http://dx.doi.org/10.7468/jksmed.2015.19.2.89
J. Korean Soc. Math. Educ., Ser. D, Res. Math. Educ.
Vol. 19, No. 2, June 2015, 89–100

Comparing U. S. and Taiwanese Pre-service Teachers' Solving Triangular Arithmagons

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(Received September 22, 2014; Revised April 16, 2015; Accepted April 16, 2015)

The study investigated pre-service teachers' knowledge and computational skills by using Triangular Arithmagon. Participants included 90 pre-service teachers from two schools in the United States and Taiwan. The Triangular Arithmagons Test (TAT) was used to measure pre-service teachers' performance in whole number, fractions, and decimals operations (i.e., addition, subtraction, multiplication, and division), each of which included level-1 (basic) and level-2 (advanced) tests. MANOVA analysis was performed to compare the performance between teachers from the United States and Taiwan. Results indicated that overall, pre-service teachers in Taiwan outperformed those in the United States, especially on the advanced-level tests. Pre-service teachers in the United States were found to have poor ability of solving complex operation problems. Different curriculum plans and teaching methods may lead to the performance gap between the two countries.

Keywords: pre-service teacher, triangular arithmagon
MESC Classification: D49
MSC2010 Classification: 97D40

INTRODUCTION

The issue of computational skills has been under vast consideration for many years. Addition, subtraction, multiplication, and division with whole numbers, decimals, and fractions all are computational skills and it is believed that advancement in mathematical education may have an impact on basic computational skills like memorizing multiplica-

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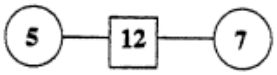
tion tables. A report on Curriculum Focal Points released by the National Council of Teachers of Mathematics in 2006 declared that the foundation for understanding, solving, and lasting learning of mathematics can be structured by constituting mathematical topics including ideas, skills, concepts, and procedures for each grade level to make a lasting impression of the fundamental of math in students' minds; and, thus, they can utilize it, too (NTCM, 2006). The report focuses on improving students' computational fluency, standard algorithm techniques, complete understanding of how mathematics work, learning basic facts, and how to implement it in practical work. The main emphasis is to make students learn through understanding.

Teachers' knowledge has been explored widely and discussed in research (Ball, Thames & Phelps, 2008; Borko et al., 1992; Confrey, Maloney & Nguyen, 2008; Hill, Schilling & Ball, 2004; Ma, 1999). However, empirical research assessing their knowledge in computational skills is limited. In the book *Knowing and Teaching Elementary Mathematics*, Ma (1999) proposed Profound Understanding of Fundamental Mathematics (PUFM) that focused on the ideal structure of elementary teachers' conceptual and procedural knowledge. She indicated a lack of conceptual knowledge for classroom teachers in her study that compares the conceptual understanding and procedural knowledge between the experienced U.S. and Chinese primary teachers. Chinese teachers showed a better conceptual understanding of the rationale of the algorithm than the U.S. teachers. For instance, when the U.S. teachers were asked to calculate 1¾ ÷ ½ , only 43% of them succeeded while all Chinese teachers got the correct answer.

Trends in International Mathematics and Science Study (TIMSS) in 2007 stated that top five countries at eighth grade level are from Asia: Chinese Taipei (Taiwan), Korea, Singapore, Hong Kong SAR, and Japan. Thus, Asian students showed a remarkable performance in cross-cultural mathematical studies as compared to those in North Americans (Mullis, Martin, & Foy, 2008). Because of the more accurate and cohesive mathematics schedule, these countries are performing better than the United States. Different researchers from all over the world have identified the factors that result in the variations in the performance (Schmidt et al., 2001; Schmidt, Wang, McKnight & Curtis, 2005). Luo, Lo & Leu (2011) identified that the difference of pre-service teachers' fraction knowledge between the United States and Taiwan is another critical element leading to performance gap among the students at the elementary level. More studies are needed to compare pre-service teachers' knowledge in computational skills and problem solving among different countries in order to gain insights into ways to strengthen teachers' knowledge in mathematics that can benefit both teachers and students.

Triangular Arithmagons refers to polygons having a number on each vertex and a box number on every side to make the number on each box equal to the sum of the circle number next to it (see Figure 1).

We begin with the rule – add the numbers in the two circles to get the number in the square.



Now find the numbers for the squares in the triangular arithmogon below.

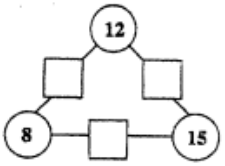


Figure 1. Triangular Arithmagons

Different arithmagons can be formed by using different arrangements and analogies of circle numbers and box numbers in the triangular arithmagons (Macintosh & Quadling, 1975; Wittmann & Muller, 2000). Simple linear algebra can be used by the students to solve triangular arithmagons. There were diverse sets of numerical patterns used, including the integer operations of addition, subtraction, multiplication, and division (see Figure 2), which helps in solving triangular arithmagons and then the square arithmagons.

The formula to find x is equal to (b – c + a)/2. Four operations, including addition, subtraction, multiplication and division are involved.

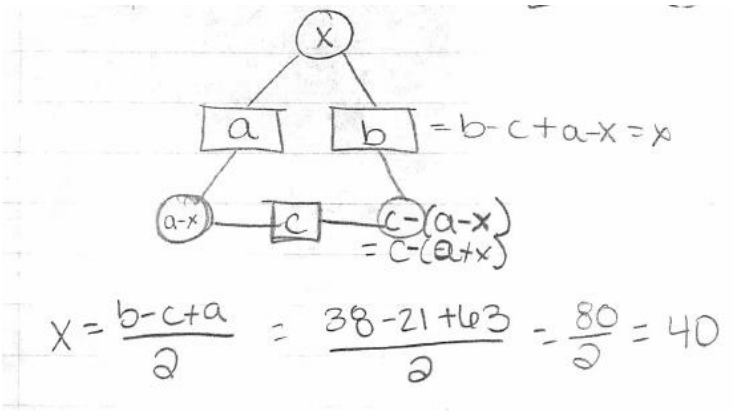


Figure 2. An example that shows student problem-solving processes

To prove algebraically the arithmagons rules, simple linear algebra lays the foundation for students. Triangular arithmagons prove to be very helpful in teaching because they enhance student's proficiency as well as problem-solving techniques. Moreover, triangular arithmagons also can be solved by establishing and implementing linear equations. Arithmagons problems provide practice in computation through problem-solving, reasoning, and mathematical thinking. National Council of Teachers of Mathematics (2000)

indicated two controversial viewpoints from two groups – back to basics and reform – where the former group placed emphasis on drill-and-practice while the latter claimed the importance of understanding through mathematical thinking. Arithmagons enables students to learn mathematics through both mathematical thinking and practice in arithmetical skills, which are critical to students' computational skills.

To provide evidence of cross-cultural differences in problem-solving and computational thinking, the purpose of this study was to compare pre-service elementary teachers' work in solving triangular arithmagons in Taiwan and the United States. The findings will help us to understand the differences of pre-service teachers' knowledge and skills in triangular arithmagons problem solving between two countries. The research questions are as follows:

1. Are there significant differences when comparing two levels of whole-number operation (addition, subtraction, multiplication, and division) between pre-service elementary teachers in the United States and Taiwan, based on their performance on the Triangular Arithmagons Test?
2. Are there significant differences when comparing two levels of fractions operation between pre-service elementary teachers in the United States and Taiwan based on their performance on the Triangular Arithmagons Test?
3. Are there significant differences when comparing two levels of decimals operation between pre-service elementary teachers in the United States and Taiwan based on their performance on the Triangular Arithmagons Test?
4. Are there any relationships between the total score of the Triangular Arithmagons Test (TAT) and the countries where the pre-service elementary teachers are from?

METHODS

Sample

Research was conducted at a Midwestern university in the United States and a university from central Taiwan. The convenience sample was utilized. Data were gathered through undergraduate students in the teacher preparation program. The sample included 90 pre-service teachers, 65 of them from Taiwan and 25 from the United States.

The participants in the U.S. sample were enrolled in a teacher-education program at a Midwestern university. Twenty-five pre-service teachers, who were preparing to teach children in grades K–8, participated in this study. Ninety-two percent of the participants were female (23 women, 2 men). The average age for this sample was 20.4 years.

The Taiwanese sample was similarly selected. The Taiwanese sample consisted of 65

pre-service teachers enrolled in a teacher education program at a university in central Taiwan who were preparing to teach children in grades K–6. They were in their third year. Eighty-six percent of the participants were female (56 women, 9 men). The average age for this sample was 20.2 years.

Instruments

The Triangular Arithmagons Test (TAT) consisted of 36 items in three dimensions: whole number, fractions and decimals. These items were designed to investigate pre-service teachers' knowledge of computational skills, and were finalized based on the feedback from three mathematics education professors. There were 12 items in each of the three dimensions (i.e., whole number, fractions, and decimals). Within each dimension, the 12 items were divided into two levels — level-1(easy) and level-2 (difficult).

The level-1 problems included the items with an unknown result of the operation (the boxes for the results of the operation in the arithmagons are empty but the circles with the operands are given) and the level-2 with unknown operands (the results are given in the boxes, but the operands are missing from the circles) (see Figure 1 and Appendices). The TAT test was timed. Students were required to complete it within an hour. Before the test began, the instructor provided verbal instructions to the test.

The instrument was piloted with forty pre-service teachers who were enrolled in the mathematics methods courses offered in an earlier semester. Three mathematics educators reviewed content validity of the test. The instrument items were modified following their suggestions. The internal reliability (Cronbach's alpha) of the test was 0.806. Examples from the Triangular Arithmagons Test are provided in Appendix A.

Data Collection and Analysis

Data were collected by researchers from the universities in the U.S. and Taiwan. The researchers gave students the TAT test and required them to complete the test within an hour. To answer the proposed questions, the one-way Multivariate Analysis of Variance (MANOVA) was performed to determine whether there were any differences between two independent groups (U.S. vs Taiwan) on more than one continuous dependent variable (i.e., whole number, fractions, decimals).

RESULTS

Descriptive Analysis

Table 1 reveals the average scores of each test and each dimension (i.e., whole number,

fractions, decimals), and the sum scores of all dimensions for pre-service teachers in the United States and Taiwan. The average scores of Taiwanese pre-service teachers were higher than those of American pre-service teachers on each sub-level test (level-1 & level-2). Generally, Taiwanese pre-service teachers received much better scores on each sub-level test than American pre-service teachers, except for the level-1 whole-number operation test (5.72 in the United States; 5.85 in Taiwan) where the difference was small. The average scores of level-2 fractions operation (1.84 in the United States; 4.93 in Taiwan) and decimals (0.68 in the United States; 5.14 in Taiwan) were much higher for Taiwanese pre-service teachers than Americans.

Table 1. Descriptive Information for the Scores from Three Types of Operation Tests

Types of Operation Tests	U.S.		Taiwan	
	M	*SD*	*M*	*SD*
Whole number operation: level-1	5.72	0.54	5.85	0.78
Whole number operation: level-2	4.16	1.80	5.86	0.46
Total whole number operation	9.88	1.96	11.71	0.93
Fractions operation: level-1	4.72	1.67	5.52	0.90
Fractions operation: level-2	1.84	2.06	4.93	1.43
Total Fractions operation	6.56	3.12	10.46	1.77
Decimals operation: level-1	3.48	2.43	5.57	0.92
Decimals operation: level-2	0.68	1.44	5.14	1.43
Total decimals operation	4.16	3.26	10.71	1.75
Sum (whole number, fractions operation & decimals operation)	20.60	6.45	32.87	3.92

MANOVA analysis

Research Question 1.

The MANOVA results (see Table 2) revealed that pre-service elementary teachers from the United States and Taiwan did not significantly differ from each other in the score of level-1 whole-number operation ($F = 0.56$, $p = .456$). However, Taiwanese pre-service teachers performed significantly better in the level-2 test of whole-number operation ($F = 50.49$, $p < .001$).

Table 2. MANOVA Analysis for Two-levels of Whole Number Operation

Variables	*F*	*p*
Whole number operation: level-1	.56	.458
Whole number operation: level-2	50.49	.001***

Research Question 2.

In terms of the test scores of two-level fractions operation (see Table 3), pre-service elementary teachers in Taiwan did better than those in the United States (Level-1 fractions operation: $F = 8.59$, $p = .004$; Level-2 fractions operation: $F = 65.44$, $p < .001$).

Table 3. MANOVA Analysis for Two-levels of Fractions Operation Tests

Variables	*F*	*p*
Fractions operation: level-1	8.59	.004**
Fractions operation: level-2	65.44	.001***

Research Question 3.

Similarly, Taiwanese pre-service elementary teachers had significantly better scores of decimals operation in both level-1 ($F = 35.35$, $p < .001$) and level-2 tests ($F = 174.31$, $p < .001$) (see Table 4).

Table 4. MANOVA Analysis for Two-levels of Decimals Operation Tests

Variables	*F*	*p*
Decimals operation: level-1	35.35	.001***
Decimals operation: level-2	174.31	.001***

Research Question 4.

One way MANOVA was used to analyze three dimensions of operation tests. According to the MANOVA test, where the total scores from three dimensions were entered as dependent variables (see Table 5), pre-service teachers from Taiwan had significantly higher scores than teachers from the United States at all three types of mathematics operation tests (Hotelling's Trace = 1.80, $F = 51.72$, $p < .001$), including whole-number operation ($F = 35.84$, $p < .001$), fractions operation ($F = 55.69$, $p < .001$), and decimals operation ($F = 151.11$, $p < .001$).

Table 5. MANOVA Analysis for Three Types of Operation Tests

Variables	*F*	*p*
Whole number operation	35.84	.001***
Fractions operation	55.69	.001***
Decimals operation	151.11	.001***

DISCUSSION

Results showed no significant difference in the performance of level-1 whole-number operation for pre-service teachers between Taiwan and the U.S.A. However, pre-service teachers in Taiwan outperformed on the level-2 whole-number operation than pre-service teachers in the United States. This implies that pre-service teachers in the United States encountered more difficulty in solving more complex whole-number operation problems than those in Taiwan.

Pre-service teachers in Taiwan performed significantly better than pre-service teachers in the United States on the level-1 and level-2 fraction operations problems. In particular, there was a big gap on the level-1 and level-2 performance scores between the U.S. and Taiwanese pre-service teachers (level-1: 4.72 vs 5.52; level-2: 1.84 vs 4.93). This means that Taiwanese pre-service teachers can solve fraction operation problems better than the U.S. pre-service teachers. It appeared that complex fraction operation problems were difficult for the pre-service teachers in the United States.

Data also show that pre-service teachers in Taiwan performed significantly better than pre-service teachers in the United States on the level-1 and level-2 decimals operation problems. In addition, the performance gap on level-2 decimals operation was more remarkable than level-1 decimals operation problems between pre-service teachers in the United States and Taiwan (level-1: 3.48 vs 5.57; level-2: 0.68 vs 5.14). This implies that the pre-service teachers in the United States had poorer ability to solve decimal operation problems than pre-service teachers in Taiwan. Results show that pre-service teachers in Taiwan performed significantly better than pre-service teachers in the United States on the sum scores of three types of tests. This result is similar to the finding of the Teacher Education and Development Study in Mathematics (TEDS-M) (Lin & Hsieh, 2008) that pre-service teachers in Taiwan (Mean score: 623) outperformed per-service teachers in the United States (Mean score: 518) on mathematical knowledge.

Whole-number operation is less complex than fractions operation and decimal operation. Fractions are a difficult topic for teachers to teach and for students to learn because they involve relations between quantities. Another reason is that one cannot count fractions because there are an infinite number of fractions between any two fractions or any two whole numbers (Lin, 2013).This may explain why American pre-service teachers had similar performance on level-1 whole-number operation as Taiwanese pre-service teacher, but not for the level-2 test. The performance gap between the U.S. and Taiwanese pre-service teachers on the triangular arithmagons tests may be due to different teaching methods and student learning styles in mathematics in the two countries. For example, Taiwanese students may spend more time practicing various operation problems

repeatedly until they could get most of the questions right in a limited time. Also, Taiwanese students may spend more time seeking potential patterns or formulas through logical analysis during the problem-solving process; however, American students may get started by trying different sets of numbers rather than finding out the underlying patterns or formulas. Based on the researchers' observation, American students appeared to apply the trial-and-error approach to problem solving. They tried a number of different solutions and rules that might not work and were time consuming. In terms of teaching methods, Taiwanese teachers may try to cover advanced skills that go beyond the textbook content during the class, even though the advanced content is not required to be included.

CONCLUSIONS AND SIGNIFICANCE

Overall, this study found that pre-service teachers in Taiwan had better computational skills in solving operational problems using triangular arithmagons than those in the United States. Compared to Taiwanese pre-service teachers, American pre-service teachers showed equivalent performance only in the basic-level whole-number operation test, but had much lower performance in the advanced-level tests, particularly in fractions and decimals. It is not surprising because the whole-number operation was deemed the easiest operation as opposed to fraction or decimal operation. This study not only confirms the outperformance of Taiwanese pre-service teachers over the U.S. teachers in computational skills and fluency for advanced operations, but also provides evidence that adds to the limited research about the performance gap of teachers' mathematics knowledge in computational skills between western (i.e., United States) and Asian countries (i.e., Taiwan). The implications of practical significance suggest that curriculum planners should pay attention to teaching methods and curriculum design when revising courses to enhance students' operation skills. Qualitative approach is suggested to understand further problem-solving processes of using triangular arithmagons among pre-service teachers in the United States and Taiwan. In addition, due to the convenience sample approach, the sample size of the teachers from the United States was smaller than that of the teachers from Taiwan, which might influence the significance results of this study.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. Jerry Becker, Dr. Der-Ching Yang and three anonymous reviewers for their helpful comments on an earlier draft of this article.

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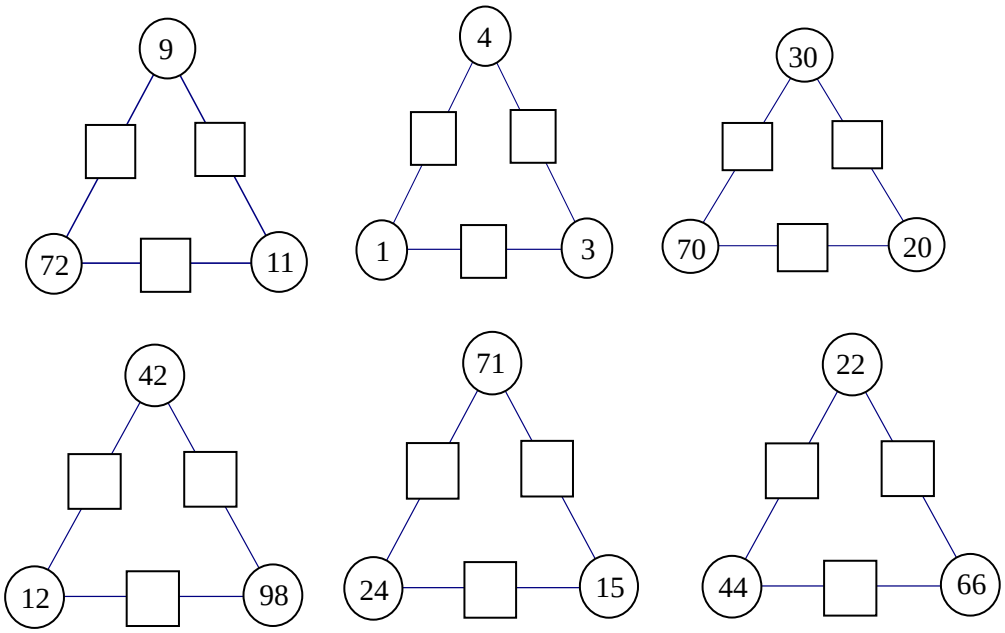
APPENDIX A. TRIANGULAR ARITHMAGONS

The instructions for the TAT test were provided verbally by the instructor.

Triangular Arithmagons refer to polygons having a number on each vertex and a box number on every side so as to make the number on each box equal to the sum of circle numbers next to it. See Figure 1 for example.

LEVEL 1



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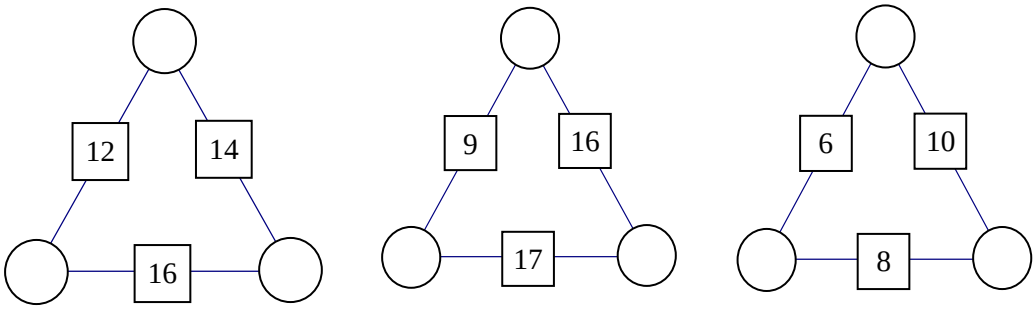

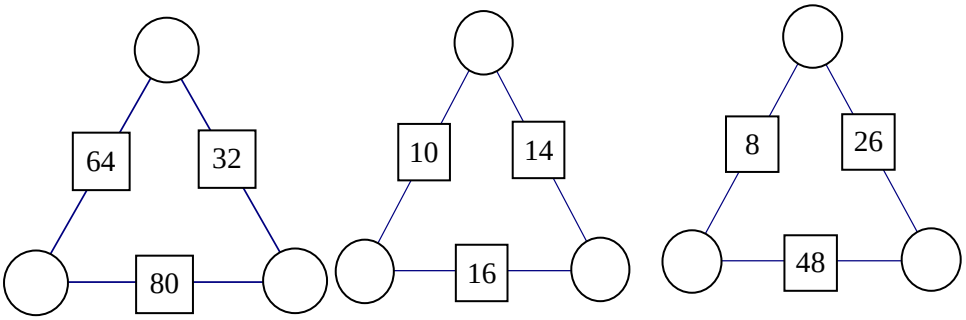
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