JOURNAL OF THE
CHUNGCHEONG MATHEMATICAL SOCIETY
Volume **28**, No. 3, August 2015
http://dx.doi.org/10.14403/jcms.2015.28.3.365

EXITSENCE OF MILD SOLUTIONS FOR SEMILINEAR MIXED VOLTERRA-FREDHOLM FUNCTIONAL INTEGRODIFFERENTIAL EQUATIONS WITH NONLOCALS

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Abstract. Of concern is the existence, uniqueness, and continuous dependence of a mild solution of a nonlocal Cauchy problem for a semilinear mixed Volterra-Fredholm functional integrodifferential equation. Our analysis is based on the theory of a strongly continuous semigroup of operators and the Banach fixed point theorem.

1. Introduction

Byszewski and Acka [5] studied the problem of existence solution of semilinear evolution equation with nonlocal conditions in Banach spaces. K.D. Kucche and M.B. Dhakne [8] established the existence of a mild solution of mixed Volterra-Fredholm functional integrodifferential equation with nonlocal condition of the form

$$x'(t) + Ax(t) = f\left(t, x_t, \int_0^t k(t,s,x_s)ds, \int_0^a h(t,s,x_s)\right)ds, t \in [0,a],$$
$$x(t) + [g(x_{t_1}, ..., x_{t_p})](t) = \phi(t), t \in [-r, 0],$$

where $0 < t_1 < ... < t_p \leq a (p \in \mathbf{N})$, $-A$ is the infinitestimal generator of a C_0 semigroup of operators $(T(t))_{t\geq 0}$ on a Banach space E and $u_t(s) = u(t+s)$ for $t \in [0,a]$, $\phi \in C([-r,0],E)$ nonlinear operators f, k, h, g are given functions satisfying some assumptions.

In this paper, we shall prove the existence and uniqueness of a mild solution for a semilinear mixed Volterra-Fredholm functional integrodifferential equation with nonlocal conditions of the form

Received January 17, 2015; Revised April 04, 2015; Accepted July 22, 2015.
2010 Mathematics Subject Classification: Primary 34K30, 34K40, 45K05.
Key words and phrases: Volterra-Fredholm equation, strongly continuous semigroup, Pachpatte's inequality, Gronwall inequality, continuous dependence, Banach fixed point theorem, nonlocal conditions.

$$(1.1)\qquad u'(t)+Au(t)=f\big(t,u_t,\int_0^t k(t,\mu)m(\mu,u_\mu)d\mu,$$
$$\int_0^a l(t,\mu)h(\mu,u_\mu)d\mu\big),t\in[0,a],$$
$$(1.2)\qquad u(s)+[g(u_{t_1},...,u_{t_p})](s)=\phi(s),s\in[-r,0]$$

where $-A$ is the infinitestimal generator of a strongly continuous semigroup of bounded linear operators $(T(t))_{t\geq 0}$ on a Banach space X and the nonlinear operators f,k,m,l,h,g are given functions satisfying some assumptions.

Equations of the form (1.1)-(1.2) and their special forms serve as an abstract formulation of many partial integrodifferential equations which arising in heat flow in materials with memory, viscoelasticity and other physical phenomena. The work in nonlocal IVP was initiated by Byszewski[3]. The nonlocal condition can be applied in physics with better effect than the classical Cauchy problem since nonlocal condition is usually more precise for physical measurements than classical condition. Theorems about the existence, uniqueness and stability of solutions of differential, integrodifferential equations and functional-differential abstract evolution equations with nonlocal conditions were studied by Byszewski [3]-[4], Balachandran and Chandrasekaran [1], Lin and Liu [10] and K.D. Kucche and M.B. Dhakne [9]. This paper is motivated by the work of K.D. Kucche and M.B. Dhakne [8], K. Balachandran and J.Y. Park [2] and Byszewski and Akca [5].

2. Preliminaries

Let X be a Banach space with norm $||\cdot||$, $-A$ is the infinitesimal generator of a C_0 semigroup $(T(t))_{t\geq 0}$ on X and $M=\sup_{t\in[0,a]}||T(t)||_{B(X)}$. In the sequel the operator norm $||\cdot||_{B(X)}$ will be denoted by $||\cdot||$. To simplify the notation let us take $I_0=[-r,0], I=[0,a]$ and $C=C([-r,0];X)$ denotes the Banach space of all continuous function $\psi:[-r,0]\to X$ with $\sup\{||\phi(\theta)||:-r\leq\theta\leq 0\}$,$B=C([-r,a]:X)$, $Z=C([0,a]:X)$. For a continuous function $w:[-r,a]\to X$, we denote w_t a function belong to C and defined by $w_t(s)=w(t+s)$ for $t\in I, s\in I_0$. Let $f:I\times C\times X\times X\to X$and $k,l:I\times I\to R$, $m,h:I\times C\to X$, are continuous functions and ϕ is a element of C. $g:[C([0,a]:X)]^p\to C$

For convenience we list the following hypotheses:

(H_1) For every $u, w, v \in C$ and $t \in I, f(., u_t, w_t, v_t) \in Z$

(H_2) There exists a constant $F > 0$

$$||f(t, \psi_t, x_t, y_t) - f(t, \chi_t, u_t, v_t)||$$
$$\leq F(||\psi - \chi||_{C([-r,t],X)} + ||x - u||_{C([-r,t],X)} + ||y - v||_{C([-r,t],X)})$$

for every $t \in I, \psi, \chi, u, v, x, y \in B$

(H_3) There exist constants $N, H > 0$ such that

$$||m(t, \psi) - m(t, \chi)|| \leq N||\psi - \chi||_{C([-r,s],X)}$$

and

$$||h(t, \psi) - h(t, \chi)|| \leq H||\psi - \chi||_{C([-r,s],X)}$$

for every $s \in I$, and $\psi, \chi \in C$.

(H_4) Let $g : X^p \to X$ and there exists a constant $G > 0$ such that

$$||[g(w_{t_1}, ..., w_{t_p})](s) - [g(u_{t_1}, ..., u_{t_p})](s)|| \leq G||w - u||_C$$

for $w, u \in B, s \in I_0$.

(H_5) $M(G + Fa + FKNa + FLHa) < 1$.

Definition 2.1. ([13]) Let$(T(t))_{t\geq 0}$ be a strongly continuous semigroup of a bounded operators in a Banach space X with infinitesimal generator A, The function is called a mild solution of the Cauchy problem $(1.1) - (1.2)$ if it satisfies the integral equations

(i) $u(t) = T(t)\phi(0) - T(t)[g(u_{t_1}, ..., u_{t_p})](0) + \int_0^t T(t - \tau)$
$f\left(\tau, u_\tau, \int_0^\tau k(\tau, \mu)m(\mu, u_\mu)d\mu, \int_0^a l(\tau, \mu)h(\mu, u_\mu)d\mu\right)d\tau, t \in [0, a]$,

(ii) $u(s) + [g(u_{t_1}, ..., u_{t_p})](s) = \phi(s), s \in I_0$.

We need the following integral inequality, often referred to as Gronwall inequality.

Lemma 2.2. ([11] p-11) *Let u and f be continuous functions defined on R_+ and c be a nonnegative constant. If*

$$u(t) \leq c + \int_0^t f(s)u(s)ds, \text{ for } t \in R_+$$

then

$$u(t) \leq c \exp\left(\int_0^t f(s)ds\right), \text{ for } t \in R_+$$

The following Pachpatte's inequality plays the crucial role in our further analysis.

LEMMA 2.3. ([12] p-47.)*Let* $z(t), u(t), v(t), w(t) \in C([\alpha,\beta], R_+)$ *and* $k \geq 0$ *be a real constant and*

$$z(t) \leq k+\int_\alpha^t u(s)\big[z(s)+\int_\alpha^s v(\sigma)z(\sigma)d\sigma+\int_\alpha^\beta w(\sigma)z(\sigma)\big]ds, \text{ for } t \in [\alpha,\beta]$$

$$r^* = \int_\alpha^\beta w(\sigma)\exp\Big(\int_\alpha^\sigma [u(\tau)+v(\tau)]d\tau\Big)d\sigma < 1$$

then

$$z(t) \leq \frac{k}{1-r^*}\exp\Big(\int_\alpha^t [u(s)+v(s)]ds\Big), \text{ for } t \in [\alpha,\beta].$$

3. Existence and uniqueness of a mild solution

THEOREM 3.1. *Assume that the functions f and g satisfy assumptions (H_1)-(H_5). Then the nonlocal Cauchy problem* $(1.1)-(1.2)$ *has a unique mild solution.*

Proof. Define an operator F on the Banach space B by the formula

$$(Fu)(t) = \begin{cases} \phi(t) - [g(u_{t_1},...,u_{t_p})](t), \ t \in [-r,0] \\ T(t)\phi(0) - T(t)[g(u_{t_1},...,u_{t_p})](0) + \int_0^t T(t-\tau) \\ \cdot\ f\Bigg(\tau, u_\tau, \int_0^\tau k(\tau,\mu)m(\mu,u_\mu)d\mu, \\ \qquad \int_0^a l(\tau,\mu)h(\mu,u_\mu)d\mu\Bigg)d\tau, \ t \in [0,a] \end{cases}$$

It is easy to see that F maps B into itself. Now, we will show that F is a contraction on B. Note that

(3.1)
$$(Fw)(t) - (Fu)(t) = [g(w_{t_1}, w_{t_2}, \ldots, w_{t_p})](t) - [g(u_{t_1}, u_{t_2}, \ldots, u_{t_p})](t), \quad w, u \in B, \ t \in [-r,0].$$

(3.2)
$$\begin{aligned}&(Fw)(t) - (Fu)(t)\\ &= T(t)[g(w_{t_1}, w_{t_2}, \ldots, w_{t_p})](0) - [g(u_{t_1}, u_{t_2}, \ldots, u_{t_p})](0)\\ &\quad + \int_0^t T(t-\tau)\Big[f\big(\tau, w_\tau, \int_0^\tau k(\tau,\mu)m(\mu,w_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu,w_\mu)d\mu\big)\\ &\quad - f\big(\tau, u_\tau, \int_0^\tau k(\tau,\mu)m(\mu,u_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu,u_\mu)d\mu\big)\Big]d\tau\end{aligned}$$

for $w, u \in B, t \in [0,a]$.

From (3.1) and (H_4)

$$||(Fw)(t) - (Fu)(t)|| \leq G||w - u||_B \text{ for } w, u \in B, t \in [-r, 0].$$

Moreover by (3.2), $(H_2\ )$, (H_3), $(H_4\ )$,(H_5) and since k, l are continuous on compact set $I \times I$ there exists constants K, L such that $|k(t,s)| \leq K$ for $t \geq s \geq 0$ and $|l(t,s)| \leq L$, for $s, t \in I$.

(3.3)

$$\begin{aligned}
&||(Fw)(t) - (Fu)(t)|| \\
&= ||T(t)|| \cdot ||[g(w_{t_1}, w_{t_2}, \dots, w_{t_p})](0) - [g(u_{t_1}, u_{t_2}, \dots, u_{t_p})](0)|| \\
&\quad + \int_0^t ||T(t-\tau)||F\bigg[||w - \mu||_{C([-r,\tau],X)} + \int_0^\tau k(\tau,\mu)||m(\mu, w_\mu) \\
&\quad - m(\mu, u_{\mu)}|| + \int_0^a l(\tau,\mu)||h(\mu, w_\mu) - h(\mu, u_\mu)||d\mu\bigg]d\tau \\
&\leq MG||w-u||_B + MFa||w-u||_B + MFKNa||w-u||_B + MFLHa||w-u||_B \\
&= M(G + Fa + FKNa + FLHa)||w-u||_B \text{ for } w, u \in Y, 0 \leq s \leq \tau \leq t \leq a.
\end{aligned}$$

From (3.3) and (3.4) we get

$$||Fw - Fu||_B \leq p||w - u||_B \text{ for } w, u \in B \tag{3.4}$$

where $p = M(G + Fa + FKNa + FLHa)$.

Since $p < 1$, (3.5) shows F is a contraction on B. Consequently, the operator$F$ satisfies all the assumptions of the Banach contraction theorem. Therefore, in space B there is a unique fixed point for$F$ and this point is the mild solution of the nonlocal Cauchy problem (1.1) − (1.2). □

4. Continuous dependence of a mild solution

In the following theorem, we study the continuous dependence of mild solutions of Cauchy problem(1.1)-(1.2) on given nonlocal conditions.

Theorem 4.1. *Suppose that the functions f and g satisfy assumptions (H_1)-(H_5). Then for each $\phi_1, \phi_2 \in C$ and for the corresponding mild solutions u_1, u_2 of the problems*

(4.1)

$$\begin{aligned}
&u'(t) + Au(t) \\
&\quad = f\bigg(t, u_t, \int_0^t k(t,\mu)m(\mu, u_\mu)d\mu, \int_0^a l(t,\mu)h(\mu, u_\mu)d\mu\bigg), t \in [0, a],
\end{aligned}$$

$$u(s) + [g(u_{t_1}, ..., u_{t_p})](s) = \phi_i(s), s \in [-r, 0], (i = 1, 2). \tag{4.2}$$

The following inequality

(4.3)
$$||u_1(t) - u_2(t)||_B$$
$$\leq \Big[M||\phi_1 - \phi_2||_C + M(G + FLHa^2)||u_1 - u_2||_C\Big] \exp\Big[aMF(1 + aKN)\Big] \text{for } t \in I$$

is true. Additionally, if $M(G + FLHa^2)\exp\Big[MFa(1 + aKN)\Big] \leq 1$ *then*

$$(4.4) \qquad ||u_1 - u_2||_B \leq \frac{M\exp\big[MFa(1 + KNa)\big]}{1 - M(G + FLHa^2)\exp\big[MFa(1 + aKN)\big]}.$$

Proof. Let $\phi_i (i = 1, 2)$ be arbitrary functions belonging to C and let $u_i (i = 1, 2)$ be the the mild solutions of problems(4.1) − (4.2).

(4.5)
$$(u_1)(t) - (u_2)(t)$$
$$= T(t)\big[\phi_1(0) - \phi_2(0)\big] - T(t)[g((u_1)_{t_1}, (u_1)_{t_2}, \ldots, (u_1)_{t_p})\big](0)$$
$$- \big[g((u_2)_{t_1}, (u_2)_{t_2}, \ldots, (u_2)_{t_p})\big](0)$$
$$+ \int_0^t T(t-\tau)\Big[f\Big(\tau, (u_1)_\tau, \int_0^\tau k(\tau,\mu)m(\mu, (u_1)_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu, (u_1)_\mu)d\mu\Big)$$
$$- f\Big(\tau, (u_2)_\tau, \int_0^\tau k(\tau,\mu)m(\mu, (u_2)_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu, (u_2)_\mu)d\mu\Big)\Big]d\tau \text{ for } t \in I,$$

and for $t \in I_0$ we have

$$(4.6) \qquad \begin{aligned} u_1(t) - u_2(t) =& [\phi_1(t) - \phi_2(t)] - [g((u_1)_{t_1}, (u_1)_{t_2}, \ldots, (u_1)_{t_p})(0) \\ & - g((u_2)_{t_1}, (u_2)_{t_2}, \ldots, (u_2)_{t_p})(0)]. \end{aligned}$$

By using (H_2)-(H_5),(4.5) and since K, L are continuous on compact set $I \times I$, then there exists constants K, L such that $|k(t,s)| \leq K$ for $t \geq s \geq 0$ and $|l(t,s)| \leq L$ for $s, t \in I$. Then we have

$$||(u_1)(\theta) - (u_2)(\theta)||$$
$$\leq M||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B$$
$$+ \int_0^\theta T(\theta-\tau)\Big[f\Big(\tau, (u_1)_\tau, \int_0^\tau k(\tau,\mu)m(\mu, (u_1)_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu, (u_1)_\mu)d\mu\Big)$$
$$- f\Big(\tau, (u_2)_\tau, \int_0^\tau k(\tau,\mu)m(\mu, (u_2)_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu, (u_2)_\mu)d\mu\Big)\Big]d\tau$$

$$
\begin{aligned}
&\le M||\phi_1-\phi_2||_C + MG||u_1-u_2||_B + \int_0^\theta MF\Big[||(u_1-u_2)_C||\\
&\quad + \int_0^\tau |k(s,t)|||m(\mu,(u_1)_\mu)-m(\mu,(u_2)_\mu||d\mu\\
&\quad + \int_0^a |l(\tau,\mu)|||h(\mu,(u_1)_\mu)-h(\mu,(u_2)_\mu||d\mu\Big]d\tau\\
&\le M||\phi_1-\phi_2||_C + MG||u_1-u_2||_B + MF\int_0^\theta \Big[||(u_1-u_2)_\tau||_C\\
&\quad + KN\int_0^\tau ||(u_1-u_2)_\mu||_C d\mu + LH\int_0^a ||(u_1-u_2)_\mu||_C d\mu\Big]\\
&\le M||\phi_1-\phi_2||_C + MG||u_1-u_2||_B + MF\int_0^\theta \Big[||(u_1-u_2)_\tau||_C\\
&\quad + NKa||u_1-u_2||_C\Big]d\tau + MFLHa^2||u_1-u_2||_C\\
&\le M||\phi_1-\phi_2||_C + M(G+FHLa^2)||u_1-u_2||_C\\
&\quad + MF(1+NKa)\int_0^\tau ||u_1-u_2||_C d\tau, \text{ for } 0\le\tau\le\theta\le t\le a.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\sup_{t\in[0,a]} ||u_1(\theta)-u_2(\theta)||\\
&\le M||\phi_1-\phi_2||_C + M(G+FHLa^2)||u_1-u_2||_C\\
&\quad + MF(1+NKa)\int_0^\tau ||u_1-u_2||_C d\tau, \text{ for } t\in I
\end{aligned} \tag{4.7}
$$

By hypothesis (H_4) and (4.6) we have

$$
||(u_1)(t)-(u_2)(t)|| \le ||\phi_1-\phi_2||_C + G||u_1-u_2||_B, \text{ for } t\in I. \tag{4.8}
$$

Since $M\ge 1$ (4.7) and (4.8) imply that

$$
\begin{aligned}
||u_1(t)-u_2(t)||_C &\le M||\phi_1-\phi_2||_C + M(G+FHLa^2)||u_1-u_2||_C\\
&\quad + MF(1+NKa)\int_0^\tau ||u_1-u_2||_C d\tau, \text{for } \ t\in I.
\end{aligned}
$$

Therefore by Gronwalls inequality we have

$$
\begin{aligned}
||u_1(t)-u_2(t)||_B &\le \Big[M||\phi_1-\phi_2||_C + M(G+FLHa^2)||u_1-u_2||_C\Big]\\
&\quad \cdot \exp\Big[aMF(1+aKN)\Big] \text{for } t\in I
\end{aligned}
$$

and therefore (4.3) holds. Finally, inequality (4.4) is a consequence of inequality (4.3). Hence the proof is complete. □

In the next theorem we show the continuous dependence of mild solution by using Pachpatte's inequality.

THEOREM 4.2. *Suppose that the functions f and g satisfy assumptions(H_1)-(H_5). Then for each $\phi_1, \phi_2 \in C$ and for the corresponding mild solutions u_1, u_2 of the problems*

(4.9)
$$u'(t) + Au(t) = f\Big(t, u_t, \int_0^t k(t,\mu)m(\mu, u_\mu)d\mu, \int_0^a l(t,\mu)h(\mu, u_\mu)d\mu\Big), \ t \in [0,a],$$

$$u(s) + [g(u_{t_1}, ..., u_{t_p})](s) = \phi_i(s), s \in [-r, 0], \ (i = 1, 2). \tag{4.10}$$

The following inequality

$$||u_1 - u_2||_B \le \frac{M \exp\big(\int_0^a [MF + KM]ds\big)}{1 - R} ||\phi_1 - \phi_2||_C \tag{4.11}$$

is true, where

$$R = \int_0^a LH \exp\Big(\int_0^\tau [MF + KM]d\sigma\Big)d\tau \le 1. \tag{4.12}$$

Proof. Let $\phi_i (i = 1, 2)$ be arbitrary functions belonging to C and let $u_i (i = 1, 2)$ be the the mild solutions of problems (4.9) – (4.10).

$$\begin{aligned}
&(u_1)(t) - (u_2)(t) \\
&= T(t)\big[\phi_1(0) - \phi_2(0)] - T(t)[g((u_1)_{t_1}, (u_1)_{t_2}, \ldots, (u_1)_{t_p})\big](0) \\
&\quad - \big[g((u_2)_{t_1}, (u_2)_{t_2}, \ldots, (u_2)_{t_p})\big](0) \\
&\quad + \int_0^t T(t-\tau)\Big[f\Big(\tau, (u_1)_\tau, \int_0^\tau k(\tau,\mu)m(\mu, (u_1)_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu, (u_1)_\mu)d\mu\Big) \\
&\quad - f\Big(\tau, (u_2)_\tau, \int_0^\tau k(\tau,\mu)m(\mu, (u_2)_\mu)d\mu, \int_0^a l(\tau,\mu)h(\mu, (u_2)_\mu)d\mu\Big)\Big]d\tau \text{ for } \in I,
\end{aligned}$$

and for $t \in I_0$ we have

$$\begin{aligned}
u_1(t) - u_2(t) = [\phi_1(t) - \phi_2(t)] &- [g((u_1)_{t_1}, (u_1)_{t_2}, \ldots, (u_1)_{t_p})(0) \\
&- g((u_2)_{t_1}, (u_2)_{t_2}, \ldots, (u_2)_{t_p})(0)]
\end{aligned}$$

By using (H_2)-(H_3) and since K, L are continuous on compact set $I \times I$, then there exist constants K, L such that $|k(t,s)| \le K$ for $t \ge s \ge 0$ and

$|l(t,s)| \le L$ for $s,t \in I$. Then we have

$$||u_1(t) - u_2(t)||$$
$$\le M||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B + \int_0^t MF\Big[||(u_1 - u_2)_\tau||_C$$
$$+ \int_0^\tau |k(s,t)|||m(\mu,(u_1)_\mu) - m(\mu,(u_2)_\mu||d\mu$$
$$+ \int_0^a |l(\tau,\mu)|||h(\mu,(u_1)_\mu) - h(\mu,(u_2)_\mu||d\mu\Big]d\tau \text{ for } \in I.$$

Define

$$\psi(t) = \sup\{||(u_1 - u_2)(t)|| : t \in [-r,s]\}, s \in I.$$

Then $||(u_1 - u_2)(t)||_C \le \psi(t)$ for all $t \in I$ and there is $t^* \in$ [-r,s] such that $\psi(t) = ||(u_1 - u_2)(t)^*||$. Hence for $t^* \in$[0,s] we have

$$\psi(t) \le M||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B + \int_0^t MF||(u_1 - u_2)_\tau||_C$$
$$+ \int_0^\tau KM||(u_1 - u_2)_\mu||d\mu + \int_0^a LH||(u_1 - u_2)_\mu||d\mu$$
$$\le M||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B + \int_0^t MF[\psi(\mu)$$
$$+ \int_0^s KM\psi(\mu)d\mu + \int_0^a LH\psi(\mu)d\mu]d\tau.$$

If $t^* \in [-r,0]$ then

$$\psi(t) = ||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B$$

and the inequality (4.13) holds obviously since $M \ge 1$. By using assumption (4.12) and applying the Pachpattes inequality in Lemma (2.3) to (4.13) (with $||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B = k$, $MF = u$, $\psi = z$, $KM = v$, $LH = w$) we obtain

$$\psi(t) \le \frac{\psi(t)M||\phi_1 - \phi_2||_C + MG||u_1 - u_2||_B}{1 - R} \exp\Big(\int_0^t [MF + KG]ds\Big), t \in I$$

and therefore, (4.11) holds. Hence the proof is complete. □

Acknowledgements

The author is deeply grateful to the referee's for the careful reading of this paper and helpful comments, which have been very useful for improving the quality of this paper.

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