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유한한 길이에서 성능이 향상된 BP 극 복호기*

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Enhanced Belief Propagation Polar Decoder for Finite Lengths

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<Abstract>

In this paper, we discuss the belief propagation decoding algorithm for polar codes. The performance of Polar codes for shorter lengths is not satisfactory. Motivated by this, we propose a novel technique to improve its performance at short lengths. We showed that the probability of messages passed along the factor graph of polar codes, can be increased by multiplying the current message of nodes with their previous message. This is like a feedback path in which the present signal is updated by multiplying with its previous signal. Thus the experimental results show that performance of belief propagation polar decoder can be improved using this proposed technique. Simulation results in binary-input additive white Gaussian noise channel (BI-AWGNC) show that the proposed belief propagation polar decoder can provide significant gain of 2 dB over the original belief propagation polar decoder with code rate 0.5 and code length 128 at the bit error rate (BER) of 10^{-4}.

Key Words : Polar Codes, Belief Propagation Decoder, Input Nodes, Output Nodes

Ⅰ. Introduction

Polar codes have drawn the attention of many researchers because of their capability to achieve the Shannon's capacity. To prove that polar codes can achieve the Shannon's limit, Arikan introduced the successive cancellation (SC) decoding of polar codes [1]. However, to achieve the Shannon's limit using SC decoding, the length of the polar code should tend to infinity, which is not practically feasible. Since the invention of polar codes, many researchers have proposed many variations in SC decoder to improve the performance of polar codes at finite lengths. Successive-cancellation list (SCL) decoder was proposed, showing that the performance of the SCL decoder is close to the maximum-likelihood (ML) decoding [2]. To further

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improve the performance of polar codes, Niu and Chen showed that, with CRC aided, the performance of SCL decoder becomes comparable to the state-of-the-art LDPC codes [3]. In order to improve the time and space complexity of the SC decoder, Niu proposed successive-cancellation stack (SCS) algorithms [4]. The successive-cancellation hybrid (SCH) algorithm combines the features of SCL and SC and approaches the performance of ML decoding with acceptable complexity [5]. These SC decoders can achieve good error-correcting capability with low complexity, however, for high-speed real-time applications, this serial decoding scheme of polar codes poses the challenge of high throughput and latency.

To overcome the issues of throughput and latecncy, researchers investigated the belief propagation (BP) algorithm for polar codes. Arikan proposed the BP decoding of polar in [1] and showed in [6] that the performance of polar codes under BP decoding is better than SC decoding.

However, the performance of polar codes using the BP decoding is still not satisfactory. Therefore, in this paper we proposed a modified BP decoder to improve the performance of polar codes for finite lengths. In the proposed BP decoder, the nodes are multiplied by their previous values instead of just using their current values. By multiplying the previous values of the nodes with their current values the reliability of the propagated messages along the factor graph is improved, which in turn increases the performance of BP decoder. The simulation results shows that the performance of BP decoders can be improved by 1-2 dB using the proposed decoder.

The remainder of this paper is organized as follows. In section II we give a short background of polar codes and introduce the belief propagation decoding of polar codes in section III. The proposed decoder is explained in section IV, with the proposed changes in the nodes of BP decoder. Section V gives the simulation results. Finally this paper is concluded in section VI.

Ⅱ. Polar Codes

Polar coding is based on the idea of channel polarization, as proposed by Arikan [1]. This effect becomes prominent when N grows large and the symmetric capacity terms, $I(W_N^{(i)}: 1 \le i \le N)$, of N independent copies, $(W_N^{(i)})$, of binary discrete memory-less channels, W, tend to 0 or 1, for all except for some indices. It means that the channel is completely noiseless when $I(W)=0$ and completely noisy when $I(W)=1$. The idea of polar coding is to transmit the information bits on these noiseless channels, while fixing the rest of the bits to a value known to both the sender and the receiver. Since we will only deal with binary symmetric channels in this paper, we will set the fixed positions to zero.

Based on the idea of channel polarization, polar codes can be constructed using a code length of $N=2^n$, a dimension k, a rate $R=k/N$, and a set of frozen bits I^c. The Bhattacharyya method is used to choose frozen bits set [1]. Thus, the source binary vector $u_1^N=(u_1,u_2,...,u_N)$, with k information bits and $N-k$ frozen bits, can be used to generate code word $x_1^N=(x_1,x_2,...,x_N)$ such that

$$x_1^N=u_1^N B_N G_N \quad (1)$$

where B_N is the bit reversal permutation matrix and $G_N=G_2^{\otimes n}$, where $\otimes$ denotes the kronecker product, $n=\log_2^N$ and $G_2=\begin{bmatrix}1&0\\1&1\end{bmatrix}$. In practice, the construction procedure of polar codes requires $n=\log_2^N$ stages as shown in figure 1. Polar code factor graph can be constructed using basic computational blocks (BCB) and for a polar code of length N, there will be $N/2$ BCB. The construction of BCB is according to the generator matrix G_N, having two inputs and outputs as shown in figure 2. We denote input nodes of jth PE in stage i by variables $v_I(i,2j)$, $v_I(i,2j+1)$ for $1\le i\le n$ and $1\le j\le N/2$, output nodes are $v_O(i,2j)$, $v_O(i,2j+1)$ respectively, and for each BCB we have

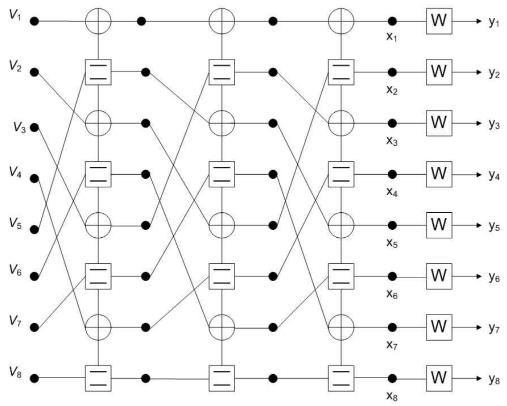
$$v_O(i,2j)=v_I(i,2j)\oplus v_I(i,2j+1) \quad (2)$$

$$v_O(i,2j+1)=v_I(i,2j+1) \quad (3)$$

where $\oplus$ denotes the modulo-two sum, $u_i=v_I(0,i)$, $1\le i\le N$, and $x_i=v_O(n,i)$, $1\le i\le N$.

Ⅲ. Belief Propagation Decoding of Polar Codes

Belief propagation decoding, also known as the sum-product algorithm, has been extensively studied for LDPC codes for many years. It is not



<Fig. 1> Construction of polar code for $N=8$, according to generator matrix G_N

only used for channel coding, but also in many other fields like computer vision, image processing, and medical research. It has been proved that the complexity of decoding polar codes under BP is $O(NlogN)$ [6-8].

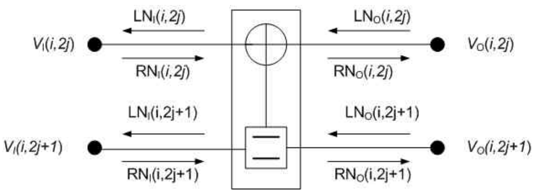
The belief propagation decoding of polar codes is based on the factor graph of the code [1]. Each node $v_I(i,2j)$ or $v_O(i,2j)$ in the BCB of polar code (see figure 2), have two types of messages: left-to-right messages and right-to-left messages. These messages are first propagated from left to right and then from right to left, which completes one round iteration of BP algorithm for polar codes. The process continues until the decoder satisfies the equation $H.\hat{x}_1^N=0$, or the decoder reaches the maximum number of iterations.

Consider the figure 2, which depicts the BCB for a polar code. Let $LN_I^t(i,2j)$ and $LN_I^t(i,2j+1)$ are the right-to-left probability messages passed to the input nodes of the jth BCB in stage i, where t is the iteration number. $RN_I^t(i,2j)$ and $RN_I^t(i,2j+1)$ are the left-to-right probability messages passed from the input nodes. Similarly, $LN_O^t(i,2j)$ and $LN_O^t(i,2j+1)$ are the right-to-left probability messages passed from the input nodes, $RN_O^t(i,2j)$ and $RN_O^t(i,2j+1)$ are the left-to-right probability messages passed to the output nodes. We will use $LN_*^t(i,j)$ to represent both $LN_I^t(i,j)$ and $LN_O^t(i,j)$, and similarly for RN_*^t. During the procedure of belief propagation the update messages for these nodes will be calculated with the help of following equations:

$$LN_I^t(i,2j) = LN_O^t(i,2j) \otimes [LN_O^t(i,2j+1) \odot RN_I^t(i,2j+1)]$$

$$LN_I^t(i,2j+1) = [RN_I^t(i,2j) \otimes LN_O^t(i,2j)] \odot LN_O^t(i,2j+1)]$$

$$RN_O^t(i,2j) = RN_I^t(i,2j) \otimes [RN_I^t(i,2j+1) \odot LN_O^t(i,2j+1)]$$



<Fig. 2> Diagram of BCB in the original BP decoder

$$RN_O^t(i,2j+1) = [RN_I^t(i,2j) \otimes LN_O^t(i,2j)] \odot RN_I^t(i,2j+1)] \quad (4)$$

The above equations can be further explained as $p_x \otimes p_y = p_{(x \oplus y)}$ and $p_x \odot p_y = p_{(x=y)}$, where $p_{(x \oplus y)}$ and $p_{(x \oplus y)}$ can be given as

$$p_{(x \oplus y)}(0) = p_x(0) * p_y(0) + p_x(1) * p_y(1)$$
$$p_{(x \oplus y)}(1) = p_x(0) * p_y(1) + p_x(1) * p_y(0)$$

$$p_{(x=y)}(0) = p_x(0) * p_y(0)$$
$$p_{(x=y)}(1) = p_x(1) * p_y(1) \quad (5)$$

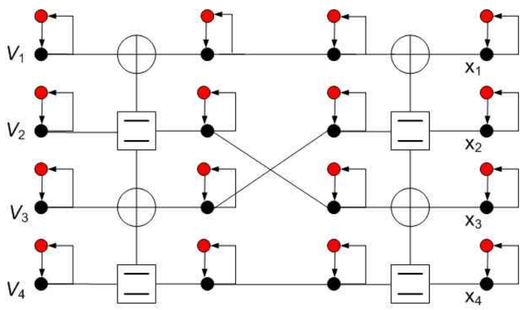
where $p_x(0)$ and $p_x(1)$ represents the probability of p_x being equal to 0 and 1, respectively. After reaching the maximum iteration number $(I_{\max})$, the BP decoder will output the decoded vector $\hat{u}_1^N = (\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_N)$ as

$$\hat{u}_i = \begin{cases} 0, & LN_{vin(0,j)}(0) > LN_{vin(0,j)}(1) \\ 1, & otherwise \end{cases} \quad (6)$$

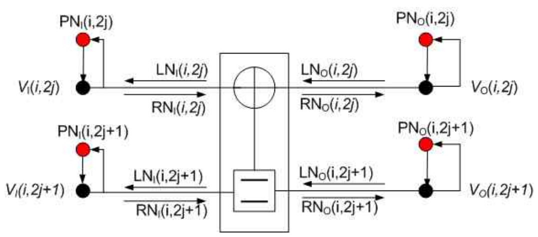
Ⅳ. Proposed Method

It is evident from [9] and [10] that the performance of polar codes of finite length is poor. Motivated by this, we investigated the polar codes and the BP decoder. In [8] and [11], it is noted that the nodes in the factor graph of polar codes have a degree of two or three, while for LDPC codes, check nodes and variable nodes have an average degree of more than three. This is the reason why polar codes do not perform well at shorter lengths.

Based on the bad performance of short polar codes, we propose a new method to improve the performance of these short codes. Due to the very small degrees of nodes in the case of polar codes, the belief that is passed between the factor graphs is distorted over multiple iterations. Moreover, if only one bit is incorrectly decoded, it will pass the wrong belief to all of the other nodes, and the correct code word cannot be decoded. We propose to multiply the nodes by their previous messages, in order to increase the reliability of propagated messages. This is like a feedback path in which the previous signal is fed to the present signal, as shown in figure 4, so that the present signal does not go out-of-bounds. The first iteration in the proposed BP decoder will remain same as that of the original BP decoder, while the subsequent iterations will be different; i. e., in the next iteration, the update equations will be multiplied by their previous messages.



<Fig. 3> Construction of the proposed BP polar decoder for $N=4$



<Fig. 4> Diagram of BCB in the proposed BP decoder

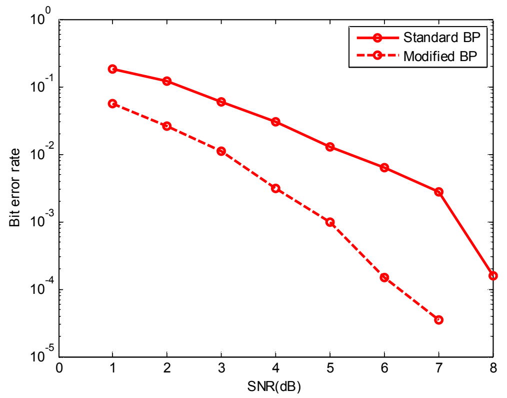
The new diagram for BCB is shown in figure 4, where the red node is the new node which stores the previous message of the current node, and $PN_*^t(i,j)$ is the messages from this new node($PN_*^t(i,j)$ represents both the input $PN_I^t(i,j)$ and output $PN_O^t(i,j)$ nodes). These new messages will be fed to the current nodes and the message update equations will be modified by '⊙' operation. Hence, the new message update equations for the BP decoder can be written as:

$$
\begin{aligned}
&\widehat{LN}_I^t(i,2j) = \widehat{LN}_O^t(i,2j)\otimes \\
&\quad [\widehat{LN}_O^t(i,2j+1)\odot\widehat{RN}_I^t(i,2j+1)] \\
&\widehat{LN}_I^t(i,2j+1) = [\widehat{RN}_I^t(i,2j)\otimes\widehat{LN}_O^t(i,2j)]\odot \\
&\quad \widehat{LN}_O^t(i,2j+1)] \\
&\widehat{RN}_O^t(i,2j) = \widehat{RN}_I^t(i,2j)\otimes \\
&\quad [\widehat{RN}_I^t(i,2j+1)\odot\widehat{LN}_O^t(i,2j+1)] \\
&\widehat{RN}_O^t(i,2j+1) = [\widehat{RN}_I^t(i,2j)\otimes\widehat{LN}_O^t(i,2j)]\odot \\
&\quad \widehat{RN}_I^t(i,2j+1)]
\end{aligned}
\tag{7}
$$

where $\widehat{LN}_*^t(i,j) = LN_*^t(i,j)\odot PN_*^t(i,j)$ $\widehat{RN}_*^t(i,j) = RN_*^t(i,j)\odot PN_*^t(i,j)$. For instance, consider that the current left messages of node $v_{*(i,j)}$ are $LN_{*(i,j)}(0)=0.297$ and $LN_{*(i,j)}(1)=0.204$, and its previous messages are $PN_{*(i,j)}(0)=0.53$ and $PN_{*(i,j)}(1)=0.47$. According to (7), the normalized new messages of the left nodes will be $\widehat{LN}_{*(i,j)}(0)=0.621$ and $LN_{*(i,j)}(1)=0.379$. This shows that the reliability of the messages is increased. This analytical result shows that the performance of the BP decoder will be improved with the new method, and this is also proved with the simulation results in the next section.

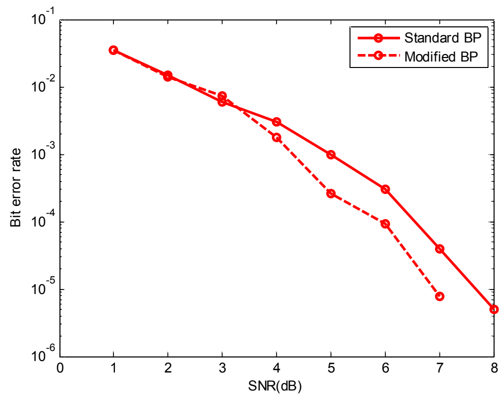
V. Simulation Results

In this section, we present the simulation results for our proposed algorithm for short polar codes. The polarized channels carrying the information bits are selected according to the method given in [1]. The code rate is 0.5, and the code word is transmitted over a binary input AWGN channel with antipodal signaling. For each value, 1,000 codewords are transmitted, and the maximum number of iterations, I_{max}, is set to 60.



<Fig. 5> Comparison of BER between the original and proposed BP Polar decoder for $N=2^7$

It can be seen from figure 5 and figure 6 that the performance of our proposed algorithm is better than the standard BP algorithm, especially at higher values of SNR. Figure 5 shows the performance of polar codes for $N=2^7$, and it can be seen that the performance of our proposed algorithm is better than the standard algorithm by 2 dB at BER of 10^{-2}. At BER of 10^{-3} we see SNR advantage of approximately 2.5 dB, which shows that the SNR advantage increases as the value of SNR increases. It is shown in figure 6 that for $N=2^8$, the performance for small values of SNR is same as the standard BP algorithm. However, at higher values of SNR, the performance of proposed algorithm is still better; e. g. at BER of 10^{-3} and 10^{-5} we observed SNR advantages of 0.6 dB and 1 dB, respectively. Hence, it can be concluded that the performance of our proposed BP decoder is better than the original decoder.



<Fig. 6> Comparison of BER between the original and proposed BP Polar decoder for $N=2^8$

VI. Conclusion

We studied the BER performance of short polar codes under belief propagation decoding. We analyzed the factor graph of polar codes and showed that by multiplying the previous messages of the nodes with their current messages, the reliability of the propagated messages can be improved. Simulation results show that the proposed method achieves a gain of 2 dB at higher SNR values over the original BP decoder.

참고문헌

[1] Arikan, E. “Channel Polarization: A method for constructing capacity achieving codes for symmetric binary-inout memoryless

channels," IEEE Trnas. Inf. Theory, Vol. 55, No. 7, 2009, pp. 3051-3073.

[2] Tal, I. and Vardy, A., "List Decoding of Polar Codes," Proceedings of the IEEE International Symposium on Information Theory (ISIT '11), St Petersburg, Russia, 2011, pp. 1-5.

[3] Niu, K. and Chen, K., "CRC-Aided Decoding of Polar Codes," IEEE Communications Letters, Vol. 16, No. 10, 2012, pp. 1668-1671.

[4] Niu, K. and Chen, K., "Stack decoding of polar codes," IET Electronics Letters, Vol. 48, No. 12, 2012, pp. 695-697.

[5] Chen, K., Niu, K. and Lin, J., "Improved Successive Cancellation Decoding of Polar Codes," IEEE Transactions on Communications, Vol. 61, No. 8, 2013, pp. 3100-3107.

[6] Arıkan, E., "A Performance Comparison of Polar Codes and Reed-Muller Codes," IEEE Communications Letters, Vol. 12, No. 6, 2008, pp. 447-449.

[7] Eslami, A. and Pishro-Nik, H., "On Bit Error Rate Performance of Polar Codes in Finite Regime," Proceedings of the 48th Annual Allerton Conference on Communication, Control, and Computing, Allerton (AACCCC '10), Allerton, Ill, USA, 2010, pp. 188-194.

[8] Hussami, N, Korada S. B. and Urbanke, R., "Performance of Polar Codes for Channel and Source Coding," IEEE International Symposium on Information Theory (ISIT'09), Seoul, Korea, 2009, pp. 1488-1492.

[9] Arıkan, E. and Telatar, E., "On the rate of channel polarization," IEEE Int. Symp. Inform. Theory, Seoul, Korea, 2009, pp. 1493-1495.

[10] Korada, S. B., Sasoglu, E. and Urbanke, R., "Polar Codes: Characterization of Exponent, Bounds, and Constructions," IEEE Transactions on Information Theory, Vol. 56, No. 12, 2010, pp. 6253-6264.

[11] Zhang, Y., Liu, A., Pan, X, He, S. and Gong, C., "A Generalization Belief Propagation Decoding Algorithm for Polar Codes Based on Particle Swarm Optimization," Mathematical Problems in Engineering, Vol. 2014, Article ID 606913,10 pages, http://dx.doi.org/10.1155/2014/606913.

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