in Fig. 6. This is because we consider the combined FS and FI IQ imbalances.

Figures 8, 9, and 10 are obtained based on multipath Rayleigh fading channels with 22-tap length. In Fig. 8, the effect of increasing the value of parameter $L^{\prime \prime}$ is investigated. By increasing the value of $L^{\prime \prime}$, the performance is improved and ISI can be completely eliminated. In this simulation, only FI IQ imbalance is considered. As shown in Figs. 9 and 10, the results of the comparisons reveal that the proposed algorithm under a PTEQ structure with $25 \%, 50 \%$, and $75 \%$ coefficient updating has approximately similar performance. In Fig. 10, both FI and FS IQ imbalance is considered. Only 150 symbols are used for the learning of the system in Figs. 8, 9 , and 10 .


Fig. 10. BER performance, 64 QAM, Full NLMS, and SCUNLMS in PTEQ structure, FI and FS IQ imbalances, Rayleigh fading channel (PTEQ length $=12$ ).

## VI. Conclusion

In this article, a new low-complexity adaptive algorithm has been suggested for the joint estimation of the transmitter and receiver IQ imbalances and channel effects. The PTEQ solution along with selective coefficient updating (SCU) is capable of compensating non-idealities efficiently under insufficient CP length. We extended the SCU of the NLMS algorithm in this situation to make this algorithm usable in real applications. In addition, one-tap FEQ equalization using setmembership filtering under sufficient CP length is derived for average computation reduction.
Generally, the results of the SM-NLMS under sufficient CP length show that there is an adequate improvement in the BER performance for the proposed algorithms and that this is very close to the BER of the ideal case. The obtained algorithms under sufficient and insufficient CP length provide a very efficient, low-complexity post-FFT compensation, and adaptive equalization, which makes these methods nearly ideal in terms of their performance.

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