Degrees of Freedom of 3-user MIMO Interference Channels with Instantaneous Relay Using Interference Alignment

Wang Qiang, Shu Yuquan, Dong Minhua, Xu Ji, Tao Xiaofeng
National Engineering Laboratory for Mobile Network Security, Beijing University of Posts and Telecommunications, Beijing, 100876, China
[email: wangq@bupt.edu.cn]
*Corresponding author: Wang Qiang

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Abstract

Instantaneous relay (relay-without-delay) using interference alignment is a promising approach to neutralizing interference and improving system capacity. In Wang Chenwei's work, a 2-user scenario required both source and relay to access the global channel state information (CSI). This paper shows a new method of interference alignment improves the degrees of freedom (DoF) prominently for the 3-user MIMO interference channel with instantaneous relay. This new method is focused on the relay node that completes the alignment interference neutralization so the global CSI is obtained only once and the pressure on the base station can be mitigated. In addition, the 3-user MIMO interference channels with instantaneous relay can achieve $2M$ DoF when source and destination have M antennas, respectively. This method shows 33% improvement over the conventional method using interference alignment which obtains $3M/2$ DoF.

Keywords: Interference Alignment, Neutralization, Instantaneous Relay, Degrees of Freedom, energy efficient.

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1. Introduction

The capacity characterization analysis of interference channels has been an unresolved problem for decades. In interference channels, source nodes and destination nodes communicate through a common medium, signals interfere with each other, and destination nodes may be unable to separate the signals from the interference, leading to degradation of capacity. Because interference is a major factor affecting system performance, interference management is crucial in a wireless system. Researchers have been investigating various techniques to deal with interference. Among these, interference alignment provides a new perspective to manage interference. Interference alignment, which has attracted great attention, was proposed by Jafar and Cadambe [1]. Various schemes based on interference alignment have been proposed in different networks.

The purpose of interference alignment is to achieve optimal degrees of freedom (DoF) and system capacity. In Cadambe and Jafar [1], the multiple-input and multiple-output (MIMO) X channel is investigated. $4M/3$ DoF is achieved by aligning interference, where M is the number of antennas at each node. This result is surprising when compared with 2-user interference channels without interference alignment which can achieve only M DoF [2]. Then Sezgin, Avestimehr, and Hassibi [3] introduce interference alignment in a K-user scenario, showing that $K/2$ DoF can be achieved in the K-user time-varying interference channel. Furthermore, in [4], authors introduce an inner-bound and an outer-bound for the total number of DoF of K-user MIMO Gaussian interference channel. However, most existing IA schemes result in significant capacity overhead in the feedback link because IA requires global channel state information (CSI) at the transmitter. In Zhang et al. [5], investigate the IA scheme using differential CSI feedback over time-correlated MIMO channels and analyze the achievable sum-rate performance using differential feedback.

Wireless relay technique has been known to provide improvements in link reliability and spectral efficiency. A great deal of research has gone into obtaining the DoF of relay networks using interference alignment [6, 7]. The DoF of two-hop interference channels equipped with sufficient antennas as relay is discussed in Tannious and Nosratinia [8], and Shomorony and Avestimehr [9]. In Al-Shatri and Weber [10], a K-user scenario with multiple single-antenna relays is considered where the number of distributed relays is large enough. Wang and Skoglund [2] show that the K-user half-duplex relay system has K DoF when the number of relay is infinitely large. The result shows that distributed processing can provide the same capacity performance as joint processing. Further, a two- way relay system has drawn much attention recently [3,4]. The alignment of interferences in a two-way relay system with a single antenna is explored in Sezgin et al. [3]. Linear beamforming is used to align interference in the system with multiple antennas[4]. The two-way relay system has been extended to different systems such as bi-directional multi-pair exchange systems [11, 12] and multi-directional multi-pair exchange systems [13].

Another concept in interference research is interference neutralization [14]. It is a technique of neutralizing interfering signals by carefully selecting of relay-forwarding strategies. The general idea is applied in deterministic channel [15] and two-hop relay channel systems[16, 17].

Because conventional relays are not shown to increase the DoF of the interference channel. Namyoon Lee[18] explores instantaneous relay in a multi-relay network. However, only a 2-user scenario has been researched for the instantaneous relay network, and 3-user communication with instantaneous relay has not been discussed in detail. The interference of users at the junction of three cells is relatively serious, and three-cell cooperation to eliminate the interference is a common scenario. Thus, we manage the interference by through instantaneous relay for a 3-user system, which is important for future wireless communication systems, and the achievable DoF can be still improved with new schemes. Therefore, it is necessary to research new methods to improve the DoF for instantaneous relay scenarios with more users.

1.1 Main Contributions

The main contribution of this paper is to introduce a new method to improve the DoF of a 3-user interference channel with instantaneous relay. The primary difference from Wang Chenwei's work is that we focus on the relay node to complete the alignment interference neutralization, so we need to obtain the CSI only once. This method can relieve the pressure on the base station. It is shown that $2M$ DoF is achievable for the scheme, with every source and destination being equipped with $M \geq 1$ antennas. The method is also different from previous methods in that interference alignment is used at destinations, not only eliminating interference but also reducing the number of antennas needed at destinations.

1.2 Organization and Notation

The rest of this paper is organized as follows. Section 2 includes a description of the system model and signal model with instantaneous relay. Section 3 addresses the achievable DoF of the proposed 3-user MIMO interference channel with instantaneous relay. Then, a method aimed at improving sum-rate is proposed in Section 4. Numerical results are proposed in Section 5. Finally, the conclusion is presented in Section 6.

First, we need to introduce a few notations. Uppercase boldface letters denote matrices and lowercase boldface letters denote vectors. Similarly, span(**X**), rank(**X**), and vec(**X**) denote the column space, rank, and vector, respectively, obtained by stacking the columns of matrix **X**. Superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ denote transpose, Hermitian transpose, and matrix inversion respectively. $(\cdot)^+$ sets the negative elements of a vector to zero, and $\mathbf{I}_n$ denotes n-D identity matrix.

2. System Model

In [18], the authors explore instantaneous relay in a multi-relay network. They choose some relays, that can gather CSI, to be intelligent relays while the rest of the relays remain dumb relays. The intelligent relays are able to estimate received signals, and dumb relays work like amplifiers. The dumb relays' links can be viewed as direct links between the sources and destinations, whereas signals passing through intelligent relays' links arrive at their destinations simultaneously, with signals passing through virtual direct links. The intelligent relays can be regarded as having the instantaneous feature. Therefore, instantaneous relay is naturally proposed in a multi-relay two-hop network.

According to Namyoon Lee's contribution in [18], the channel model consists of three sources, three destinations, and an instantaneous relay, as shown in **Fig. 1**. Global channel knowledge is assumed at all seven nodes, with the notable exception that the source nodes need not know the channel coefficients of the direct channels to the destination nodes. If each source and destination has M antennas and each source sends d data symbols, considering the relay can detect all data symbols sent by sources, the relay needs $3d$ antennas at least. The relay can receive signals from all sources and forward signals to all destinations instantaneously.

During the process of transmission, source i sends signal vector $\mathbf{x}^{[i]}(k)$ to its corresponding destination at time slot k. $\mathbf{x}^{[i]}(k)$ is precoded as $\mathbf{x}^{[i]}(k)=\mathbf{V}^{[i]}(k)\mathbf{s}^{[i]}(k)$, where $\mathbf{V}^{[i]}$ represents the beamforming matrix $\mathbf{V}^{[i]}(\mathrm{k})=[\mathbf{v}_1^{[i]}(k)\,\mathbf{v}_2^{[i]}(k)...\mathbf{v}_d^{[i]}(k)]$ and $\mathbf{s}^{[i]}(k)$ denotes the d data symbol vectors $\mathbf{s}^{[\mathrm{i}]}(\mathrm{k})=[\mathbf{s}_1^{[\mathrm{i}]}(\mathrm{k})\,\mathbf{s}_2^{[\mathrm{i}]}(\mathrm{k})...\mathbf{s}_d^{[\mathrm{i}]}(\mathrm{k})]^{\mathrm{T}}$. Assuming that each data symbol has unit variance and each user has the average power constraint $\mathrm{E}\left\{\mathrm{tr}[\mathbf{x}^{[i]}(k)\mathbf{x}^{[i]}(k)^{\mathrm{H}}]\right\}\le P$, where P is the constraint power of transmission, the received signal at the relay at time slot k is obtained as follows:

$$\mathbf{y}^{[R]}(k)=\sum_{i=1}^{3}\mathbf{H}^{[R,i]}(k)\mathbf{x}^{[i]}(k)+\mathbf{n}^{[R]} \qquad (1)$$

where $\mathbf{H}^{[R,i]}(k)$ denotes the channel matrix from source i to relay and $\mathbf{n}^{[R]}$ stands for the additive white Gaussian noise (AWGN) vector with zero mean and unit variance at relay. Because it is assumed that the relay has the ability to decode and forward all received signals, after receiving the signal $\mathbf{y}^{[R]}(k)$, the relay generates the new transmitting signal $\mathbf{x}^{[R]}(k)$ as $\mathbf{x}^{[R]}(k)=\mathbf{V}^{[R]}(k)\mathbf{s}^{[R]}(k)$, where $\mathbf{V}^{[R]}(k)=[\mathbf{v}_1^{[R]}(k)\,\mathbf{v}_2^{[R]}(k)...\mathbf{v}_{3d}^{[R]}(k)]$ and $\mathbf{s}^{[R]}(k)=[\mathbf{s}_1^{[R]}(k)\,\mathbf{s}_2^{[R]}(k)...\mathbf{s}_{3d}^{[R]}(k)]^{\mathrm{T}}$. $\mathbf{x}^{[R]}(k)$ must satisfy the relay power constraint $\mathrm{E}\left\{\mathrm{tr}[\mathbf{x}^{[R]}(k)\mathbf{x}^{[R]}(k)^{\mathrm{H}}]\right\}\le P$. Because of the feature of the interference channel, the received signal at base stations is denoted as follows

$$\mathbf{y}^{[j]}(k)=\sum_{i=1}^{3}\mathbf{H}^{[j,i]}(k)\mathbf{x}^{[i]}(k)+\mathbf{H}^{[j,R]}(k)\mathbf{x}^{[R]}(k)+\mathbf{n}^{[j]}\,. \qquad (2)$$

where $\mathbf{H}^{[j,i]}(k)$ denotes an $\mathrm{M}\times\mathrm{M}$ channel matrix from source i to destination j. $\mathbf{H}^{[j,R]}(k)$ denotes an $\mathrm{M}\times 3d$ channel matrix from relay to destination j. $\mathbf{n}^{[j]}$ represents the additive white Gaussian noise (AWGN) vector with zero mean and unit variance at destination j.

It is also assumed that the decoding matrix of destination j is $\mathbf{W}^{[j]}(k)$, so the output signal is

$$\hat{\mathbf{x}}^{[j]}(k)=\mathbf{W}^{[j]}(k)^{\mathrm{H}}\mathbf{y}^{[j]}(k) . \tag{3}$$

In this paper, it is assumed that the generic channel matrices have the feature that all channel elements are generated from an independently and identically distributed complex Gaussian distribution with zero mean and unit variance. It is also assumed that the CSI is perfectly known at all nodes.

3. Achievable DoF of The Proposed Scheme

The proposed scheme is shown in **Fig. 2**, where half of the interference signals are neutralized and the other half are aligned in a small subspace.

To achieve M DoF, an equal DoF is assigned to each pair of sources, as $d_1=d_2=d_3=d=\frac{2\mathrm{M}}{3}$. Thus, $2\mathrm{M}$ antennas are needed in the relay. For arbitrary M, we consider a 3-symbol extension, so that we obtain a $3\mathrm{M}\times 3\mathrm{M}$ effective channel from sources to destinations. Each $\mathrm{M}\times\mathrm{M}$ channel matrix is repeated 3 times to provide a $3\mathrm{M}\times 3\mathrm{M}$ block-diagonal matrix. The $3\mathrm{M}\times 3\mathrm{M}$ channel matrix after the 3-symbol extension is defined as follows.

$$\bar{\mathbf{H}}^{[j,i]}=\begin{bmatrix}\mathrm{H}^{[j,i]} & 0 & 0\\ 0 & \mathrm{H}^{[j,i]} & 0\\ 0 & 0 & \mathrm{H}^{[j,i]}\end{bmatrix} . \tag{4}$$

Thus, we can obtain the $6\mathrm{M}\times 3\mathrm{M}$ effective channel matrix from the source i to relay, which is $\bar{\mathbf{H}}^{[R,i]}$, and the $3\mathrm{M}\times 6\mathrm{M}$ effective channel matrix from relay to the destination j, which is $\bar{\mathbf{H}}^{[j,R]}$.

At time slot k, the source i sends message $\mathbf{x}^{[i]}(k)$ which is a $3\mathrm{M}\times 1$ transmitted vector to its corresponding destination. The 3-symbol extension vector $\mathbf{x}^{[i]}$ is extended to

$$\bar{\mathbf{x}}^{[i]}=\begin{bmatrix}\mathbf{x}^{[i]}(3k)\\ \mathbf{x}^{[i]}(3k+1)\\ \mathbf{x}^{[i]}(3k+2)\end{bmatrix} . \tag{5}$$

Then the transmitted message is divided into two sub-messages $\bar{\mathbf{x}}_1^{[i]},\bar{\mathbf{x}}_2^{[i]}$. Each sub-message is a $3\mathrm{M}\times 1$ transmitted vector. An $\bar{\mathbf{x}}_m^{[i]}$ is encoded by $\mathbf{s}_{m,1}^{[i]},\ldots,\mathbf{s}_{m,d}^{[i]}$ using the

Gaussian codebook. The beamforming vector of $\mathbf{s}_{mp}^{i}$ is described as $\mathbf{v}_{mp}^{i}$. Then the signal transmitted by source i is

$$\begin{aligned}\overline{\mathbf{x}}^{[i]} &= \sum_{p=1}^{d} \mathbf{v}_{1,p}^{[i]}\mathbf{s}_{1,p}^{[i]} + \sum_{p=1}^{d} \mathbf{v}_{2,p}^{[i]}\mathbf{s}_{2,p}^{[i]} \\ &= \mathbf{V}_{1}^{[i]}\mathbf{s}_{1}^{[i]} + \mathbf{V}_{2}^{[i]}\mathbf{s}_{2}^{[i]} \end{aligned} \quad (6)$$

where $\mathbf{V}_{m}^{[i]} = [\mathbf{v}_{m,1}^{[i]},\dots,\mathbf{v}_{m,d}^{[i]}]$, $\mathbf{s}_{m}^{[i]} = [\mathbf{s}_{m,1}^{[i]},\dots,\mathbf{s}_{m,d}^{[i]}]^{\mathrm{T}}$, p denote the the number of transmitted signals at the source. $p=1,2,\dots\dots,d$, and $m \in \{1,2\}$, $i \in \{1,2,3\}$, $d = \frac{2\mathrm{M}}{3}$.

3.1 Constraints at Relay

The received signal at instantaneous relay is

$$\mathbf{y}^{[R]} = \sum_{i=1}^{3}\sum_{m=1}^{2} \overline{\mathrm{H}}^{[R,i]}\mathrm{V}_{m}^{[i]}\mathbf{s}_{m}^{[i]} + \mathrm{n}^{[R]}. \quad (7)$$

Because the instantaneous relay is equipped with $2\mathrm{M}$ antennas, it can decode all the $6\mathrm{M}$ data symbols from the three sources by zero forcing over the 3-symbol extension channel. After estimating transmitting symbols, the relay generates data symbols which will send at relay as

$$\begin{aligned} &\mathbf{s}_{1,r}^{[R]} = \mathbf{s}_{1,r}^{[1]} \quad \mathbf{s}_{2,r}^{[R]} = \mathbf{s}_{2,r}^{[1]} \\ &\mathbf{s}_{3,r}^{[R]} = \mathbf{s}_{1,r}^{[2]} \quad \mathbf{s}_{4,r}^{[R]} = \mathbf{s}_{2,r}^{[2]} \\ &\mathbf{s}_{5,r}^{[R]} = \mathbf{s}_{1,r}^{[3]} \quad \mathbf{s}_{6,r}^{[R]} = \mathbf{s}_{2,r}^{[3]} \end{aligned} \quad (8)$$

The transmitted signals at relay are denoted as

$$\begin{aligned}\mathbf{x}^{[R]} &= \sum_{r=1}^{3d}\sum_{q=1}^{6} \mathbf{v}_{q,r}^{[R]}\mathbf{s}_{q,r}^{[R]} \\ &= \sum_{q=1}^{6} \mathbf{V}_{q}^{[R]}\mathbf{s}_{q}^{[R]} \end{aligned} \quad (9)$$

where $\mathbf{V}_{n}^{[R]} = [\mathbf{v}_{n,1}^{[R]},\dots,\mathbf{v}_{n,3d}^{[R]}]$, $\mathbf{s}_{n}^{[i]} = [\mathbf{s}_{n,1}^{[R]},\dots,\mathbf{s}_{n,3d}^{[R]}]^{\mathrm{T}}$, and $q \in \{1,2,3,4,5,6\}$, $r \in \{1,2,\dots\dots,3d\}$, $3d = 2\mathrm{M}$. The proposed scheme uses an instantaneous relay to neutralize half of the interfering data symbols, Through the designed relay we expect to achieve the following goals

1. Neutralize $\mathbf{s}_{1}^{[2]}$, $\mathbf{s}_{1}^{[3]}$ at D_1.
2. Neutralize $\mathbf{s}_{1}^{[1]}$, $\mathbf{s}_{2}^{[3]}$ at D_2.
3. Neutralize $\mathbf{s}_{2}^{[1]}$, $\mathbf{s}_{2}^{[2]}$ at D_3.

Thus, beamforming vectors should satisfy the following constraints

At D_1:

$$\begin{aligned} \mathbf{v}_{3,r}^{[R]} &= -(\bar{\mathbf{H}}^{[1,R]})^{-1}\bar{\mathbf{H}}^{[1,2]}\mathbf{v}_{1,r}^{[2]} \\ \mathbf{v}_{5,r}^{[R]} &= -(\bar{\mathbf{H}}^{[1,R]})^{-1}\bar{\mathbf{H}}^{[1,3]}\mathbf{v}_{1,r}^{[3]} , \end{aligned} \quad (10)$$

At D_2 :

$$\begin{aligned} \mathbf{v}_{1,r}^{[R]} &= -(\bar{\mathbf{H}}^{[2,R]})^{-1}\bar{\mathbf{H}}^{[2,1]}\mathbf{v}_{1,r}^{[1]} \\ \mathbf{v}_{6,r}^{[R]} &= -(\bar{\mathbf{H}}^{[2,R]})^{-1}\bar{\mathbf{H}}^{[2,3]}\mathbf{v}_{2,r}^{[3]} , \end{aligned} \quad (11)$$

At D_3 :

$$\begin{aligned} \mathbf{v}_{2,r}^{[R]} &= -(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[3,1]}\mathbf{v}_{2,r}^{[1]} \\ \mathbf{v}_{4,r}^{[R]} &= -(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[3,2]}\mathbf{v}_{2,r}^{[2]} . \end{aligned} \quad (12)$$

3.2 Source Node

Destinations can receive signals not only from sources but also from relays. After neutralization, we remove $2M$ interfering symbols at each destination, and the remaining $2M$ interfering symbols at destinations are as follows.

At D_1 :

$\bar{\mathbf{H}}^{[1,2]}\mathbf{V}_2^{[2]}\mathbf{s}_2^{[2]}, \bar{\mathbf{H}}^{[1,3]}\mathbf{V}_2^{[3]}\mathbf{s}_2^{[3]}, \bar{\mathbf{H}}^{[1,R]}\mathbf{V}_4^{[R]}\mathbf{s}_4^{[R]}, \bar{\mathbf{H}}^{[1,R]}\mathbf{V}_6^{[R]}\mathbf{s}_6^{[R]}$.

At D_2 :

$\bar{\mathbf{H}}^{[2,1]}\mathbf{V}_2^{[1]}\mathbf{s}_2^{[1]}, \bar{\mathbf{H}}^{[2,3]}\mathbf{V}_1^{[3]}\mathbf{s}_1^{[3]}, \bar{\mathbf{H}}^{[2,R]}\mathbf{V}_2^{[R]}\mathbf{s}_2^{[R]}, \bar{\mathbf{H}}^{[2,R]}\mathbf{V}_5^{[R]}\mathbf{s}_5^{[R]}$.

At D_3 :

$\bar{\mathbf{H}}^{[3,1]}\mathbf{V}_1^{[1]}\mathbf{s}_1^{[1]}, \bar{\mathbf{H}}^{[2,2]}\mathbf{V}_1^{[2]}\mathbf{s}_1^{[2]}, \bar{\mathbf{H}}^{[2,R]}\mathbf{V}_1^{[R]}\mathbf{s}_1^{[R]}, \bar{\mathbf{H}}^{[2,R]}\mathbf{V}_3^{[R]}\mathbf{s}_3^{[R]}$.

By using (10)-(12), the interfering symbols at destinations can be written as follows.

At D_1 :

$(\bar{\mathbf{H}}^{[1,2]} - \bar{\mathbf{H}}^{[1,R]}(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[3,2]})\mathbf{V}_2^{[2]}\mathbf{s}_2^{[2]}$ and

$(\bar{\mathbf{H}}^{[1,3]} - \bar{\mathbf{H}}^{[1,R]}(\bar{\mathbf{H}}^{[2,R]})^{-1}\bar{\mathbf{H}}^{[3,2]})\mathbf{V}_2^{[3]}\mathbf{s}_2^{[3]}$.

At D_2 :

$(\bar{\mathbf{H}}^{[2,1]} - \bar{\mathbf{H}}^{[2,R]}(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[3,1]})\mathbf{V}_2^{[1]}\mathbf{s}_2^{[1]}$ and

$(\bar{\mathbf{H}}^{[2,3]} - \bar{\mathbf{H}}^{[2,R]}(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[1,3]})\mathbf{V}_1^{[3]}\mathbf{s}_1^{[3]}$.

At D_3 :

$(\bar{\mathbf{H}}^{[3,1]} - \bar{\mathbf{H}}^{[3,R]}(\bar{\mathbf{H}}^{[2,R]})^{-1}\bar{\mathbf{H}}^{[2,1]})\mathbf{V}_1^{[1]}\mathbf{s}_1^{[1]}$ and

$(\bar{\mathbf{H}}^{[3,2]} - \bar{\mathbf{H}}^{[3,R]}(\bar{\mathbf{H}}^{[1,R]})^{-1}\bar{\mathbf{H}}^{[1,2]})\mathbf{V}_1^{[2]}\mathbf{s}_1^{[2]}$.

Reducing signal dimensions of the interfering symbols, by designing $\mathbf{V}_m^1$, $\mathbf{V}_m^2$, $\mathbf{V}_m^3$, $m \in \{1, 2\}$, the proposed scheme aligns interference into a smaller signal dimension at D_j,

$j \in \{1, 2, 3\}$. The source strategy is to select beamforming vectors to satisfy the following constrains:

$$\begin{aligned} &\text{span}[(\bar{\mathbf{H}}^{[1,2]} - \bar{\mathbf{H}}^{[1,R]}(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[3,2]})\mathbf{V}_2^{[2]}] \\ &= \text{span}[(\bar{\mathbf{H}}^{[1,3]} - \bar{\mathbf{H}}^{[1,R]}(\bar{\mathbf{H}}^{[2,R]})^{-1}\bar{\mathbf{H}}^{[3,2]})\mathbf{V}_2^{[3]}] \\ &\text{span}[(\bar{\mathbf{H}}^{[2,1]} - \bar{\mathbf{H}}^{[2,R]}(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[3,1]})\mathbf{V}_2^{[1]}] \\ &= \text{span}[(\bar{\mathbf{H}}^{[2,3]} - \bar{\mathbf{H}}^{[2,R]}(\bar{\mathbf{H}}^{[3,R]})^{-1}\bar{\mathbf{H}}^{[1,3]})\mathbf{V}_1^{[3]}] \\ &\text{span}[(\bar{\mathbf{H}}^{[3,1]} - \bar{\mathbf{H}}^{[3,R]}(\bar{\mathbf{H}}^{[2,R]})^{-1}\bar{\mathbf{H}}^{[2,1]})\mathbf{V}_1^{[1]}] \\ &= \text{span}[(\bar{\mathbf{H}}^{[3,2]} - \bar{\mathbf{H}}^{[3,R]}(\bar{\mathbf{H}}^{[1,R]})^{-1}\bar{\mathbf{H}}^{[1,2]})\mathbf{V}_1^{[2]}]. \end{aligned} \tag{13}$$

3.3 Destination Node

Because the beamforming vectors designed by (10)-(12) at instantaneous relay can neutralize $2M$ interference, and beamforming vectors obtained by using (13) at the source to align the rest of the interferences into M-dimension space, all the data symbols at the destinations can occupy $3M$ signal dimensions. Because all channel metrics are linearly independent, each destination can obtain $2M$ desired symbols by applying the zero-forcing method over a 3-symbol extension channel. Therefore, each user can obtain $2M/3$ DoF. Then, the system can attain $2M$ DoF. The resulting DoF is higher than $2M/3$, a desired result, and can be achieved by 3-user MIMO interference channels without instantaneous relay.

4. The Algorithm Using Interference Neutralization and Alignment

Based on the proposed scheme, a joint transmitting and receiving vectors design scheme for interference channel systems is proposed, the aim of which is to improve the sum-rate. The system has 3-pair sources and a 6-antenna relay, and each source is equipped with three antennas. The vectors design can be generally separated into two parts. One is interference management, in which interference neutralization and alignment will be used. The other is sum-rate improvement, in which a maximum chordal distance criterion will be used.

4.1 Interference Neutralization and Alignment

Because a relay can receive signals from all sources, the signal received at the relay is given by

$$\mathbf{y}_R = \mathbf{H}_1\mathbf{P}_1\mathbf{S}_1 + \mathbf{H}_2\mathbf{P}_2\mathbf{S}_2 + \mathbf{H}_3\mathbf{P}_3\mathbf{S}_3 + \mathbf{n}, \tag{14}$$

where $\mathbf{n}$ denotes the i.i.d. complex Gaussian noise vector at relay with zero mean and $\mathrm{E}\left(\mathbf{nn}^{\mathrm{H}}\right) = \sigma_{\mathbf{n}}^2 \mathrm{I}$. H_i is the 6×3 full rank channel matrix whose elements are i.i.d. Distributed as complex Gaussian. $\mathbf{P}_i$ is the 3×2 precoding matrix which satisfies the transmitting power constraint $\mathrm{Tr}(\mathbf{P}_i^{\mathrm{H}}\mathbf{P}_i) \le p$. We recognize the singular value decomposition (SVD) of $\mathbf{H}_3\mathbf{P}_3$ as

$$\mathbf{H}_3\mathbf{P}_3=\mathbf{U}_1\mathbf{\Lambda}_1\mathbf{V}_1^{\mathrm{H}}, \tag{15}$$

We define the matrix $\mathbf{U}_1$ as

$$\mathbf{U}_1=\begin{bmatrix}\mathbf{U}_{6\times2}^1 & \mathbf{U}_{6\times4}^2\end{bmatrix}. \tag{16}$$

where the matrix $\mathbf{U}_{6\times2}^1$ is composed of the first two left singular vectors and the matrix $\mathbf{U}_{6\times4}^2$ holds the last four left singular vectors. Then, the matrix that completely eliminates the signal from the third source becomes $\mathbf{M}_1=\begin{bmatrix}\mathbf{U}_{6\times4}^2\end{bmatrix}^{\mathrm{H}}$. Because $\mathbf{U}_1$ is a unitary matrix, we obtain the following results:

$$\mathbf{M}_1\mathbf{U}_1=[\mathbf{0}_{4\times2} \quad \mathbf{I}_{4\times4}], \tag{17}$$

where $\mathbf{0}_{4\times2}$ is a zero matrix. Then, we can easily obtain

$$\mathbf{M}_1\mathbf{U}_1\mathbf{\Lambda}_1=\mathbf{0}. \tag{18}$$

Multiplying (18) and (14), the received signal can be written as

$$\mathbf{y}_R^1=\mathbf{M}_1\mathbf{y}_R=\mathbf{M}_1\mathbf{H}_1\mathbf{P}_1S_1+\mathbf{M}_1\mathbf{H}_2\mathbf{P}_2S_2+\mathbf{M}_1\mathbf{n}. \tag{19}$$

Next, the SVD of $\mathbf{M}_1\mathbf{H}_2\mathbf{P}_2$ is denoted as $\mathbf{U}_2\mathbf{\Lambda}_2\mathbf{V}_2^{\mathrm{H}}$, and $\mathbf{U}_2$ is denoted as

$$\mathbf{U}_2=\begin{bmatrix}\mathbf{U}_{4\times2}^1 & \mathbf{U}_{4\times2}^2\end{bmatrix}. \tag{20}$$

In the same manner as how we deal with $\mathbf{M}_1$, we multiply $\mathbf{M}_2$ to $\mathbf{y}_R^1$, where $\mathbf{M}_2=\begin{bmatrix}\mathbf{U}_{4\times2}^2\end{bmatrix}^{\mathrm{H}}$, obtaining

$$\mathbf{y}_R^2=\mathbf{M}_2\mathbf{y}_R^1=\mathbf{M}_2\mathbf{M}_1\mathbf{H}_1\mathbf{P}_1\mathbf{S}_1+\mathbf{M}_2\mathbf{M}_1\mathbf{n}, \tag{21}$$

Then, we decompose **QR** decomposition to $\mathbf{M}_2\mathbf{M}_1\mathbf{H}_1\mathbf{P}_1$,

$$\mathbf{M}_2\mathbf{M}_1\mathbf{H}_1\mathbf{P}_1=\mathbf{QR}, \tag{22}$$

where **Q** is a unitary matrix and **R** is an upper triangular matrix. We multiply $\mathbf{Q}^{\mathrm{H}}$ by (21):

$$\mathbf{Q}^{\mathrm{H}}\mathbf{y}_R^2=\mathbf{R}\mathbf{S}_1+\mathbf{Q}^{\mathrm{H}}\mathbf{M}_2\mathbf{M}_1\mathbf{n}. \tag{23}$$

Because of the characteristics of the upper triangular matrix, the second element of S_1 can easily be found. Then we use it to calculate the first element. Finally, symbols transmitted by the first source node are obtained. Use S_1 to (19)

$$\mathbf{y}_R^3=\mathbf{y}_R^1-\mathbf{M}_1\mathbf{H}_1\mathbf{P}_1\mathbf{S}_1=\mathbf{M}_1\mathbf{H}_2\mathbf{P}_2\mathbf{S}_2+\mathbf{M}_1\mathbf{n}, \tag{24}$$

The symbols sent by the second source can also be calculated using the same method. Then, we use $\mathbf{S}_1$ and $\mathbf{S}_2$ with (14)

$$\mathbf{y}_R^4=\mathbf{y}_R-\mathbf{H}_1\mathbf{P}_1\mathbf{S}_1-\mathbf{H}_2\mathbf{P}_2\mathbf{S}_2=\mathbf{H}_3\mathbf{P}_3\mathbf{S}_3+\mathbf{n}. \tag{25}$$

By using the **QR**-method, S_3 can be found.

Finally, symbols sent by sources can all be calculated. Then, the relay sends the symbols to destinations with proper transmitting vectors to neutralize interference, as discussed in Section 3.

As noted in Section 3, the purpose of the algorithm is to improve the sum-rate, which is calculated as follows. Making the transmitting vectors of the relay $\mathbf{V}_n^{[R]}$ $n = 1, 2 \cdots 6$ satisfies (10)-(12), two interferences can be neutralized in every destination, so after interference-neutralization the following is received by the destinations

$$\begin{aligned}\mathbf{y}^{[1]} = (\mathbf{H}^{[1,1]}\mathbf{V}_1^{[1]} + \mathbf{H}^{[1,R]}\mathbf{V}_1^{[R]})\mathbf{s}_1^{[1]} + (\mathbf{H}^{[1,1]}\mathbf{V}_2^{[1]} + \\ \mathbf{H}^{[1,R]}\mathbf{V}_2^{[R]})\mathbf{s}_2^{[1]} + (\mathbf{H}^{[1,2]}\mathbf{V}_2^{[2]} + \mathbf{H}^{[1,R]}\mathbf{V}_4^{[R]})\mathbf{s}_2^{[2]} \\ +(\mathbf{H}^{[1,3]}\mathbf{V}_2^{[3]} + \mathbf{H}^{[1,R]}\mathbf{V}_6^{[R]})\mathbf{s}_2^{[3]} + \mathbf{n}_1\end{aligned}, \quad (26)$$

$$\begin{aligned}\mathbf{y}^{[2]} = (\mathbf{H}^{[2,2]}\mathbf{V}_1^{[2]} + \mathbf{H}^{[2,R]}\mathbf{V}_3^{[R]})\mathbf{s}_1^{[2]} + (\mathbf{H}^{[2,2]}\mathbf{V}_2^{[2]} + \\ \mathbf{H}^{[2,R]}\mathbf{V}_4^{[R]})\mathbf{s}_2^{[2]} + (\mathbf{H}^{[2,1]}\mathbf{V}_2^{[1]} + \mathbf{H}^{[2,R]}\mathbf{V}_2^{[R]})\mathbf{s}_2^{[1]} \\ +(\mathbf{H}^{[2,3]}\mathbf{V}_1^{[3]} + \mathbf{H}^{[2,R]}\mathbf{V}_5^{[R]})\mathbf{s}_1^{[3]} + \mathbf{n}_2\end{aligned}, \quad (27)$$

$$\begin{aligned}\mathbf{y}^{[3]} = (\mathbf{H}^{[3,3]}\mathbf{V}_1^{[3]} + \mathbf{H}^{[3,R]}\mathbf{V}_5^{[R]})\mathbf{s}_1^{[3]} + (\mathbf{H}^{[3,3]}\mathbf{V}_2^{[3]} + \\ \mathbf{H}^{[3,R]}\mathbf{V}_6^{[R]})\mathbf{s}_2^{[3]} + (\mathbf{H}^{[3,1]}\mathbf{V}_1^{[1]} + \mathbf{H}^{[3,R]}\mathbf{V}_1^{[R]})\mathbf{s}_1^{[1]} \\ +(\mathbf{H}^{[3,2]}\mathbf{V}_1^{[2]} + \mathbf{H}^{[3,R]}\mathbf{V}_3^{[R]})\mathbf{s}_1^{[2]} + \mathbf{n}_3\end{aligned}, \quad (28)$$

where $\mathbf{n}_i$ denotes the i.i.d complex Gaussian noise vector with zero mean and $\mathrm{E}(\mathbf{n}\mathbf{n}^{\mathrm{H}}) = \sigma_{\mathbf{n}}^2\mathbf{I}$, which is combined with noise from the relay-destination channel and the source-destination channel. Therefore, $\mathbf{n}_i$ is also an i.i.d complex Gaussian noise vector with zero mean, except that $\mathrm{E}(\mathbf{n}_i\mathbf{n}_i^{\mathrm{H}}) = \sigma_{\mathbf{n}_i}^2\mathbf{I}$, $\sigma_{\mathbf{n}_i}^2 = 2\sigma_{\mathbf{n}}^2$. As shown in (26)-(28), there are still two interferences left at each destination, that is, the last two terms in each equation except the noise term. As we have specified, each destination node has three antennas, so we must align the two interferences into the same dimensionality to obtain the desired signals, giving us the following:

$$\mathrm{span}(\mathbf{H}^{[1,2]}\mathbf{V}_2^{[2]} + \mathbf{H}^{[1,R]}\mathbf{V}_4^{[R]}) = \mathrm{span}(\mathbf{H}^{[1,3]}\mathbf{V}_2^{[3]} + \mathbf{H}^{[1,R]}\mathbf{V}_6^{[R]}), \quad (29)$$

$$\mathrm{span}(\mathbf{H}^{[2,1]}\mathbf{V}_2^{[1]} + \mathbf{H}^{[2,R]}\mathbf{V}_2^{[R]}) = \mathrm{span}(\mathbf{H}^{[2,3]}\mathbf{V}_1^{[3]} + \mathbf{H}^{[2,R]}\mathbf{V}_5^{[R]}), \quad (30)$$

$$\mathrm{span}(\mathbf{H}^{[3,1]}\mathbf{V}_1^{[1]} + \mathbf{H}^{[3,R]}\mathbf{V}_1^{[R]}) = \mathrm{span}(\mathbf{H}^{[3,2]}\mathbf{V}_1^{[2]} + \mathbf{H}^{[3,R]}\mathbf{V}_3^{[R]}). \quad (31)$$

Reshaping (10)-(12) gives the following.

At D_1:

$$\mathbf{V}_1^{[2]} = -(\mathbf{H}^{[1,2]})^{-1}\mathbf{H}^{[1,R]}\mathbf{V}_3^{[R]}, \quad (32)$$

$$\mathbf{V}_1^{[3]} = -(\mathbf{H}^{[1,3]})^{-1}\mathbf{H}^{[1,R]}\mathbf{V}_5^{[R]}, \quad (33)$$

At D_2:

$$\mathbf{V}_1^{[1]} = -(\mathbf{H}^{[2,1]})^{-1}\mathbf{H}^{[2,R]}\mathbf{V}_1^{[R]}, \quad (34)$$

$$\mathbf{V}_{2}^{[3]} = -(\mathbf{H}^{[2,3]})^{-1}\mathbf{H}^{[2,R]}\mathbf{V}_{6}^{[R]}, \tag{35}$$

At D_3:

$$\mathbf{V}_{2}^{[1]} = -(\mathbf{H}^{[3,1]})^{-1}\mathbf{H}^{[3,R]}\mathbf{V}_{2}^{[R]}, \tag{36}$$

$$\mathbf{V}_{2}^{[2]} = -(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]}\mathbf{V}_{4}^{[R]}. \tag{37}$$

Applying (32)-(37) to equations (29)-(31) and simplifying them, gives the following:

$$\begin{aligned}&\mathrm{span}((\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_{4}^{[R]})\\ &= \mathrm{span}((\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,3]}(\mathbf{H}^{[2,3]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_{6}^{[R]}),\end{aligned} \tag{38}$$

$$\begin{aligned}&\mathrm{span}((\mathbf{H}^{[2,R]} - \mathbf{H}^{[2,1]}(\mathbf{H}^{[3,1]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_{2}^{[R]})\\ &= \mathrm{span}((\mathbf{H}^{[2,R]} - \mathbf{H}^{[2,3]}(\mathbf{H}^{[1,3]})^{-1}\mathbf{H}^{[1,R]})\mathbf{V}_{5}^{[R]}),\end{aligned} \tag{39}$$

$$\begin{aligned}&\mathrm{span}((\mathbf{H}^{[3,R]} - \mathbf{H}^{[3,1]}(\mathbf{H}^{[2,1]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_{1}^{[R]})\\ &= \mathrm{span}((\mathbf{H}^{[3,R]} - \mathbf{H}^{[3,2]}(\mathbf{H}^{[1,2]})^{-1}\mathbf{H}^{[1,R]})\mathbf{V}_{3}^{[R]}).\end{aligned} \tag{40}$$

At this point, we have succeeded in eliminating the precoding vector $\mathbf{V}_{m}^{[i]}\ m = 1,2; i = 1,2,3$, so it is unnecessary to obtain precoding information from sources. Then, we need only to design the $\mathbf{V}_{q}^{[R]}\ q = 1,2,\cdots,6$ of the relay to align interferences and improve the sum-rate.

Taking $y^{[1]}$ at D_1, for example, with the computational process of other destinations being the same as D_1, eliminating $\mathbf{V}_{m}^{[i]}, m = 1,2; i = 1,2,3$ $\mathbf{y}^{[1]}$ gives the following:

$$\begin{aligned}\mathbf{y}^{[1]} = &(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,1]}(\mathbf{H}^{[2,1]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_{1}\mathbf{p}_{1}\mathbf{s}_{1}^{[1]} +\\ &(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,1]}(\mathbf{H}^{[3,1]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_{2}\mathbf{p}_{2}\mathbf{s}_{2}^{[1]} +\\ &(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_{4}\mathbf{p}_{4}\mathbf{s}_{2}^{[2]} +\\ &(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,3]}(\mathbf{H}^{[2,3]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_{6}\mathbf{p}_{6}\mathbf{s}_{2}^{[3]} + \mathbf{n}_{1},\end{aligned} \tag{41}$$

where $\mathbf{V}_q \mathrm{p}_q\ q = 1,2\cdots 6$ takes the place of $\mathbf{V}_{q}^{[R]}\ q = 1,2,\cdots,6$. The transmitting vector $\mathbf{V}_{q}^{[R]}$ must satisfy the transmitting power constraint $\mathrm{Tr}((\mathbf{V}_{q}^{[R]})^{\mathrm{H}}\mathbf{V}_{q}^{[R]}) \le \mathrm{p}_q$, so we make $\mathbf{V}_q$ the unit vector and $\mathrm{p}_q{}^2$ the transmitting power of relay. Then zero-forcing is used to obtain the desired signal. Because of (38), $(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_4$ and $(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,3]}(\mathbf{H}^{[2,3]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_6$ span the same space. One of these matrices is chosen arbitrarily, and the singular value decomposition (SVD) of the matrix can be described as:

$$(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_4 = [\mathbf{U}^1\mathbf{U}^0]\mathbf{\Lambda}\mathbf{V}, \tag{42}$$

where matrix $\mathbf{U}^0$ is composed of the last two singular vectors on the left and $\mathbf{U}^1$ holds the left singular vectors.

Then, from (42), the interference nulling matrix at D_1, which completely eliminates the interference, becomes $\mathbf{M}_1=(\mathbf{U}^0)^{\mathrm{H}}$. Multiplying this by (41), the non-interfering received signal vector $\mathbf{y}_1$ can be written as

$$\begin{aligned}\mathbf{y}_1 &= \mathbf{M}_1\mathbf{y}^{[1]} \\ &= \mathbf{M}_1(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,1]}(\mathbf{H}^{[2,1]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_1\mathbf{p}_1\mathbf{s}_1^{[1]} \\ &+\mathbf{M}_1(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,1]}(\mathbf{H}^{[3,1]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_2\mathbf{p}_2\mathbf{s}_2^{[1]} + \mathbf{M}_1\mathbf{n}_1 .\end{aligned} \tag{43}$$

We combine the two signals of a 2×1 vector and reshape (43) as follows:

$$\begin{aligned}\mathbf{y}_1 = [&\mathbf{M}_1(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,1]}(\mathbf{H}^{[2,1]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_1 \\ &\mathbf{M}_1(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,1]}(\mathbf{H}^{[3,1]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_2] \\ &\times\begin{pmatrix} \mathrm{p}_1 & 0 \\ 0 & \mathrm{p}_2 \end{pmatrix} \times \begin{pmatrix} \mathbf{s}_1^{[1]} \\ \mathbf{s}_2^{[1]} \end{pmatrix} + \mathbf{M}_1\mathbf{n}_1 ,\end{aligned} \tag{44}$$

$\mathbf{H}_1, \mathbf{P}_1, \mathbf{S}_1$ denote the corresponding part

$$\mathbf{y}_1 = \mathbf{H}_1\mathbf{P}_1\mathbf{S}_1 + \mathbf{M}_1\mathbf{n}_1 . \tag{45}$$

We use **QR**-method to arrive at $\mathbf{s}_1^{[1]}$ and $\mathbf{s}_2^{[1]}$.

The same method can be used for other destinations; thus, we can obtain the sum-rate:

$$\text{sum}-\text{rate} = \sum_{\mathrm{j}=1}^{3} \log_2 \left| \mathbf{I} + \mathbf{P}_j{}^2 \mathbf{H}_j \mathbf{H}_j{}^{\mathrm{H}} (\sigma_{\mathbf{n}_j}^2 \mathbf{M}_j \mathbf{M}_j{}^{\mathrm{H}})^{-1} \right| , \tag{46}$$

where $\mathbf{P}_j$ is obtained by using the water-filling method.

4.2 Choosing Proper Transmitting Vectors Using a Maximum Chordal Distance Criterion

In this subsection, the method for obtaining the transmitting vector $\mathbf{V}_{\mathrm{i}}$ focused on simplifying the complexity of the calculation will be discussed. D_1 is taken as an example. $\mathbf{V}_{\mathrm{i}}$ is restricted to interference alignment equation (29)-(31) (because $\mathbf{V}_q\mathrm{p}_q = \mathbf{V}_q^{[R]}$).

Substituting (35) and (37) into (29), we obtain

$$\begin{aligned}&\mathrm{span}((\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})\mathbf{V}_4^{[R]}) \\ &= \mathrm{span}((\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,3]}(\mathbf{H}^{[2,3]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_6^{[R]}) .\end{aligned} \tag{47}$$

To simplify the calculation-complexity, we let $\mathbf{V}_4^{[R]}$ be the eigenvectors of $(\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})$ and strengthen the constraint by eliminating the "span":

$$\mathbf{V}_4^{[R]} = (\mathbf{H}^{[1,R]} - \mathbf{H}^{[1,3]}(\mathbf{H}^{[2,3]})^{-1}\mathbf{H}^{[2,R]})\mathbf{V}_6^{[R]} . \tag{48}$$

Then, $\mathbf{V}_6^{[R]}$ can be solved for. After expanding this method to other destinations, $\mathbf{V}_q^{[R]}$ can be obtained. Then, we normalize $\mathbf{V}_q^{[R]}$ to get $\mathbf{V}_q$ through $\mathbf{V}_q^{[R]} / \left|\mathbf{V}_q^{[R]}\right|$.

Apparently, matrix $(\mathbf{H}^{[1,R]}-\mathbf{H}^{[1,2]}(\mathbf{H}^{[3,2]})^{-1}\mathbf{H}^{[3,R]})$ has more than one eigenvector and to one chosen can influence the capacity of the system. Here, a maximum chordal distance criterion [19] is used to determine the favorable $\mathbf{V}_q^{[R]}$. We defines the chordal distance between an $m\times n_1$ matrix $\mathbf{X}_1$ and $m\times n_2$ matrix $\mathbf{X}_2$ for $m\geq n_1,n_2$ as in [19]:

$$\begin{aligned}\mathrm{d}_{cd}(\mathbf{X}_1,\mathbf{X}_2)&=\frac{1}{\sqrt{2}}\left\|\mathbf{Q}(\mathbf{X}_1)\mathbf{Q}(\mathbf{X}_1)^{\mathrm{H}}-\mathbf{Q}(\mathbf{X}_2)\mathbf{Q}(\mathbf{X}_2)^{\mathrm{H}}\right\|_{\mathrm{F}}\\&=\sqrt{\frac{n_1+n_2}{2}-\left\|\mathbf{Q}(\mathbf{X}_1)^{\mathrm{H}}\mathbf{Q}(\mathbf{X}_2)\right\|_{\mathrm{F}}^2},\end{aligned}\tag{49}$$

where $\mathbf{Q}(\mathbf{X})$ is defined as a matrix consisting of orthonormal basis vectors that span the column space of $\mathbf{X}$ and achieved by **QR**-decomposition. Because the interference signal spaces have been aligned, we consider only the chordal distance between the desired signals and one of the interference signals. Taking D_1, for instance, $\mathbf{H}_m^{[i]}$ denotes the channel coefficient of $\mathbf{s}_m^{[i]}$ in (41) and the chordal distance between the desired signals and the interference signals of D_1 is

$$\mathrm{P}_{D,1}=\mathrm{d}_{cd}(\mathbf{H}_1^{[1]}\mathbf{V}_1^{[R]},\mathbf{H}_2^{[2]}\mathbf{V}_4^{[R]})+\mathrm{d}_{cd}(\mathbf{H}_2^{[1]}\mathbf{V}_2^{[R]},\mathbf{H}_2^{[2]}\mathbf{V}_4^{[R]}),\tag{50}$$

If we expand this method to the other destinations, the maximum chordal distance expression can be calculated as follows:

$$\mathrm{P}=\max_{\mathbf{V}_1^{[R]},\mathbf{V}_2^{[R]},\mathbf{V}_4^{[R]}}(\mathrm{P}_{D,1},\mathrm{P}_{D,2},\mathrm{P}_{D,3}).\tag{51}$$

By using the maximum chordal distance method, we can select favorable transmitting vectors that can enlarge the chordal distance between the desired signals and the interferences, thereby improving the capacity of the system.

In this section, the detailed calculation process of Section 3 is discussed in terms of the sum-rate. A zero-forcing method is used to obtain the desired signals one by one in relay, and the signals are retransmitted to the destinations to neutralize half of the interferences. Then interference alignment is used to align the remaining interferences to one space. Afterwards, the zero-forcing method is used to work out the desired signals at each destination node. Also, a maximum chordal distance method is introduced to choose better transmitting vectors in relay. In the next section, the numerical result will be shown.

5. Numerical Result

In this section, simulations are used to evaluate the proposed scheme in terms of the sum-rate. We compare our scheme with conventional schemes and discuss the performance when relay nodes or destination nodes are equipped with more antennas than the minimum configuration. The simulation is based on the system discussed in Section 4. The transmission power at relay is assumed to be equal to the total transmission power at the three sources. It is also assumed that the elements of all the channel matrices follow i.i.d. complex Gaussian distribution with zero mean and unit variance.

Fig. 3 shows the comparison of the relay system sum-rate discussed in Section 3 and the 3-user system. Each node of the 3-user system is equipped with M antennas. For a K-user system without relay the DoF outer bound is $MK/2$. We take M as equal to 4, then the 3-user system can obtain the same DoF as the system proposed in Section 3. We use the interference alignment scheme recommended in [19] and give the sum-rate curves derived from using the ZF method and the MMSE method, respectively, to decode. The curve "3-user IA ZF-method-selection" is the sum-rate using the selection criterion proposed in [19]. The sum-rate of the relay system (the "3-user with instantaneous relay 3 6 3" curve) is calculated by using the method recommended in Sections 3 and 4, and the "3-user with instantaneous relay 3 6 3-selection" curve is drawn by using the maximum chordal distance criterion noted in Section 4.2. Apparently, using the maximum chordal distance criterion can improve the sum-rate. The simulation results show, when the same DoF is obtained, our scheme performs better than a conventional K-user system.

Fig. 4 shows the variation of the sum-rate when the number of antennas at the relay node changes from six to nine. At the first glance, the lines are very close to each other and cannot be easily separated. As shown, the higher rate can be obtained with more antennas. However, improvement of the sum-rate is insignificant, so it is unnecessary to equip too many antennas at the relay when using the proposed scheme.

In **Fig. 5**, all four lines are drawn based on the same configuration in the sources, relay, interference neutralization and alignment algorithm; the only difference lies in the number of antennas at the destinations. The black line is the same as that of the system mentioned in Section 3, which uses exactly the method recommended in Section 4. The blue line has four antennas at each destination because there are four symbols left after neutralization. Every destination node has enough dimensionality to solve the desired signals, so using interference alignment is unnecessary. The blue line and black line nearly coincide; this phenomenon indicates the IA method is effective in eliminating interference. However, because we are increasing the number of antennas to five, the sum-rate is going down, as shown by the red dotted line. Thus, using all five antennas is unnecessary, and we can choose the best four antennas based on channel status, resulting in the red full line. The result shows interference alignment used to reduce signal dimension is effective and can obtain the same sum-rate if one antenna is added at each destination. However, with the number of antennas at destinations increasing, the sum-rate declines because more interference is brought into the system. A reasonable way to solve this problem is to select antennas according to the channel status.

6. Conclusion

In this paper, we analyzed the DoF of the 3-user interference channel with an instantaneous relay for multiple antennas systems. We propose a linear beamforming scheme and an interference-forward-relaying scheme, inspired by aligned interference neutralization. This new method is focused on the relay node, which completes the alignment interference neutralization so that we need to obtain the global CSI only once. This method can relieve the pressure of base station. In addition, we show that a total of $2M$ DoF can be achieved when all sources and destinations have $M \geq 1$ multiple antennas. The simulation results showe that interference alignment used to reduce signal dimension is effective. The same sum-rate can be obtained when the destination has one more antenna than the source. However, with the number of antennas at destinations increasing, the sum-rate declines, and antenna selection or another method should be used.

7. Figures

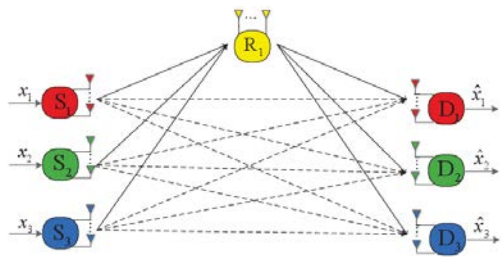

Fig. 1. 3-user interference channel with instantaneous relay.

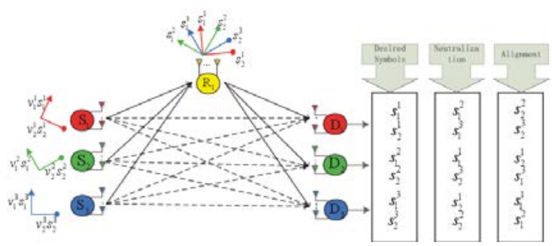

Fig. 2. The proposed scheme for the 3-user interference channel with the instantaneous relay.

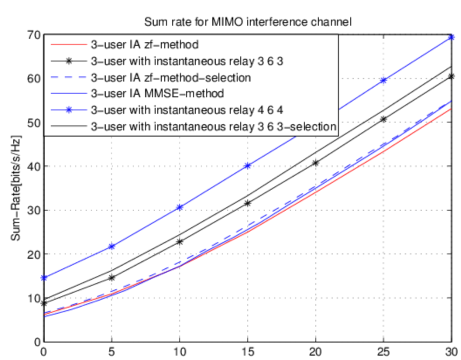

Fig. 3. Comparison of the sum rate for 3-user system equipped with and without relay, with each source and destination node having 4 antennas and relay having 6.

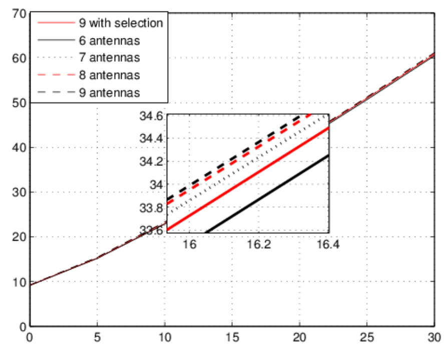

Fig. 4. Comparison of the sum rate for different numbers of antennas in relay.

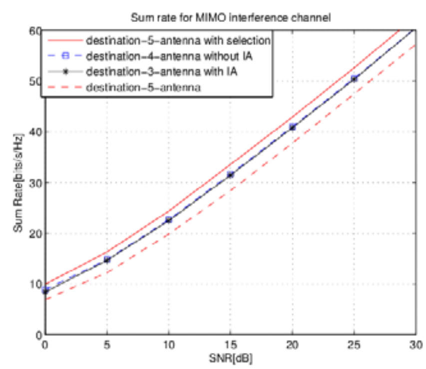

Fig. 5. Comparison of the sum rate for different numbers of antennas at destination.

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Wang Qiang Associate professor, Master supervisor in National Engineering Laboratory for Mobile Network Security, Beijing University of Posts & Telecommunications. He received Ph.D in 2008. His research mainly focuses on radio transmit techniques of wireless network based on novel network architecture (e.x. CoMP, Relay etc.), and key technologies of wireless network for 3GPP LTE-Advanced and ITU IMT-Advanced. He participated in many national projects such as NSFC, 863 etc.

Shu Yuquan is a Master candidate in Wireless Technology Innovation Institute and National Engineering Laboratory for Mobile Network Security, BUPT. His research focus is on precoder and decoder design for interference alignment (IA) in wireless interference networks.

Dong Minhua is a Master candidate in Wireless Technology Innovation Institute and National Engineering Laboratory for Mobile Network Security, BUPT. Her research focus is on signal processing and interference alignment (IA) in wireless interference networks.

Xu Ji is a Master candidate in Wireless Technology Innovation Institute and National Engineering Laboratory for Mobile Network Security, BUPT. His research focus is on VLSI implementation, interference alignment (IA) in wireless interference networks.

Tao Xiaofeng received his B.S degree in electrical engineering from Xi'an Jiaotong University, China, in 1993, and M.S. and Ph.D. in telecommunication engineering from BUPT in 1999 and 2002, respectively. He is an IET Fellow. He was chief architect of the Chinese National Future 4G TDD working group from 2003 to 2006 and established the 4G TDD CoMP trial network in 2006. He is currently a professor at BUPT, director of National Engineering Laboratory for Mobile Network Security, inventor or co-inventor of 50 patents, and author or coauthor of 120 papers on 4G and beyond.