Spectrum Allocation based on Auction in Overlay Cognitive Radio Network

Wenhao Jiang*, Wenjiang Feng and Yang Yu
College of Communication Engineering, Chongqing University
Chongqing, China, 400044
[e-mail: 20121201007{fengwj, 20131202028}@cqu.edu.cn]
*Corresponding author: Wenhao Jiang

Received March 10, 2015; revised June 14, 2015; accepted July 18, 2015; published September 30, 2015

Abstract

In this paper, a mechanism for spectrum allocation in overlay cognitive radio networks is proposed. In overlay cognitive radio networks, the secondary users (SUs) must first sense the activity of primary users (PUs) to identify unoccupied spectrum bands. Based on their different contributions for the spectrum sensing, the SUs get payoffs that are computed by the fusion center (FC). The unoccupied bands will be auctioned and SUs are asked to bid using payoffs they earned or saved. Coalitions are allowed to form among SUs because each SU may only need a portion of the bands. We formulate the coalition forming process as a coalition forming game and analyze it by game theory. In the coalition formation game, debtor-creditor relationship may occur among the SUs because of their limited payoff storage. A debtor asks a creditor for payoff help, and in return provides the creditor with a portion of transmission time to relay data for the creditor. The negotiations between debtors and creditors can be modeled as a Bayesian game because they lack complete information of each other, and the equilibria of the game is investigated. Theoretical analysis and numerical results show that the proposed auction yields data rate improvement and certain fairness among all SUs.

Keywords: Cognitive radio, Spectrum allocation, Auction, Game theory, Bayesian equilibria

http://dx.doi.org/10.3837/tiis.2015.09.003 **ISSN : 1976-7277**

1. Introduction

Cognitive radio (CR) technology is a promising solution to solve the problem of spectrum scarcity and low spectrum utilization associated to classical fixed spectrum assignment schemes [1]. Due to higher priority of primary users (PUs), secondary users (SUs) need to perceive the behavior of PUs in their assigned frequency bands and perform opportunistic spectrum access without or with minimal interference to them. Usually a user or a node trusted by all SUs (or a third party which shares no common interest with any SU) is introduced as an FC to collect the results of SUs' perception. In this way the FC has a list of vacant spectrum bands which are available for a period of time.

The SUs want to access the vacant bands for transmission. How to allocate vacant bands among the SUs is one common problem in cognitive radios. To improve the spectrum efficiency, many works have been done [2, 3]. In [2] the authors present a game theoretic model for spectrum sharing, where users seek to satisfy their quality of service demands in a distributed fashion. They also extend their model by considering the frequency spatial reuse, and consider the user interactions as a game upon a graph where players only contend with their neighbors. There are a large number of works that attempt to solve the spectrum allocation problem with economical tools such as game theory [4], contract theory [5, 6], auction [7], pricing [4, 8]. Pricing is often used when the seller knows precisely the value of the resource being sold. In [4] the authors formulate an oligopoly market consisting of a few firms and a consumer to investigate the pricing problem. By using a Bertrand game model, they analyze the impacts of several system parameters such as spectrum substitutability and channel quality on the Nash equilibrium and propose a distributed algorithms to obtain the solution for this dynamic game. In cases where the seller only knows limited information of the buyers' valuations of the resource, contract is more effective because by motivating the buyers truthfully reveal their private valuations, the seller can optimally allocate the resource. In [5] the authors proposed a quality-price contract for the spectrum trading to allocate the spectrum. [6] exploits the incentive effect of cooperative spectrum sharing where SUs relay traffics PUs in exchange for dedicated spectrum access time for SUs' own communications. The PU-SU interaction is modeled as a labor market and analyzed by contract theory. It must be stressed that these mechanisms should guarantee the spectrum bands are allocated according to SUs true valuations, which is called truthfulness [9]. Truthfulness is important because if SUs can lie to obtain benefits, efficiency and fairness cannot be achieved. Auction mechanism is a suitable approach when the seller has no knowledge about the value of the resource. Many auction mechanisms [7, 10] have been studied because they guarantee truthfulness. However fairness is often ignored or simplified [10]. The authors of this paper believe that SUs' contribution to the CR network (such as contribution of cooperative sensing) should be rewarded and assigning higher priority of spectrum bands to more contributed SUs is a kind of fairness [11].

In this paper, we propose a spectrum allocation mechanism similar to auction mechanisms. We first quantify the contribution of every SU in the cooperative sensing and reward it with tokens. Then the FC holds an auction for the vacant bands. An SU may use its tokens to bid for spectrum bands in the following period or it may save the tokens for future use. Since the entire vacant bands are considered as one commodity, the SUs may need to cooperate and form coalitions. During the coalition forming process, debtor-creditor relationship may occur among the SUs because of their limited payoff storage. A debtor may ask a creditor for payoff help, and in return provides the creditor with a portion of transmission time to relay data for the creditor. We model the coalition forming process as a coalition formation game and the negotiations between debtors and creditors as a non-cooperative Bayesian game. We further investigate these games with the help of coalitional game theory [12] and non-cooperative game theory, respectively.

The remainder of the paper is organized as follows: Section 2 presents the system model and the proposed auction. Section 3 presents the investigation of coalition formation in the auction through a coalitional game theoretic perspective. Section 4 introduces the potential cooperation among SUs and analyzes the Bayesian equilibria within. In Section 5 we validate our auction mechanism and our theoretical analysis through simulations. And Section 6 concludes the paper.

2. System model and proposed auction

2.1 Network model

We consider a CR network consists of N SUs and M potential unoccupied channels. Each SU detects the PU activities on one or more channels according to its different detection performances on different channels. SUs sense the spectrum bands cooperatively and periodically for infinite iterations. An FC is introduced in the network and we assume it could communicate with all CUs via a perfect common control channel (CCC). The FC categorizes the sensing results and obtains global decisions on the channel status. In this work we also assume that the FC is the manager of all vacant channels and it is in charge of spectrum allocation. The FC also adopts some algorithm (such as algorithm in [11], or reputation-based algorithm) to quantify each SU's contribution to the spectrum sensing, then pays tokens accordingly. If there are any available channels for the SUs in a sensing-transmission round, we address this round as an episode. In an episode each SU either demands some spectrum for transmission or has no data to transmit. The valuations of these spectrum for different SUs may vary due to the fact that even the same bandwidth may produce different transmission performances to different SUs. Here we assume the spectrum valuation of an SU is based on its data rate. We focus on the spectrum allocation problem in one episode. We denote spectrum demand and valuation of SU i by w_i and v_i respectively, and denote the total available spectrum in this episode by B. We also assume that for any i, $w_i \leq B$ (i.e., the total available spectrum can satisfy at least one SU). A critical problem here is how to allocate the spectrum among the SUs according to their demand and valuation, while satisfying some fairness criterion.

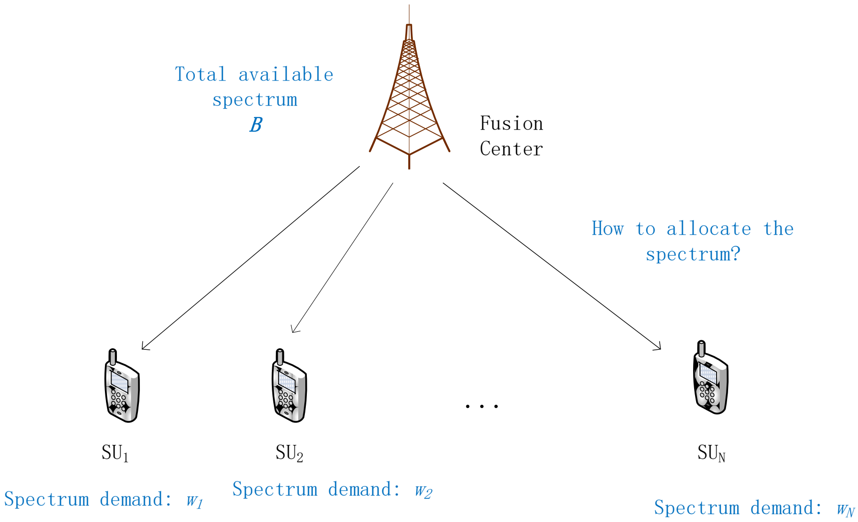

Fig. 1. Transmission time division in cooperation

2.2 Token spectrum auction

We propose a token spectrum auction to address this problem. In this auction, the SUs bid for the spectrum using the tokens they earned in this episode or they saved from previous episodes. Because the currency here is the token which represents the contribution to cooperative sensing, the SUs always have a limited budget and thus they cannot always pay according to their valuations. We denote the token storage of SU_i by α_i. The token spectrum auction is deployed as follows.

Step 1: The FC announces there is a total bandwidth of B and it will be auctioned as one commodity.

Step 2: Each SU reports its spectrum demand w_i and its first bid b_i, where b_i is calculated according to its valuation.

Step 3: The FC tells each SU the information of all SUs including: w_i, b_i and token storage α_i. Then the FC asks the SUs to form coalitions.

Step 4: The SUs try to form coalitions. There might be negotiations between the SUs.

Step 5: All coalitions report their bid information to the FC. In particular, coalition S_k reports its final bid b_k^u as well as the exact division of this bid among its members.

Step 6: The FC allocates the available spectrum to a coalition in a statistical manner where coalition S_k obtains the spectrum with probability $p(S_k) = \frac{b_k^u}{\sum b_j^u}$. The FC then charges all members of the winner.

We provide some notes on the proposed auction: In step 2, first bid b_i reflects real spectrum valuation of SU_i in form of token. b_i may exceed its token storage α_i and is not necessarily identical to the final payment of SU_i. For instance, if $b_i > \alpha_i$ and SU_i failed to get help from other SUs, the final payment of SU_i $b_i^m = \alpha_i$. In step 5, coalition S_k report both its final bid b_k^u and the division in order to tell the FC how to calculate its winning probability and how to charge its members if S_k wins. Obviously $b_k^u = \sum_{i \in S_k} b_i^m$. It can be observed that coalitions with higher final bids will have higher probabilities to obtain the spectrum. The potential negotiations in step 4 will be discussed in later sections.

We propose such an auction for the following reasons:

1) Coalitions with higher final bids will have higher probabilities to obtain the spectrum. Therefore in general, this auction improves the spectrum efficiency.
2) Unlike the first-price or second-price auction, there is a chance to win the spectrum for SUs with low valuation or low token storage. They have an incentive to form coalitions and compete with others. Therefore the auction encourages every SU to take part in the competition.
3) By using tokens as currency instead of money, the auction guarantees certain fairness. SUs which make more contribution earn more tokens, therefore hold better positions in the spectrum competition. This motivates SUs to make more effort and it is positive for the network.
4) The proposed auction allows SUs to negotiate and cooperate in the coalition formation. This introduces flexibility to the spectrum allocation and further improves spectrum efficiency.

With the token spectrum auction given, we will investigate the behavior of SUs in the auction. We formulate the coalition formation process as a coalition formation game and study it in Section 3. The negotiations and cooperation between SUs are modeled as a non-cooperative game and will be investigated in Section 4.

3. Coalition formation game

In this section we formulate and analyze the coalition formation using coalition game theory. We do not consider the potential negotiations between SUs here through these negotiations might introduce some impact to the coalition formation. Before we start, some coalition game theory concepts must be introduced.

3.1 Coalition formation concepts

Researchers have been highly interested in coalition formation game [13-15]. The goal is to find algorithms for characterizing the coalitional structures that form in a network where the grand coalition is not optimal. In [14], the authors present a generic framework for coalition formation where coalitions form and break through two simple merge-and-split rules. Follow their works, we present some definitions.

Definition 1: A coalitional game (N, v) is said to have a *transferable utility* (TU) if the value v (S) can be arbitrarily apportioned between the coalition's players. Otherwise, the coalitional game has a *non-transferable utility* (NTU) and each player will have its own utility within coalition S.

Definition 2: Let $N = \{1,2,\dots,n\}$ be a fixed set of players called the *grand coalition*. Non-empty subsets of N are called *coalitions*. A *collection* is any family $C = \{C_1, C_2, \dots, C_l\}$ of mutually disjoint coalitions, and l is called its size. If additionally $\bigcup_{j=1}^{l} C_j = N$, the collection C is called a *partition* of N.

Definition 3: A *comparison relation* $\rhd$ is defined to compare two collections $R = \{R_1, R_2, \dots, R_l\}$ and $T = \{T_1, T_2, \dots, T_m\}$ that are partitions of the same set A (same players in R and T). $R \rhd T$ implies that the way R partitions A is preferred to the way T partitions A based on a criterion.

Many criteria (referred to as *orders*) can be adopted to compare collections or partitions. In general they are divided into two types: coalition value orders and individual value orders. Coalition value orders compare two collections by using the value of the coalitions inside these collections. For instance, the utilitarian order [16] compares two collections where $R \rhd T$ indicates $\sum_{i=1}^{l} v(R_l) > \sum_{i=1}^{m} v(T_m)$. On the other hand individual value orders compare two collections using the player's utility, not the coalition value. An important individual order is the Pareto order. For a collection R, denote the utility of a player i in a coalition $R_i \in R$ by $u_i(R)$. Pareto order is defined as

$$R \rhd T \text{ iff } u_i(R) \geq u_i(T)\ \forall\, i \in R, T \tag{1}$$

with at least one strict inequality for a player j. Pareto order is an adequate comparison relation for NTU games and we will show later the coalition formation here is a NTU game in subsection 3.2 A.

We further introduce some concepts that will later be used to analyze the stability of a partition.

Definition 4: A *defection function* $\mathbb{D}$ is a function which associates with each partition R some partitioned subsets of the grand coalition N. A partition $P = \{P_1, P_2, \dots, P_l\}$ of N is $\mathbb{D}$ -stable if no group of players have an incentive to leave P when they can only form the collections allowed by $\mathbb{D}$.

[13] presents some defection functions such as $\mathbb{D}_c$ and $\mathbb{D}_{hp}$. $\mathbb{D}_c$ allows formation of all collections in the grand coalition while $\mathbb{D}_{hp}$ allows formation of all P- homogeneous partitions in the grand coalition. In other words, a partition P is $\mathbb{D}_{hp}$-stable, if no players in P have an incentive to leave P through merge-and-split (we will introduce this rule later) to

form other partitions, while a partition P is $\mathbb{D}_c$-stable, if no players in P have an incentive to leave P through any operation to form other collections in the grand coalition.

3.2 Coalition formation in token spectrum auction

A. Game formation

To investigate the behavior of the SUs in the token spectrum auction, we formulate the coalition formation process as a coalition formation game so that we can refer to coalition game theory. The process can be modeled as a (N, v) where N is the set of players and v is the utility function or value of a coalition.

In the proposed auction we aim to encourage players (i.e., SUs) to form coalitions to compete the bandwidth B. Each coalition is supposed to use the spectrum as much as possible, however the total spectrum demand should not exceed available bandwidth B. So we expect $\sum_{i\in S} w_i$ to be large as possible while $\sum_{i\in S} w_i \leq B$ holds, for every coalition S. According to the auction mechanism, a suitable utility function is given by

$$v(S) = p(S)R(S) - Res(S) \tag{2}$$

where $p(S)$ is the probability that coalition S wins the spectrum, $R(S)$ is the total data rate of coalition S and $Res(S)$ is a restriction function. Again we emphasize that in this section we do not consider the potential negotiations and deals in the coalition formation. Therefore

$$p(S) = \frac{\sum_{j\in S} \min\{b_j, \alpha_j\}}{\sum_{i=1}^{N} \min\{b_i, \alpha_i\}} \tag{3}$$

$$R(S) = \sum_{i\in S} R_i \tag{4}$$

where R_i is the date rate of SU_i. If the total demand of a coalition exceeds available bandwidth B, the division among the coalition members cannot be done. To avoid this, we define the restriction function as follows

$$Res(S) = \begin{cases} 0, & if\ \sum_{i\in S} w_i \leq B \\ +\infty, & if\ \sum_{i\in S} w_i > B \end{cases} \tag{5}$$

Obviously $v(S)$ represents the expected total data rate of coalition S in the auction if the bandwidth division can be done among its members. Coalition S will always welcome new members as long as they do not violate the bandwidth demand restriction, since new members increase $v(S)$ by increasing $p(S)$ and $R(S)$. We define utility of SU_i in coalition S as

$$u_i(S) = \begin{cases} p(S)R_i, & if\ \sum_{i\in S} w_i \leq B \\ -\infty, & if\ \sum_{i\in S} w_i > B \end{cases} \tag{6}$$

$u_i(S)$ represents the expected data rate of SU_i when SU_i is in coalition S. Utility $-\infty$ indicates that SU_i will not have a positive data rate when bandwidth division cannot be done in S. $-\infty$ is used to emphasize this circumstance should never occur and to match with the definition of $v(S)$. In this way utility of each SU in coalition S is a portion of value of S, and $v(S) = \sum_{i\in S} u_i(S)$. It can be easily observed that any rational coalition would like to invite an SU to join it as long as this SU does not violate the bandwidth demand restriction, and any SU prefers to accept one of the invitations than compete alone. We also notice that $v(S)$

cannot be arbitrarily apportioned between the members because S cannot alter any member's bandwidth demand. Therefore the game we formulated is a NTU game and Pareto order is suitable here.

B. Coalition formation algorithm

We propose a coalition formation algorithm based on two simple rules addressed as "merge" and "split" which modify a partition T of SUs set N [14].

Definition 5: Merge Rule: $\{T_1, T_2, \dots, T_k\} \rightarrow \{\bigcup_{i=1}^{k} T_i\}$ where $\{\bigcup_{i=1}^{k} T_i\} \rhd \{T_1, T_2, \dots, T_k\}$

Definition 6: Split Rule: $\{\bigcup_{i=1}^{k} T_i\} \rightarrow \{T_1, T_2, \dots, T_k\}$ where $\{T_1, T_2, \dots, T_k\} \rhd \{\bigcup_{i=1}^{k} T_i\}$ With these rules, multiple coalitions can merge into one coalition, and one coalition can split into multiple coalitions if merging/splitting operation yields a better collection based on the order $\rhd$. Here $\rhd$ is the Pareto order, therefore coalitions will merge (split) only if at least one SU strictly improves its utility through this merging (splitting) without harming other SUs' utilities.

Remark 1: Suppose $\rhd$ is a comparison relation. Then every iteration of the merge and split rules terminates.

With the support of Remark 1, an algorithm can be designed based on merge and split rules without considering if it terminates. The coalition formation algorithm works as follows: At the beginning, every SU is a coalition. The FC picks an SU (referred to as the head) to start the merging process. For instance, FC may appoint the SU with the highest "bid per unit bandwidth" (i.e., $\frac{b_i}{w_i}$) to start so that this SU is given some advantage in competition. The head invites other SUs to join in its coalition S_h in a certain order. If an invited SU rejects the invitation, the head turns to the next SU. The merging ends once S_h reaches bandwidth demand restriction (i.e., $\sum_{i \in S_h} w_i = B$) or no SU can join in S_h with a mutual benefit. Then the FC appoints an SU $j \in N \backslash S_h$ as the new head and start the merging among $N \backslash S_h$ again. The algorithm is repeated until every SU has made its merging decisions, i.e., an SU has either played the role of head or accepted an invitation. A final partition P^{final} of the set of all SUs is formed when the algorithm terminates.

We present some notes on the algorithm. The $v(S)$ and $u_i(S)$ we proposed have a significant fall at the edge $\sum_{i \in S} w_i = B$, leading to an important property of the coalitions that once two coalitions decide to merge, any of their members will not be split from the large coalition. For instance, if $S_1 = \{SU_1, SU_2\}$ and $S_2 = \{SU_3, SU_4\}$ decide to merge into$S_3 = \{SU_1, SU_2, SU_3, SU_4\}$, S_3 will not exclude any of its member. Otherwise S_3 would not be formed in the first place. This property implies essentially the proposed algorithm has deployed a complete merge-and-split because even if we allow coalitions to split after P^{final} is formed, no change will occur in P^{final}. So all conclusions of merge-and-split can apply to our algorithm. It should also be noticed that the merging can be simplified because the head can filter the SUs according to bandwidth demand restriction after every acceptance of its invitation. The head only invites qualified SUs and the invited SUs have no incentive to reject in accordance with merge rule. Therefore the time of coalition formation can be significantly reduced.

In practice, every head forms a coalition S_h that maximizes its own utility. Denoting the set of SUs which belong to previous heads' coalitions by S_{pre}, the current head aims to find

$$S_h = arg\max_{S_h \subseteq N \setminus S_{pre}} \sum_{i \in S_h} \min\{b_i, \alpha_i\} \text{ subject to } \sum_{i \in S_h} w_i \leq B \quad (7)$$

Solving this optimization problem is NP-complete and the complexity increases significantly with the number of SUs N. For small N, (7) may be solved by exhaustive search or iteration method. However for large N there is no practical method to find optimal solutions. We introduce a simple method to help the head with this problem. First the head arranges the available SUs (i.e., $\forall\, i \in N \setminus S_{pre}$) decreasingly with regard to payment per unit bandwidth ($\frac{b_i}{w_i}$). Without loss of generality, we assume $\frac{b_1}{w_1} \geq \frac{b_2}{w_2} \geq \cdots \geq \frac{b_l}{w_l}$. The head checks if an SU satisfies the bandwidth demand restriction in the above order and invites the first qualified SU (assume it is SU_j). Then the head updates its bandwidth demand restriction and continue the search from SU_j. This process repeats until the bandwidth demand restriction cannot be satisfied by any SU. The method may not obtain optimal solution for (7), but it significantly reduce time and communication overheads while yielding an acceptable performance in many scenarios. It must be stressed that the method does not take into account the circumstances where SUs negotiate and make deals. Modified method for those scenarios will be introduced in Section 4.

C. Stability analysis

The result of the proposed algorithm is a network partition P^{final} composed of disjoint independent coalitions of SUs. The stability of P^{final} can be investigated using the concept of defection function $\mathbb{D}$ in **Definition 4**.

Theorem 1: Every partition obtained from our proposed coalition formation algorithm is $\mathbb{D}_{hp}$-stable.

Proof: A $\mathbb{D}_{hp}$-stable can be considered as a state where no coalition has an incentive to alter the partition through merge-and-split. The termination of any merge-and-split is evidently $\mathbb{D}_{hp}$-stable. And we have explained that essentially our algorithm has deployed a complete merge-and-split in subsection B. Therefore the resulting partition P^{final} is $\mathbb{D}_{hp}$-stable.

Before we discuss $\mathbb{D}_c$-stability, we provide some properties of a $\mathbb{D}_c$-stable partition proved by the authors of [14].

1) A $\mathbb{D}_c$-stable partition may not exist.
2) If a $\mathbb{D}_c$-stable partition exists, it is $\mathbb{D}_{hp}$-stable. And it is the unique outcome of every iteration of the merge and split rules.
3) If it exists, the $\mathbb{D}_c$-stable partition P is $\rhd$-maximal, i.e., $P \rhd R, \forall R \neq P$.

Theorem 2: If the $\mathbb{D}_c$-stable partition exists, our proposed coalition formation algorithm will converge to this unique $\mathbb{D}_c$-stable partition. Otherwise, the proposed algorithm converges to one of the $\mathbb{D}_{hp}$-stable partitions.

Proof: It can be proved by **Theorem 1** and property 2).

To demonstrate the $\mathbb{D}_c$-stability and $\mathbb{D}_{hp}$-stability explicitly, we introduce two examples.

Example 1: We assume there are 3 SUs. Their bandwidth demands w_1=4, w_2=3, w_3=2. If total available bandwidth is B=4, the $\mathbb{D}_c$ -stable partition exists, i.e., $P = \{\{SU_1\},\{SU_2\},\{SU_3\}\}$. If B=5, the $\mathbb{D}_c$ -stable partition exists, i.e., $P = \{\{SU_1\},\{SU_2, SU_3\}\}$. Our proposed algorithm guarantees to converge to P^{final}=P.

Example 2: We assume there are 4 SUs. w_1=3, w_2=3, w_3=3, w_4=2, and b_1=4, b_2=3, b_3=2, b_4=1. We also assume the SUs can afford their bids. If B=5, $\mathbb{D}_c$-stable partition does not exist and we have three $\mathbb{D}_{hp}$-stable partitions. They are $P_1 = \{\{SU_1, SU_4\},\{SU_2\},\{SU_3\}\}$, $P_2 = \{\{SU_1\},\{SU_2, SU_4\},\{SU_3\}\}$ and $P_3 = \left\{\{SU_1\},\{SU_2\},\{SU_3, SU_4\}\right\}$ respectively. Our proposed algorithm converges to P^{final}=P_1.

4. Potential cooperation between SUs

4.1 Cooperation model

Potential cooperation may exist in coalition formation. Some SUs may have insufficient token storage to afford their spectrum valuation and some SUs may have large token storage but suffer bad channel gains between their transmitters and receivers in this episode. These two types of SUs may cooperate to improve their transmission performance. Assume SU_i has extra tokens and suffers a bad channel gain. According to Shannon formula,

$$R_i = w_i \log_2(1 + \gamma_i) \tag{8}$$

where R_i is data rate of SU_i, γ_i is signal-noise-ratio. With a low γ_i, w_i increment leads to very limited data rate improvement. Therefore it is not wise for SU_i to invest more tokens for more spectrum. On the other hand, we assume SU_j is seeking token support from other SUs to improve its probability of obtaining its demand spectrum. If SU_j is willing to share a portion of its transmission time to relay for SU_i, then SU_i might want to lend SU_j some tokens in return. If the relay time slot and lent tokens satisfy both SU_i and SU_j, this cooperation will bring mutual benefits to both SUs. The cooperation is shown in **Fig. 2**.

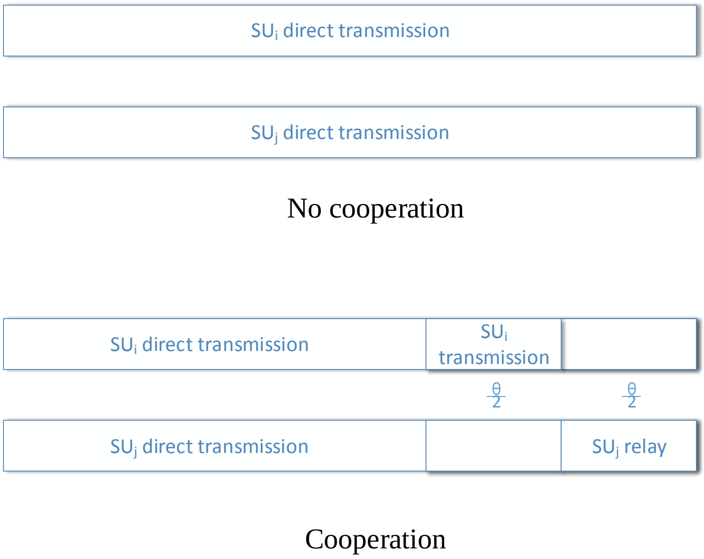

Fig. 2. Transmission time division in cooperation

We assume SU$_i$ and SU$_j$ transmit in the amplified and forward (AF) mode, SU$_i$ achieves a data rate (bits/s/Hz) [17]:

$$R_r^{per} = \frac{1}{2}\log_2(1 + P_i h_i + \frac{P_i P_j h_{ij} h_{ji}}{P_i h_{ij} + P_j h_{ji} + 1}) \tag{9}$$

where P_i, P_j are transmission power of SU$_i$ and SU$_j$ respectively and h_i, h_j, h_{ij}, h_{ji} are channel gains shown in **Fig. 3**. Notice that in the relay time slot SU$_i$ and SU$_j$ share w_i+w_j spectrum in their separate transmissions, therefore SU$_i$ actually achieves a data rate (bits/s):

$$R_r = (w_i + w_j)\,\frac{1}{2}\log_2(1 + P_i h_i + \frac{P_i P_j h_{ij} h_{ji}}{P_i h_{ij} + P_j h_{ji} + 1}) \tag{10}$$

In this paper we assume both SU$_i$ and SU$_j$ know the relay data rate R_r through some mechanism, however SU$_j$ does not know direct transmission data rate of SU$_i$ (R_i).

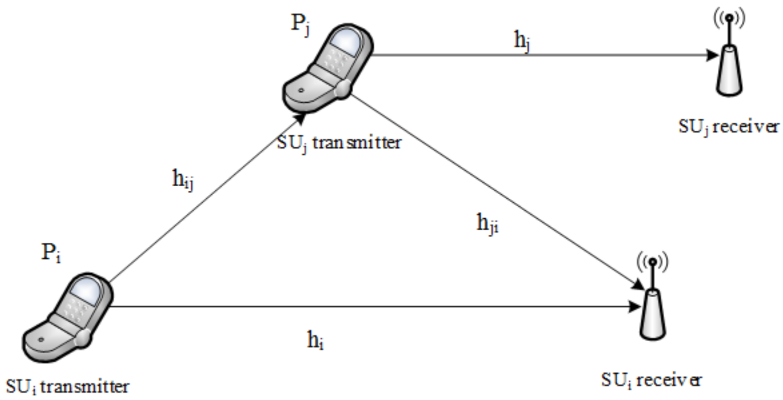

Fig. 3. Cooperation between SU$_i$ and SU$_j$

Evidently, relay time fraction θ determines the benefit of both SU_i and SU_j, and a bargaining about θ will be introduced. Under the proposed auction mechanism, the form of this bargaining varies depending on whether SU_i and SU_j are in the same coalition when bargaining occurs. It is assumed SU_i would not negotiate with members of other coalitions since financing them harms its own interest, however SU_i tends to finance members of the same coalition (partners) or potential partners. We will discuss the cases where SU_i bargains with partners and potential partners in later subsections, respectively.

4.2 Token consumption strategy

We investigate the token consumption strategy before discussing the bargaining cases because the lenders must take into account the value of the tokens they will lend. For different SUs, same tokens may have different values since earning powers are usually different. In the proposed auction mechanism, tokens represent contributions to the network and can only be used to purchase spectrum utilization opportunity. Consequently every SU has a function that maps its achievable data rate in an episode to respective estimated tokens (i.e., first bid). Denote this function of SU_k by $f_k(R_k)$, where R_k is the data rate. If SU_k assumes it has a probability of p_k^{win} to win in the auction, $f_k(R_k)$ must satisfy the following equations due to the fact that each SU prefers to consume all tokens it earns if the episodes end.

$$f_k(0) = 0 \quad and \quad f_k\left(R_k^{avg}\right)p_k^{win} = \beta_k \tag{11}$$

where R_k^{avg} is the expected data rate of SU_k during all episodes and β_k is the average tokens SU_k earns between two episodes. We assume SU_k has knowledge on the distributions of spectrum demand w_k and SNR γ_k, then R_k^{avg} can be easily obtained through Shannon formula and statistical method. Although there are many functions qualified, we adopt a simple linear function for the “data rate to token” mapping as follows:

$$f_k(R_k) = \frac{\beta_k}{R_k^{avg} p_k^{win}} \times R_k \tag{12}$$

In an episode, SU_k determines spectrum demand w_k, estimates R_k and uses (12) to decide its first bid. Also, when SU_k bargain with others, it validates whether data rate improvement introduced by cooperation is worth the tokens lent based on (12).

4.3 Bargain with a partner

If SU_j is already a partner of SU_i, they share common interest, i.e., spectrum winning probability. In this case, it is appropriate to assume that SU_j offers a deal and SU_i decides whether to accept. In particular, SU_j proposes a relay time slot θ and SU_i validates its utility and decides whether to accept. This is a non-cooperative game and non-cooperative game theory can be applied.

Rewrite the notation $p(S)$ as $p(T)$, where T is the total token amount coalition S pays.

The utilities of the SU_i and SU_j based on data rate increment are defined as follows:

$$u_i(\theta) = \begin{cases} P(T^m)[(1-\theta)R_i + \theta R_r] - P(T)R_i - f_i^{-1}(T^m - T) & \textit{if offer accepted} \\ 0 & \textit{if offer rejected} \end{cases} \quad (13)$$

$$u_j(\theta) = \begin{cases} P(T^m)(1-\theta)R_j - P(T)R_j & \textit{if offer accepted} \\ 0 & \textit{if offer rejected} \end{cases} \quad (14)$$

where T^m is the total token amount of coalition S will pay if the offer is accepted. Notice that $T^m - T = b_j - \alpha_j$. $f_i^{-1}(\cdot)$ is the inverse function of $f_i(\cdot)$ and $f_i^{-1}(T^m - T)$ can be replaced by $\frac{R_i(T^m-T)}{b_i}$. The expected utility of SU_j can be written

$$u_j(\theta) = \left[P(T^m)(1-\theta)R_j - P(T)R_j\right] \cdot \text{Prob(offer accepted)} \quad (15)$$

The offer SU_j proposes will be accepted if and only if $u_i(\theta) \geq 0$, i.e.,

$$u_i^{acpt}(\theta) = P(T^m)[(1-\theta)R_i + \theta R_r] - P(T)R_i - \frac{R_i(T^m-T)}{b_i} \geq 0 \quad (16)$$

And SU_j attempts to maximize its utility $u_j(\theta)$, i.e., proposes the optimal θ^*

$$\theta^* = arg \max_{\theta\in[0,1]} u_j(\theta)$$

$$= arg \max_{\theta\in[0,1]} [P(T^m)(1-\theta)R_j - P(T)R_j] \cdot \text{Prob}(P(T^m)[(1-\theta)R_i + \theta R_r] - P(T)R_i - \frac{R_i(T^m-T)}{b_i} \geq 0) \quad (17)$$

The problem here is SU_j does not know SU_i direct transmission data rate R_i. Without any prior knowledge, SU_j assumes R_i follows a distribution in [K_1, K_2], where $0 \leq K_1 < K_2 < R_r$. We have the following theorem:

Theorem 3: Let $A = p(T^m) - p(T) - \frac{1}{b_i}(T^m - T)$, when $A \geq 0$, $\theta^* = 0$ and SU_i will accept this offer. When $A < 0$, θ^* can be obtained as follows:

1) If $\theta_p \leq \underline{\theta}$, for any $\theta \in [0,1]$ the offer will never be accepted.
2) If $\theta_p \in (\underline{\theta}, \overline{\theta})$, $\theta^* = \theta_p$.
3) If $\theta_p \geq \overline{\theta}$, SU_j checks if $P(T^m)\left(1-\overline{\theta}\right)R_j - P(T)R_j > 0$. If it holds, $\theta^* = \overline{\theta}$; otherwise θ^* does not exist.

where

$$\underline{\theta} = \frac{-AK_1}{p(T^m)(R_r - K_1)} \quad (18)$$

$$\overline{\theta} = \frac{-AK_2}{p(T^m)(R_r - K_2)} \quad (19)$$

$$\theta_p = \frac{A}{p(T^m)} - \frac{\sqrt{(R_r - K_1)R_r A \cdot \frac{1}{b_i}(T - T^m)}}{p(T^m)(K_1 - R_r)} \quad (20)$$

Proof: Write $u_i^{acpt}(\theta)$ as

$$u_i^{acpt}(\theta) = R_i\left[p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)\right] + \theta \cdot p(T^m)R_r \tag{21}$$

Let $u_i^{acpt}(\theta) \geq 0$, we have

$$R_i\left[p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)\right] \geq -\theta \cdot p(T^m)R_r \tag{22}$$

Obviously if $p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T) \geq 0$, (22) always holds. In that case

$$\theta \leq \frac{A}{p(T^m)} \tag{23}$$

Therefore if $A \geq 0$, for any $\theta \in [0, \frac{A}{p(T^m)}]$, $\text{Prob}(\text{offer accepted}) = 1$. Evidently SU_j wants θ to be as small as possible. Thus $\theta^* = 0$. In essence, this is a circumstance where SU_j reminds SU_i that increasing SU_i's bid by $T^m - T$ will significantly increase the winning probability of the coalition (as well as its own) and SU_i takes this reminding.

If $A < 0$, $p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T) < 0$. Then

$$\text{Prob}(\text{offer accepted}) = \text{Prob}\left(R_i \leq \frac{-\theta p(T^m)R_r}{p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)}\right) \tag{24}$$

SU_j assumes R_i follows a uniform distribution in [K_1, K_2]. Denote by $p^{acpt}(\theta)$ the probability of offer is accepted. We have

$$p^{acpt}(\theta) = \begin{cases} 0, & if\ \frac{-\theta p(T^m)R_r}{p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)} \leq K_1 \\ \frac{-\theta p(T^m)R_r - K_1[p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)]}{[p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)](K_2 - K_1)}, & if\ \frac{-\theta p(T^m)R_r}{p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)} \in (K_1, K_2) \\ 1, & if\ \frac{-\theta p(T^m)R_r}{p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)} \geq K_2 \end{cases} \tag{25}$$

It is a piece-wise function. Notice that $0 \leq K_1 < K_2 < R_r$, therefore

$$\frac{-\theta p(T^m)R_r}{p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)} \in (K_1, K_2) \Leftrightarrow \frac{-AK_1}{p(T^m)(R_r - K_1)} < \theta < \frac{-AK_2}{p(T^m)(R_r - K_2)} \Leftrightarrow \underline{\theta} < \theta < \overline{\theta} \tag{26}$$

When $\underline{\theta} < \theta < \overline{\theta}$, the utility of SU_j is

$$u_j(\theta) = [(1-\theta)p(T^m)R_j - p(T)R_j]\, p^{acpt}(\theta)$$

$$= R_j[(1-\theta)p(T^m) - p(T)] \cdot \frac{-\theta p(T^m)R_r - K_1[p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)]}{[p(T^m)(1-\theta) - p(T) - \frac{1}{b_i}(T^m - T)](K_2 - K_1)} \tag{27}$$

Differentiating (27) with respect to θ, we have

$$p^2(T^m)(K_1 - R_r)\theta^2 + 2p(T^m)A(R_r - K_1)\theta + \left[K_1A^2 - R_rA\big(p(T^m) - p(T)\big)\right] = 0 \tag{28}$$

Evidently θ_p in (20) is one of the two solutions of (28) and it maximizes $u_j(\theta)$ (without considering the boundary values). We can prove $\theta_p > 0$ and $u_j(\theta_p) > 0$ althrough we omit the proof here due to space limitation. However the relationship between $\underline{\theta}, \theta_p$ and $\overline{\theta}$ is not constant. Notice that for any $\theta \in [0, \theta_p)$, $u_j'(\theta) > 0$ and for any $> \theta_p$, $u_j'(\theta) < 0$. Thus we have: if $\theta_p \leq \underline{\theta}$, the offer will never be accepted because $u_j(\underline{\theta}) = 0$ and it decreases with θ; if $\theta_p \in (\underline{\theta}, \overline{\theta})$, $\theta^* = \theta_p$; and if $\theta_p \geq \overline{\theta}, p^{acpt}(\theta) = 1$, $u_j(\theta)$ decreases with θ, SU_j checks if $P(T^m)\left(1-\overline{\theta}\right)R_j - P(T)R_j > 0$. If it holds, $\theta^* = \overline{\theta}$; otherwise θ^* does not exist. Therefore **Theorem 3** is obtained.

4.4 Bargain with a potential partner

If the first bid of SU_j is high but its token storage is relatively low, coalitions will consider whether to invite SU_j carefully. If SU_j cannot get token fund from others, its contribution to the coalition is negligible. In this circumstance, a coalition *S* must first figure out if any of its member would fund SU_j before it decides whether to invite SU_j. Therefore for any coalition member SU_i, SU_j is a potential partner.

For SU_j, its low token storage introduces great disadvantage in its spectrum competition. First, coalition *S* is not complete when SU_j bargains with it, so SU_j does not know its final form. Second, SU_j does not know if it will be invited by subsequent coalitions as well as their competition power. These uncertainties put SU_j in a bad position in the bargaining, therefore it is appropriate to assume in this type of bargaining, SU_i will propose an offer and SU_j will decide whether to acept. It is natural to assume SU_j has a minimum normalized data rate R_j^{min} to meet its basic communication demand. If SU_i proposes a θ that results in a normalized data rate R_j^{nor} lower than R_j^{min}, SU_j will reject this offer. SU_i knows this R_j^{min} exists, but does not know the value. Define utility of SU_j by

$$u_j = \begin{cases} \left(p(T^m) - p(\alpha_j)\right) \cdot \left(R_j^{nor} - R_j^{min}\right) & if\ offer\ accepted \\ 0 & if\ \ offer\ rejected \end{cases} \tag{29}$$

where

$$R_j^{nor} = R_j(1-\theta) \tag{30}$$

Notice the coalition is not yet in its complete form, therefore T^m represents the total payment of its current members if SU_j is funded. Substituting (30) into (29) we obtain

$$u_j(\theta) = \begin{cases} \left(p(T^m) - p(\alpha_j)\right) \cdot \left[R_j(1-\theta) - R_j^{min}\right] & if\ offer\ accepted \\ 0 & if\ \ offer\ rejected \end{cases} \tag{31}$$

Utility of SU_i remains the same as in (13). SU_i attempts to maximize its utility without prior knowledge of R_j^{min}. Let $\left(p(T^m) - p(\alpha_j)\right) \cdot \left[R_j(1-\theta) - R_j^{min}\right] > 0$, we obtain

$$\theta < 1 - \frac{R_j^{min}}{R_j} \quad (32)$$

Denote $1 - \frac{R_j^{min}}{R_j}$ by θ^{max}. Evidently for any $\theta \geq \theta^{max}$, $R_j(1-\theta) - R_j^{min} \leq 0$, thereby offer will be rejected. SU_i assumes θ^{max} follows a uniform distribution in [K_1, K_2], where $0 \leq K_1 < K_2 < 1$. Now we rewrite utility of SU_i as

$$u_i(\theta) = \left\{p(T^m)[(1-\theta)R_i + \theta R_r] - p(T)R_i - \frac{R_i(T^m - T)}{b_i}\right\} \cdot \text{Prob(offer accepted)}$$

$$= \left[\left(p(T^m) - p(T) - \frac{T^m - T}{b_i}\right)R_i + p(T^m)(R_r - R_i)\theta\right] Prob(\theta^{max} > \theta) \quad (33)$$

where T represents the total payment of current members (except SU_j) of the coalition and

$$Prob(\theta^{max} > \theta) = \begin{cases} 1 & if\ \theta \leq K_1 \\ \frac{K_2 - \theta}{K_2 - K_1} & if\ K_1 < \theta < K_2 \\ 0 & if\ \theta \geq K_2 \end{cases} \quad (34)$$

Here we assume $R_r > R_i$ because a potential partner is not yet a partner. If SU_j cannot improve data rate of SU_i through relaying, SU_i will never fund SU_j. We have

1) If $\theta \leq K_1$, $u_i'(\theta) = p(T^m)(R_r - R_i) > 0$, therefore the optimal value $\theta^* = K_1$.
2) If $K_1 < \theta < K_2$,

$$u_i'(\theta) = \frac{1}{K_2 - K_1}\left[-AR_i + K_2 p(T^m)(R_r - R_i) - 2p(T^m)(R_r - R_i)\theta\right] \quad (35)$$

   where A is defined in **Theorem 3**. Let $u_i'(\theta) = 0$, we obtain $\theta_p = \frac{K_2}{2} - \frac{AR_i}{2p(T^m)(R_r - R_i)}$ that maximizes $u_i(\theta)$. If $\theta_p \leq K_1$, $\theta^* = K_1$. If $K_1 < \theta_p < K_2$, $\theta^* = \theta_p$. However $\theta_p \geq K_2$ requires both $A < 0$ and $K_2 \leq \frac{-AR_i}{p(T^m)(R_r - R_i)}$, which results in $u_i(K_2) \leq 0$. Notice $u_i(\theta)$ increases with θ, so SU_i would not assume θ^{max} to be such a value. Therefore the estimation of θ^{max} by SU_i must satisfy $0 < \frac{-AR_i}{p(T^m)(R_r - R_i)} < K_1 < K_2 < 1$.

Based on the above discussion, we have the following theorem:

Theorem 4: SU_j assumes θ^{max} follows a uniform distribution in [K_1, K_2]. If $A \geq 0$, then $0 \leq K_1 < K_2 < 1$; otherwise $0 < \frac{-AR_i}{p(T^m)(R_r - R_i)} < K_1 < K_2 < 1$ Then the optimal value of proposed relay time fraction θ^* can be obtained as follows: calculate $\theta_p = \frac{K_2}{2} -$

$\frac{AR_i}{2p(T^m)(R_r - R_i)}$. If $\theta_p \leq K_1$, $\theta^* = K_1$; otherwise $\theta^* = \theta_p$.

4.5 Modified coalition formation and bidding

In Section 3 we introduced coalition formation without considering potential negotiations and cooperation among SUs. Now we take into account these factors and modify the coalition formation and bidding process. An instance shown in **Table 1** is used to illustrate, where total available bandwidth $B = 8$.

Table 1. Instance for modified coalition formation and bidding

	SU_1	SU_2	SU_3	SU_4	SU_5	SU_6	SU_7
demand w_i	3	3	3	3	2	2	2
first bid b_i	10	9	8	7	6	5	3
storage α_i	20	20	20	20	1	4	3

Apparently, SU_1 is the first head that picks its coalition members. Denote the coalition of SU_1 by S_1. After SU_2 joins S_1, S_1 needs one of SU_5, SU_6 and SU_7. The member of S_1 (SU_1 and SU_2) negotiates with SU_5 respectively. If SU_5 makes a deal with either SU_1 or SU_2, it joins S_1; otherwise it remains uninvited. We assume SU_5 fails to join S_1, so S_1 will invite SU_6 and its final form will be $S_1 = \{SU_1, SU_2, SU_6\}$. In the next SU_3 will be the new head and picks SU_4 to join its coalition S_2. Again S_2 tries to negotiate with SU_5. We assume SU_4 and SU_5 have a deal, hence SU_5 joins S_2. The coalition formation terminates with $S_1 = \{SU_1, SU_2, SU_6\}$, $S_2 = \{SU_3, SU_4, SU_5\}$ and $S_3 = \{SU_7\}$. It must be stressed that potential partners will not be chosen as heads. For instance, if SU_1, SU_2, SU_3 and SU_4 form coalition S_1, then SU_5 will not be the head of S_2. Instead SU_6 will be the head because it has the highest payment per bandwidth even if it is not funded. Before coalitions submit their bids, a negotiation inside S_1 (assume it is between SU_1 and SU_6, and assume the negotiation is successful) is done. Therefore the final payments of coalitions are: S_1 pays 24 tokens, S_2 pays 21 and S_3 pays 3. The payment division inside S_1 is: SU_1 pays 11, SU_2 pays 9 and SU_6 pays 4. The payment division inside S_2 is: SU_3 pays 8, SU_4 pays 12 and SU_5 pays 1. In the above, SU_5 was a potential partner for SU_4 therefore discussion in **subsection 4.4** applies in their negotiation. SU_6 is a partner for SU_1 therefore discussion in **subsection 4.3** applies.

5. Simulation results

In this section we validate the proposed auction mechanism through simulations. We first validate the mutual benefit in the cooperation, then simulate the spectrum allocation.

5.1 Mutual benefit in cooperation

We simulate and illustrate the mutual benefit in cooperation between two partners. We set $R_i = 1$, $R_j = 10$, $R_r = 10$, $b_i = 10$, $p(T^m) = 0.56$, $p(T) = 0.375$, $T^m = 25$, $T = 12$. **Fig. 4** shows the utilities with different relay time fraction θ proposed by SU_j. The blue curves are utilities of SU_j estimated by itself with different parameters K_1 and K_2 ([1 1.6] represents $K_1 = 1$ and $K_2 = 1.6$). Different estimations of R_i result in different θ^* therefore influence the actual utilities of SU_i and SU_j. We see θ^* of three different estimations are 0.2739, 0.2128 and 0.2490, respectively. The corresponding actual utilities are 0.25, -0.0553 and 0.1258 for SU_i, 0.2840, 0.6231 and 0.4220 for SU_j. Accurate estimations of R_i may produce relatively large data rate increment of SU_j while inaccurate estimations may lead to larger data rate increment of SU_i. Notice the cooperation will only be done when both $u_i > 0$ and $u_j > 0$, therefore when SU_j assumes $R_i \in [0.5\ 1.5]$, the negotiation will fail. So the cooperation will always introduce mutual benefit for SU_i and SU_j.

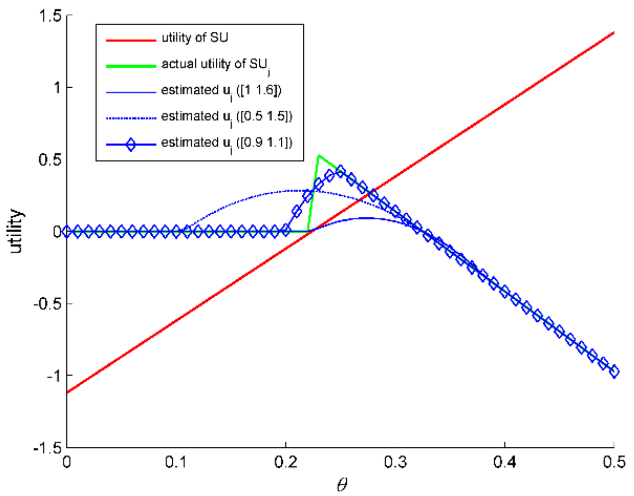

Fig. 4. Utilities of cooperation between partners

To investigate the cooperation between a member of a coalition and its potential partner, we set $R_i = 1$, $R_j = 1$, $R_r = 10$, $R_j^{min} = 0.8$, $p(T^m) = 0.55$, $p(T) = 0.444$, $p(\alpha_j) = 0.022$, b_i=20, $T^m = 50$, $T = 40$. **Fig. 5** shows the utilities with different relay time fraction θ proposed by SU_i. The blue curves are utilities of SU_i estimated by itself with different parameters K_1 and K_2. Here $A = p(T^m) - p(T) - \frac{T^m - T}{b_i} = -0.444 < 0$, therefore $\frac{-AR_i}{p(T^m)(R_r - R_i)} = 0.0988 < K_1 < K_2$. Different estimations of θ^{max} yield different θ^* leading to different utilities of SU_i and SU_j. Three estimations provide three θ^* at 0.1494, 0.1744 and 0.2 respectively. The corresponding actual utilities are 0.2278, 0.3403 and 0.4556 for SU_i, 0.0242, 0.0122 and 0 for SU_j. The offer with $\theta^* = 0.2$ will be rejected because $u_j = 0$. Once the negotiation succeeds, the cooperation will lead to a win-win situation.

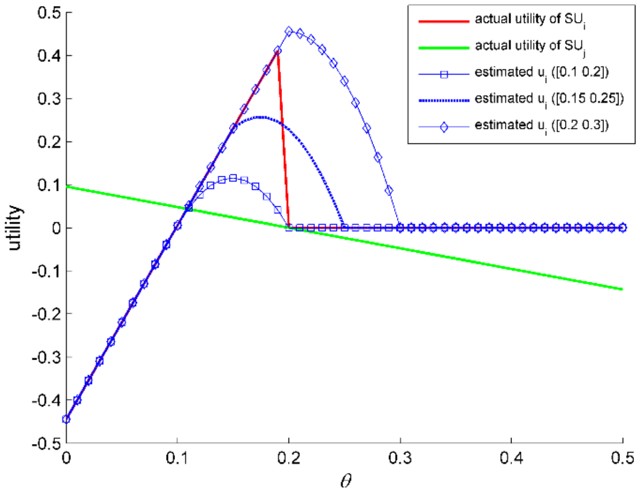

Fig. 5. Utilities of cooperation between a member and a potential partner

5.2 Token spectrum auction

We investigate the proposed token spectrum auction by studying a specific case with 10 SUs. The parameters of SUs are set as in **Table 2**. In **Table 2**, notation [5, 7] presents the corresponding variable follows a uniform distribution in [5, 7]. In every episode w_i, R_i/w_i and θ_i^{max} randomly generate a value in corresponding intervals. We also randomly generate relay data rate shown in **Table 3**. We run the simulation 10 times and each simulation lasts 100 episodes.

Table 2. Parameters of SUs

SU	1	2	3	4	5	6	7	8	9	10
β_i	20	12	9	8	5	4	3	2	6	2
w_i	[5, 7]	[3, 6]	[3, 4]	[4, 6]	[3, 5]	[2, 5]	[1, 4]	[1, 3]	[4, 7]	[1, 4]
R_i/w_i	[1, 2]	[1, 3]	[2, 4]	[1, 2]	[2, 3]	[3, 5]	[2, 4]	[1, 2]	[3, 4]	[2, 3]
θ_i^{max}	0	0	0	0	[0,0.2]	[0,0.3]	[0.1,0.25]	[0.2,0.4]	[0,0.1]	[0.2,0.4]
p_i^{win}	0.4	0.4	0.55	0.45	0.35	0.45	0.4	0.45	0.3	0.4

Table 3. Data rate for relay transmission

SU_j \ SU_i	1	2	3	4	5	6	7	8	9	10
1	0	705	465	240	585	525	30	495	570	405
2	90	0	120	150	240	240	675	540	420	150
3	600	150	0	150	195	165	180	405	390	360
4	225	210	165	0	360	240	345	255	120	360
5	465	600	480	15	0	225	510	435	480	285

6	735	375	375	45	165	0	495	105	375	90
7	330	585	300	360	285	195	0	510	285	360
8	525	300	165	225	450	300	30	0	435	405
9	570	210	330	405	315	225	135	75	0	285
10	600	555	525	465	180	240	405	180	300	0

Two contrastive spectrum allocation mechanisms are introduced, named random allocation and token optimal allocation. The random allocation randomly chooses a set of SUs under the condition that the total spectrum demand does not exceed the total available spectrum *B* in current episode. The token optimal allocation is deployed at the FC. The FC allocates the available spectrum to a set of SUs that maximize its payoff (i.e., the FC aims to collect the maximum tokens in current episode).

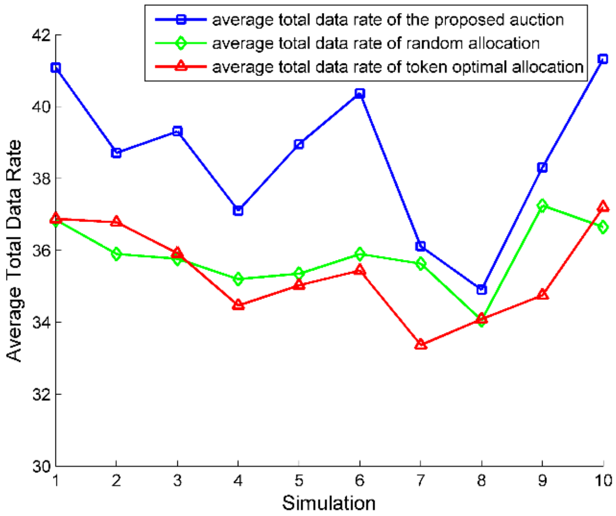

Fig. 6. Average total data rate

Fig. 6 shows the average total data rate in every simulation. In our proposed auction mechanism, due to the randomness in the winner decision, it is possible that an "inefficient" coalition obtains the spectrum resulting in low data rates in some episodes. However this mechanism motivates the SUs with weak competition power. And the significant data rate gain in the relay transmissions in cooperation compensates the aforementioned data rate loss. In fact, averaging the "average total data rate" over 10 simulations we obtain 38.61, 35.85 and 35.39 for three mechanisms respectively. The proposed auction outperforms the other two.

Fig. 7 shows spectrum obtaining frequency of each SU. In token optimal allocation, SU_1, SU_2 and SU_3 have very high obtaining frequencies because they earn more tokens every episode (i.e., high β_i). With low β_i and relatively high spectrum demand w_i, SU_6 and SU_{10} show low frequencies (12.4 and 11.9 respectively), and may lose their incentive to participate the spectrum competition. In random allocation, SU_8, SU_{10} and SU_7 show high frequencies because w_8, w_{10} and w_7 are significantly lower than others. The random allocation neglects the contributions of SUs represented by tokens and obliterates the SUs' incentive to contribute the network. The proposed token auction shows balanced allocation where SU_1 obtains the highest frequency 52.3 and SU_{10} obtains the lowest frequency 27.4. SUs with low β_i improve their frequencies through cooperation. The data indicate all SUs have a fair and acceptable chance to obtain the spectrum in accordance with their contributions.

6. Conclusions

This paper proposes a spectrum allocation mechanism based on auction in overlay cognitive radio network. The proposed mechanism quantities contribution of each SU and pays them tokens. The SUs form coalitions and pay tokens to compete the available spectrum in an episode. SUs with low token storage may borrow tokens from other SUs, in return providing relay transmission for the lenders. We model the negotiation between the borrowers and the lenders as Bayesian games because of the incomplete information, and investigate the behaviors and equilibria in these games. Theoretical analysis and simulation results indicate both borrower and lender benefit from cooperation. Simulation results also show our proposed auction yields satisfactory data rate and fairness.

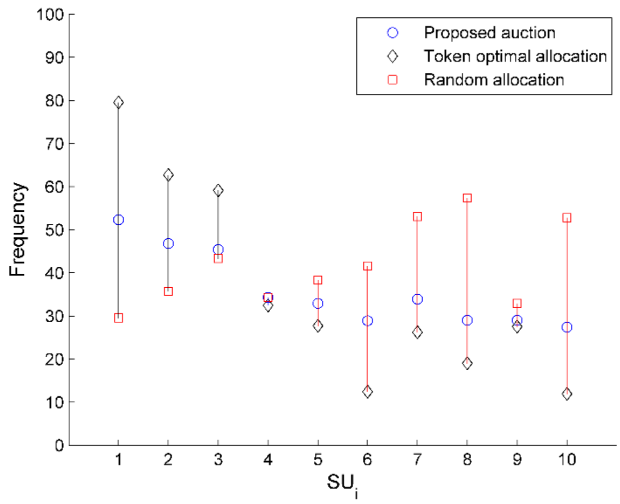

Fig. 7. Spectrum obtaining frequency of each SU

References

[1] I.F. Akyildiz, W.Y. Lee, M.C. Vuran and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol.50, no.13, pp. 2127-2159, Sep 15 2006. Article (CrossRef Link)

[2] R. Southwell, X. Chen and J.W. Huang, "Quality of service games for spectrum sharing," *IEEE*

Journal on Selected Areas In Communications, vol.32, no.3, pp. 589-600, Mar 2014. Article (CrossRef Link)

[3] D.B. Rawat, S. Shetty and K. Raza, "Geolocation-aware resource management in cloud computing-based cognitive radio networks," *International Journal of Cloud Computing*, vol.3, no.3, pp. 267-287 2014. Article (CrossRef Link)

[4] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of nash equilibrium, and collusion," *IEEE Journal on Selected Areas In Communications*, vol.26, no.1, pp. 192-202, Jan 2008. Article (CrossRef Link)

[5] L. Gao, X.B. Wang, Y.Y. Xu and Q.A. Zhang, "Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach," *IEEE Journal on Selected Areas In Communications*, vol.29, no.4, pp. 843-855, Apr 2011. Article (CrossRef Link)

[6] L.J. Duan, L. Gao and J.W. Huang, "Cooperative spectrum sharing: A contract-based approach," *IEEE Transactions on Mobile Computing*, vol.13, no.1, pp. 174-187, Jan 2014. Article (CrossRef Link)

[7] J. Huang, R.A. Berry and M.L. Honig, "Auction-based spectrum sharing," *Mobile Networks & Applications*, vol.11, no.3, pp. 405-418, Jun 2006. Article (CrossRef Link)

[8] F. Wang, M. Krunz and S.G. Cui, "Price-based spectrum management in cognitive radio networks," *IEEE Journal Of Selected Topics In Signal Processing*, vol.2, no.1, pp. 74-87, Feb 2008. Article (CrossRef Link)

[9] C.L. Li, Z. Liu, X.Y. Geng, M. Dong, F. Yang, X.Y. Gan, X.H. Tian and X.B. Wang, "Two dimension spectrum allocation for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol.13, no.3, pp. 1410-1423, Mar 2014. Article (CrossRef Link)

[10] Y. Zhang, C. Lee, D. Niyato and P. Wang, "Auction approaches for resource allocation in wireless systems: A survey," *IEEE Communications Surveys And Tutorials*, vol.15, no.3, pp. 1020-1041 2013. Article (CrossRef Link)

[11] J. Rajasekharan and V. Koivunen, "Cooperative game-theoretic approach to spectrum sharing in cognitive radios," *Signal Processing*, vol.106, pp. 15-29, Jan 2015. Article (CrossRef Link)

[12] W. Saad, Z. Han, M. Debbah, A. Hjorungnes and T. Basar, "Coalitional game theory for communication networks," *IEEE Signal Processing Magazine*, vol.26, no.5, pp. 77-97, Sep 2009. Article (CrossRef Link)

[13] K.R. APT and A. WITZEL, "A generic approach to coalition formation," *International Game Theory Review (IGTR)*, no.03, pp. 347-367 2009. Article (CrossRef Link)

[14] K.R. Apt and T. Radzik, "Stable partitions in coalitional games," in eprint arXiv:cs/0605132, pp. 5132, 2006. Article (CrossRef Link)

[15] T. Sandholm, K. Larson, M. Andersson, O. Shehory and F. Tohme, "Coalition structure generation with worst case guarantees," *Artificial Intelligence*, vol.111, no.1-2, pp. 209-238, Jul 1999. Article (CrossRef Link)

[16] W. Saad, Z. Han, M. Debbah, A. Hjorungnes and T. Basar, "Coalitional games for distributed collaborative spectrum sensing in cognitive radio networks," *IEEE Infocom 2009 - IEEE Conference on Computer Communications, Vols 1-5*, pp. 2114-2122 2009. Article (CrossRef Link)

[17] J.N. Laneman, D.N.C. Tse and G.W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol.50, no.12, pp. 3062-3080, Dec 2004. Article (CrossRef Link)

Wenhao Jiang received his BS degree in electrical and information engineering and MS degree in electrical and communication engineering from College of Communication Engineering, Chongqing University (CQU), Chongqing, China in 2009 and 2012 respectively. He is currently studying for his Ph.D. degree in communication and information system at College of Communication Engineering, CQU. His research interests include cognitive radio network and wireless network.

Wenjiang Feng received his Ph.D. degree in electrical engineering from Chongqing University in 2000. Currently, he is a professor at the college of communication engineering in Chongqing University. His research interests fall into the broad areas of communication theory, wireless communication.

Yang Yu received his BS degree in electrical and information engineering Chongqing University in June 2012. He is currently pursuing MS degree at College of Communication Engineering, CQU. His research fields include wireless network communication and full-duplex communication.