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INT-SOFT POSITIVE IMPLICATIVE FILTERS IN BE-ALGEBRAS

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ABSTRACT. The notion of int-soft implicative filters of a BE-algebra is introduced, and related properties are investigated. The problem of classifying int-soft positive implicative by their γ-inclusive filter is solved. We provide conditions for a soft set to be an int-soft filter. We make a new int-soft implicative filter from old one.

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1. Introduction

In 1966, Imai and Iséki [3] and Iséki [4] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. As a generalization of a BCK-algebra, Kim and Kim [7] introduced the notion of a BE-algebra, and investigated several properties. In [2], Ahn and So introduced the notion of ideals in BE-algebras. They gave several descriptions of ideals in BE-algebras.

Various problems in system identification involve characteristics which are essentially non-probabilistic in nature [14]. In response to this situation Zadeh [15] introduced *fuzzy set theory* as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [16]. To solve complicated problem in economics, engineering, and environment, we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt

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with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [11]. Maji et al. [10] and Molodtsov [11] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [11] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [10] described the application of soft set theory to a decision making problem. Maji et al. [9] also studied several operations on the theory of soft sets.

Ahn et al. [1] introduced the notion of an implicative vague filter in BE-algebras, and investigate some properties of it. Jun et al. [6] defined the notion of an int-soft implicative filter and related properties are studied.

In this paper, we introduce the notion of int-soft positive implicative filter of a BE-algebra, and investigate their properties. We solve the problem of classifying int-soft subalgebras by their γ-inclusive positive implicative filters. We provide conditions for a soft set to be an int-soft filter. We make a new int-soft implicative filter from old one. We discuss characterizations of int-soft positive implicative filters.

2. Preliminaries

An algebra $(X; *, 1)$ of type (2, 0) is called a *BE-algebra* [7] if

(BE1) $x * x = 1$ for all $x \in X$;
(BE2) $x * 1 = 1$ for all $x \in X$;
(BE3) $1 * x = x$ for all $x \in X$;
(BE4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$ (*exchange*)

We introduce a relation "$\leq$" on a BE-algebra X by $x \leq y$ if and only if $x * y = 1$. A non-empty subset S of a BE-algebra X is said to be a *subalgebra* of X if it is closed under the operation "$*$". Noticing that $x * x = 1$ for all $x \in X$, it is clear that $1 \in S$. A BE-algebra $(X; *, 1)$ is said to be *self distributive* [7] if $x * (y * z) = (x * y) * (x * z)$ for all $x, y, z \in X$.

Definition 2.1. Let $(X; *, 1)$ be a BE-algebra and let F be a non-empty subset of X. F is called a *filter* [5] of X if

(F1) $1 \in F$;
(F2) $x * y \in F$ and $x \in F$ imply $y \in F$ for all $x, y \in X$.

F is called an *implicative filter* [12] of X if it satisfies (F1) and

(F3) $x * (y * z) \in F$ and $x * y \in F$ imply $x * z \in F$ for all $x, y, z \in X$.

Definition 2.2. Let $(X; *, 1)$ be a BE-algebra and let F be a non-empty subset of X. Then F is called a *positive implicative filter* of X if it satisfies (F1) and

(F4) $x * ((y * z) * y) \in F$ and $x \in F$ imply $y \in F$ for all $x, y \in X$.

Proposition 2.3. *Let $(X; *, 1)$ be a BE-algebra and let F be a filter of X. If $x \leq y$ and $x \in F$ for any $y \in X$, then $y \in F$.*

Proposition 2.4. *Let $(X; *, 1)$ be a self distributive BE-algebra. Then following hold: for any $x, y, z \in X$,*

(i) *if $x \leq y$, then $z * x \leq z * y$ and $y * z \leq x * z$.*
(ii) *$y * z \leq (z * x) * (y * z)$.*
(iii) *$y * z \leq (x * y) * (x * z)$.*

A BE-algebra $(X; *, 1)$ is said to be *transitive* if it satisfies Proposition 2.4(iii).

A soft set theory is introduced by Molodtsov [11]. In what follows, let U be an initial universe set and X be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A, B, C, \cdots \subseteq X$.

Definition 2.5. A *soft set* (f, A) of X over U is defined to be the set of ordered pairs

$$(f, A) := \{(x, f(x)) : x \in X,\ f(x) \in \mathscr{P}(U)\},$$

where $f : X \to \mathscr{P}(U)$ such that $f(x) = \emptyset$ if $x \notin A$.

For a soft set (f, A) of X and a subset γ of U, the γ-*inclusive set* of (f, A), denoted by $i_A(f; \gamma)$, is defined to be the set

$$i_A(f; \gamma) := \{x \in A \mid \gamma \subseteq f(x)\}.$$

For any soft sets (f, X) and (g, X) of X, we call (f, X) a *soft subset* of (g, X), denoted by $(f, X) \tilde{\subseteq} (g, X)$, if $f(x) \subseteq g(x)$ for all $x \in X$. The *soft union* of (f, X) and (g, X), denoted by $(f, X) \tilde{\cup} (g, X)$, is defined to be the soft set $(f \tilde{\cup} g, X)$ of X over U in which $f \tilde{\cup} g$ is defined by

$$(f \tilde{\cup} g)(x) = f(x) \cup g(x) \text{ for all } x \in M.$$

The *soft intersection* of (f, X) and (g, X), denoted by $(f, X) \tilde{\cap} (g, X)$, is defined to be the soft set $(f \tilde{\cap} g, M)$ of X over U in which $f \tilde{\cap} g$ is defined by

$$(f \tilde{\cap} g)(x) = f(x) \cap g(x) \text{ for all } x \in S.$$

3. Int-soft positive implicative filters

In what follows, we take a BE-algebra X, as a set of parameters unless specified.

Definition 3.1 ([1])**.** A soft set (f, X) of X over U is called an *intersection-soft filter* (briefly, *int-soft filter*) over U if it satisfies:

(IS1) $(\forall x \in X)\,(f(1) \supseteq f(x))$,
(IS2) $(\forall x, y \in X)\,(f(x * y) \cap f(x) \subseteq f(y))$.

Proposition 3.2 ([1])**.** *Every int-soft filter (f, X) of X over U satisfies the following properties:*

(i) $(\forall x, y \in X)\,(x \leq y \Rightarrow f(x) \subseteq f(y))$.
(ii) $(\forall x, y, z \in X)\,(f(x * z) \supseteq f(x * (y * z)) \cap f(y))$.

Proposition 3.3 ([6])**.** *Let (f, X) be a soft set of X over U. Then (f, X) is an int-soft filter of X over U if and only if it satisfies*

$$(\forall x, y, z \in X)(z \leq x * y \Rightarrow f(y) \supseteq f(x) \cap f(z))$$

.

Definition 3.4. A soft set (f, X) of X over U is called an *intersection-soft implicative filter* (briefly, *int-soft implicative filter*) [6] over U if it satisfies (IS1) and

(IS3) $(\forall x, y, z \in X)\,(f(x * (y * z)) \cap f(x * y) \subseteq f(x * z))$.

A soft set (f, X) of X over U is called an *intersection-soft positive implicative filter* (briefly, *int-soft positive implicative filter*) over U if it satisfies (IS1) and

(IS4) $(\forall x, y, z \in X)\,(f(x * ((y * z) * y)) \cap f(x) \subseteq f(y))$.

Example 3.5. Let $E = X$ be the set of parameters and $U = X$ be the initial universe set where $X := \{1, a, b, c, d, 0\}$ is a BE-algebra [5] with the following Cayley table:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

Let (f, X) be a soft set of X over U defined as follows:

$$f : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \gamma_2 & \text{if } x \in \{1, a, b\} \\ \gamma_1 & \text{if } x \in \{c, d\}, \end{cases}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. It is easy to check that (f, X) is both an int-soft implicative filter of X over U and an int-soft positive implicative filter of X. Let (g, X) be a soft set of X over U defined as follows:

$$g : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \gamma_2 & \text{if } x \in \{1, b\} \\ \gamma_1 & \text{if } x \in \{a, c, d\}, \end{cases}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. It is easy to check that (g, X) is an int-soft implicative filter of X over U. But it is not an int-soft positive implicative filter of X over U since $g(b * ((a * d) * a)) = g(1) \cap g(b) = \gamma_2 \nsubseteq \gamma_1 = g(a)$.

Proposition 3.6. *Every int-soft positive implicative filter over U is an int-soft filter over U.*

Proof. Let (f, X) be an int-soft positive implicative filter over U. Putting $z := y$ in (IS4), we have $f(x * y) \cap f(x) \subseteq f(y)$. Hence (IS2) holds. Therefore (f, X) is an int-soft filter over U. $\square$

The converse of Proposition 3.6 is not true in general as seen in the following example.

Example 3.7. Let $E = X$ be the set of parameters and $U = X$ be the initial universe set where $X := \{1, a, b, c, d, 0\}$ is a BE-algebra [5] with the following Cayley table:

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Let (f, X) be a soft set of X over U defined as follows:

$$f : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \gamma_2 & \text{if } x = 1 \\ \gamma_1 & \text{if } x \in \{a, b, c, d, 0\}, \end{cases}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. It is easy to check that (f, X) is an int-soft filter of X. But it is not an int-soft positive implicative filter over U, since $f(1 * ((a * 0) * a)) \cap f(1) = \gamma_2 \nsubseteq \gamma_1 = f(a)$.

We provide conditions for an int-soft filter to be an int-soft positive implicative filter.

Theorem 3.8. *Let X be a BE-algebra. Then an int-soft filter (f, X) is an int-soft positive implicative filter of X if and only if*

$$(\forall x, y \in X)(f(x) \supseteq f((x * y) * x)). \tag{3.1}$$

Proof. Assume that (f, X) is an int-soft positive implicative filter of X and let $x, y \in X$. Using (IS4) and (IS1), we have $f(x) \supseteq f(1 * ((x * y) * x)) \cap f(1) = f((x * y) * x)$. Hence (3.1) holds.

Conversely, suppose that (f, X) is an int-soft filter of X satisfying (3.1). It follows from (3.1) and (IS2) that $f(y) \supseteq f((y*z)*y) \supseteq f(x*((y*z)*y)) \cap f(x)$ for any $x, y, z \in X$. Hence (IS4) holds. Thus (f, X) is an int-soft positive implicative filter of X. □

Definition 3.9. Let X be a BE-algebra. X is said to be *commutative* if the following identity holds:

(C) $(x * y) * y = (y * x) * x$, i.e., $x \vee y = y \vee x$, where $x \vee y = (y * x) * x$ $\forall x, y \in X$.

Theorem 3.10. *Let X be a commutative self distributive BE-algebra. Every int-soft positive implicative filter over U is an int-soft implicative filter over X.*

Proof. Let (f, X) be an int-soft positive implicative filter over U. By Proposition 3.6, it is an int-soft filter over U. Using (BE4) and Proposition 2.4(iii), we obtain $(x * (y * z)) * [(x * y) * (x * (x * z))] = 1$ for any $x, y, z \in X$. It follows from

Proposition 3.3 that $f(x*(x*z)) \supseteq f(x*(y*z)) \cap f(x*y)$. On the other hand, using $(BE4)$ and (C) we have

$$\begin{aligned}((x*z)*z)*(x*z) =& x*(((x*z)*z)*z)\\ =& x*((z*(x*z))*(x*z))\\ =& x*(1*(x*z))\\ =& x*(x*z)\end{aligned}$$

By Theorem 3.8, we get $f(x*z) \supseteq f(((x*z)*z)*(x*z)) = f(x*(x*z)) \supseteq f(x*(y*z)) \cap f(x*y)$. This completes the proof. □

Theorem 3.11. *A soft set (f, X) of X over U is an int-soft positive implicative filter of X over U if and only if the γ-inclusive set $i_X(f;\gamma)$ is a positive implicative filter of X over U for all $\gamma \in \mathscr{P}(U)$ with $i_X(f;\gamma) \neq \emptyset$.*

The filter $i_X(f;\gamma)$ in Theorem 3.11 is called the *inclusive filter* of X over U.

Proof. Assume that (f, X) is an int-soft positive implicative filter over U. Let $x, y, z \in X$ and $\gamma \in \mathscr{P}(U)$ be such that $x*((y*z)*y) \in i_X(f;\gamma)$ and $x \in i_X(f;\gamma)$. Then $\gamma \subseteq f(x*((y*z)*y))$ and $\gamma \subseteq f(x)$. It follows from (IS1) and (IS4) that $\gamma \subseteq f(1)$ and $\gamma \subseteq f(x*((y*z)*y) \cap f(x) \subseteq f(y)$ for $x, y, z \in X$. Hence $1 \in i_X(f;\gamma)$ and $y \in i_X(f;\gamma)$. Thus $i_X(f;\gamma)$ is a positive implicative filter of X over U.

Conversely, suppose that $i_X(f;\gamma)$ is a positive implicative filter of X over U for all $\gamma \in \mathscr{P}(U)$ with $i_X(f;\gamma) \neq \emptyset$. For any $x \in X$, let $f(x) = \gamma$. Since $i_X(f;\gamma)$ is a positive implicative filter of X, we have $1 \in i_X(f;\gamma)$ and so $f(x) = \gamma \subseteq f(1)$. For any $x, y \in X$, let $f(x*((y*z)*y)) = \gamma_{x*((y*z)*y)}$ and $f(x) = \gamma_x$. Take $\gamma = \gamma_{x*((y*z)*y)} \cap \gamma_x$. Then $x*((y*z)*y) \in i_X(f;\gamma)$ and $x \in i_X(f;\gamma)$ which imply that $y \in i_X(f;\gamma)$. Hence

$$f(y) \supseteq \gamma = \gamma_{x*((y*z)*y)} \cap \gamma_x = f(x*((y*z)*y)) \cap f(x).$$

Thus (f, X) is a int-soft positive implicative filter of X over U. □

We make a new int-soft positive implicative filter from one.

Theorem 3.12. *Let $(f, X) \in S(U)$ and define a soft set (f^*, X) of X over U by*

$$f^* : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} f(x) & \text{if } x \in i_X(f;\gamma),\\ \delta & \text{otherwise}\end{cases}$$

where γ is any subset of U and δ is a subset of U satisfying $\delta \subsetneq \bigcap_{x \notin i_X(f;\gamma)} f(x)$. If (f, X) is an int-soft positive implicative filter of X, then so is (f^*, X).

Proof. Assume that (f, X) is an int-soft positive implicative filter of X. Then $i_X(f;\gamma)(\neq \emptyset)$ is a positive implicative filter of X over U for all $\gamma \subseteq U$ by Theorem 3.11. Hence $1 \in i_X(f;\gamma)$, and so $f^*(1) = f(1) \supseteq f(x) \supseteq f^*(x)$ for all $x \in X$.

Let $x, y, z \in X$. If $x * ((y * z) * y) \in i_X(f; \gamma)$ and $x \in i_X(f; \gamma)$, then $y \in i_X(f; \gamma)$. Hence

$$\begin{aligned} f^*(y) = f(y) &\supseteq f(x * ((y * z) * y)) \cap f(x) \\ &= f^*(x * ((y * z) * y)) \cap f^*(x). \end{aligned}$$

If $x * ((y * z) * y) \notin i_X(f; \gamma)$ or $x \notin i_X(f; \gamma)$, then $f^*(x * ((y * z) * y)) = \delta$ or $f^*(x) = \delta$. Thus

$$f^*(y) \supseteq \delta = f^*(x * ((y * z) * y)) \cap f^*(x).$$

Therefore (f^*, X) is an int-soft positive implicative filter of X. □

Theorem 3.13. *Every filter of a BE-algebra can be represented as a γ-inclusive set of an int-soft positive implicative filter.*

Proof. Let F be a filter of a BE-algebra X. For a subset γ of U, define a soft set (f, X) over U by

$$f : X \to \mathscr{P}(U), \ x \mapsto \begin{cases} \gamma & \text{if } x \in F, \\ \emptyset & \text{if } x \notin F. \end{cases}$$

Obviously, $F = i_X(f; \gamma)$. We now prove that (f, X) is an int-soft positive implicative filter of X. Since $1 \in F = i_X(f; \gamma)$, we have $f(1) = \gamma$, $f(1) \supseteq f(x)$ for all $x \in X$. Let $x, y, z \in X$. If $x * ((y * z) * y), x \in F$, then $y \in F$ because F is a positive implicative filter of X. Hence $f(x * ((y * z) * y)) = f(x) = f(y) = \gamma$, and so $f(x * ((y * z) * y)) \cap f(x) \subseteq f(y)$. If $x * ((y * z) * y) \in F$ and $x \notin F$, then $f(x * ((y * z) * y)) = \gamma$ and $f(x) = \emptyset$ which imply that

$$f(x * ((y * z) * y)) \cap f(x) = \gamma \cap \emptyset = \emptyset \subseteq f(y).$$

Similarly, if $x * ((y * z) * y) \notin F$ and $x \in F$, then $f(x * ((y * z) * y)) \cap f(x) \subseteq f(y)$. Obviously, if $x * ((y * z) * y) \notin F$ and $x \notin F$, then $f(x * ((y * z) * y)) \cap f(x) \subseteq f(y)$. Therefore (f, X) is an int-soft positive implicative filter of X. □

Theorem 3.14. *If (f, X) and (g, X) are int-soft positive implicative filters of X, then the soft intersection $(f, X) \tilde{\cap} (g, X)$ of (f, X) and (g, X) is an int-soft positive implicative filter of X.*

Proof. For any $x \in X$, we have

$$(f \tilde{\cap} g)(1) = f(1) \cap g(1) \supseteq f(x) \cap g(x) = (f \tilde{\cap} g)(x).$$

Let $x, y, z \in X$. Then

$$\begin{aligned} (f \tilde{\cap} g)(y) &= f(y) \cap g(y) \\ &\supseteq (f(x * ((y * z) * y)) \cap f(x)) \cap (g(x * ((y * z) * y)) \cap g(x)) \\ &= (f(x * ((y * z) * y)) \cap g(x * ((y * z) * y))) \cap (f(x) \cap g(x)) \\ &= (f \tilde{\cap} g)(x * ((y * z) * y)) \cap (f \tilde{\cap} g)(x). \end{aligned}$$

Hence $(f, X) \tilde{\cap} (g, X)$ is an int-soft positive implicative filter of X. □

The following example shows that the soft union of int-soft positive implicative filters of X may not be an int-soft positive implicative filter of X.

Example 3.15. Let $E = X$ be the set of parameters and $U = X$ be the initial universe set, where $X = \{1, a, b, c, d\}$ is a BE-algebra as in Example 3.5. Let (f, X) and (g, X) be soft sets of X over U defined, respectively, as follows:

$$f : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \gamma_3 & \text{if } x \in \{1, c\} \\ \gamma_1 & \text{if } x \in \{a, b, d\} \end{cases}$$

and

$$g : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \gamma_4 & \text{if } x \in \{1\} \\ \gamma_2 & \text{if } x \in \{a, b, c, d\} \end{cases}$$

where γ_1, γ_2, γ_3, and γ_4 are subsets of U with $\gamma_1 \subsetneq \gamma_2 \subsetneq \gamma_3 \subsetneq \gamma_4$. It is easy to check that (f, X) and (g, X) are int-soft positive implicative filters of X over U. But $(f, X) \tilde{\cup} (g, X) = (f \tilde{\cup} g, X)$ is not an int-soft positive implicative filter of X over U, since

$$\begin{aligned}(f \tilde{\cup} g)(c * ((a * b) * a)) \cap (f \tilde{\cup} g)(c) &= (f \tilde{\cup} g)(1) \cap (f \tilde{\cup} g)(c) \\ &= (f(1) \cup g(1)) \cap (f(c) \cup g(c)) \\ &= \gamma_4 \cap \gamma_3 = \gamma_3 \nsubseteq \gamma_2 = \gamma_1 \cup \gamma_2 \\ &= f(a) \cup g(a).\end{aligned}$$

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