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Pseudo Jacket Matrix and Its MIMO SVD Channel

Pseudo Jacket 행렬을 이용한 MIMO SVD Channel

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요 약 Jacket Matrices: Construction and Its Application for Fast Cooperative Wireless signal Processing[27]에 소개된 Jacket 행렬로부터 일반화된 의사 Jacket 행렬에 대한 특성과 생성에 관한 정리가 발표됐다. 본 논문에서는 MIMO 채널과 같이 2x4, 3x6 같은 비정방 행렬에서의 의사 Jacket 역행렬에 대한 예제를 제안했다. 또한 의사 MIMO 특이값 분해 (SVD, Singular Value Decomposition) channel을 추론하여 적용하였으며 안테나 어레이를 분할하여 추정하는 채널을 기반으로 SVD를 활용하는데 적용하였다. 이것은 MIMO 채널 및 고유값 분해 (EVD, Eigen Value decomposition) 등에 사용할 수 있다.

Abstract Some characters and construction theorems of Pseudo Jacket Matrix which is generalized from Jacket Matrix introduced by Jacket Matrices: Construction and Its Application for Fast Cooperative Wireless signal Processing[27] was announced. In this paper, we proposed some examples of Pseudo inverse Jacket matrix, such as 2x4, 3x6 non-square matrix for the MIMO channel. Furthermore we derived MIMO singular value decomposition (SVD) pseudo inverse channel and developed application to utilize SVD based on channel estimation of partitioned antenna arrays. This can be also used in MIMO channel and eigen value decomposition (EVD).

Key Words : Jacket matrix, Pseudo Jacket Matrix, Pseudo inverse, element-wise inverse

Ⅰ. Introduction

A MIDST numerous matrices that are being utilized in engineering applications[1][2][3], structured matrices such as Orthogonal[4], Hadamard[5], Conference[6], Toeplitz[7], Unitary[8], Circulant[9], Hankel[10], Jacket[11], etc. matrices play an important role in signal processing. Hadamard matrix and its generalizations are orthogonal matrices with many applications for signal sequence transform and data processing[12]. Hadamard transform and its generalizations such as weighted Hadamard transform have been used for audio and video coding because of the high practical value of these transformations for representing signal and images[13].

Jacket matrix, motivated by the center weighted Hadamard matrix with an inverse-constraint and introduced by Lee in 1989, is a class of matrices with their inverse matrices being determined by the element-wise of matrices[14]. They are closely related to

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various famous mathematical objects such as Turyn and Butson type Hadamard matrices, orthogonal designs, etc., which have numerous applications to many mathematical and theoretical physics problems. The class of Jacket matrices contains the class of hadamard and complex Hadamard matrices[15]. it has a large overlap, but does not coincide, with the class of generalized Hadamard matrices[16][17]. Especially, several interesting matrices, such as Hadamard matrix, Haar matrix, Fourier matrix and Slant matrix, belong to Jacket matrix family[18][19]. Since the inverse matrix of Jacket matrix can be determined easily, Jacket matrix and its transforms have been extensively investigated [19][20][21]. In addition, Jacket matrix is related to many useful matrices, such as unitary matrices and Hermitian matrices that have been potentially applied in digital signal processing, wireless communications, cryptography, and so on[22][23][24][25][26].

But Jacket matrix requires it is a square matrix and has inverse matrix, and many matrices in practice can not satisfy these conditions. They are always matrix $A=(a_{ij})_{m\times n}$ or not inverse. In this paper, we will generalize the definition of Jacket matrix to Pseudo Jacket Matrix, study the existence and its construction.

This paper is organized as follows, In Section Ⅱ, the definition of Pseudo Jacket Matrix and some examples are given. In Section Ⅲ, we will prove some construction Theorems of Pseudo Jacket Matrix. In Section Ⅳ, we derived MIMO SVD Pseudo Inverse Channel and it's applications. Finally, conclusions are drawn in Section Ⅳ.

Ⅱ. The Definition of Pseudo Jacket Matrix

Jacket Matrix was introduced by Prof. Moon Ho Lee[27] as follows:

Definition 1: For a square matrix $A=(a_{ij})_{n\times n}$, if its inverse matrix can obtained simply by an element-wise inverse, such as

$$A^{-1}=\frac{1}{c}(a'_{ij})^T;\ a'_{ij}=\begin{cases}\frac{1}{a_{ij}}, & a_{ij}\neq 0\\ 0, & a_{ij}=0\end{cases}$$

where c is a non-zero constant then, we call matrix $A=(a_{ij})_{n\times n}$ as a jacket matrix, and if $a_{ij}\neq 0$ for all $1\le i,\ j\le n$ then $c=n$.

There are many types of Jacket Matrix, such as Hadamard matrix[28], Sylvester-Hadamard Matrix of Rank 2^k [29], block Jacket matrix[30], etc., and many applications[27]. In this paper, we will extend Jacket Matrix to Pseudo Jacket Matrix which will have much more applications in engineering such as MIMO wireless communications, signal processing, quantum computations and image processing etc.

Definition 2: For a matrix $A=(a_{ij})_{m\times n}$, if its pseudo inverse matrix can obtained simply by an element-wise inverse, such as

$$A^{+}=\frac{1}{c}(a'_{ij})^T;\ a'_{ij}=\begin{cases}\frac{1}{a_{ij}}, & a_{ij}\neq 0\\ 0, & a_{ij}=0\end{cases} \tag{1}$$

where c is a non-zero constant, and A^+ satisfy:

$$\begin{cases}AA^+A=A\\ A^+AA^+=A^+\\ (AA^+)^T=AA^+\\ (A^+A)^T=A^+A\end{cases} \tag{2}$$

then, we call matrix $A=(a_{ij})_{m\times n}$ as a pseudo jacket matrix.

Remark 1: In Definition 2, if $a_{ij}\neq 0$ for all $1\le i\le m$, $1\le j\le n$, $C=\max(m,n)$.

Remark 2: In Definition 2, for matrix $A=(a_{ij})_{m\times n}$, if $m=n$, then $A^+=A^{-1}$, so the common (honest) Jacket matrix A is also a pseudo Jacket matrix.

Are there pseudo Jacket matrices? We first see the following examples and theorems.

Example 1: Obviously, each zero matrix $O=(0)_{m\times n}$ is Pseudo Jacket Matrix, and $O^+=(0)_{n\times m}$.

Example 2: Obviously, the diagonal matrix

$$\sum_{m\times n}=\begin{pmatrix} \sigma_1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \sigma_t & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{pmatrix}_{m\times n}$$

where $\sigma_i \neq 0$, $1 \le i$ is Pseudo Inverse Jacket Matrix, and

$$\sum^{+}_{n\times m}=\begin{pmatrix} \frac{1}{\sigma_1} & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\sigma_2} & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{\sigma_t} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{pmatrix}_{n\times m}$$

Example 3: Let $A=\begin{pmatrix}1\\1\end{pmatrix}$, we take $B=\frac{1}{2}(1 \quad 1)$, then

(i)

$$ABA=\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{2}(1 \quad 1)\begin{pmatrix}1\\1\end{pmatrix}$$
$$=\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}=A$$

(ii)

$$BAB=\frac{1}{2}(1 \quad 1)\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{2}(1 \quad 1)$$
$$=\frac{1}{4}(2)(1 \quad 1)=\frac{1}{2}(1 \quad 1)=B$$

(iii)

$$(AB)^T=\left(\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{2}(1 \quad 1)\right)^T$$
$$=\left(\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\right)^T=\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}=AB$$

(iv)

$$(BA)^T=\left(\frac{1}{2}(1 \quad 1)\begin{pmatrix}1\\1\end{pmatrix}\right)^T=\left(\frac{1}{2}(1)\right)^T$$
$$=(1)=BA$$

So $B=A^+$ and A is a Pseudo Jacket Matrix.

Example 4: Symmetrically, matrix $A_1=A^T=(1 \quad 1)$ is also a Pseudo Jacket Matrix and

$$A_1^+=\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}$$

Ⅲ. The Construction of Pseudo Jacket Matrix

In this Section, we will prove some construction theorems of Pseudo Jacket Matrix, and find that many types of matrices belong to the set of Pseudo Jacket matrix. In fact, in a general way, we have the following construction theorems.

Theorem 1: The honest Jacket Matrix is also Pseudo Jacket Matrix.

Proof: For reversible square Jacket matrix $A=(a_{ij})_{n\times n}$, $A^+=A^{-1}$ and A^{-1} is element-wise inverse, So matrix A is a Pseudo Jacket matrix. ■

Theorem 2: For matrix

$$A=\begin{pmatrix}a_1\\a_2\\\vdots\\a_n\end{pmatrix}, \tag{3}$$

if $a_1^2=a_2^2=\cdots=a_n^2\neq 0$, then matrix A is a Pseudo Jacket Matrix.

Proof: As $a_i \neq 0$ for all $1\le i\le n$, we take

$$B=\frac{1}{\max(n,1)}\begin{pmatrix}\frac{1}{a_1} & \frac{1}{a_2} & \dots & \frac{1}{a_n}\end{pmatrix}$$
$$=\frac{1}{n}\begin{pmatrix}\frac{1}{a_1} & \frac{1}{a_2} & \dots & \frac{1}{a_n}\end{pmatrix}$$

Then, we have

(i)

$$ABA=\begin{pmatrix}a_1\\a_2\\\vdots\\a_n\end{pmatrix}\frac{1}{n}\begin{pmatrix}\frac{1}{a_1} & \frac{1}{a_2} & \dots & \frac{1}{a_n}\end{pmatrix}\begin{pmatrix}a_1\\a_2\\\vdots\\a_n\end{pmatrix}$$
$$=\frac{1}{n}\begin{pmatrix}1 & \frac{a_1}{a_2} & \dots & \frac{a_1}{a_n}\\ \frac{a_2}{a_1} & 1 & \dots & \frac{a_2}{a_n}\\ \dots & \dots & \dots & \dots\\ \frac{a_n}{a_1} & \frac{a_n}{a_2} & \dots & 1\end{pmatrix}\begin{pmatrix}a_1\\a_2\\\vdots\\a_n\end{pmatrix}$$

$$= \frac{1}{n}\begin{pmatrix} na_1 \\ na_2 \\ \vdots \\ na_n \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = A$$

(ii)

$$BAB = \frac{1}{n}\begin{pmatrix} \frac{1}{a_1} \\ \frac{1}{a_2} \\ \vdots \\ \frac{1}{a_n} \end{pmatrix}^T \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \frac{1}{n}\begin{pmatrix} \frac{1}{a_1} \\ \frac{1}{a_2} \\ \vdots \\ \frac{1}{a_n} \end{pmatrix}^T$$

$$= \frac{1}{n^2}(n)\begin{pmatrix} \frac{1}{a_1} & \frac{1}{a_2} & \cdots & \frac{1}{a_n} \end{pmatrix} = B$$

(iii) As $a_1^2 = a_2^2 = \cdots = a_n^2 \neq 0$, then $a_i^2 = a_j^2$, we have

$$\frac{a_i}{a_j} = \frac{a_j}{a_i}$$

and

$$(AB)^T = \left(\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \frac{1}{n}\begin{pmatrix} \frac{1}{a_1} & \frac{1}{a_2} & \cdots & \frac{1}{a_n} \end{pmatrix}\right)^T$$

$$= \left(\frac{1}{n}\begin{pmatrix} 1 & \frac{a_1}{a_2} & \cdots & \frac{a_1}{a_n} \\ \frac{a_2}{a_1} & 1 & \cdots & \frac{a_2}{a_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{a_n}{a_1} & \frac{a_n}{a_2} & \cdots & 1 \end{pmatrix}\right)^T$$

$$= \frac{1}{n}\begin{pmatrix} 1 & \frac{a_1}{a_2} & \cdots & \frac{a_1}{a_n} \\ \frac{a_2}{a_1} & 1 & \cdots & \frac{a_2}{a_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{a_n}{a_1} & \frac{a_n}{a_2} & \cdots & 1 \end{pmatrix} = AB$$

(iv)

$$(BA)^T = \left(\frac{1}{n}\begin{pmatrix} \frac{1}{a_1} & \frac{1}{a_2} & \cdots & \frac{1}{a_n} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}\right)^T$$

$$= (1)^T = BA$$

So $B = A^+$ and A is a Pseudo Jacket Matrix. ■

Theorem 3: Symmetrically, for matrix

$$A_1 = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}$$

if $a_1^2 = a_2^2 = \cdots = a_n^2 \neq 0$, then matrix A_1 is a Pseudo Jacket Matrix and

$$A_1^+ = \frac{1}{n}\begin{pmatrix} \frac{1}{a_1} \\ \frac{1}{a_2} \\ \vdots \\ \frac{1}{a_n} \end{pmatrix} \tag{4}$$

Theorem 4: Let A is a Pseudo Jacket Matrix, matrix D comes from exchanging two rows i and j of A, then D is also a Pseudo Jacket Matrix.

Proof: Let P_{ij} is the permutation matrix that comes from exchanging two rows i and j of identity matrix I, then

$$D = P_{ij}A = (d_{ij})_{m\times n}$$

and

$$P_{ij}^+ = P_{ij}^{-1} = P_{ij} = P_{ij}^T$$

$$A^+ = \frac{1}{c}(a'_{ij})^T;\ a'_{ij} = \begin{cases} \frac{1}{a_{ij}}, & a_{ij} \neq 0 \\ 0, & a_{ij} = 0 \end{cases}$$

We take

$$D_1 = A^+P_{ij}$$

then

$$D_1 = \frac{1}{c}(d'_{ij})^T;\ d'_{ij} = \begin{cases} \frac{1}{d_{ij}}, & d_{ij} \neq 0 \\ 0, & d_{ij} = 0 \end{cases}$$

and

(i)

$$DD_1D = P_{ij}AA^+P_{ij}P_{ij}A = P_{ij}A = D$$

(ii)

$$D_1DD_1 = A^+P_{ij}P_{ij}AA^+P_{ij} = A^+P_{ij} = D_1$$

(iii)

$$(DD_1)^T = (P_{ij}AA^+P_{ij})^T = DD_1$$

(iv)

$$(D_1 D)^T = \left(A^+ P_{ij} P_{ij} A\right)^T = D_1 D.$$

So the matrix D is a Pseudo Jacket Matrix.

■

Theorem 5: Symmetrically, let A is a Pseudo Jacket Matrix, matrix D comes from exchanging two columns i and j of A, then D is also a Pseudo Jacket Matrix.

Theorem 6:, Let $A_{m_1 \times n}, B_{m_2 \times n}$ are Pseudo Jacket Matrix, if $AB^+ = BA^+$, $BA^+A = B$ and $AB^+B = A$ then matrix

$$D = \begin{pmatrix} A \\ B \end{pmatrix}_{(m_1+m_2)\times n} \tag{5}$$

is also a Pseudo Matrix.

Proof: As $A_{m_1 \times n}, B_{m_2 \times n}$ are Pseudo Jacket Matrix, then

$$A^+ = \frac{1}{c_1}\left(a'_{ij}\right)^T;\ a'_{ij} = \begin{cases} \frac{1}{a_{ij}}, & a_{ij} \neq 0 \\ 0, & a_{ij} = 0 \end{cases}$$

and

$$B^+ = \frac{1}{c_2}\left(b'_{ij}\right)^T;\ b'_{ij} = \begin{cases} \frac{1}{b_{ij}}, & b_{ij} \neq 0 \\ 0, & b_{ij} = 0 \end{cases}$$

We take

$$D_1 = \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}$$

$$= \frac{1}{c_1 + c_2}\left(d'_{ij}\right)^T.$$

$$d'_{ij} = \begin{cases} \frac{1}{d_{ij}}, & d_{ij} \neq 0 \\ 0, & d_{ij} = 0 \end{cases}$$

$$d_{ij} = \begin{cases} a_{ij}, & 1 \le i \le m_1 \\ b_{ij}, & 1 \le i \le m_2 \end{cases}$$

then, we have

(i)

$$DD_1 D = \begin{pmatrix} A \\ B \end{pmatrix}\frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

$$= \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 AA^+ & c_2 AB^+ \\ c_1 BA^+ & c_2 BB^+ \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

$$= \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 AA^+A + c_2 AB^+B \\ c_1 BA^+A + c_2 BB^+B \end{pmatrix}$$

$$= \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A + c_2 A \\ c_1 B + c_2 B \end{pmatrix} = D$$

(ii)

$$D_1 D D_1 = \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

$$\frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}$$

$$= \frac{1}{(c_1 + c_2)^2}\left(c_1 A^+ A + c_2 BB^+\right)\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}$$

$$= \frac{1}{(c_1 + c_2)^2}\begin{pmatrix} c_1^2 A^+ + c_1 c_2 A^+ & c_1 c_2 B^+ + c_2^2 B^+ \end{pmatrix}$$

$$= \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix} = D_1$$

(iii)

$$(DD_1)^T = \left(\begin{pmatrix} A \\ B \end{pmatrix}\frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}\right)^T$$

$$= \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 AA^+ & c_2 AB^+ \\ c_1 BA^+ & c_2 BB^+ \end{pmatrix}^T = DD_1$$

(iv)

$$(D_1 D)^T = \left(\frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}\right)^T$$

$$= \frac{1}{(c_1 + c_2)^2}\left(c_1 A^+ A + c_2 BB^+\right)^T = D_1 D.$$

So the matrix

$$D = \begin{pmatrix} A \\ B \end{pmatrix}_{(m_1+m_2)\times n}$$

is a Pseudo Jacket Matrix and

$$D^+ = \frac{1}{c_1 + c_2}\begin{pmatrix} c_1 A^+ & c_2 B^+ \end{pmatrix}.$$

■

Example 5: Let

$$A = B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

are common Jacket Matrix and also Pseudo Jacket Matrix and

$$A^{+}=A^{-1}=B^{+}=B^{-1}=\frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}.$$

Obviously,

$$AB^{+}=BA^{+};\ BA^{+}A=B;\ AB^{+}B=A$$

So, from Theorem 6, we have matrix

$$D=\begin{pmatrix}A\\ B\end{pmatrix}=\begin{pmatrix}1 & 1\\ 1 & -1\\ 1 & 1\\ 1 & -1\end{pmatrix}$$

is a Pseudo Jacket Matrix and the pseudo inverse

$$D^{+}=\frac{1}{4}\left(\begin{pmatrix}1 & 1\\ 1 & -1\\ 1 & 1\\ 1 & -1\end{pmatrix}\right)^{T}=\frac{1}{4}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\end{pmatrix}.$$

Theorem 7: Symmetrically, Let $A_{m\times n_1}$, $B_{m\times n_2}$ are Pseudo Jacket Matrix, if $A^{+}B=B^{+}A$, $A^{+}AB=B$ and $B^{+}BA=A$ then matrix

$$D=\begin{pmatrix}A & B\end{pmatrix}_{m\times(n_1+n_2)} \tag{6}$$

is also a Pseudo Matrix.

Theorem 8: For matrix

$$A=\begin{pmatrix}a_1\\ a_2\\ \vdots\\ a_n\end{pmatrix}$$

if $a_i=0$ or $a_i^2=k$, k is a constant, then matrix A is a Pseudo Jacket Matrix.

Proof: Based on Theorem 4, we can exchange the rows of matrix A into matrix

$$B=\begin{pmatrix}0\\ \vdots\\ 0\\ b_1\\ \vdots\\ b_t\end{pmatrix}=\begin{pmatrix}0\\ B_1\end{pmatrix},$$

where $b_1^2=b_2^2=\cdots=b_t^2\neq 0$, and from Theorem 2, B_1 is a Pseudo Jacket Matrix. Let $D=\begin{pmatrix}0 & B_1^{+}\end{pmatrix}$, we have

(i)

$$BDB=\begin{pmatrix}0\\ B_1\end{pmatrix}\begin{pmatrix}0 & B_1^{+}\end{pmatrix}\begin{pmatrix}0\\ B_1\end{pmatrix}=B$$

(ii)

$$DBD=\begin{pmatrix}0 & B_1^{+}\end{pmatrix}\begin{pmatrix}0\\ B_1\end{pmatrix}\begin{pmatrix}0 & B_1^{+}\end{pmatrix}=D$$

(iii)

$$(BD)^{T}=\left(\begin{pmatrix}0\\ B_1\end{pmatrix}\begin{pmatrix}0 & B_1^{+}\end{pmatrix}\right)^{T}$$

$$=\left(\begin{pmatrix}0 & 0\\ 0 & B_1B_1^{+}\end{pmatrix}\right)^{T}=BD$$

(iv)

$$(DB)^{T}=\left(\begin{pmatrix}0 & B_1^{+}\end{pmatrix}\begin{pmatrix}0\\ B_1\end{pmatrix}\right)^{T}=DB.$$

So matrix

$$B=\begin{pmatrix}0\\ B_1\end{pmatrix}$$

is a Pseudo Jacket Matrix, and $D=B^{+}=\begin{pmatrix}0 & B_1^{+}\end{pmatrix}$. So matrix A is also a Pseudo Jacket Matrix. ■

Theorem 9: Symmetrically, for matrix

$$A=\begin{pmatrix}a_1 & a_2 & \cdots & a_n\end{pmatrix}, \tag{7}$$

if $a_i=0$ or $a_i^2=k$, k is a constant, then matrix A is a Pseudo Jacket Matrix.

Are there any other types of Pseudo Jacket Matrix? We see the following theorems and examples.

Theorem 10: Let

$$H=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \tag{8}$$

is a honest Jacket Matrix, also a Pseudo Jacket Matrix, if $A=(a_{ij})_{m\times n}$ is a Pseudo Jacket Matrix then

$$B=H\otimes A=\begin{pmatrix}A & A\\ A & -A\end{pmatrix}=(b_{ij})_{2m\times 2n}$$

is also a Pseudo Jacket Matrix.

Proof: As $A=(a_{ij})_{m\times n}$ is a Pseudo Jacket Matrix, then

$$A^{+}=\frac{1}{c}\left(a'_{ij}\right)^{T};\ a'_{ij}=\begin{cases}\frac{1}{a_{ij}}, & a_{ij}\neq 0\\ 0, & a_{ij}=0\end{cases}$$

We take

$$D=\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}=\frac{1}{2c}(b'_{ij})^{T};\ b'_{ij}=\begin{cases}\frac{1}{b_{ij}}, & b_{ij}\neq 0\\ 0, & b_{ij}=0\end{cases}$$

then we have

(i)

$$BDB=\begin{pmatrix}A & A\\ A & -A\end{pmatrix}\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}\begin{pmatrix}A & A\\ A & -A\end{pmatrix}$$

$$=\frac{1}{2}\begin{pmatrix}2AA^{+} & 0\\ 0 & -2AA^{+}\end{pmatrix}\begin{pmatrix}A & A\\ A & -A\end{pmatrix}=B$$

(ii)

$$DBD=\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}\begin{pmatrix}A & A\\ A & -A\end{pmatrix}\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}$$

$$=\frac{1}{4}\begin{pmatrix}2A^{+}A & 0\\ 0 & -2A^{+}A\end{pmatrix}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}=D$$

(iii)

$$(BD)^{T}=\left(\begin{pmatrix}A & A\\ A & -A\end{pmatrix}\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}\right)^{T}$$

$$=\left(\frac{1}{2}\begin{pmatrix}2AA^{+} & 0\\ 0 & -2AA^{+}\end{pmatrix}\right)^{T}=BD$$

(iv)

$$(DB)^{T}=\left(\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}\begin{pmatrix}A & A\\ A & -A\end{pmatrix}\right)^{T}$$

$$=\left(\frac{1}{2}\begin{pmatrix}2A^{+}A & 0\\ 0 & -2A^{+}A\end{pmatrix}\right)=DB.$$

So matrix $B=H\otimes A$ is a Pseudo Jacket Matrix, and

$$B^{+}=D=\frac{1}{2}\begin{pmatrix}A^{+} & A^{+}\\ A^{+} & -A^{+}\end{pmatrix}$$

Example 6: Let

$$A=\begin{pmatrix}1\\ 1\end{pmatrix}$$

is a Pseudo Jacket Matrix and

$$H=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

is a honest Jacket Matrix then

$$B=H\otimes A=\begin{pmatrix}1 & 1\\ 1 & 1\\ 1 & -1\\ 1 & -1\end{pmatrix}$$

is a Pseudo Jacket Matrix.

Example 7: Let $A=(1\quad 1)$ is a Pseudo Jacket Matrix and

$$H=\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

is a honest Jacket Matrix, then

$$B=H\otimes A=\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\end{pmatrix}$$

is a Pseudo Jacket Matrix and

$$B^{+}=D=\frac{1}{4}\begin{pmatrix}1 & 1\\ 1 & 1\\ 1 & -1\\ 1 & -1\end{pmatrix}$$

Further, we have the following theorem.

Theorem 11: Let

$$J=\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^{2}\\ 1 & \omega^{2} & \omega\end{pmatrix}\tag{9}$$

is a honest Jacket Matrix, also a Pseudo Jacket Matrix, where $\omega\neq 1$, $\omega^{3}=1$. if $A=(a_{ij})_{m\times n}$ is a Pseudo Jacket Matrix, then

$$B=J\otimes A=\begin{pmatrix}A & A & A\\ A & \omega A & \omega^{2}A\\ A & \omega^{2}A & \omega A\end{pmatrix}=(b_{ij})_{3m\times 3n}$$

is also a Pseudo Jacket Matrix.

Proof: As $A=(a_{ij})_{m\times n}$ is a Pseudo Jacket Matrix then

$$A^{+}=\frac{1}{c}\left(a'_{ij}\right)^{T};\ a'_{ij}=\begin{cases}\frac{1}{a_{ij}}, & a_{ij}\neq 0\\ 0, & a_{ij}=0\end{cases}$$

We take

$$D=\frac{1}{3}\begin{pmatrix}A^{+} & A^{+} & A^{+}\\ A^{+} & \omega^{2}A^{+} & \omega A^{+}\\ A^{+} & \omega A^{+} & \omega^{2}A^{+}\end{pmatrix}=\frac{1}{3c}(b'_{ij})^{T};$$

$$b'_{ij}=\begin{cases}\frac{1}{b_{ij}}, & b_{ij}\neq 0\\ 0, & b_{ij}=0\end{cases}$$

then we have

(i)

Because $\omega \neq 1,\ \omega^3 = 1,$ then we have $1+\omega+\omega^2 = 0$, and

BDB

$$=\begin{pmatrix} A & A & A \\ A & \omega A & \omega^2 A \\ A & \omega^2 A & \omega A \end{pmatrix} \frac{1}{3}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix}\begin{pmatrix} A & A & A \\ A & \omega A & \omega^2 A \\ A & \omega^2 A & \omega A \end{pmatrix}$$

$$=\frac{1}{3}\begin{pmatrix} 3AA^+ & 0 & 0 \\ 0 & 3AA^+ & 0 \\ 0 & 0 & 3AA^+ \end{pmatrix}\begin{pmatrix} A & A & A \\ A & \omega A & \omega^2 A \\ A & \omega^2 A & \omega A \end{pmatrix} = B$$

(ii)

DBD

$$=\frac{1}{3}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix}\begin{pmatrix} A & A & A \\ A & \omega A & \omega^2 A \\ A & \omega^2 A & \omega A \end{pmatrix}\frac{1}{3}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix}$$

$$=\frac{1}{9}\begin{pmatrix} 3A^+A & 0 & 0 \\ 0 & 3A^+A & 0 \\ 0 & 0 & 3A^+A \end{pmatrix}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix} = D$$

(iii)

$$(BD)^T = \left(\begin{pmatrix} A & A & A \\ A & \omega A & \omega^2 A \\ A & \omega^2 A & \omega A \end{pmatrix}\frac{1}{3}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix}\right)^T$$

$$=\left(\frac{1}{3}\begin{pmatrix} 3AA^+ & 0 & 0 \\ 0 & 3AA^+ & 0 \\ 0 & 0 & 3AA^+ \end{pmatrix}\right)^T = BD$$

(iv)

$(DB)^T$

$$=\left(\frac{1}{3}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix}\begin{pmatrix} A & A & A \\ A & \omega A & \omega^2 A \\ A & \omega^2 A & \omega A \end{pmatrix}\right)^T$$

$$=\left(\frac{1}{3}\begin{pmatrix} 3A^+A & 0 & 0 \\ 0 & 3A^+A & 0 \\ 0 & 0 & 3A^+A \end{pmatrix}\right)^T = DB.$$

So matrix $B = J \otimes A$ is a Pseudo Jacket Matrix, and

$$B^+ = D = \frac{1}{3}\begin{pmatrix} A^+ & A^+ & A^+ \\ A^+ & \omega^2 A^+ & \omega A^+ \\ A^+ & \omega A^+ & \omega^2 A^+ \end{pmatrix} \qquad \blacksquare$$

Example 8: Let $A = (1 \quad 1)$ is a Pseudo Jacket Matrix, and

$$J = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{10}$$

is a honest Jacket Matrix then

$$B = J \otimes A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & \omega & \omega & \omega^2 & \omega^2 \\ 1 & 1 & \omega^2 & \omega^2 & \omega & \omega \end{pmatrix}$$

is a Pseudo Jacket Matrix and

$$B^+ = D = \frac{1}{6}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \\ 1 & \omega & \omega^2 \end{pmatrix}$$

Ⅳ. MIMO SVD Pseudo Inverse Channel

In the section, we derive MIMO SVD pseudo $\boldsymbol{H} \in \mathbb{C}^{N_T \times N_R}$ inverse channel. The MIMO channel matrix is decomposed by the singular value decomposition (SVD), that is, we have

$$\boldsymbol{H} = \boldsymbol{U\Sigma V}^H \tag{11}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\boldsymbol{\Sigma}$ is a rectangular diagonal matrix with non-negative real elements which means the element-wise inverse Jacket matrix. The diagonal elements of $\boldsymbol{\Sigma}$ are the singular values of the channel matrix H, denoting by $\sigma_1, \sigma_2, \cdots, \sigma_{N_{\min}}$, where $N_{\min} = \min(N_T, N_R)$. In case of $N_{\min} = N_T$, SVD in Eq. (11) can be expressed as

$$\boldsymbol{H} = \mathbf{U\Sigma V}^H = \underbrace{\left[\mathbf{U}_{N_{\min}}\ \mathbf{U}_{N_R - N_{\min}}\right]}_{\mathbf{U}} \underbrace{\begin{bmatrix} \boldsymbol{\Sigma}_{N_{\min}} \\ \mathbf{0}_{N_R - N_{\min}} \end{bmatrix}}_{\Sigma} \mathbf{V}^H = \mathbf{U}_{N_{\min}} \boldsymbol{\Sigma}_{N_{\min}} \mathbf{V}^H, \tag{12}$$

where $\mathbf{U}_{N_{\min}}$ is composed of $N_{\min}$ left-singular vectors and $\mathbf{\Sigma}_{N_{\min}}$ is a square matrix. In case of $N_{\min} = N_R$, SVD in Eq. (11) can be expressed as

$$\mathbf{H} = \mathbf{U}\underbrace{\begin{bmatrix}\mathbf{\Sigma}_{N_{\min}} & \mathbf{0}_{N_T-N_{\min}}\end{bmatrix}}_{\mathbf{\Sigma}}\underbrace{\begin{bmatrix}\mathbf{V}^H_{N_{\min}} \\ \mathbf{V}^H_{N_T-N_{\min}}\end{bmatrix}}_{\mathbf{V}^H} = \mathbf{U}\mathbf{\Sigma}_{N_{\min}}\mathbf{V}^H_{N_{\min}} \quad . \qquad (13)$$

where $\mathbf{V}_{N_{\min}}$ is composed of $N_{\min}$ right-singular vectors. Then we get eigenvalue decomposition,

$$\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{\Sigma}\mathbf{\Sigma}^H\mathbf{U}^H = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H, \qquad (14)$$

where $\mathbf{U}^H\mathbf{U} = \mathbf{I}_{N_R}$ and $\mathbf{\Lambda} \in \mathbb{C}^{N_R \times N_R}$ is a diagonal matrix.

The transmitted signal vector is defined as

$$\boldsymbol{Q} = \boldsymbol{E}\left\{\boldsymbol{x}\boldsymbol{x}^H\right\} \qquad (15)$$

Then, the channel capacity of MIMO channel is expressed as

$$C = \max_{\mathrm{tr}(Q)=N_T} \log_2 \det\left(\boldsymbol{I}_{N_R} + \frac{E_x}{N_T N_0}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right) \qquad (16)$$

The channel capacity is given as

$$C = \log_2 \det\left(\boldsymbol{I}_{N_R} + \frac{E_x}{N_T N_0}\boldsymbol{H}\boldsymbol{H}^H\right) \qquad (17)$$

Using the eigen decomposition $\boldsymbol{H}\boldsymbol{H}^H = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ and the identity $\det(\boldsymbol{I}_m + \boldsymbol{A}\boldsymbol{B}) = \det(\boldsymbol{I}_m + \boldsymbol{B}\boldsymbol{A})$, where $\boldsymbol{A} \in \mathbb{C}^{m \times n}$ and , $\boldsymbol{B} \in \mathbb{C}^{n \times m}$ the channel capacity in Equation (17) is expressed as

$$\begin{aligned} C &= \log_2 \det\left(\boldsymbol{I}_{N_R} + \frac{E_x}{N_T N_0}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H\right) \\ &= \log_2 \det\left(\boldsymbol{I}_{N_R} + \frac{E_x}{N_T N_0}\mathbf{\Lambda}\right) \\ &= \sum_{i=1}^{r} \log_2 \det\left(\boldsymbol{I}_{N_R} + \frac{E_x}{N_T N_0}\lambda_i\right) \end{aligned} \qquad (18)$$

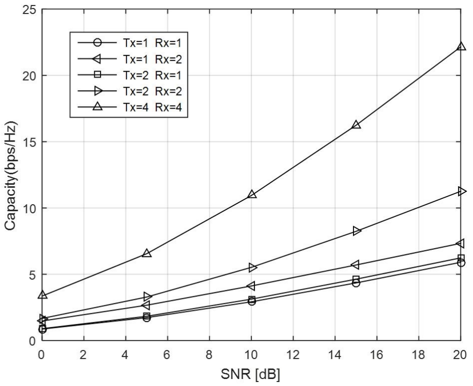

그림 1. SNR변화에 따른 MIMO 채널 용량
Fig. 1. MIMO channel capacity with SNR.

We can compute the capacity of the MIMO channel as SNR is varied, when CSI is not known at the transmitter side. Fig. 1 shows the channel capacity as varying the number of antennas. From Fig. 1 that the MIMO channel capacity improves with increasing the number of transmit and receive antennas.

V. Conclusion

In this paper, we extended the definition of Jacket Matrix to Pseudo Inverse Jacket Matrix, proved some construction theorems of Pseudo Inverse Jacket Matrix, and presented some examples of Pseudo Inverse Jacket matrix. Furthermore we derived MIMO SVD pseudo inverse channel and developed application. We discuss mainly research the applications of Pseudo Inverse Jacket Matrix in many subjects, as MIMO Channel, SVD, and EVD decomposition.

References

[1] G. Golub and C. Van Loan, Matrix computations. Johns Hopkins University Press, 1996, vol. 3.
[2] G. Strang, “Linear algebra and its applications, thomson-brooks,” Cole, Belmont, CA, USA, 2005.
[3] R. A. Horn and C. R. Johnson, Topics in Matrix Analysis. Cambridge Univ. Press Cambridge etc, 1991.

[4] A. Duran and F. Griinbaum, "A survey on orthogonal matrix polynomials satisfying second order differential equations," Journal of computational and applied mathematics, vol. 178, no. 1, pp. 169-190, 2005.

[5] M. Lee and M. Kaveh, "Fast hadamard transform based on a simple matrix factorization," Acoustics, Speech and Signal Processing, IEEE Transactions on, vol. 34, no. 6, pp. 1666-1667, 1986.

[6] J. Seberry and A. Whiteman, "New hadamard matrices and conference matrices obtained via mathon's construction, ', Graphs and Combinatorics, vol. 4, no. 1, pp. 355-377, 1988.

[7] W. Bajwa, J. Haupt, G. Raz, S. Wright, and R. Nowak, "Toeplitz-structured compressed sensing matrices," in Statistical Signal Processing, 2007. SSP'07. IEEE/SP 14th Workshop on. IEEE, 2007, pp. 294-298.

[8] T. Abrudan, J. Eriksson, and V. Koivunen, "Steepest descent algorithms for optimization under unitary matrix constraint," Signal Processing, IEEE Transactions on, vol. 56, no. 3, pp. 1134-1147, 2008.

[9] W. Ji-ke and S. Xiu-ming, "The circulant matrix and its applications in the computation of structures, " Mathematicae Numericae Sinica, vol. 2, pp. 144-153, 1979.

[10] H. Park, L. Zhang, and J. Rosen, "Low rank approximation of a hankel matrix by structured total least norm," BIT Numerical Mathematics, vol. 39, no. 4, pp. 757-779, 1999.

[11] J. Hou and M. Lee, "Information hided jacket matrix and its fast factorization algorithm, " Journal of Convergence Information Technology, vol. 5, no. 6, 2010.

[12] A. Ahmed and D. Day, "Orthogonal transforms for digital signal processing," 1975.

[13] K. Horadam, Hadamard matrices and their applications. Princeton university press, 2011.

[14] M. Lee, "The center weighted hadamard transform," Circuits and Systems, IEEE Transactions on, vol. 36, no. 9, pp. 1247-1249, 1989.

[15] W. Tadej and K. Zyczkowski, "A concise guide to complex hadamard matrices," Open Systems & Information Dynamics, vol. 13, no. 02, pp. 133-177, 2006.

[16] D. Drake, "Partial λ-geometries and generalized hadamard matrices over groups," Canad. J. Math, vol. 31, no. 3, pp. 617-627, 1979.

[17] R. Mathon, A. Rosa, J. Abel, M. Greig, D. Kreher, C. Colbourn, T. van Trung, S. Furino, R. Mullin, H. Gronau et al., "The crc handbook of combinatorial designs," CRC Press, Boca Raton FL, Part, vol. 1, pp. 7-10, 1996.

[18] M. Lee and Y. Borissov, "A proof of non-existence of bordered jacket matrices of odd order over some fields," Electronics letters, vol. 46, no. 5, pp. 349-351, 2010.

[19] M. Lee, "A new reverse jacket transform and its fast algorithm, " Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on, vol. 47, no. 1, pp. 39-47, 2000.

[20] M. Lee and G. Zeng, "Family of fast jacket transform algorithms," Electronics Letters, vol. 43, no. 11, pp. 651-651, 2007.

[21] G. Zeng and M. Lee, "A generalized reverse block jacket transform, " Circuits and Systems I: Regular Papers, IEEE Transactions on, vol. 55, no. 6, pp. 1589-1600, 2008.

[22] K. Horadam, "A generalized hadamard transform," in Information Theory, 2005. ISIT 2005. Proceedings. International Symposium on. IEEE, 2005, pp. 1006-1008.

[23] S. Lee and J. Yi, "Fast reverse jacket transform as an alternative representation of the n-point fast fourier transform, " Journal of Mathematical Imaging and Vision, vol. 16, no. 1, pp. 31-39, 2002.

[24] C. Lin, W. Zhang, Y. Guo, and Y. Xu, "Co-cyclic jacket matrix based optimal training design and placement for MIMO OFDM channel estimation," in Circuits and Systems for Communications, 2008. ICCSC 2008. 4th IEEE International

Conference on. IEEE, 2008, pp. 392-396.

[25] W. Ma, “The jacket matrix and cryptography,” Technical Report, Chon-buk National University, Tech. Rep., 2004.

[26] Z. Li, X. Xia, and M. Lee, “A simple orthogonal space-time coding scheme for asynchronous cooperative systems for frequency selective fading channels,” Communications, IEEE Transactions on, vol. 58, no. 8, pp. 2219-2224, 2010.

[27] M. H. Lee, Jacket Matrices: Construction and Its Application for Fast Cooperative Wireless signal Processing. LAP LAMBERT Academic Publishing, 2012.

[28] S. Lee and M. Lee, “On the reverse jacket matrix for weighted hadamard transform,” Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on, vol. 45, no. 3, pp. 436-441, 1998.

[29] M. Lee, “Jacket matrix and its fast algorithms for cooperative wireless signal processing,” 2012 11.30, MIT Seminar PPT.

[30] M. Lee and J. Hou, “Fast block inverse jacket transform,” Signal Processing Letters, IEEE, vol. 13, no. 8, pp. 461-464, 2006.

[31] Y. S. Im, E. Y. Kang, "MPEG-2 Video Watermarking in Quantized DCT Domain," The Journal of The Institute of Internet, Broadcasting and Communication(JIIBC), Vol. 11, No. 1, pp. 81-86, 2011.

[32] I. Jeon, S. Kang, H. Yang, “Development of Security Quality Evaluate Basis and Measurement of Intrusion Prevention System,” Journal of the Korea Academia-Industrial cooperation Society(JKAIS), Vol. 11, No. 1, pp. 81-86, 2010.

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