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WEAK CONVERGENCE THEOREMS FOR GENERALIZED MIXED EQUILIBRIUM PROBLEMS, MONOTONE MAPPINGS AND PSEUDOCONTRACTIVE MAPPINGS

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Abstract. In this paper, we introduce a new iterative algorithm for finding a common element of the set of solutions of a generalized mixed equilibrium problem related to a continuous monotone mapping, the set of solutions of a variational inequality problem for a continuous monotone mapping, and the set of fixed points of a continuous pseudocontractive mapping in Hilbert spaces. Weak convergence for the proposed iterative algorithm is proved. Our results improve and extend some recent results in the literature.

1. Introduction

Let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and induced norm $\| \cdot \|$, and let C be a nonempty closed convex subset of H. Let $B : C \to H$ be a nonlinear mapping, let $\varphi : C \to \mathbb{R}$ be a function, and let Θ be a bifunction of $C \times C$ into $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers.

The generalized mixed equilibrium problem (for short, GMEP) of finding $x \in C$ such that

$$\Theta(x, y) + \langle Bx, y - x\rangle + \varphi(y) - \varphi(x) \geq 0, \quad \forall y \in C, \tag{1.1}$$

was introduced by Peng and Yao [26] (also see [34]). The set of solutions of the GMEP is denoted by $GMEP(\Theta, \varphi, B)$.

The GMEP is very general in the sense that it includes, as special cases, the generalized equilibrium problem (for short, GEP) in case that $\varphi = 0$ in (1.1) ([30]), the mixed equilibrium problem (for short, MEP) in case that $B = 0$ in (1.1) ([6, 32]), the equilibrium problem (for short, EP) in case that $B = 0$ and $\varphi = 0$ in (1.1) ([3, 10, 11]) and others. In particular, if $\varphi = 0$ and $\Theta(x, y) = 0$

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for all $x, y \in C$ in (1.1), the GMEP reduces the following variational inequality problem (for short, VIP) of finding $x \in C$ such that

$$\langle Bx, y - x\rangle \geq 0, \quad \forall y \in C.$$

The set of solutions of the VIP is denoted by $VI(C, B)$.

A mapping F of C into H is called *monotone* if

$$\langle x - y, Fx - Fy\rangle \geq 0, \quad \forall x,\ y \in C.$$

A mapping F of C into H is called *α-inverse-strongly monotone* (see [12]) if there exists a positive real number α such that

$$\langle x - y, Fx - Fy\rangle \geq \alpha\|Fx - Fy\|^2, \quad \forall x,\ y \in C.$$

If F is an α-inverse-strongly monotone mapping of C into H, then it is obvious that F is $\frac{1}{\alpha}$-Lipschitz continuous, that is, $\|Fx-Fy\| \leq \frac{1}{\alpha}\|x-y\|$ for all $x, y \in C$. Clearly, the class of monotone mappings includes the class of α-inverse-strongly monotone mappings.

Recall that a mapping $T : C \to H$ is said to be *pseudocontractive* if

$$\langle x - y, Tx - Ty\rangle \leq \|x - y\|^2,$$

and T is said to be *k-strictly pseudocontractive* if there exists a constant $k \in [0, 1)$ such that

$$\langle x - y, Tx - Ty\rangle \leq \|x - y\|^2 - k\|(I - T)x - (I - T)y\|^2, \quad \forall x,\ y \in C,$$

where I is the identity mapping. A mapping T of C into itself is called *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$, $\forall x,\ y \in C$. Obviously, the class of k-strictly pseudocontractive mappings includes the class of nonexpansive mappings as a subclass, and the class of pseudocontractive mappings includes the class of strictly pseudocontractive mappings as a subclass. Moreover, this inclusion is strict due to an example in [8] (see, also Example 5.7.1 and Example 5.7.2 in [2]). Fixed point problems for pseudocontractive mappings and strictly pseudocontractive mappings were studied by many authors, see, for example, [1, 9, 14, 16, 19, 22, 23, 24, 35] and the references therein.

Recently, many authors have introduced some iterative algorithms for finding a common element of the set of the solutions of the GMEP, the GEP, the MEP, the EP, and the VIP and the set of fixed points of a countable family of nonexpansive mappings, and have proved strong convergence of the sequences generated by the proposed iterative algorithms; see [6, 13, 15, 17, 25, 26, 27, 29, 30, 31, 32] and the references therein. Also we refer to [4, 5, 7, 18, 21] for the GMEP, the GEP, the EP, and the VIP combined with the fixed point problem for nonexpansive semigroups and strictly pseudocontractrive mappings.

In particular, in 2007, Tada and Takahashi [29] introduced an iterative algorithm for finding a common element of the set of solutions of the EP and the set of fixed points of a nonexpansive mapping, and proved weak convergence of the sequence generated by the proposed iterative algorithm. In 2008, Moudafi [25] proposed an iterative algorithm for finding a common element of the set of solutions of the GEP related to an α-inverse-strongly monotone mapping B and the set of fixed points of a nonexpansive mapping, and obtained weak convergence of the sequence generated by the proposed iterative algorithm. In 2009, Ceng *et al.* [5] presented an iterative algorithm for finding a common element of the set of solutions of the EP and the set of fixed points of a k-strictly pseudocontractive mapping, and showed weak convergence of the sequence generated by the proposed iterative algorithm. In 2012, Jung [18] considered an iterative algorithm for finding a common element of the set of solutions of the GMEP related to α-inverse-strongly monotone mapping B, the set of solutions of the VIP for β-inverse-strongly monotone mapping F and the set of fixed points of a k-strictly pseudocontractive mapping, and established weak convergence of the sequence generated by the proposed iterative algorithm.

On the other hand, in 2003, Takahashi and Toyoda [31] proposed an iterative algorithm for finding a common element of the set of solutions of the VIP for α-inverse-strongly monotone mapping F and the set of fixed points of a nonexpansive mapping, and proved weak convergence of the sequence generated by the proposed iterative algorithm. In 2009, Plubtieng and Kumam [27] extended the result of Takahashi and Toyoda [31] to the case of a countable family of nonexpansive mappings, and as an application, they obtained weak convergence of an iterative algorithm for finding a common element of the set of solutions of the VIP for α-inverse-strongly monotone mapping F and the set of solutions of the EP.

In this paper, motivated and inspired by the above mentioned results, we introduce a new iterative algorithm for finding a common element of the set of solutions of the GMEP related to a continuous monotone mapping B, the set of solutions of the VIP for a continuous monotone mapping F and the set of fixed points of a continuous pseudocontractive mapping T in a Hilbert space. We prove weak convergence of the sequence generated by the proposed iterative algorithm to a common element of three sets. As direct consequences, we obtain the results for the GEP related to a continuous monotone mapping B, the MEP and the EP, combined with the VIP for a continuous monotone mapping F and the fixed point problem for a continuous pseudocontractive mapping T. Our results extend, improve, and develop some recent results in the literature.

2. Preliminaries and lemmas

In the following, we denote by $Fix(T)$ the set of fixed points of the mapping T, and we denote the strong convergence and the weak convergence of $\{x_n\}$ to x by $x_n \to x$ and $x_n \rightharpoonup x$, respectively.

Let H be a real Hilbert space, and let C be a nonempty closed convex subset of H. In H, we have

$$\|\lambda x + (1-\lambda)y\|^2 = \lambda\|x\|^2 + (1-\lambda)\|y\|^2 - \lambda(1-\lambda)\|x-y\|^2$$

for all $x, y \in H$ and $\lambda \in \mathbb{R}$. For every point $x \in H$, there exists a unique nearest point in C, denoted by $P_C(x)$, such that

$$\|x - P_C(x)\| \leq \|x - y\|$$

for all $y \in C$. P_C is called the *metric projection* of H onto C. $P_C(x)$ is characterized by the property:

$$u = P_C(x) \Longleftrightarrow \langle x - u, u - y\rangle \geq 0 \quad \text{for all } x \in H,\ y \in C. \tag{2.1}$$

It is also well known that H satisfies the *Opial condition*, that is, for any sequence $\{x_n\}$ with $x_n \rightharpoonup x$, the inequality

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|$$

holds for every $y \in H$ with $y \neq x$.

For solving the GMEP, the GEP, the MEP, and the EP for a bifunction $\Theta : C \times C \to \mathbb{R}$, let us assume that Θ satisfies the following conditions:

(A1) $\Theta(x, x) = 0$ for all $x \in C$;
(A2) Θ is monotone, that is, $\Theta(x, y) + \Theta(y, x) \leq 0$ for all $x,\ y \in C$;
(A3) for each $x, y, z \in C$,

$$\limsup_{t\downarrow 0} \Theta(tz + (1 - t)x, y) \leq \Theta(x, y);$$

(A4) for each $x \in C, y \mapsto \Theta(x, y)$ is convex and lower semicontinuous.

We can prove the following lemma by using the same method as in [20, 34], and so we omit its proof.

Lemma 2.1. *Let C be a nonempty closed convex subset of H. Let Θ be a bifunction form $C \times C$ to $\mathbb{R}$ satisfies* (A1)–(A4)*, and let $\varphi : C \to \mathbb{R}$ be a proper lower semicontinuous and convex function. Let $B : C \to H$ be a continuous monotone mapping. Then, for $r > 0$ and $x \in H$, there exists $u \in C$ such that*

$$\Theta(u, y) + \langle Bu, y - u\rangle + \varphi(y) - \varphi(u) + \frac{1}{r}\langle y - u, u - x\rangle \geq 0, \quad \forall y \in C.$$

Define a mapping $K_r : H \to C$ as follows:

$$K_r x = \Big\{u \in C : \Theta(u, y) + \langle Bu, y - u\rangle + \varphi(y) - \varphi(u) + \frac{1}{r}\langle y - u, u - x\rangle \geq 0,\ \forall y \in C\Big\}$$

for all $x \in H$ and $r > 0$. Then, the following hold:

(i) *For each $x \in H$, $K_r(x) \neq \emptyset$;*
(ii) *K_r is single-valued*;
(iii) *K_r is firmly nonexpansive, that is, for any $x,\ y \in H$,*

$$\|K_r x - K_r y\|^2 \leq \langle K_r x - K_r y, x - y\rangle;$$

(iv) $Fix(K_r) = GMEP(\Theta, \varphi, B)$;
(v) *$GMEP(\Theta, \varphi, B)$ is closed and convex.*

We also need the following lemmas for the proof of our main results.

Lemma 2.2 ([28]). *Let H be a real Hilbert space, let $\{\alpha_n\}$ be a sequence of real numbers such that $0 < a \le \alpha_n \le b < 1$ for all $n \ge 1$, and let $\{v_n\}$ and $\{w_n\}$ be sequences in H such that, for some c*

$$\limsup_{n\to\infty} \|v_n\| \le c, \quad \limsup_{n\to\infty} \|w_n\| \le c, \quad \text{and} \quad \limsup_{n\to\infty} \|\alpha_n v_n + (1-\alpha_n) w_n\| = c.$$

Then $\lim_{n\to\infty} \|v_n - w_n\| = 0$.

Lemma 2.3 ([31]). *Let C be a nonempty closed convex subset of a real Hilbert spaces H, and let $\{x_n\}$ be a sequence in H. If*

$$\|x_{n+1} - x\| \le \|x_n - x\|, \quad \forall x \in C \text{ and } \forall n \ge 1,$$

then $\{P_C x_n\}$ converges strongly to some $z \in C$, where P_C stands for the metric projection of H onto C.

The following lemmas are Lemma 2.3 and Lemma 2.4 of Zegeye [33], respectively.

Lemma 2.4 ([33]). *Let C be a closed convex subset of a real Hilbert space H. Let $F : C \to H$ be a continuous monotone mapping. Then, for $r > 0$ and $x \in H$, there exists $z \in C$ such that*

$$\langle Fz, y - z\rangle + \frac{1}{r}\langle y - z, z - x\rangle \ge 0, \quad \forall y \in C.$$

For $r > 0$ and $x \in H$, define $F_r : H \to C$ by

$$F_r x = \left\{ z \in C : \langle Fz, y - z\rangle + \frac{1}{r}\langle y - z, z - x\rangle \ge 0, \quad \forall y \in C \right\}.$$

Then the following hold:

(i) *F_r is single-valued*;

(ii) *F_r is firmly nonexpansive, that is,*

$$\|F_r x - F_r y\|^2 \le \langle F_r x - F_r y, x - y\rangle, \quad \forall x,\ y \in H;$$

(iii) $Fix(F_r) = VI(C, F)$;

(iv) *$VI(C, F)$ is a closed convex subset of C.*

Lemma 2.5 ([33]). *Let C be a closed convex subset of a real Hilbert space H. Let $T : C \to H$ be a continuous pseudocontractive mapping. Then, for $r > 0$ and $x \in H$, there exists $z \in C$ such that*

$$\langle Tz, y - z\rangle - \frac{1}{r}\langle y - z, (1 + r)z - x\rangle \le 0, \quad \forall y \in C.$$

For $r > 0$ and $x \in H$, define $T_r : H \to C$ by

$$T_r x = \left\{ z \in C : \langle Tz, y - z\rangle - \frac{1}{r}\langle y - z, (1 + r)z - x\rangle \le 0, \quad \forall y \in C \right\}.$$

Then the following hold:

(i) *T_r is single-valued*;

(ii) T_r *is firmly nonexpansive, that is,*

$$\|T_r x - T_r y\|^2 \leq \langle T_r x - T_r y, x - y\rangle, \quad \forall x, \ y \in H;$$

(iii) $Fix(T_r) = Fix(T)$;
(iv) $Fix(T)$ *is a closed convex subset of* C.

3. Iterative algorithms

Throughout the rest of this paper, we always assume the following:

- H is a real Hilbert space;
- C is a nonempty closed convex subset of H;
- Θ is a bifunction form $C \times C$ to $\mathbb{R}$ satisfies (A1)–(A4);
- $\varphi : C \to \mathbb{R}$ is a proper lower semicontinuous and convex function;
- $B : C \to H$ is a continuous monotone mapping;
- $GMEP(\Theta, \varphi, B)$ is the set of solutions of the GMEP related to B:
- $K_{r_n} : H \to C$ is a mapping defined by

$$K_{r_n} x = \Big\{ u \in C : \Theta(u, y) + \langle Bu, y - u\rangle + \varphi(y) - \varphi(u) + \frac{1}{r_n}\langle y - u, u - x\rangle \geq 0, \ \forall y \in C \Big\}$$

  for all $x \in H$ and for $r_n \in (0, \infty)$ and $\liminf_{n\to\infty} r_n > 0$;
- $F : C \to H$ is a continuous monotone mapping;
- $VI(C, F)$ is the set of solutions of the VIP for F;
- $T : C \to C$ is a continuous pseudocontractive mapping;
- $F_{r_n} : H \to C$ is a mapping defined by

$$F_{r_n} x = \Big\{ z \in C : \langle Fz, y - z\rangle + \frac{1}{r_n}\langle y - z, z - x\rangle \geq 0, \quad \forall y \in C \Big\}$$

  for all $x \in H$ and for $r_n \in (0, \infty)$ and $\liminf_{n\to\infty} r_n > 0$;
- $T_{r_n} : H \to C$ is a mapping defined by

$$T_{r_n} x = \Big\{ z \in C : \langle Tz, y - z\rangle - \frac{1}{r_n}\langle y - z, (1 + r_n)z - x\rangle \leq 0, \quad \forall y \in C \Big\}$$

  for all $x \in H$ and for $r_n \in (0, \infty)$ and $\liminf_{n\to\infty} r_n > 0$;
- $\Omega_1 := GMEP(\Theta, \varphi, B) \cap VI(C, F) \cap Fix(T) \neq \emptyset$.

By Lemma 2.1, Lemma 2.4 and Lemma 2.5, we note that K_{r_n}, F_{r_n} and T_{r_n} are nonexpansive, and $Fix(K_{r_n}) = GMEP(\Theta, \varphi, B)$, $Fix(F_{r_n}) = VI(C, F)$ and $Fix(T_{r_n}) = Fix(T)$.

Now, we propose a new iterative algorithm for finding a common point of the set of solutions of the GMEP related to a continuous monotone mapping B, the set of solutions of the VIP for a continuous monotone mapping F, and the set of fixed points of a continuous pseudocontractive mapping T.

Algorithm 3.1. For an arbitrarily chosen $x_1 \in C$, let the iterative sequences $\{x_n\}$ and $\{u_n\}$ be generated by

$$\begin{cases} \Theta(u_n, y) + \langle Bu_n, y - u_n\rangle + \varphi(y) - \varphi(u_n) \\ \qquad\qquad\qquad + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \ \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_{r_n}F_{r_n}K_{r_n}x_n, \quad \forall n \geq 1, \end{cases} \tag{3.1}$$

where $\{\alpha_n\} \subset [a, b]$ for some $a,\ b \in (0,1)$ and $r_n \in (0,\infty)$ and $\liminf_{n\to\infty} r_n > 0$.

Theorem 3.1. *The sequences $\{x_n\}$ and $\{u_n\}$ generated by Algorithm 3.1 converge weakly to $z \in \Omega_1$, where $z = \lim_{n\to\infty} P_{\Omega_1}(x_n)$.*

Proof. From now, we put $u_n = K_{r_n}x_n$, $z_n = F_{r_n}u_n$ and $w_n = T_{r_n}z_n$. Without loss of generality, we assume $r_n > c > 0$ for $\forall n \geq 1$ and some $c \in \mathbb{R}$.

We divide the proof into several steps.

Step 1. We show that $\{x_n\}$ is bounded. To this end, let $p \in \Omega_1 := GMEP(\Theta, \varphi, B) \cap VI(C, F) \cap Fix(T)$. Then, from Lemma 2.1(iv), Lemma 2.4(iii) and Lemma 2.5(iii), it follows that $p = K_{r_n}p$, $p = F_{r_n}p$ and $p = T_{r_n}p$. From $z_n = F_{r_n}u_n$ and the nonexpansivity of F_{r_n}, we get

$$\|z_n - p\| = \|F_{r_n}u_n - F_{r_n}p\| \leq \|u_n - p\|. \tag{3.2}$$

Also, by $u_n = K_{r_n}x_n \in C$ and the nonexpansivity of K_{r_n},

$$\|u_n - p\| = \|K_{r_n}x_n - K_{r_n}p\| \leq \|x_n - p\|,$$

and so

$$\|z_n - p\| \leq \|x_n - p\|. \tag{3.3}$$

By using the convexity of $\|\cdot\|^2$, we also obtain

$$\begin{aligned} \|x_{n+1} - p\|^2 &= \|\alpha_n(x_n - p) + (1-\alpha_n)(T_{r_n}z_n - p)\|^2 \\ &\leq \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\|T_{r_n}z_n - p\|^2 \\ &\leq \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\|z_n - p\|^2 \\ &\leq \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\|x_n - p\|^2 \\ &= \|x_n - p\|^2, \end{aligned} \tag{3.4}$$

and hence

$$\|x_{n+1} - p\| \leq \|x_n - p\|. \tag{3.5}$$

So, there exists $r \in \mathbb{R}$ such that

$$r = \lim_{n\to\infty} \|x_n - p\|.$$

Therefore $\{x_n\}$ is bounded, and so are $\{u_n\}$ and $\{z_n\}$ by (3.2) and (3.3). Moreover, from

$$\|w_n - p\| = \|T_{r_n}z_n - p\| \leq \|z_n - p\|,$$

$\{w_n\}$ is also bounded.

Step 2. We show that $\lim_{n\to\infty}\|x_n - u_n\| = 0$. To this end, let $p \in \Omega_1$. Since K_{r_n} is firmly nonexpansive and $u_n = K_{r_n}x_n$, we have

$$\begin{aligned}\|u_n - p\|^2 &= \|K_{r_n}(x_n) - K_{r_n}(p)\|^2\\ &\le \langle K_{r_n}(x_n) - K_{r_n}(p), x_n - p\rangle\\ &= \langle u_n - p, x_n - p\rangle\\ &= \frac{1}{2}(\|u_n - p\|^2 + \|x_n - p\|^2) - \frac{1}{2}\|(x_n - p) - (u_n - p)\|^2\\ &= \frac{1}{2}(\|u_n - p\|^2 + \|x_n - p\|^2 - \|x_n - u_n\|^2),\end{aligned}$$

and hence

$$\|u_n - p\|^2 \le \|x_n - p\|^2 - \|x_n - u_n\|^2. \tag{3.6}$$

On the other hand, by using the convexity of $\|\cdot\|^2$, (3.2) and (3.6), we get

$$\begin{aligned}\|x_{n+1} - p\|^2 &\le \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\|z_n - p\|^2\\ &\le \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\|u_n - p\|^2\\ &\le \alpha_n\|x_n - p\|^2 + (1-\alpha_n)(\|x_n - p\|^2 - \|x_n - u_n\|^2).\end{aligned}$$

This implies that

$$\begin{aligned}(1-b)\|x_n - u_n\|^2 &\le (1-\alpha_n)\|x_n - u_n\|^2\\ &\le \|x_n - p\|^2 - \|x_{n+1} - p\|^2.\end{aligned}$$

Since $\lim_{n\to\infty}\|x_{n+1} - p\|^2 = \lim_{n\to\infty}\|x_n - p\|^2$, we obtain

$$\lim_{n\to\infty}\|x_n - u_n\| = 0.$$

Step 3. We show that $\lim_{n\to\infty}\|x_n - w_n\| = \lim_{n\to\infty}\|x_n - T_{r_n}z_n\| = 0$. Indeed, let $p \in \Omega_1$ and $r = \lim_{n\to\infty}\|x_n - p\|$. Since T_{r_n} nonexpansive and $Fix(T) = Fix(T_{r_n})$, it follows from (3.3) that

$$\|T_{r_n}z_n - p\| \le \|z_n - p\| \le \|x_n - p\|,$$

and hence $\limsup_{n\to\infty}\|T_{r_n}z_n - p\| \le r$. By (3.4), we also get

$$\begin{aligned}&\limsup_{n\to\infty}\|\alpha_n(x_n - p) + (1-\alpha_n)(T_{r_n}z_n - p)\|\\ =\ &\limsup_{n\to\infty}\|x_{n+1} - p\|\\ \le\ &\limsup_{n\to\infty}\|x_n - p\| = \lim_{n\to\infty}\|x_n - p\| = r.\end{aligned}$$

By Lemma 2.2, we obtain $\lim_{n\to\infty}\|x_n - w_n\| = \lim_{n\to\infty}\|x_n - T_{r_n}z_n\| = 0$.

Step 4. We show that $\lim_{n\to\infty}\|u_n - z_n\| = 0$.. Since F_{r_n} is firmly nonexpansive, using $z_n = F_{r_n}u_n$ and $p = F_{r_n}p$, we have

$$\begin{aligned}\|z_n - p\|^2 &= \|F_{r_n}u_n - F_{r_n}p\|^2\\ &\le \langle F_{r_n}u_n - F_{r_n}p, u_n - p\rangle\\ &= \ \langle z_n - p, u_n - p\|\end{aligned}$$

$$= \frac{1}{2}(\|z_n - p\|^2 + \|u_n - p\|^2) - \frac{1}{2}\|z_n - u_n\|^2,$$

and hence

$$\begin{aligned}\|z_n - p\|^2 &\le \|u_n - p\|^2 - \|z_n - u_n\|^2 \\ &\le \|x_n - p\|^2 - \|z_n - u_n\|^2.\end{aligned}$$

So, we obtain

$$\begin{aligned}\|x_{n+1} - p\|^2 &\le \alpha\|x_n - p\|^2 + (1 - \alpha_n)\|z_n - p\|^2 \\ &\le \alpha_n\|x_n - p\|^2 + (1 - \alpha_n)(\|x_n - p\|^2 - \|z_n - u_n\|^2) \\ &= \|x_n - p\|^2 - (1 - \alpha_n)\|z_n - u_n\|^2.\end{aligned}$$

From conditions $\alpha_n \in [a, b] \subset (0, 1)$, it follows

$$\begin{aligned}(1 - b)\|z_n - u_n\|^2 &\le (1 - \alpha_n)\|z_n - u_n\|^2 \\ &\le \|x_n - p\|^2 - \|x_{n+1} - p\|^2.\end{aligned}$$

By $r = \lim_{n\to\infty} \|x_n - p\|$, we conclude

$$\lim_{n\to\infty} \|z_n - u_n\| = 0.$$

Step 5. We show that $\lim_{n\to\infty} \|w_n - z_n\| = 0$. Indeed, from Step 2, Step 3 and Step 4, it follows that

$$\|w_n - z_n\| \le \|w_n - x_n\| + \|x_n - u_n\| + \|u_n - z_n\| \to 0.$$

Step 6. We show that any of its weak cluster point z of $\{x_n\}$ belongs in Ω_1 and $x_n \rightharpoonup z$. In this case, there exists a subsequence $\{x_{n_i}\}$ which converges weakly to z.

We will show that $z \in \Omega_1$. First, by the same argument as in the proof of [26, Theorem 3.1], we can obtain that $z \in GMEP(\Theta, \varphi, B)$. For the sake of completeness, we include its proof. From $u_n = K_{r_n} x_n$, it follows that

$$\Theta(u_n, y) + \langle Bu_n, y - u_n\rangle + \varphi(y) - \varphi(u_n) + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \ge 0, \quad \forall y \in C.$$

By (A2), we deduce

$$\langle Bu_n, y - u_n\rangle + \varphi(y) - \varphi(u_n) + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \ge \Theta(y, u_n), \quad \forall y \in C,$$

and hence

$$\begin{aligned}&\langle Bu_{n_i}, y - u_{n_i}\rangle + \varphi(y) - \varphi(u_{n_i}) \\ &\qquad + \frac{1}{r_{n_i}}\langle y - u_{n_i}, u_{n_i} - x_{n_i}\rangle \ge \Theta(y, u_{n_i}), \quad \forall y \in C.\end{aligned} \tag{3.7}$$

Set $y_t = ty + (1 - t)z$ for all $t \in (0, 1]$ and $y \in C$. Since $y \in C$ and $z \in C$, we get $y_t \in C$. Thus, it follows from (3.7) that

$$\begin{aligned}\langle By_t, y_t - u_{n_i}\rangle &\ge \langle By_t, y_t - u_{n_i}\rangle - \varphi(y_t) + \varphi(u_{n_i}) - \langle Bu_{n_i}, y_t - u_{n_i}\rangle \\ &\quad - \langle y_t - u_{n_i}, \frac{u_{n_i} - x_{n_i}}{r_{n_i}}\rangle + \Theta(y_t, u_{n_i})\end{aligned}$$

$$= \langle By_t - Bu_{n_i}, y_t - u_{n_i}\rangle - \varphi(y_t) + \varphi(u_{n_i}) - \langle y_t - u_{n_i}, \frac{u_{n_i} - x_{n_i}}{r_{n_i}}\rangle + \Theta(y_t, u_{n_i}).$$

From the fact that $\|u_n - x_n\| \to 0$ by Step 2, we obtain that $\frac{u_{n_i}-x_{n_i}}{r_{n_i}} \to 0$ and $u_{n_i} \rightharpoonup z$ as $i \to \infty$. Moreover, from the monotonicity of B, we get $\langle y_t - u_{n_i}, By_t - Bu_{n_i}\rangle \geq 0$. So, from (A4) and the weak lower semicontinuity of φ, it follows that

$$\langle By_t, y_t - z\rangle \geq -\varphi(y_t) + \varphi(z) + \Theta(y_t, z) \ \text{ as } i \to \infty. \tag{3.8}$$

By (A1), (A4) and (3.8), we also have

$$\begin{aligned} 0 &= \Theta(y_t, y_t) + \varphi(y_t) - \varphi(y_t) \\ &\leq t\Theta(y_t, y) + (1-t)\Theta(y_t, z) + t\varphi(y) + (1-t)\varphi(z) - \varphi(y_t) \\ &\leq t[\Theta(y_t, y) + \varphi(y) - \varphi(y_t)] + (1-t)\langle y_t - z, By_t\rangle \\ &= t[\Theta(y_t, y) + \varphi(y) - \varphi(y_t)] + (1-t)t\langle y - z, By_t\rangle, \end{aligned}$$

and hence

$$0 \leq \Theta(y_t, y) + \varphi(y) - \varphi(y_t) + (1-t)\langle By_t, y - z\rangle. \tag{3.9}$$

Letting $t \to 0$ in (3.9) yields that for each $y \in C$,

$$\Theta(z, y) + \langle Bz, y - z\rangle + \varphi(y) - \varphi(z) \geq 0.$$

This implies that $z \in GMEP(\Theta, \varphi, B)$.

Second, we show that $z \in VI(C, F)$. In fact, from the definition of $z_{n_i} = F_{r_{n_i}} u_{n_i}$, we have

$$\langle Fz_{n_i}, y - z_{n_i}\rangle + \langle y - z_{n_i}, \frac{z_{n_i} - u_{n_i}}{r_{n_i}}\rangle \geq 0, \quad \forall y \in C. \tag{3.10}$$

Set $v_t = tv + (1-t)z$ for all $t \in (0, 1]$ and $v \in C$. Then, $v_t \in C$. So, from (3.10), it follows that

$$\begin{aligned} &\langle Fv_t, v_t - z_{n_i}\rangle \\ &\geq \langle Fv_t, v_t - z_{n_i}\rangle - \langle Fz_{n_i}, v_t - z_{n_i}\rangle - \langle v_t - z_{n_i}, \frac{z_{n_i} - u_{n_i}}{r_{n_i}}\rangle \\ &= \langle Fv_t - Fz_{n_i}, v_t - z_{n_i}\rangle - \langle v_t - z_{n_i}, \frac{z_{n_i} - u_{n_i}}{r_{n_i}}\rangle. \end{aligned} \tag{3.11}$$

By Step 2 and Step 4, we obtain that $\frac{z_{n_i}-u_{n_i}}{r_{n_i}} \to 0$ and $z_{n_i} \rightharpoonup z$ as $i \to \infty$. Since F is monotone, we also have that $\langle Fv_t - Fz_{n_i}, v_t - z_{n_i}\rangle \geq 0$. Thus, it follows from (3.11) that

$$0 \leq \lim_{n\to\infty} \langle Fv_t, v_t - z_{n_i}\rangle = \langle Fv_t, v_t - z\rangle,$$

and hence

$$\langle Fv_t, v - z\rangle \geq 0, \quad \forall v \in C.$$

If $t \to 0$, the continuity of F yields that

$$\langle Fz, v-z, \rangle \geq 0, \quad \forall v \in C.$$

This implies that $z \in VI(C,F)$.

Thirdly, we prove that $z \in Fix(T)$. In fact, from the definition of $w_{n_i} = T_{r_{n_i}} z_{n_i}$, we have

$$\langle Tw_{n_i}, y - w_{n_i}\rangle - \frac{1}{r_{n_i}}\langle y - w_{n_i}, (1+r_{n_i})w_{n_i} - z_{n_i}\rangle \leq 0, \quad \forall y \in C. \tag{3.12}$$

Put $v_t = tv + (1-t)z$ for all $t \in (0,1]$ and $v \in C$. Then, $v_t \in C$, and from (3.12) and pseudocontractivity of T, it follows that

$$\begin{aligned}
&\langle Tv_t, w_{n_i} - v_t\rangle \\
&\geq \langle Tv_t, w_{n_i} - v_t\rangle + \langle Tw_{n_i}, v_t - w_{n_i}\rangle \\
&\quad - \frac{1}{r_{n_i}}\langle v_t - w_{n_i}, (1+r_{n_i})w_{n_i} - z_{n_i}\rangle \\
&= -\langle Tv_t - Tw_{n_i}, v_t - w_{n_i}\rangle - \frac{1}{r_{n_i}}\langle v_t - w_{n_i}, w_{n_i} - z_{n_i}\rangle \\
&\quad - \langle v_t - w_{n_i}, w_{n_i}\rangle \\
&\geq -\|v_t - w_{n_i}\|^2 - \frac{1}{r_{n_i}}\langle v_t - w_{n_i}, w_{n_i} - z_{n_i}\rangle - \langle v_t - w_{n_i}, w_{n_i}\rangle \\
&= -\langle v_t - w_{n_i}, v_t\rangle - \langle v_t - w_{n_i}, \frac{w_{n_i} - z_{n_i}}{r_{n_i}}\rangle.
\end{aligned} \tag{3.13}$$

By Step 3 and Step 5, we get that $\frac{w_{n_i}-z_{n_i}}{r_{n_i}} \to 0$ and $w_{n_i} \rightharpoonup z$ as $i \to \infty$. Therefore, as $i \to \infty$ in (3.13), it follows that

$$\langle Tv_t, z - v_t, \rangle \geq \langle z - v_t, v_t\rangle,$$

and hence

$$-\langle Tv_t, v - z\rangle \geq -\langle v - z, v_t\rangle, \quad \forall v \in C.$$

Letting $t \to 0$ and using the fact that T is continuous, we get

$$-\langle Tz, v - z, \rangle \geq -\langle v - z, z\rangle, \quad \forall v \in C.$$

Now, let $v = Tz$. Then we obtain $z = Tz$ and hence $z \in Fix(T)$. Therefore, $z \in \Omega_1$.

Now, we prove that $x_n \rightharpoonup z \in \Omega_1$. To this end, let $\{x_{n_j}\}$ be another subsequence of $\{x_n\}$ such that $x_{n_j} \rightharpoonup z'$. Then, by the above argument, $z' \in \Omega_1$. If $z \neq z'$, then the Opial condition yields

$$\begin{aligned}
\lim_{n\to\infty}\|x_n - z\| &= \liminf_{i\to\infty}\|x_{n_i} - z\| \\
&< \liminf_{i\to\infty}\|x_{n_i} - z'\| \\
&= \lim_{n\to\infty}\|x_n - z'\| \\
&= \liminf_{j\to\infty}\|x_{n_j} - z'\|
\end{aligned}$$

$$< \liminf_{j\to\infty} \|x_{n_j} - z\|$$
$$= \lim_{n\to\infty} \|x_n - z\|,$$

which is a contradiction. So, $z = z'$. Thus, we conclude that

$$x_n \rightharpoonup z \in \Omega_1.$$

Also from Step 2, it follows that $u_n \rightharpoonup z \in \Omega_1$.

Step 7. We show that $z = \lim_{n\to\infty} P_{\Omega_1}(x_n)$. For this purpose, let $v_n = P_{\Omega_1}(x_n)$. Since $z \in \Omega_1$, it follows from (2.1) that

$$\langle x_n - v_n, v_n - z\rangle \geq 0.$$

Since $\|x_{n+1} - p\| \leq \|x_n - p\|$ for $p \in \Omega_1$, Lemma 2.3 implies that $v_n \to z_0$ for some $z_0 \in \Omega_1$. Since $x_n \rightharpoonup z$ by Step 6, we obtain

$$\langle z - z_0, z_0 - z\rangle \geq 0,$$

and hence $z = z_0 = \lim_{n\to\infty} P_{\Omega_1}(x_n)$. This completes the proof. □

If we take $C \equiv H$ in Theorem 3.1, then we obtain the following result.

Corollary 3.2. *Let $\Omega_2 := GMEP(\Theta, \varphi, B) \cap F^{-1}(0) \cap Fix(T)$. The sequences $\{x_n\}$ and $\{u_n\}$ generated by Algorithm 3.1 converge weakly to $z \in \Omega_2$, where $z = \lim_{n\to\infty} P_{\Omega_2}(x_n)$.*

Proof. Since $D(F) = H$, we have $VI(H, F) = F^{-1}(0)$. Thus the result follows from Theorem 3.1. □

Now, in order to obtain direct consequences of Theorem 3.1, we recall special cases of the GMEP again.

If $\varphi = 0$ in (1.1), then the GMEP reduces the following generalized equilibrium problem (for short, GEP) of finding $x \in C$ such that

$$\Theta(x, y) + \langle Bx, y - x\rangle \geq 0, \quad \forall y \in C.$$

The set of solutions of the GEP is denoted by $GEP(\Theta, B)$.

If $B = 0$ in (1.1), then the GMEP reduces the following mixed equilibrium problem (for short, MEP) of finding $x \in C$ such that

$$\Theta(x, y) + \varphi(y) - \varphi(x) \geq 0, \ \ \forall y \in C.$$

The set of solutions of the MEP is denoted by $MEP(\Theta, \varphi)$.

If $B = 0$ and $\varphi = 0$ in (1.1), then the GMEP reduces the following equilibrium problem (for short, EP) of finding $x \in C$ such that

$$\Theta(x, y) \geq 0, \ \ \forall y \in C.$$

The set of solutions of the EP is denoted by $EP(\Theta)$.

If we take $\varphi \equiv 0$ in Theorem 3.1, then we obtain the following result.

Corollary 3.3. *Let* $\Omega_3 := GEP(\Theta, B) \cap VI(C,F) \cap Fix(T) \neq \emptyset$*, and let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by* $x_1 \in C$ *and*

$$\begin{cases} \Theta(u_n, y) + \langle Bu_n, y - u_n\rangle + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_{r_n}F_{r_n}u_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ *and* $\{u_n\}$ *converge weakly to* $z \in \Omega_3$*, where* $z = \lim_{n\to\infty} P_{\Omega_3}(x_n)$.

If $F \equiv 0$, then F_r in Lemma 2.4 is the identity mapping. Thus, from Corollary 3.3, we have the following corollary.

Corollary 3.4. *Let* $\Omega_4 := GEP(\Theta, B) \cap Fix(T) \neq \emptyset$*, and let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by* $x_1 \in C$ *and*

$$\begin{cases} \Theta(u_n, y) + \langle Bu_n, y - u_n\rangle + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_{r_n}u_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ *and* $\{u_n\}$ *converge weakly to* $z \in \Omega_4$*, where* $z = \lim_{n\to\infty} P_{\Omega_4}(x_n)$.

It we take $B \equiv 0$ and $\varphi \equiv 0$ in Theorem 3.1, we obtain the following result.

Corollary 3.5. *Let* $\Omega_5 := EP(\Theta) \cap VI(C,F) \cap Fix(T) \neq \emptyset$*, and let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by* $x_1 \in C$ *and*

$$\begin{cases} \Theta(u_n, y) + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_{r_n}F_{r_n}u_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ *and* $\{u_n\}$ *converge weakly to* $z \in \Omega_5$*, where* $z = \lim_{n\to\infty} P_{\Omega_5}(x_n)$.

It we take $F \equiv 0$ in Corollary 3.5, we obtain the following corollary.

Corollary 3.6. *Let* $\Omega_6 := EP(\Theta) \cap Fix(T) \neq \emptyset$*, and let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by* $x_1 \in C$ *and*

$$\begin{cases} \Theta(u_n, y) + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_{r_n}u_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ *and* $\{u_n\}$ *converge weakly to* $z \in \Omega_6$*, where* $z = \lim_{n\to\infty} P_{\Omega_6}(x_n)$.

If we take $B \equiv 0$ in Theorem 3.1, we get the following result.

Corollary 3.7. *Let* $\Omega_7 := MEP(\Theta, \varphi) \cap VI(C,F) \cap Fix(T) \neq \emptyset$*, and let* $\{x_n\}$ *and* $\{u_n\}$ *be sequences generated by* $x_1 \in C$ *and*

$$\begin{cases} \Theta(u_n, y) + \varphi(y) - \varphi(u_n) + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1-\alpha_n)T_{r_n}F_{r_n}u_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ *and* $\{u_n\}$ *converge weakly to* $z \in \Omega_7$*, where* $z = \lim_{n\to\infty} P_{\Omega_7}(x_n)$.

If we take $F \equiv 0$ in Corollary 3.7, we obtain the following corollary.

Corollary 3.8. *Let $\Omega_8 := MEP(\Theta, \varphi) \cap Fix(T) \neq \emptyset$, and let $\{x_n\}$ and $\{u_n\}$ be sequences generated by $x_1 \in C$ and*

$$\begin{cases} \Theta(u_n, y) + \varphi(y) - \varphi(u_n) + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \geq 0, \quad \forall y \in C, \\ x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T_{r_n} u_n, \quad \forall n \geq 1. \end{cases}$$

Then $\{x_n\}$ and $\{u_n\}$ converge weakly to $z \in \Omega_8$, where $z = \lim_{n\to\infty} P_{\Omega_8}(x_n)$.

Remark 3.9. 1) For finding a common element of $GMEP(\Theta, \varphi, B) \cap VI(C, F) \cap Fix(T)$, where B is a continuous monotone mapping, F is a continuous monotone mapping, and T is a continuous pseudocontractive mapping, Theorem 3.1 is a new ones different from previous those introduced by several authors. Consequently, in the sense that our convergence is for the more general class of continuous monotone and continuous pseudocontractive mappings, our results improve, develop and complement the corresponding results, which were obtained recently by several authors in references; for example, see [5, 18, 25, 29, 31] and references therein.

2) We recall some special cases of the GMEP as follows:

(i) If $\Theta(x, y) = 0$ for all $x, y \in C$ in (1.1), the GMEP reduces the following generalized variational inequality problem (for short, GVI) of finding $x \in C$ such that

$$\langle Bx, y - x\rangle + \varphi(y) - \varphi(x) \geq 0, \quad \forall y \in C.$$

(ii) If $B = 0$ and $\Theta(x, y) = 0$ for all $x, y \in C$ in (1.1), the GMEP reduces the following minimization problem (for short, MP) finding $x \in C$ such that

$$\varphi(y) - \varphi(x) \geq 0, \quad \forall y \in C.$$

Applying Theorem 3.1, we can also establish the new corresponding results for the GVI and the MP.

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