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영확률에 기반한 적응 이퀄라이져의 최적조건

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Optimum Conditions of Adaptive Equalizers Based on Zero-Error Probability

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요 약

신호처리에서 영확률을 성능기준으로 하는 적응 알고리듬들은 충격성 잡음 환경에서 우수한 성능과 안정된 수렴을 보인다. 이 논문에서는 MSE 성능기준과 비교분석을 통해 영확률 성능기준이 MSE와 동일한 최적해를 가진다는 것을 증명한다. 또한 이 연구를 통해, 영확률 기반 알고리듬의 크기 조정된 입력이 충격성 잡음으로부터 최적해가 방해 받지 않도록 유지해주는 역할을 하고 있음을 보인다.

Key Words : zero-error, probability, impulsive noise, optimum weight, magnitude controlled, equalizer

ABSTRACT

In signal processing, the zero-error probability (ZEP) criterion and related algorithm (MZEP) outperforms MSE-based algorithms and yields superior and stable convergence in impulsive noise environment. In this paper, the analysis of the relationship with MSE criterion proves that ZEP criterion has equivalent optimum solution of MSE criterion. Also this work reveals that the magnitude controlled input of MZEP algorithm plays the role in keeping the optimum solution undisturbed from impulsive noise.

Ⅰ. Introduction

Besides the harsh problems such as multipath propagation and severe fading in wireless network communication environment, impulsive noise from a variety of sources affects the links[1-3]. Many signal processing algorithms designed on the basis of MSE criterion may fail when impulsive noise is present[4].

As an alternative to the MSE, the zero-error probability (ZEP) criterion has been introduced in [5]. By maximization of ZEP (MZEP) and steepest descent method, the MZEP algorithm has been developed for communication systems with impulsive noise and channel distortions. For application for underwater communication channels, the nonlinear MZEP has been proposed to compensate for ISI without error propagation[6].

One drawback of MZEP in which weights are calculated based on block processing method is a heavy computational burden. In the work in [7] a method utilizing the current gradient in estimation of the next gradient has been proposed and shown that its computational complexity can be significantly reduced.

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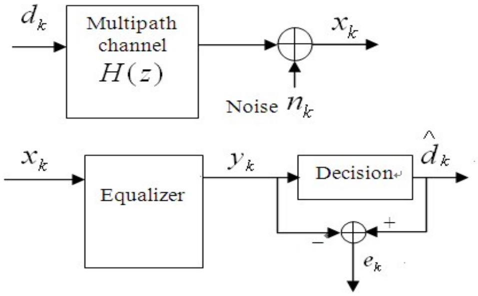

Fig. 1. Base-band communication system model

Though the ZEP based adaptive algorithms have improved so as to be better suited to practical situations and problems, any analysis of its optimum solutions and their behavior has not been carried out.

In this paper, through the analysis of its optimum weight behavior being compared with the MSE criterion, we will uncover some properties and factors that play the role in robustness against impulsive noise.

Ⅱ. System Model and MSE Criterion

In communication systems, a symbol point is transmitted and distorted through the wireless channel, and noise n_k is added to the channel output as depicted in baseband model in Fig. 1. The multipath channel $H(z)$ can be expressed as $H(z)=\sum h_i z^{-i}$ in z-transform. With the noise being added to the distorted channel output, the equalizer input becomes as $x_k=\sum h_i d_{k-i}+n_k$ [8]. When we assume that the equalizer structure is TDL (tapped delay line) with L weights, the output y_k at time k can be expressed as $y_k=\mathbf{W}_k^T\mathbf{X}_k$ with the input $\mathbf{X}_k=[x_k,x_{k-1},...,x_{k-j},...,x_{k-L+1}]^T$ and weight $\mathbf{W}_k=[w_{0,k},w_{1,k},..,w_{j,k},...,w_{L-1,k}]^T$.

The difference between the transmitted symbol point d_k and the output y_k is defined as error e_k as in (1). Then the MSE criterion P_{MSE} is defined as the expectation of error power[8].

$$e_k=d_k-y_k=d_k-\mathbf{W}_k^T\mathbf{X}_k \tag{1}$$

$$P_{MSE}=E[e_k{}^2] \tag{2}$$

Instead of taking (2), minimization of the instant power of system error e_k with respect to weight is employed as a cost function which is efficient for implementation in the well-known LMS algorithm. While the influence of the Gaussian noise can be mitigated owing to the expectation or mean operation $E[\cdot]$ in (2), impulsive noise may defeat the averaging operation since a single large impulse can dominate the mean operation. Therefore impulsive noise can lead algorithms based on the MSE criterion to become unstable. This indicates that the instant error power e_k^2 without being averaged may cause worse instability in the system.

The gradient of the MSE criterion becomes

$$\frac{\partial P_{MSE}}{\partial \mathbf{W}}=2E[\mathbf{X}_k\mathbf{X}_k^T]\mathbf{W}_k-2E[d_k\mathbf{X}_k] \tag{3}$$

Letting the gradient be zero, we obtain the optimum weight vector for MSE criterion as

$$\mathbf{W}_{MSE}^o=\frac{E[d_k\mathbf{X}_k]}{E[\mathbf{X}_k\mathbf{X}_k^T]} \tag{4}$$

On the other hand, the correlation between error

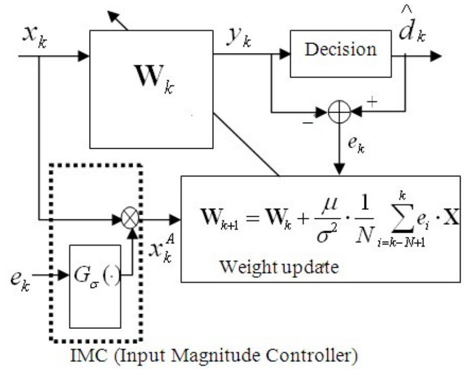

Fig. 2. ZEP-based equalizer with IMC (Input Magnitude Controller)

and input is

$$E[e_k \mathbf{X}_k] = E[d_k \mathbf{X}_k] - \mathbf{W}_k E[\mathbf{X}_k \mathbf{X}_k^T] \quad (5)$$

In the minimum MSE with $\mathbf{W}^o$, we have the following two conditions.

$$\frac{\partial P_{MSE}}{\partial \mathbf{W}} = 0 \quad \text{and} \quad E[e_k \mathbf{X}_k] = 0 \quad (6)$$

Ⅲ. ZEP Criterion and Optimum Weight

As another cost function to be minimized, P_{ZEP} the criterion of zero-error probability is defined as

$$P_{ZEP} = f_E(e=0) \quad (7)$$

The maximization of $f_E(e=0)$ forces error samples to be concentrated on zero. More importantly, the error PDF $f_E(e_{CME})$ is significantly less sensitive to strong impulses estimation method using Gaussian kernel (with the kernel size σ) and a block of N error samples $\{e_k, e_{k-1}, ..., e_i, ..., e_{k-N+1}\}$ as described in the work [9].

$$P_{ZEP} = \frac{1}{N} \sum_{i=k-N+1}^{k} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-(e-e_i)^2}{2\sigma^2}] \quad (8)$$

For maximization of $f_E(e=0)$ the steepest descent method is employed and it leads to the following MZEP (maximum ZEP) algorithm with the step-size μ_{MZEP} that controls the system stability.

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \mu_{MZEP} \frac{\partial P_{ZEP}}{\partial \mathbf{W}} \quad (9)$$

The gradient in (9) can be expressed as

$$\frac{\partial P_{ZEP}}{\partial \mathbf{W}} = \frac{1}{\sigma^2} \cdot \frac{1}{N} \sum_{i=k-N+1}^{k} e_i \cdot G_\sigma(e_i) \cdot \mathbf{X}_i \quad (10)$$

Since the right term in (10) can be considered as a time-averaged version of $E[e_k \cdot G_\sigma(e_k) \cdot \mathbf{X}_k]$, we may have

$$\frac{\partial P_{ZEP}}{\partial \mathbf{W}} = \frac{1}{\sigma^2} \cdot E[e_k \cdot G_\sigma(e_k) \cdot \mathbf{X}_k] \quad (11)$$

In the steady state, we have the following two conditions as

$$\frac{\partial P_{ZEP}}{\partial \mathbf{W}} = 0 \quad \text{or} \quad E[e_k \cdot G_\sigma(e_k) \cdot \mathbf{X}_k] = 0 \quad (12)$$

Comparing $E[e_k \mathbf{X}_k] = 0$ in (6) for MSE criterion and $E[e_k \cdot G_\sigma(e_k) \cdot \mathbf{X}_k] = 0$ in (12) for ZEP criterion, we may consider that $G_\sigma(e_k) \cdot \mathbf{X}_k$ in (12) implies that input $\mathbf{X}_k$ is attenuated by $G_\sigma(e_k)$. This leads us to define the magnitude-controlled input $\mathbf{X}_k^A$ as

$$\mathbf{X}_k^A = G_\sigma(e_k) \cdot \mathbf{X}_k \quad (13)$$

Figure 2 depicts the MZEP equalizer with the input magnitude controller (IMC) for $\mathbf{X}_k^A$. The kernel $G_\sigma(e_k) = \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-e_k^2}{2\sigma^2}]$ is always positive and is a function of an exponential decay with the error power e_k^2. An excessively large value of error power means the presence of a strong impulse within the input $\mathbf{X}_k$. This kind of large input can cause instability in the system.

Through the magnitude control process $G_\sigma(e_k) \cdot \mathbf{X}_k$ in (16), the magnitude of the input $\mathbf{X}_k$ is controlled for the weight update by a scale factor $G_\sigma(e_k)$ which is inversely proportional to the error power.

Then the gradient (10) and weight update

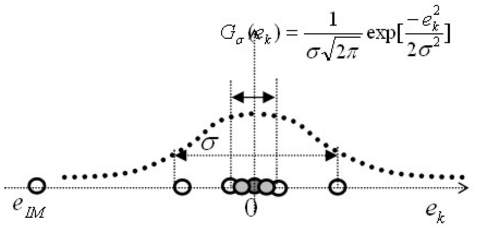

Fig. 3. Gaussian kernel and error samples gathered around zero.

equation (9) can be rewritten as

$$\frac{\partial P_{ZEP}}{\partial \mathbf{W}} = \frac{1}{\sigma^2}\cdot\frac{1}{N}\sum_{i=k-N+1}^{k}(d_k - \mathbf{X}_i^T\mathbf{W}_i)\cdot\mathbf{X}_i^A \qquad (14)$$

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \frac{\mu_{MZEP}}{\sigma^2}\cdot\frac{1}{N}\sum_{i=k-N+1}^{k} e_i\cdot\mathbf{X}_i^A \qquad (15)$$

Letting the gradient (14) be zero, we obtain the optimum weight vector $\mathbf{W}_{ZEP}^o$ for ZEP criterion as

$$\mathbf{W}_{ZEP}^o = \frac{\frac{1}{N}\sum_{i=k-N+1}^{k} d_k\mathbf{X}_i^A}{\frac{1}{N}\sum_{i=k-N+1}^{k}\mathbf{X}_i^A\mathbf{X}_i^T} = \frac{\frac{1}{N}\sum_{i=k-N+1}^{k} G_\sigma(e_i)d_k\mathbf{X}_i}{\frac{1}{N}\sum_{i=k-N+1}^{k} G_\sigma(e_i)\mathbf{X}_i\mathbf{X}_i^T} \qquad (16)$$

In the steady state, we may assume that most of the error samples are located at around zero as depicted in Fig. 3. This assumption leads us to treat $G_\sigma(e_k)$ as a constant $1/\sigma\sqrt{2\pi}$.

As a typical algorithm using the MSE criterion, LMS (least mean square) is to minimize the instant error power e_k^2 instead $E[e_k^2]$ for practical reasons [8]. Then the gradient of LMS becomes

$$\frac{\partial e_k^2}{\partial \mathbf{W}} = -2e_k\mathbf{X}_k \qquad (17)$$

Letting (17) be zero, we obtain the optimum weight vector

$$\mathbf{W}_{LMS}^o = \frac{d_k\mathbf{X}_k}{\mathbf{X}_k\mathbf{X}_k^T} \qquad (18)$$

The two optimum weights (16) and (18) as in practical algorithms become equivalent by taking the statistical average $E[\cdot]$ to them as

$$E[\mathbf{W}_{ZEP}^o] = \frac{E[d_k\mathbf{X}_k]}{E[\mathbf{X}_k\mathbf{X}_k^T]} \qquad (19)$$

The equation (19) indicates

$$E[\mathbf{W}_{ZEP}^o] = E[\mathbf{W}_{LMS}^o] = \mathbf{W}_{MSE}^o \qquad (20)$$

In the aspect of computational complexity, the MZEP has $O(N)$ operations for the weight update due to the summation as in (15) while the LMS algorithm in (17) has no such summations at all. Since the recursive gradient estimation proposed in [7] can be employed in MZEP, its computational complexity can become similar to LMS because the summation operation is not needed.

On the other hand, the equation (16) reveals another important property in the situation of large error occurrence such as impulsive noise. Since the steady state weight vectors can be considered to be reached the optimum state, it is worthwhile to investigate whether the steady state weight vector can keep the optimum weight value under impulsive noise situations. From this point of view, it is

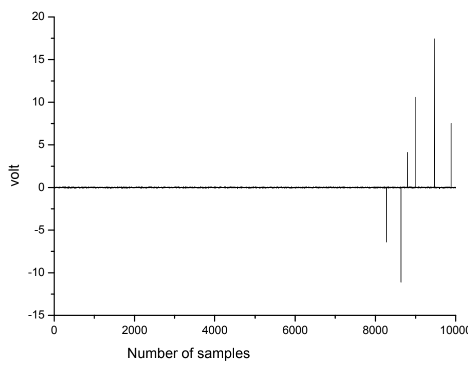

Fig. 4. Impulsive noise for the examination of weight behavior in the steady state (after convergence).

reasonable to see that $\mathbf{W}_{ZEP}^{SteadyState}$ can stay in the value of $\mathbf{W}_{ZEP}^{o}$ thanks to magnitude-controlling $G_{\sigma}(e_k)$ and time-average process $\frac{1}{N}\sum_{i=k-N+1}^{k}$, whereas the steady state weight of LMS can be fluctuated since a single large impulse can afflict (17) directly. These properties are examined by observing the behavior of steady state weight vectors under impulsive noise situations in the following section.

Ⅳ. Simulation Results and Discussion

In this section, it will be investigated how the steady state weight vectors behave under impulsive noise situations. We use the same signal processing environment as in [7] except that impulse noise is applied after convergence, that is, in the steady state. The symbol point set to be transmitted is $\{d_1=-3, d_2=-1, d_3=1, d_4=3\}$. The random symbol point d_k at time k is transmitted through the multipath channel $H(z)=0.304+0.903z^{-1}+0.304z^{-2}$. Then the additive Gaussian white noise (AWGN) is added to the channel output. The impulse noise add ed after convergence (from k = 8000) is generated according to the work [4] with $\sigma_2^2=50.001$ and incident rate $\varepsilon=0.01$. The variance of the background AWGN is 0.001 and the distribution

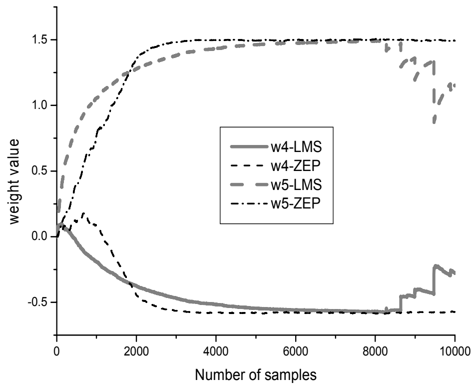

Fig. 5. Behavior of weight values in steady state under impulsive noise.

function of overall noise n_k is (21) and is depicted in Fig. 4.

$$f_N(n_k)=\frac{\varepsilon}{\sigma_2\sqrt{2\pi}}\exp[\frac{-n_k^2}{2\sigma_2^2}]+\frac{1-\varepsilon}{\sigma_1\sqrt{2\pi}}\exp[\frac{-n_k^2}{2\sigma_1^2}] \quad (21)$$

This channel–distorted and noise-added signal is used as an input x_k to the TDL equalizer with L=11. The sample size N for the MZEP algorithm is 20, the kernel size σ is 0.8. The convergence step-sizes are μ_{MZEP} = 0.004 for MZEP algorithm and μ_{LMS} = 0.001for LMS algorithm. All the parameter values are chosen to produce the lowest steady state MSE in this simulation.

Figure 4 shows that the AWGN is present throughout the sample time and the impulses added after k = 8000. The trace of $w_{4,k}$ and $w_{5,k}$ (the other tap weights are not included just for the page-limit) in Fig. 5. It is observed that the steady state weight of LMS algorithm shows abrupt changes at the exact time of each impulse occurrence by the amount being proportional to its impulse intensity. In the weight traces of MZEP, each tap weight presents no fluctuations under the strong impulses. This result shows that the dominant role in the robustness against impulsive noise is the IMC.

Ⅴ. Conclusion

The MZEP algorithm outperforms MSE-based algorithms in supervised signal processing in most equalization applications. Particularly in impulsive noise environment, its performance is superior. In this paper, through analysis of the relationship with MSE-based optimum solution and behavior of optimum weight, it has been proven that the optimum solution of ZEP criterion is equivalent to the one of MSE criterion. This work has also revealed that the magnitude controlled input of MZEP plays the role in keeping the optimum solution undisturbed from impulsive noise. Investigation of the detailed characteristics of the magnitude controlled input in future study is demanded to lead the ZEP-related adaptive algorithms to finding more enhanced methods and their application fields.

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