Soft-Input Soft-Output Multiple Symbol Detection for Ultra-Wideband Systems

Chanfei Wang, Hui Gao, Tiejun Lv
School of Information and Communication Engineering,
Beijing University of Posts and Telecommunications (BUPT), Beijing, China 100876
[e-mail: {wangchanfei, huigao, lvtiejun}@bupt.edu.cn]
*Corresponding author: Tiejun Lv

Received February 5, 2015; revised May 19, 2015; accepted April 8, 2015; published July 31, 2015

Abstract

A multiple symbol detection (MSD) algorithm is proposed relying on soft information for ultra-wideband systems, where differential space-time block code is employed. The proposed algorithm aims to calculate a posteriori probabilities (APP) of information symbols, where a forward and backward message passing mechanism is implemented based on the BCJR algorithm. Specifically, an MSD metric is analyzed and performed for serving the APP model. Furthermore, an autocorrelation sampling is employed to exploit signals dependencies among different symbols, where the observation window slides one symbol each time. With the aid of the bidirectional message passing mechanism and the proposed sampling approach, the proposed MSD algorithm achieves a better detection performance as compared with the existing MSD. In addition, when the proposed MSD is exploited in conjunction with channel decoding, an iterative soft-input soft-output MSD approach is obtained. Finally, simulations demonstrate that the proposed approaches improve detection performance significantly.

Keywords: Differential space-time block code (DSTBC), ultra-wideband (UWB), multiple symbol detection (MSD), a posteriori probabilities (APP), soft-input soft-output (SISO).

This work is supported by the National Natural Science Foundation of China (NSFC) (Grant No. 61271188, 61401041), the Fundamental Research Funds for the Central Universities (Grant No. 2014RC0106), and Beijing Municipal Science and Technology Commission Research Fund Project (Grant No. D151100000115002).

http://dx.doi.org/10.3837/tiis.2015.07.016 **ISSN : 1976-7277**

1. Introduction

As a potential solution for short range wireless communications, impulse radio ultra-wideband (UWB) is an attractive technology without acquiring additional spectrum resources. UWB offers some potentials, including low probability of interception, precise positioning capability, and coexistence with existing services [1]. These features have motivated interest in UWB technology for a variety of state-of-the-art wireless applications in recent years. The successful deployment of UWB systems, however, is encumbered by the harsh propagation environments. Each transmitted pulse arrives at the receiver over hundreds of delayed paths, with possibly severe shape distortion affected by diffraction and scattering effects. Moreover, due to the high frequency selectivity of UWB channels [2], coherent receivers face many practical challenges, including intensive computational cost and extremely high sampling rate to estimate channel state information. These challenges make it difficult and costly to realize the optimal coherent receivers for UWB communications. Therefore, non-coherent UWB systems are preferred in view of their low implementation complexity [3]. In single antenna systems, the typical non-coherent detection is based on differential detection (DD), which employs the autocorrelation receivers (AcR) [4]. Although no explicit channel estimation is required for AcR, the non-coherent detection brings noise due to the use of noise template. Thus, DD results in severe bit error rate (BER) performance degradation as compared to the coherent receivers. To alleviate this issue, multiple symbol detection (MSD) is designed to suppress noise [5]. As an effective means to improve the BER performance of single antenna systems, multi-antennas non-coherent UWB systems are emerging recently. As a special form of DD, differential space-time block code (DSTBC) encoded AcR is investigated in UWB channels [6], where additional diversity gain has been achieved. In order to further enhance the detection performance, MSD is introduced in the DSTBC UWB systems. It has pointed out in [7] that MSD outperforms DD in terms of the BER performance. Furthermore, in order to fulfill a low-complexity MSD, decision-feedback is considered in [8]. However, all of these MSD are all about hard decision algorithm, which results in limited BER performance improvement. Therefore, soft information detection is inevitable, and needed to ensure detection accuracy in non-coherent UWB systems.

For many application scenarios, soft decision detection has been perceived as a promising approach to improve the systems performances [9]. In [10], a soft-input soft-output (SISO) MSD algorithm is proposed for the single antenna UWB systems. As an appealing benefit, trellis-based bidirectional information transmission is promising compared with that of the unidirectional transmission in [10]. An soft information detection has been proposed in [11] in Rayleigh fading channel, where DSTBC is concatenated with error correction code, e.g., convolutional code [12], and Turbo code [13]. The receiver in [11] employs a posteriori probabilities (APP) decoder, where linear prediction is presented to utilize the temporal correlation of fading for detection. Furthermore, a low-complexity iterative detection has been addressed in [14]. In order to avoid estimation for channel parameters, and further improve the detection performance, there is a need to investigate SISO MSD relying on the bidirectional message passing based on the DSTBC trellis [15].

In this paper, an effective SISO MSD algorithm is proposed in the DSTBC UWB systems, where the forward and bacward message passing mechanism [16] is implemented. The proposed algorithm aims to evaluate the APP of the information bits. In particular, a mathematical framework for MSD metric is analyzed for serving the APP model. To further exploit signals dependencies among DSTBC symbols, an AcR architecture is developed. For ease of explanation, in the following, signals sampling will be referred to as "observation window". Most of the MSD algorithms employ block-by-block sampling [7], where the observation window of size $M+1$symbols durations slides forwards M symbols when the current M symbols have been detected. We develop a different sampling architecture, where the observation window slides only one symbol duration each time. Benefitting from the proposed sampling and forward and backward message passing, the potential of the SISO MSD for improving the BER performance is achieved. In addition, the proposed SISO MSD can be exploited in conjunction with channel decoding to obtain an iterative detection approach. Simulations are presented to validate the effectiveness of the proposed appraoaches. To be more specific, the main contributions of this paper are summarized as follows.

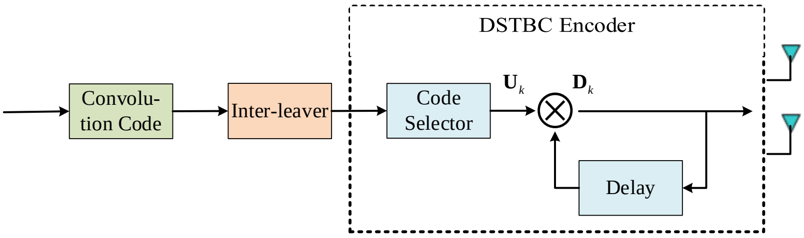
1. An SISO MSD is proposed to calculate APP of information symbols. In contrast to hard decision MSD in DSTBC UWB systems [7], the proposed MSD provides more reliable detection due to using soft information. It is noted that, the proposed MSD is particularly attractive for the non-coherent UWB systems, since it is capable of exploiting accurate detection information without requiring channel estimation.
2. Inspired by the forward and backward message passing, the proposed SISO MSD facilitates the bidirectional message passing implementation by means of DSTBC trellis. In contrast to soft information detection provided in [15], our SISO MSD exploits more accurate detection information with the aid of the forward and backward message passing. Therefore, it exhibits much better BER performance.
3. A mathematical model for MSD metric is analyzed for the proposed SISO MSD in DSTBC UWB systems. From the theoretical analysis perspective, we can deduce that the metric is capable of judging the transmitted information from the received signals accurately. Therefore, the proposed algorithm carries out detection effectively.

The remainder of this paper is organized as follows. System description of a DSTBC UWB system is introduced in Section 2. In Section 3, an MSD metric is analyzed based on the DSTBC UWB system, and then an SISO MSD is proposed relying on the MSD metric. Simulation results are demonstrated in Section 4. Finally, conclusions are drawn in Section 5.

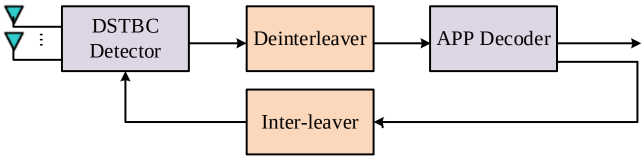
Notations: Lower-case (upper-case) boldface symbols denote vectors (matrices); $\mathbf{I}_K$ represents the $K \times K$ identity matrix; for any matrix $\mathbf{A}$, $\mathrm{Tr}(\mathbf{A})$ and $\mathbf{A}^{\mathrm{T}}$ denote the trace, and the transpose of $\mathbf{A}$, respectively; $\mathrm{E}[\cdot]$ and $\mathrm{Var}[\cdot]$ denote expectation and variance, respectively; $\otimes$ stands for convolution; $\propto$ represents the proportion relationship.

2. System Model

We consider a DSTBC system, which is equipped with Q $(Q \geq 1)$ receive antennas. Note that each DSTBC code applied has to be an orthogonal unitary matrix. When the orthogonal unitary matrices are designed according to the DSTBC scheme [17], the proposed algorithm can be extended to the systems with more than two transmit antennas. In order to facilitate the encoding scheme, we consider two transmit antennas at the transmitter. The basic system structure is shown in **Fig. 1**.



(a) A system with two transmit antennas.



(b) A system with multiple receiver antennas.

Fig. 1. DSTBC UWB system, where (a) and (b) denote the transmitter and the receiver, respectively

The information bits are encoded with a forward error correction code, e.g., convolutional code. Then, the coded bits are interleaved and fed into the DSTBC encoder. As the encoding strategy mentioned in [6], the encoded information are divided into blocks of two bits. Finally, every block is mapped onto a information symbol from the code book $\Omega = \{\mathbf{U}^0, \mathbf{U}^1, \mathbf{U}^2, \mathbf{U}^3\}$, where the code $\mathbf{U}^x \in \Omega, (x = 0, 1, 2, 3)$ is an orthogonal unitary matrix and has the form $\mathbf{U}^0 : \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $\mathbf{U}^1 : \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$; $\mathbf{U}^2 : \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\mathbf{U}^3 : \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then, the code selector determines the information symbol according to the rule, $00 \to \mathbf{U}^0$; $01 \to \mathbf{U}^1$; $10 \to \mathbf{U}^2$; $11 \to \mathbf{U}^3$. In the differential encoded system, the transmitted code for the k-th information symbol $\mathbf{U}_k \in \Omega$ is obtained by $\mathbf{D}_k = \mathbf{D}_{k-1}\mathbf{U}_k$ where $k = 1, 2, \ldots, K$, and K represents the number of transmitted symbols; the 2×2 matrix $\mathbf{D}_k$ is sent over two antennas during two frame durations.The reference matrix is set as $\mathbf{D}_0 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Let $d_{p,2k+n-1}$ $(p = 1, 2; n = 1, 2)$ denotes the p-th row and n-th column of $\mathbf{D}_k$. Then, the transmitted signal from the p-th antenna can be expressed by

$$\begin{aligned} s_p(t) &= \sum_{k=0}^{K} \sum_{n=1}^{2} d_{p,2k+n-1}\,\omega(t - (n-1)T_f - kT_s) \\ &= \sum_{k=0}^{K} \sum_{n=1}^{2} d_{p,2k+n-1}\,\omega(t - (2k+n-1)T_f) = \sum_{j=0}^{2K+1} d_{p,j}\,\omega(t - jT_f), \end{aligned} \tag{1}$$

where $\omega(t)$ denotes the monocycle pulse waveform; T_f is frame duration; one transmitted symbol duration is $T_s = 2T_f$; the single time index $j = 2k + n - 1$ is introduced to replace the double index (k,n), and $d_{p,2k+n-1}$ is rewritten by $d_{p,j}$, correspondingly. Considering a quasi-static dense multipath fading environment [18], UWB channel impulse response between the p-th transmit antenna and the q-th ($1 \le q \le Q$) receive antenna is given by

$$h_{p,q}(t) = \sum_{l=1}^{L_{p,q}} \alpha_l^{p,q} \delta(t - \tau_l^{p,q}), \tag{2}$$

where $L_{p,q}$ denotes the number of propagation paths; $\alpha_l^{p,q}$ and $\tau_l^{p,q}$ are the gain coefficient and the delay associated with the l-th path, respectively; $\delta(t)$ is Delta function. The overall channel response $g_{p,q}(t)$ between the p-th transmit antenna and the q-th receive antenna can be described as

$$g_{p,q}(t) = \omega(t) \otimes h_{p,q}(t) = \sum_{l=1}^{L_{p,q}} \alpha_l^{p,q} \omega(t - \tau_l^{p,q}). \tag{3}$$

Then, the signal at the q-th receive antenna is obtained as

$$y_q(t) = \sum_{p=1}^{2} s_p(t) \otimes h_{p,q}(t) + n_q(t) = \sum_{p=1}^{2} \sum_{j=0}^{2K+1} d_{p,j} g_{p,q}\left(t - jT_f\right) + n_q(t), \tag{4}$$

where $n_q(t)$ denotes additive white Gaussian noise (AWGN) with zero mean, and two-sided power spectral density $N_0/2$. Based on (4), several MSD algorithms have been developed [6-8]. However, all these investigations result in limited performances improvement due to their hard decision detection. Therefore, in order to further improve the BER performance of the existing MSD, it is necessary to investigate an efficient algorithm relying on soft decision. In the next section, we will attempt to analyze an SISO MSD algorithm.

3. SISO MSD for DSTBC UWB Systems

In this section, an SISO MSD mathematical model will be derived for the DSTBC UWB systems. Firstly, by employing an AcR structure sampling, an MSD metric will be analyzed. Furthermore, an SISO MSD will be proposed with the aids of the MSD metric and BCJR approach in message passing mechanism.

3.1 MSD Metric for SISO MSD

In the following, an MSD metric will be presented for the DSTBC UWB systems. Given the transmitted symbols matrices, the conditional probabilities metric of the received signals matrices will be referred to as MSD metric. An AcR architecture is developed to further exploit signals dependencies among information symbols. For explanation convenience, the observation window size is set to $M+1$ symbol durations, during which we assume that the channel remains invariant [5]. The observation window in [7] slides M symbols each time. As a result, the AcR ignores information dependencies among different symbols blocks. Different from the previous approaches, in our sampling, the observation window slides forwards only one symbol duration each time. To this end, the proposed sampling contributes to exploiting dependencies among different symbols blocks.

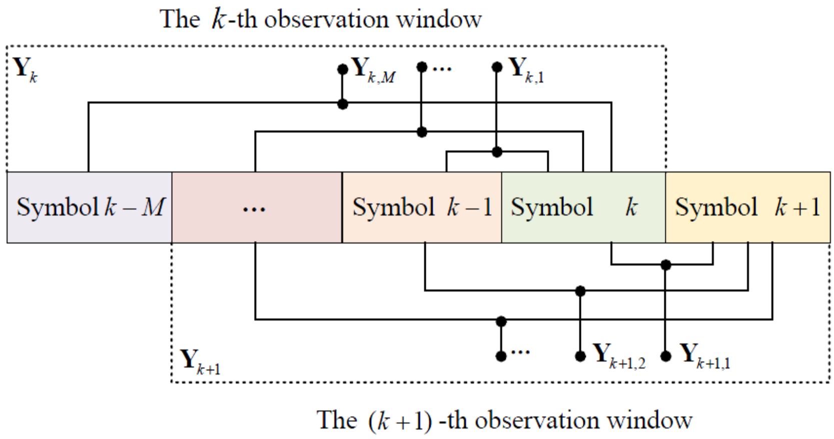


Fig. 2. The proposed AcR sampling mechanism for the k-th and ($k+1$)-th observation window

Fig. 2 illustrates the details of the proposed sampling for two adjacent observation windows. In the k-th observation window $(k-M)T_s \le t \le kT_s$, the receivr samplings are obtained as $\mathbf{Y}_k = [\mathbf{Y}_{k,1}, \mathbf{Y}_{k,2}, \cdots, \mathbf{Y}_{k,M}]$, where 2×2 matrix $\mathbf{Y}_{k,m}(m=1,\cdots,M)$ is the correlation function given by

$$\mathbf{Y}_{k,m} = \sum_{q=1}^{Q} \int_0^{T_i} \left(\mathbf{y}_k^q(t)\right)^{\mathrm{T}} \mathbf{y}_{k-m}^q(t)dt, \tag{5}$$

where $T_i \le T_s$ is integration interval, $\mathbf{y}_k^q(t) = [y_q(t+2kT_f)\ \ y_q(t+(2k+1)T_f)]$ is the received signals waveforms at the q-th antenna. The (u,v)-entry of $\mathbf{Y}_{k,m}$ is given by

$$Y_{k,m}(u,v) = \sum_{q=1}^{Q} \int_0^{T_i} y_q\left(t+2kT_f+(u-1)T_f\right) y_q\left(t+2(k-m)T_f+(v-1)T_f\right)dt \tag{6}$$

with $u,v=1,2$.

Remark 1: The proposed AcR-based MSD is specifically tailored to the distinct signal structure of UWB impulse radio. It requires correlation operations between different segments of the received signals. The computed correlations will be fed into a digital processor. This can be implemented by using analog delay line, multiplier, and integrator; which avoids analog to digital converter with ultra high sampling rate. With $Q=2$ as an example, **Fig. 3** illustrates the AcR structure for k-th and $(k-m)$-th matrixes.

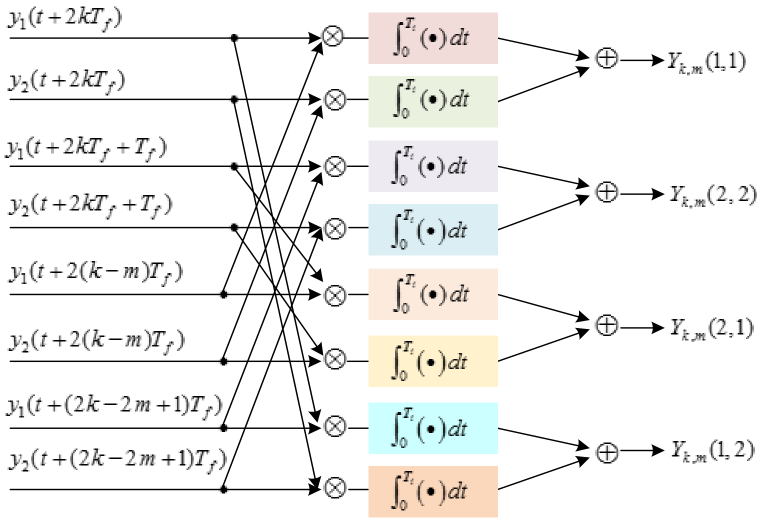


Fig. 3. AcR structure of the k-th and ($k-m$)-th received matrixes in UWB system with two receive antennas

In order to facilitate analysis for MSD metric, substituting (4) into (6), we have

$$Y_{k,m}(u,v)=\sum_{q=1}^{Q}\int_0^{T_i}\left(\sum_{p=1}^{2}d_{p,2k+u-1}g_{p,q}(t)+n_{k,u}^{q}(t)\right)\left(\sum_{p=1}^{2}d_{p,2(k-m)+v-1}g_{p,q}(t)+n_{(k-m),v}^{q}(t)\right)dt, \quad (7)$$

where

$$n_{k,u}^{q}(t)=n_q\left(t+2kT_f+(u-1)T_f\right), \quad (8)$$

and

$$n_{(k-m),v}^{q}(t)=n_q\left(t+2(k-m)T_f+(v-1)T_f\right) \quad (9)$$

denote the time-shifted noises. For convenience of description, we divide $Y_{k,m}(u,v)$ into signal component $S_{k,m}(u,v)$ and noise component $N_{k,m}(u,v)$. Then, (7) is reformulated as

$$Y_{k,m}(u,v)=S_{k,m}(u,v)+N_{k,m}(u,v). \quad (10)$$

The specific expression of each term in (10) is given by Appendix A. In order to simplify the analysis, we assume that $n_q(t)$ in (4) is a wideband AWGN process with bandwidth $W>1/T_i$ [18]. Thus, $N_{k,m}(u,v)$ can be considered as a noise with zero mean, and conditional variance σ^2. A detailed derivation of σ^2 can be found in Appendix A. Now, $\mathbf{Y}_{k,m}$ in (5) can be rewritten as a matrix form

$$\mathbf{Y}_{k,m} = \mathbf{S}_{k,m} + \mathbf{N}_{k,m}, \tag{11}$$

where

$$\mathbf{S}_{k,m} = \begin{pmatrix} S_{k,m}(1,1) & S_{k,m}(1,2) \\ S_{k,m}(2,1) & S_{k,m}(2,2) \end{pmatrix}, \tag{12}$$

and

$$\mathbf{N}_{k,m} = \begin{pmatrix} N_{k,m}(1,1) & N_{k,m}(1,2) \\ N_{k,m}(2,1) & N_{k,m}(2,2) \end{pmatrix} \tag{13}$$

represent the signal component and noise component, respectively. Based on the differential modulation $\mathbf{D}_k = \mathbf{D}_{k-1}\mathbf{U}_k$ and $\mathbf{D}_k\mathbf{D}_k^{\mathrm{T}} = 2\mathbf{I}_2$, we have

$$\mathbf{D}_k\mathbf{D}_{k-m}^{\mathrm{T}} = \mathbf{D}_{k-1}\mathbf{U}_k\mathbf{D}_{k-m}^{\mathrm{T}} = 2\prod_{z=k-m+1}^{k}\mathbf{U}_z. \tag{14}$$

Let $\varepsilon_{p,q} = \int_0^{T_i} h_{pq}^2(t)dt$, and $\xi = \sum_{p=1}^{2}\sum_{q=1}^{Q}\varepsilon_{p,q}$. Combining (11) and (14), $\mathbf{Y}_{k,m}$ is given by

$$\mathbf{Y}_{k,m} = (\mathbf{D}_k\mathbf{D}_{k-m}^{\mathrm{T}})\ \xi + \mathbf{N}_{k,m}. \tag{15}$$

Based on (15), the MSD metric can be expressed by

$$p(\mathbf{Y}_k \mid \mathbf{U}_k, \mathbf{U}_{k-1}, \ldots, \mathbf{U}_{k-M+1}) \propto \prod_{m=1}^{M}\exp\left(-\frac{|\mathbf{Y}_{k,m} - (2\prod_{z=k-m+1}^{k}\mathbf{U}_z)\xi|^2}{\sigma^2}\right). \tag{16}$$

Now, MSD metric has been evaluated [18].

Remark 2: The mathematical model for MSD metric has been analyzed in DSTBC UWB systems. From the theoretical analysis, it can be seen that the proposed metric is capable of exploiting the transmitted symbols information from the received signals accurately. With the aid of MSD metric, the SISO MSD algorithm will be formulated in the next subsection.

Remark 3: From the MSD metric in (16), we can explore the correlations among multiple symbols to get information assisting the detection of the current symbol. By increasing the observation window size M, detection performance can be enhanced. Note that there is a trade-off between detection performance and practical implementations. Thus, from the perspective of detection performance, the BER performance is becoming better and better at the cost of a higher and higher computational complexity when M increases.

3.2 BCJR-Based SISO MSD Algorithm

The target of the SISO MSD is to calculate APP of information bits as $\Lambda\left(d_k(i)\right) = p\{d_k(i) = z \mid \mathbf{Y}\}$, where $d_k(i)$ is the i-th bit corresponding to the information symbol $\mathbf{U}_k$ with $i = 1, 2,$ and $z = 0,1$. In order to obtain soft information by implementing message passing with trellis diagram, the information source can be modeled as a Markov finite-state process [19]. Correspondingly, the state transitions of the information source are characterized by the transition probabilities, from which we can obtain the APP. In the following, we will develop a trellis representation and apply it to the BCJR-based message passing approach. The differential modulation can be represented by a DSTBC trellis structure [10], which is shown in **Fig. 4**.

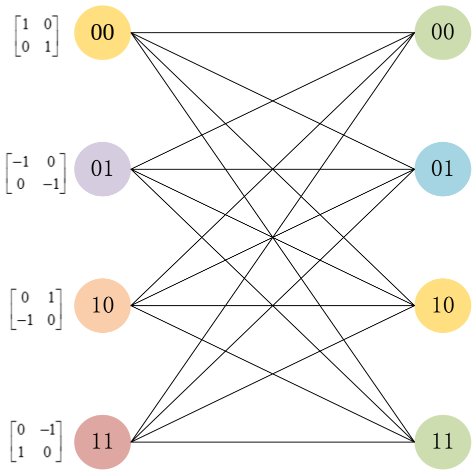


Fig. 4. DSTBC trellis diagram, where the nodes denote states, and the branches represent states transitions

The trellis diagram represents the time progression of the state sequences, where the nodes denote the states, and the branches represent the transitions having non-zeros probabilities. For every state sequence, there is a unique path through the trellis diagram, and vice versa. In order to integrate the trellis diagram with MSD effectively, we define the trellis as follows. When the source state group is $\mathbf{S}_{k-1} = [\mathbf{U}_{k-1}, \ldots, \mathbf{U}_{k-M}]^{\mathrm{T}}$, inputting information symbol $\mathbf{U}_k$, then the output is $\mathbf{X}_k = [\mathbf{U}_k, \mathbf{U}_{k-1}, \ldots, \mathbf{U}_{k-M+1}]^{\mathrm{T}}$. Set $\mathcal{A}_{d_k(i)=z} = \{(\mathbf{S}_{k-1} \rightarrow \mathbf{S}_k) : d_k(i) = z\}$ denotes all the state transitions $\mathbf{S}_{k-1} \rightarrow \mathbf{S}_k$ associated with input $\mathbf{U}_k$ containing $d_k(i) = z$. Let $\mathbf{Y} = \{\mathbf{Y}_k\}$ be the total received signals matrixes. Consequently, the associated trellis nodes correspond to the probabilities $p(\mathbf{S}_k \mid \mathbf{Y})$, and the associated trellis branches correspond to the probabilities $p(\mathbf{S}_{k-1}, \mathbf{S}_k \mid \mathbf{Y})$. Given $\mathbf{Y}$, $p(\mathbf{Y})$ is a constant, then we can derive the joint probabilities to obtain the probabilities $p(\mathbf{S}_k \mid \mathbf{Y})$ and $p(\mathbf{S}_{k-1}, \mathbf{S}_k \mid \mathbf{Y})$, respectively. Therefore, the APP of information bits can be rewritten in terms of the trellis diagram as

$$p\{d_k(i)=z \mid \mathbf{Y}\} \propto \sum\nolimits_{(\mathbf{S}_{k-1}\to\mathbf{S}_k)\in \mathrm{A}_{d_k(i)=z}} p(\mathbf{Y},\mathbf{S}_{k-1},\mathbf{S}_k). \tag{17}$$

Note that the received signals matrixes $\mathbf{Y}$ can be split into three sections, namely, the present received matrix $\mathbf{Y}_k$ corresponding to the present state, the matrixes $\mathbf{Y}_{m<k}$ before the present, and matrixes $\mathbf{Y}_{m>k}$ after the present. Consequently, the joint probability associated with the transition $\mathbf{S}_{k-1} \to \mathbf{S}_k$ can be further expressed as

$$\begin{aligned} p(\mathbf{Y},\mathbf{S}_{k-1},\mathbf{S}_k) &= p(\mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{Y}_{m>k},\mathbf{S}_{k-1},\mathbf{S}_k) \\ &= p(\mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{S}_{k-1},\mathbf{S}_k)\,p(\mathbf{Y}_{m>k} \mid \mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{S}_{k-1},\mathbf{S}_k). \end{aligned} \tag{18}$$

According to the properties of Markov process [20], if $\mathbf{S}_k$ is known, then the events after k do not depend on $\mathbf{Y}_{m<k}$. Thus, these conditional probabilities in (18) can be obtained as

$$p(\mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{S}_{k-1},\mathbf{S}_k) = p(\mathbf{Y}_{m<k},\mathbf{S}_{k-1})\,p(\mathbf{S}_k,\mathbf{Y}_k \mid \mathbf{Y}_{m<k},\mathbf{S}_{k-1}) = p(\mathbf{Y}_{m<k},\mathbf{S}_{k-1})\,p(\mathbf{S}_k,\mathbf{Y}_k \mid \mathbf{S}_{k-1}), \tag{19}$$

and

$$p(\mathbf{Y}_{m>k} \mid \mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{S}_{k-1},\mathbf{S}_k) = p(\mathbf{Y}_{m>k} \mid \mathbf{S}_k). \tag{20}$$

Substituting (19) and (20) into (18), we have

$$p(\mathbf{Y},\mathbf{S}_{k-1},\mathbf{S}_k) = p(\mathbf{Y}_{m<k},\mathbf{S}_{k-1})\,p(\mathbf{Y}_k,\mathbf{S}_k \mid \mathbf{S}_{k-1})\,p(\mathbf{Y}_{m>k} \mid \mathbf{S}_k). \tag{21}$$

For brevity, we define the probabilities functions as follows.

$$\begin{aligned} \alpha(\mathbf{S}_k) &= p(\mathbf{Y}_{m<k+1},\mathbf{S}_k), \\ \beta(\mathbf{S}_k) &= p(\mathbf{Y}_{m>k} \mid \mathbf{S}_k), \\ \gamma(k) &= p(\mathbf{Y}_k,\mathbf{S}_k \mid \mathbf{S}_{k-1}). \end{aligned} \tag{22}$$

Based on (22), (21) is simplified as

$$p(\mathbf{Y},\mathbf{S}_{k-1},\mathbf{S}_k) = \alpha(\mathbf{S}_{k-1})\gamma(k)\beta(\mathbf{S}_k), \tag{23}$$

where $\alpha(\mathbf{S}_{k-1})$ is the forward state transition probability, $\beta(\mathbf{S}_k)$ denotes the backward state transition probability, and $\gamma(k)$ represents the branch transition probability. $\alpha(\mathbf{S}_{k-1})$ and $\beta(\mathbf{S}_k)$ can be calculated through the forward and backward recursions based on the DSTBC trellis structure. In order to implement the bidirectional message passing mechanism with MSD, we need to reformulate an expression for the branch transition probability. Note that, $\gamma(k)$ can be further reformulated as

$$\gamma(k) = p(\mathbf{Y}_k, \mathbf{S}_k \mid \mathbf{S}_{k-1}) = \frac{p(\mathbf{Y}_k, \mathbf{S}_{k-1}, \mathbf{S}_k) p(\mathbf{S}_{k-1}, \mathbf{S}_k)}{p(\mathbf{S}_{k-1}, \mathbf{S}_k) p(\mathbf{S}_{k-1})} = p(\mathbf{Y}_k \mid \mathbf{S}_{k-1}, \mathbf{S}_k) p(\mathbf{S}_k \mid \mathbf{S}_{k-1}), \quad (24)$$

where $p(\mathbf{S}_k \mid \mathbf{S}_{k-1})$ represents the state transition probability, which can be considered as the local check function for the state transition $\mathbf{S}_{k-1} \to \mathbf{S}_k$ given by

$$p(\mathbf{S}_k \mid \mathbf{S}_{k-1}) = \mathrm{T}_k(\mathbf{U}_k, \mathbf{X}_k, \mathbf{S}_{k-1}, \mathbf{S}_k) = \begin{cases} 1, & \text{possible event} \\ 0, & \text{otherwise;} \end{cases} \quad (25)$$

and $p(\mathbf{Y}_k \mid \mathbf{S}_{k-1}, \mathbf{S}_k)$ corresponds to the conditional probability with output $\mathbf{X}_k$ expressed as

$$p(\mathbf{Y}_k \mid \mathbf{S}_{k-1}, \mathbf{S}_k) = p(\mathbf{Y}_k \mid \mathbf{X}_k) = p(\mathbf{Y}_k \mid \mathbf{U}_k, \mathbf{U}_{k-1}, \ldots, \mathbf{U}_{k-M+1}). \quad (26)$$

From (24)-(26), we have

$$\gamma(k) \propto \mathrm{T}_k(\mathbf{U}_k, \mathbf{X}_k, \mathbf{S}_{k-1}, \mathbf{S}_k) p(\mathbf{Y}_k \mid \mathbf{U}_k, \mathbf{U}_{k-1}, \ldots, \mathbf{U}_{k-M+1}), \quad (27)$$

where $p(\mathbf{Y}_k \mid \mathbf{U}_k, \mathbf{U}_{k-1}, \ldots, \mathbf{U}_{k-M+1})$ is given by (16). Assume that channel is memoryless [16], $\alpha(\mathbf{S}_k)$ in (23) can be obtained by the forward message passing as

$$\alpha(\mathbf{S}_k) = \sum\nolimits_{\mathbf{S}_{k-1}} \alpha(\mathbf{S}_{k-1}) \gamma(k). \quad (28)$$

Similarly, $\beta(\mathbf{S}_k)$ is given by the backward message passing as

$$\beta(\mathbf{S}_k) = \sum\nolimits_{\mathbf{S}_{k+1}} \beta(\mathbf{S}_{k+1}) \gamma(k+1). \quad (29)$$

A detailed derivation of (28) and (29) can be found in Appendix B. Combining (17) and (23), APP is obtained as

$$p\{d_k(i) = z \mid \mathbf{Y}\} \propto \sum\nolimits_{(\mathbf{S}_{k-1} \to \mathbf{S}_k) \in \mathrm{A}_{d_k(i)=z}} \alpha(\mathbf{S}_{k-1}) \beta(\mathbf{S}_k) \gamma(k), \quad (30)$$

where the summation is over all possible output associated with $\mathbf{U}_k$ containing $d_k(i) = z$; $\gamma(k)$ is given by (27), $\alpha(\mathbf{S}_k)$ and $\beta(\mathbf{S}_k)$ are obtained from (28) and (29), respectively. In order to reveal the intrinsic message passing pattern and visualize the proposded SISO MSD algorithm, we employ a delivery representation to explain the implications. The message passing model between the adjacent states is demonstrated by **Fig. 5**, where circles with ring are state variables, the circles represent variables nodes, and squares denote the transition function nodes. The resultant message for APP consists of forward and backward message.

During the specific implementation, firstly, we calculate $\gamma(k)$ from the received signals. Then, the forward passing message $\alpha(\mathbf{S}_k)$, and the backward passing message $\beta(\mathbf{S}_k)$ are recursively calculated, respectively.

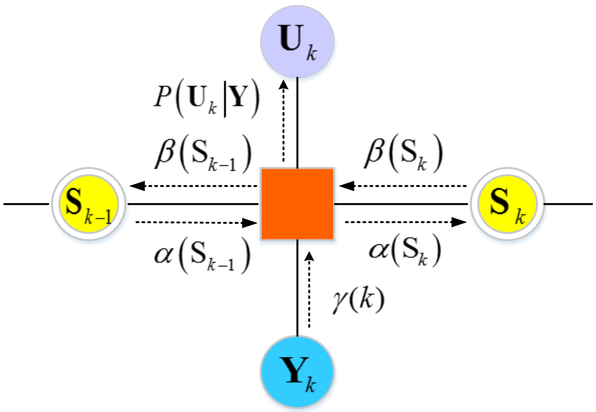


Fig. 5. A forward and backward message passing representation of the proposed algorithm. Circles with ring are state variables; the circles represent variables nodes; squares denote transition function node

Finally, APP of information symbols will be obtained from (30). For clarity, the calculation details are summarized in **Table 1**.

Table 1. Algorithm: Calculating (30) Based on the Trellis Diagram.

Step 1) Initialize the forward and backward passing as $\alpha(\mathbf{S}_0)=1$, and $\beta(\mathbf{S}_0)=1$, respectively.
Step 2) Calculate branch transition $\gamma(k)$ using (27) after MSD metric in (19) has been calculated.
Step 3) According to DSTBC trellis, calculate $\alpha(\mathbf{S}_k)$ and $\beta(\mathbf{S}_k)$ from (28) and (29), respectively.
Step 4) Obtain the APP of the information bits from (30) based on $\alpha(\mathbf{S}_{k-1})$, $\beta(\mathbf{S}_k)$, and $\gamma(k)$.

Remark 4: When the proposed SISO MSD is combined with channel decoding, an iterative detection and decoding approach is obtained. In particular, the messages exchanged between the detector and the decoders are known as extrinsic information [21]. During the iterative decoding process, the inputs to DSTBC detector are a priori or the extrinsic information of the input symbols, which will be delivered to channel decoder; the decoder computes the APP from (30), and then sends back to the detector as a priori information for the next iteration [22-24].

4. Simulations

In this section, simulations are conducted to validate the BER performance of the proposed SISO MSD algorithm. A DSTBC UWB system is considered for the case of receiver antennas $Q=1, 2$, and 4, respectively. The channel is generated according to the IEEE 802.15.3a CM1 model [5], and the monocycle waveform is the normalized second derivative of a Gaussian function $\omega(t)=[1-4\pi(t/T_m)^2]\exp[-2\pi(t/T_m)^2]$, where the pulse duration

$T_m = 0.287$ ns. In order to eliminate the inter-symbol interference, the frame duration is chosen as $T_f = 100$ ns, such that $T_f > T_n$ [5], where $T_n = 50$ ns denotes the maximum excess delay of the channel.

Firstly, we investigate the BER performance of the proposed algorithm without considering channel coding. **Fig. 6** illustrates the performance comparisons of the BCJR-based MSD and MSD for different observation window size M with $Q = 1$. It is clear that the proposed algorithm has a better BER performance than the MSD in [6]. This is attributed to more accurate detection information provided by the bidirectional message passing of the SISO MSD when compared to MSD in [6]. Furthermore, performance gain becomes larger when M increases. Then, we consider the BER performance for different Q with $M = 2$. It is shown in **Fig. 7** that a performance improvement can be achieved with the proposed MSD when compared to MSD in [6].

Next, an iterative SISO MSD algorithm is further investigated. The generator for the rate 1/2, (5, 7) octal convolution code with a constraint length of 3 is employed [14]. For the iterative SISO MSD, 4 iterations between the detector and the decoder are performed. **Fig. 8** compares the performance of the proposed SISO MSD, and the existing MSD in [6]. It can be seen that gain can be achieved at the cost of extra iterations between the SISO detector and the SISO channel decoder as compared to MSD in [6]. These results clearly highlight the great benefits of iterative SISO MSD in conjunction with channel decoding.

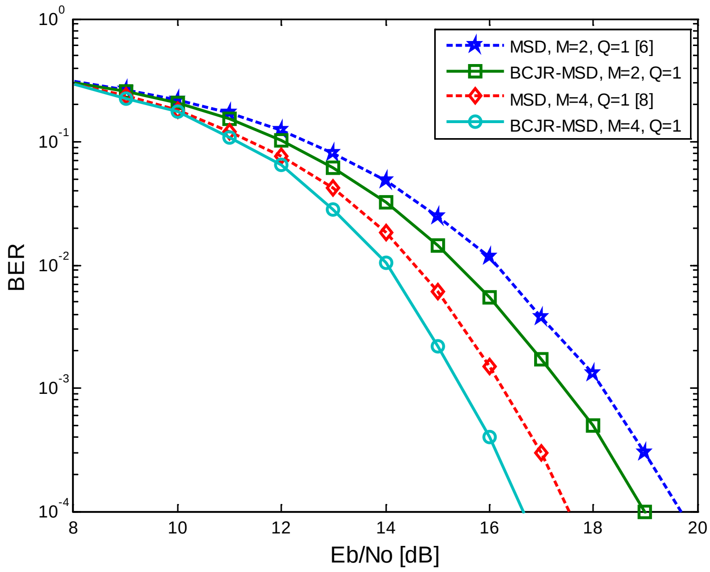


Fig. 6. BER performance comparisons of the BCJR-based SISO MSD and MSD with Q = 1 and different M

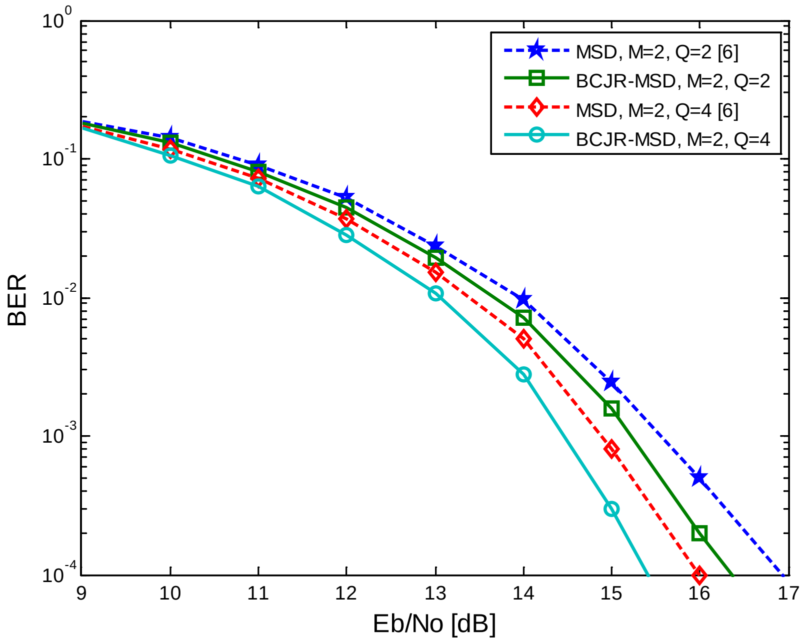


Fig. 7. BER performance comparisons of the BCJR-based SISO MSD and MSD with $M = 2$ and different Q

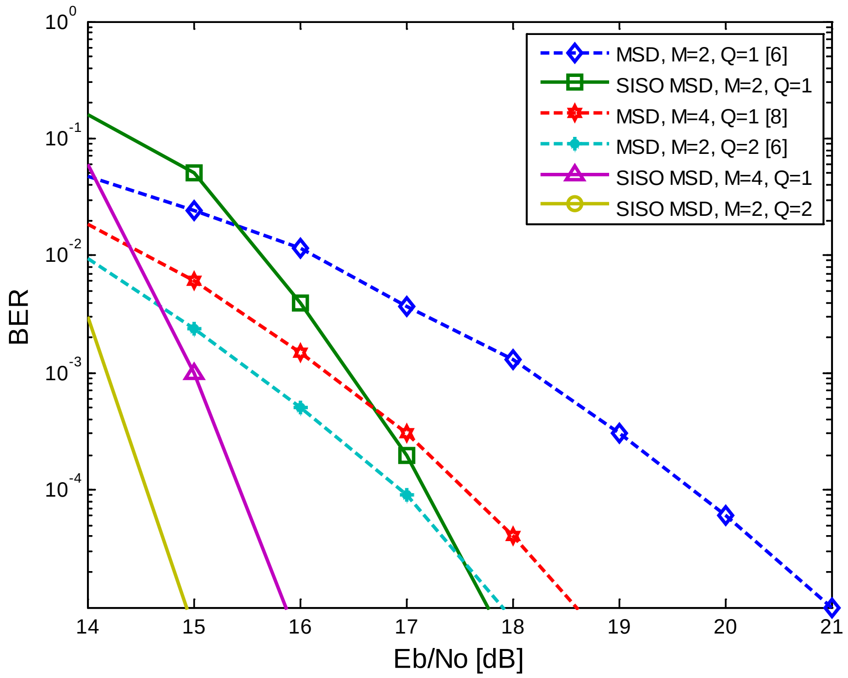


Fig. 8. BER performance comparisons of iterative SISO MSD and MSD with different M and Q for 4 iterations

5. Conclusion

An SISO MSD algorithm has been proposed for the DSTBC UWB systems. Specifically, we have constructed an MSD metric to serve the SISO MSD, where the forward and backward message passing mechanism is implemented. In addition, an iterative detection and decoding approach is developed. Simulations illustrate that the proposed SISO MSD can offer significant gains as compared with the existing MSD, and iterative detection approach further improves the BER performance.

Appendix A

Derivation of Parameters in (10)

In (10), the signal component $S_{k,m}(u,v)$ is given by

$$S_{k,m}(u,v) = \sum_{q=1}^{Q} \int_0^{Ti} \Big(\sum_{p=1}^{2} d_{p,2k+u-1} g_{p,q}(t) \sum_{p=1}^{2} d_{p,2(k-m)+v-1} g_{p,q}(t) \Big) dt, \tag{31}$$

and the noise component can be expressed as

$$N_{k,m}(u,v) = N_1 + N_2 + N_3, \tag{32}$$

where

$$N_1 = \sum_{p=1}^{2} d_{p,2k+u-1} \sum_{q=1}^{Q} \int_0^{Ti} g_{p,q}(t) n^q_{(k-m),v}(t) dt, \tag{33}$$

$$N_2 = \sum_{p=1}^{2} d_{p,2(k-m)+v-1} \sum_{q=1}^{Q} \int_0^{Ti} g_{p,q}(t) n^q_{k,u}(t) dt, \tag{34}$$

and

$$N_3 = \sum_{q=1}^{Q} \int_0^{Ti} n^q_{k,u}(t) n^q_{(k-m),v}(t) dt. \tag{35}$$

Assume that $n_q(t)$ is a wideband AWGN process with bandwidth $W > 1/T_i$. Given the channel realizations $h = \{h_{p,q}(t)\}$, clearly, N_1 and N_2 are Gaussian random variables; N_3 can also be considered as approximately Gaussian distributed [18]. Then, according to the central limit theorem, $N_{k,m}(u,v)$ is zero mean, and its conditional variance is

$$\sigma^2 = \mathrm{Var}[N_{k,m}(u,v)\,|\,h] = \mathrm{E}[N_1^2\,|\,h] + \mathrm{E}[N_2^2\,|\,h] + \mathrm{E}[N_3^2]. \tag{36}$$

According to (33), (34), and (35), the conditional variance of N_1 and N_2 can be described as

$$\mathrm{E}[N_1^2\,|\,h] = \mathrm{E}[N_2^2\,|\,h] = \frac{N_0}{2}\sum_{p=1}^{2}\sum_{q=1}^{Q}\varepsilon_{p,q}, \tag{37}$$

where $\varepsilon_{p,q} = \int_0^{T_i} h_{pq}^2(t)dt,$ and

$$\mathrm{E}[N_3^2] = \frac{QWT_iN_0^2}{2}. \tag{38}$$

Substituting (37) and (38) into (36), we have

$$\sigma^2 = N_0\sum_{p=1}^{2}\sum_{q=1}^{Q}\varepsilon_{p,q} + \frac{QWT_iN_0^2}{2}. \tag{39}$$

Appendix B

Derivation of $\alpha(\mathbf{S}_k)$ and $\beta(\mathbf{S}_k)$

The forward message passing in (22) can be re-expressed as

$$\begin{aligned}\alpha(\mathbf{S}_k) &= p(\mathbf{Y}_{m<k+1},\mathbf{S}_k) = p(\mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{S}_k) = \sum\nolimits_{\mathbf{S}_{k-1}} p(\mathbf{Y}_{m<k},\mathbf{Y}_k,\mathbf{S}_k,\mathbf{S}_{k-1}) \\ &= \sum\nolimits_{\mathbf{S}_{k-1}} p(\mathbf{S}_{k-1},\mathbf{Y}_{m<k})\,p(\mathbf{Y}_k,\mathbf{S}_k\,|\,\mathbf{S}_{k-1},\mathbf{Y}_{m<k}).\end{aligned} \tag{40}$$

Based on the properties of the Markov process [25], when $\mathbf{S}_k$ is known, the events after k do not depend on $\mathbf{Y}_{m<k}$. Therefore, we have

$$p(\mathbf{Y}_k,\mathbf{S}_k\,|\,\mathbf{S}_{k-1},\mathbf{Y}_{m<k}) = p(\mathbf{Y}_k,\mathbf{S}_k\,|\,\mathbf{S}_{k-1}). \tag{41}$$

Substituting (41) into (40), then the forward message passing probability is reformulated as

$$\alpha(\mathbf{S}_k) = \sum\nolimits_{\mathbf{S}_{k-1}} p(\mathbf{S}_{k-1},\mathbf{Y}_{m<k})\,p(\mathbf{Y}_k,\mathbf{S}_k\,|\,\mathbf{S}_{k-1}) = \sum\nolimits_{\mathbf{S}_{k-1}} \alpha(\mathbf{S}_{k-1})\gamma(k). \tag{42}$$

Based on (22), the backward message passing is given by

$$\begin{aligned}\beta(\mathbf{S}_k) &= p(\mathbf{Y}_{m>k} \mid \mathbf{S}_k) = \frac{p(\mathbf{S}_k, \mathbf{Y}_{m>k})}{p(\mathbf{S}_k)} = \frac{p(\mathbf{S}_k, \mathbf{Y}_{k+1}, \mathbf{Y}_{m>k+1})}{p(\mathbf{S}_k)} \\ &= \sum_{\mathbf{S}_{k+1}} \frac{p(\mathbf{S}_{k+1}, \mathbf{S}_k, \mathbf{Y}_{k+1}, \mathbf{Y}_{m>k+1})}{p(\mathbf{S}_k)} = \sum_{\mathbf{S}_{k+1}} \frac{p(\mathbf{S}_{k+1}, \mathbf{Y}_{m>k+1})\, p(\mathbf{S}_k, \mathbf{Y}_{k+1} \mid \mathbf{S}_{k+1}, \mathbf{Y}_{m>k+1})}{p(\mathbf{S}_k)}.\end{aligned} \tag{43}$$

Similarly, if $\mathbf{S}_{k+1}$ is known, then the events before $k+1$ do not depend on $\mathbf{Y}_{m>k+1}$. Thus,

$$p(\mathbf{S}_k, \mathbf{Y}_{k+1} \mid \mathbf{S}_{k+1}, \mathbf{Y}_{m>k+1}) = p(\mathbf{S}_k, \mathbf{Y}_{k+1} \mid \mathbf{S}_{k+1}). \tag{44}$$

Substituting (44) into (43), the backward message passing is obtained as

$$\begin{aligned}\beta(\mathbf{S}_k) &= \sum_{\mathbf{S}_{k+1}} \frac{p(\mathbf{S}_{k+1}, \mathbf{Y}_{m>k+1})\, p(\mathbf{S}_k, \mathbf{Y}_{k+1} \mid \mathbf{S}_{k+1})}{p(\mathbf{S}_k)} \\ &= \sum_{\mathbf{S}_{k+1}} p(\mathbf{Y}_{m>k+1} \mid \mathbf{S}_{k+1})\, p(\mathbf{S}_{k+1}, \mathbf{Y}_{k+1} \mid \mathbf{S}_k) = \sum_{\mathbf{S}_{k+1}} \beta(\mathbf{S}_{k+1})\gamma(k+1).\end{aligned} \tag{45}$$

References

[1] L. Yang, G. Giannakis, "Ultrawideband communications: An idea whose time has come," *IEEE Signal Process. Mag.*, vol. 21, no. 6, pp. 26-54, 2004. Article (CrossRef Link)

[2] U. Demir, C. Bas, S. Ergen, "Engine compartment UWB channel model for intravehicular wireless sensor networks," *IEEE Trans. Veh. Technol.*, vol. 63, pp. 2497-2505, 2014. Article (CrossRef Link)

[3] H. Gao, X. Su, T. Lv, S. Yang, and Y. Lu, "IFI and ISI premitigation for block-code-modulated non-coherent UWB impulse radio: A code optimization approach," *IEEE Trans. Veh. Technol.*, vol. 61, pp. 1635-1648, 2012. Article (CrossRef Link)

[4] K. Witrisal, G. Leus, M. Pausini, and C. Krall, "Equivalent system model and equalization of differential impulse radio UWB systems," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 9, pp. 1851-1862, 2005. Article (CrossRef Link)

[5] V. Lottici and Z. Tian, "Multiple symbol differential detection for UWB communications," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1656-1666, 2008. Article (CrossRef Link)

[6] Q. Zhang and C. Ng, "DSTBC impulse radios with autocorrelation receiver in ISI-free UWB channels," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 806-811, 2008. Article (CrossRef Link)

[7] T. Wang, T. Lv, and H. Gao, "Sphere decoding based multiple symbol detection for differential space-time block coded ultra-wideband systems," *IEEE Commun. Lett.*, vol. 15, no. 3, pp. 269-271, 2011. Article (CrossRef Link)

[8] T. Wang, T. Lv, H. Gao, and Y. Lu, "BER analysis of decision-feedback multiple symbol detection in non-coherent MIMO ultra-wideband systems," *IEEE Trans. Veh. Technol.*, vol. 62, pp. 1-6, 2013. Article (CrossRef Link)

[9] Y. Zhu, D. Guo, and M. Honig, "A message-passing approach for joint channel estimation, interference mitigation, and decoding," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 6008-6018, 2009. Article (CrossRef Link)

[10] Q. Zhou, X. Ma, "Soft-input soft-output multiple symbol differential detection for uwb communications," *IEEE Commun. Lett.*, vol. 16, no. 8, pp. 1296-1299, 2012. Article (CrossRef Link)

[11] A. V. Nguyen and M. A. Ingram, “Iterative demodulation and decoding of differential space-time block codes,” in *Proc. of IEEE Vehicular Technology Conf. (VTC 2000)*, pp. 2394-2400, 2000. Article (CrossRef Link)
[12] B. Hochwald and S. Brink, “Achieving near-capacity on a multiple-antenna channel,” *IEEE Trans. Commun.*, vol. 51, pp. 389-399, 2003. Article (CrossRef Link)
[13] A. Worm, P. Hoeher, and N. Wehn, “Turbo-decoding without SNR estimation,” *IEEE Commun. Lett.*, vol. 4, no. 6, pp. 193-195, 2000. Article (CrossRef Link)
[14] K. J. Han and J. H. Lee, “Iterative decoding of a differential space-time block code with low complexity,” in *Proc. of IEEE Vehicular Technology Conf. (VTC 2002)* pp. 1322-1325, 2002. http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=1002830
[15] A. Steiner, M. Peleg, and S. Shamai, “Iterative decoding of space-time differentially coded unitary matrix modulation,” *IEEE Trans. Signal Process*., vol. 50, pp. 2385-2395, 2002. Article (CrossRef Link)
[16] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Trans. Inf. Theory*., vol. 20, pp. 284-287, 1974. Article (CrossRef Link)
[17] H. Jafarkhani and V. Tarokh, “Multiple transmit antenna differential detection from generalized orthogonal designs,” *IEEE Trans. Inf. Theory*, vol. 47, pp. 2626-2631, 2001. Article (CrossRef Link)
[18] T. Quek and M. Win, “Analysis of UWB transmitted-reference communication systems in dense multipath channels,” *IEEE J. Sel. Areas Commun.*, vol. 23, no. 9, pp. 1863-1874, 2005. Article (CrossRef Link)
[19] C. Tan and N. Beaulieu, “On first-order Markov modeling for the Rayleigh fading channel,” *IEEE Trans. Commun.*, vol. 48, pp. 2032-2040, 2000. Article (CrossRef Link)
[20] Y. Ephraim and N. Merhav, “Hidden Markov processes,” *IEEE Trans. Inf. Theory*., vol. 48, pp. 1518-1569, 2002. Article (CrossRef Link)
[21] S. Park and Y. Lee, “Iterative noncoherent receiver for differential STBC in fast fading channels,” *IEEE Trans. Veh. Technol.*, vol. 58, pp. 4654-4657, 2009. Article (CrossRef Link)
[22] N. Wu, O. Alamri, S. Ng, and L. Hanzo, “Precoded sphere-packing-aided bit-interleaved differential space-time coded modulation using iterative decoding,” *IEEE Trans. Veh. Technol.*, vol. 57, pp. 1311-1316, 2008. Article (CrossRef Link)
[23] O. Alamri, B. Yeap, and L. Hanzo, “A turbo detection and sphere packing-modulation-aided space-time coding scheme,” *IEEE Trans. Veh. Technol.*, vol. 56, pp. 575-582, 2007. Article (CrossRef Link)
[24] K. Narayanan and G. Stuber, “A serial concatenation approach to iterative demodulation and decoding,” *IEEE Trans. Commun.*, vol. 47, pp. 956-961, 1999. Article (CrossRef Link)
[25] L. Shue, S. Dey, and B. Anderson, “On state-estimation of a two-state hidden Markov model with quantization,” *IEEE Trans. Signal Process.*, vol. 49, pp. 202-208, 2001. Article (CrossRef Link)

Chanfei Wang is now working towards the Ph.D. degree in information engineering from Beijing University of Posts and Telecommunications (BUPT), Beijing, China. Her research interests focus on signal processing techniques in ultra-wideband wireless communucations.

Hui Gao received his B. Eng. degree in Information Engineering and Ph.D. degree in Signal and Information Processing from Beijing University of Posts and Telecommunications (BUPT), Beijing, China, in July 2007 and July 2012, respectively. From May 2009 to June 2012, he also served as a research assistant for the Wireless and Mobile Communications Technology R&D Center, Tsinghua University, Beijing, China. From Apr. 2012 to June 2012, he visited Singapore University of Technology and Design (SUTD), Singapore, as a research assistant. From July 2012 to Feb. 2014, he was a Postdoc Researcher with SUTD. He is now with the School of Information and Communication Engineering, Beijing University of Posts and Telecommunications (BUPT), as an assistant professor. His research interests include massive MIMO systems, cooperative communications, ultra-wideband wireless communications.

Tiejun Lv received the M.S. and Ph.D. degrees in electronic engineering from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 1997 and 2000, respectively. From January 2001 to December 2002, he was a Postdoctoral Fellow with Tsinghua University, Beijing, China. From September 2008 to March 2009, he was a Visiting Professor with the Department of Electrical Engineering, Stanford University, Stanford, CA. He is currently a Full Professor with the School of Information and Communication Engineering, Beijing University of Posts and Telecommunications (BUPT). He is the author of more than 200 published technical papers on the physical layer of wireless mobile communications. His current research interests include signal processing, communications theory and networking. Dr. Lv is also a Senior Member of the Chinese Electronics Association. He was the recipient of the Program for New Century Excellent Talents in University Award from the Ministry of Education, China, in 2006.