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NEW FAMILY OF ITERATIVE METHODS FOR SOLVING NON-LINEAR EQUATIONS USING NEW ADOMIAN POLYNOMIALS

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Abstract. We suggest and analyze a family of multi-step iterative methods for solving nonlinear equations using the decomposition technique mainly due to Rafiq et al. [13].

1. Introduction

In recent years, much attention has been given to develop several iterative methods for solving nonlinear equations (see for example [1, 6-12, 14]). These methods can be classified as one-step and two-step methods.
Abbasbandy [1] and Chun [6] have proposed and studied several one-step and two-step iterative methods with higher order convergence by using the decomposition technique of Adomian [2].
In [11], Noor developed two-step and three-step iterative methods by using the Adomian decomposition technique and by combining the well-known Newton method with other one-step and two-step methods.
In [1, 11-12], the authors have used the higher order derivatives which is a drawback. To overcome this drawback, following the lines of [11], we suggest and analyze a family of multi-step iterative methods which do not involve the high-order derivatives of the function for solving nonlinear equations using the decomposition technique mainly due to Rafiq et al. [13]. We also discuss the convergence of the new proposed methods. Several numerical examples are given to illustrate the efficiency and the performance of the new iterative method. Our results can be considered as an improvement and refinement of the previous results.

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2. Iterative Methods

Consider the nonlinear equation

$$f(x) = 0. \tag{1}$$

We assume that α is a simple root of (1) and β is an initial guess sufficiently close to α. We can rewrite (1) as a coupled system using the Taylor series.

$$f(\beta) + (x-\beta)f'(\beta) + \frac{(x-\beta)^2}{2!}f''(\beta) + g(x) = 0, \tag{2}$$

$$g(x) = f(x) - f(\beta) - (x-\beta)f'(\beta) - \frac{(x-\beta)^2}{2!}f''(\beta). \tag{3}$$

We can rewrite (3) in the following form as

$$x = \beta - \frac{f(\beta)}{f'(\beta)} - \frac{(x-\beta)^2}{2!}\frac{f''(\beta)}{f'(\beta)} - \frac{g(x)}{f'(\beta)}. \tag{4}$$

We can rewrite (4) in the following equivalent form as

$$x = c + N(x), \tag{5}$$

where

$$c = \beta - \frac{f(\beta)}{f'(\beta)} \tag{6}$$

and

$$N(x) = -\frac{(x-\beta)^2}{2!}\frac{f''(\beta)}{f'(\beta)} - \frac{g(x)}{f'(\beta)}. \tag{7}$$

In order to prove the multi-step iterative methods, He [8] and Lao [10] have considered the case with the definition that

$$g(x_0) = 0 \tag{8}$$

and Noor and Noor [12] have considered the case

$$f(x_0) = g(x_0), \tag{9}$$

which is actually

$$f\Big(\sum_{i=0}^{\infty} x_i\Big) = g\Big(\sum_{i=0}^{\infty} x_i\Big), \tag{10}$$

and is the stronger one. We rectify this error and also remove such kind of conditions. For this purpose, we substituting (3) into (7) to obtain

$$N(x) = x - \beta - \frac{f(x)}{f'(\beta)} + \frac{f(\beta)}{f'(\beta)} \tag{11}$$

with

$$N'(x) = 1 - \frac{f'(x)}{f'(\beta)}. \tag{12}$$

We now construct a new family of iterative methods by using the following decomposition method mainly due to Rafiq et al. [13]. This decomposition of the nonlinear operator $N(x)$ is quite different than that of Adomian decomposition. The main idea of this technique is to look for a solution of (5) having the series form

$$x = \sum_{i=0}^{\infty} x_i. \tag{13}$$

The nonlinear operator N can be decomposed as

$$N(x) = N\Big(\sum_{i=0}^{\infty} x_i\Big) = \sum_{i=0}^{\infty} A_i, \tag{14}$$

where A_i are the functions which are known as the new Adomian polynomials depending on $x_0, x_1, \cdots$; given by a formula

$$A_i = \frac{1}{i!} \cdot \frac{d^i}{d\lambda^i}\Big[N\Big(\sum_{j=0}^{i} \lambda^j x_j\Big) - N\Big(\sum_{j=0}^{i-1} \lambda^j x_j\Big)\Big]. \tag{15}$$

First few new Adomian polynomials are as follows

$$\begin{aligned} A_0 &= N(x_0), \\ A_1 &= x_1 N'(x_0), \\ A_2 &= x_2 N'(x_0), \\ A_3 &= x_3 N'(x_0), \\ &\vdots \\ A_i &= x_i N'(x_0), \\ &\vdots \end{aligned} \tag{16}$$

Substituting (12) and (13) into (5), we obtain

$$\sum_{i=0}^{\infty} x_i = c + \sum_{i=0}^{\infty} A_i. \tag{17}$$

It follows from (6), (12) and (17), that

$$x_0 = c = \beta - \frac{f(\beta)}{f'(\beta)}. \tag{18}$$

This allows us to suggest the following one-step iterative method for solving (1).

Algorithm 1.

For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},\ f'(x_n) \neq 0,\ n = 0, 1, 2, \cdots .$$

which is known as *"Newton's Method"* and it has the *second order convergence.* From (10) and (17), we have

$$\begin{aligned} x_1 = N(x_0) &= x_0 - \beta - \frac{f(x_0)}{f'(\beta)} + \frac{f(\beta)}{f'(\beta)} \\ &= -\frac{f(x_0)}{f'(\beta)}. \end{aligned} \tag{19}$$

Again using (12), (15), (16), (17) and (18) , we conclude that

$$\begin{aligned} x \approx c + x_1 &= x_0 + N(x_0) \\ &= \beta - \frac{f(\beta)}{f'(\beta)} - \frac{f(x_0)}{f'(\beta)}. \end{aligned} \tag{20}$$

Using this relation, we can suggest the following two-step iterative methods for solving (1).

Algorithm 2.

For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme
Predictor-Step

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},\ f'(x_n) \neq 0,$$

Corrector-Step

$$x_{n+1} = y_n - \frac{f(y_n)}{f'(x_n)}.$$

This Algorithm is commonly known as *"Double-Newton Method"* with the *third order convergence.*
Again

$$\begin{aligned} x_2 = A_1 &= x_1 N'(x_0) \\ &= N(x_0)N'(x_0) \\ &= -\frac{f(x_0)}{f'(\beta)} \cdot [1 - \frac{f'(x_0)}{f'(\beta)}] \\ &= -\frac{f(x_0)}{f'(\beta)} + \frac{f(x_0)f'(x_0)}{f'^2(\beta)}. \end{aligned} \tag{21}$$

From (11), (17)-(21), we conclude that

$$\begin{aligned} x &\approx c + x_1 + x_2 \\ &= \beta - \frac{f(\beta)}{f'(\beta)} - \frac{f(x_0)}{f'(\beta)} - \frac{f(x_0)}{f'(\beta)} + \frac{f(x_0)f'(x_0)}{f'^2(\beta)}. \end{aligned}$$

Using this, we can suggest and analyze the following two-step iterative method for solving (1).

Algorithm 3. (AP)

For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme
Predictor-Step

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}\ ; f'(x_n) \neq 0,\ n = 0, 1 \cdots , \tag{22}$$

Corrector-Step

$$x_{n+1} = y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)f'(y_n)}{f'^2(x_n)}\ ; n = 0, 1, 2, \cdots .$$

Algorithm 3 is called the two-step iterative method for solving (1).
Again using (11) and (16), we have

$$x_3 = A_2 = x_2 N'(x_0) = N(x_0)N'^2(x_0).$$

From (11), (18) – (20), we have

$$\begin{aligned} x &\approx c + x_1 + x_2 + x_3 \\ &= c + N(x_0) + N(x_0)N'(x_0) + N(x_0)N'^2(x_0), \\ &= \beta - \frac{f(\beta)}{f'(\beta)} - 3\frac{f(x_0)}{f'(\beta)} + 3\frac{f(x_0)f'(x_0)}{f'^2(\beta)} - \frac{f(x_0)f'^2(x_0)}{f'^3(\beta)}. \end{aligned}$$

Using this, we can suggest and analyze the following iterative method for solving (1).

Algorithm 4.

For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme
Predictor-Step

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},\ f'(x_n) \neq 0,\ n = 0, 1, \cdots , \tag{23}$$

Corrector-Step

$$x_{n+1} = y_n - 3\frac{f(y_n)}{f'(x_n)} + 3\frac{f(y_n)f'(y_n)}{f'^2(x_n)} - \frac{f(y_n)f'^2(y_n)}{f'^3(x_n)}.$$

If $f'(y_n) = 0$ then Algorithm 3 reduces to the following method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$x_{n+1} = y_n - 2\frac{f(y_n)}{f'(x_n)}.$$

Now using finite difference approximation, we obtain

$$f'(y_n) \cong \frac{f(y_n) - f(x_n)}{y_n - x_n}. \tag{24}$$

Combining (22) and (24), we suggest the following new iterative method for solving (1) as follows

$$\begin{aligned} x_{n+1} &= y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)}{f'^2(x_n)} \cdot \frac{f(y_n) - f(x_n)}{y_n - x_n} \\ &= y_n - 2\frac{f(y_n)}{f'(x_n)} + \frac{f(y_n)}{f'^2(x_n)} \cdot \frac{f(y_n) - f(x_n)}{-\frac{f(x_n)}{f'(x_n)}} \\ &= y_n - \frac{f(y_n)}{f'(x_n)} - \frac{f^2(y_n)}{f(x_n)f'(x_n)}. \end{aligned}$$

Also we can suggest and analyze the following iterative method for solving (1).

Algorithm 5. (A)

For a given x_0, compute the approximate solution x_{n+1} by the iterative scheme

Predictor-Step

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \; f'(x_n) \neq 0, \; n = 0, 1, \cdots , \tag{25}$$

Corrector-Step

$$x_{n+1} = y_n - \frac{f(y_n)}{f'(x_n)} - \frac{f^2(y_n)}{f(x_n)f'(x_n)}. \tag{26}$$

3. Convergence Analysis

Theorem 1. *Let $\beta \in I$ be a simple zero of sufficiently differentiable function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ for an open interval I. If x_0 is sufficiently close to β, then the two-step iterative method defined by Algorithm 3 has the fourth-order convergence.*

Proof. Let $\beta \in I$ be a simple zero of f. Since f is sufficiently differentiable function, by expanding $f(x_n)$ and $f'(x_n)$ about β, we get

$$f(x_n) = f'(\beta)[e_n + c_2 e_n^2 + c_3 e_n^3 + \cdots], \tag{27}$$

$$f'(x_n) = f'(\beta)[1 + 2c_2e_n + 3c_3e_n^2 + \cdots], \tag{28}$$

where $c_k = \frac{f_k(\beta)}{k!f'(\beta)}$, $k = 1, 2, 3, \cdots$ and $e_n = x_n - \beta$.
Now from (27) and (28), we have

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 - 2(c_3 - c_2^2)e_n^3 - (3c_4 - 7c_2c_3 + 4c_2^3)e_n^4 + \cdots . \tag{29}$$

From (18) and (29), we get

$$y_n = \beta + c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 4c_2^3)e_n^4 + \cdots . \tag{30}$$

Now expanding $f(y_n)$ about β, we have

$$f(y_n) = f'(\beta)[c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 4c_2^3)e_n^4 + \cdots]. \tag{31}$$

From (28) and (31), we have

$$\frac{f(y_n)}{f'(x_n)} = c_2e_n^2 + 2(c_3 - 2c_2^2)e_n^3 + (3c_4 - 14c_2c_3 + 9c_2^3)e_n^4 + \cdots . \tag{32}$$

Now expanding $f'(y_n)$ about β and using (32), we have

$$f'(y_n) = f'(\beta)[1 + 2c_2e_n^2 + 4(c_3 - c_2^2)e_n^3 + (6c_2c_4 - 11c_3c_2^2 + 8c_2^4)e_n^4 + \cdots]. \tag{33}$$

From (31) and (34), we get

$$f(y_n)f'(y_n) = f'(\beta)^2[c_2e_n^2 + (2c_3 - 2c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 7c_2^3)e_n^4 + \cdots]. \tag{34}$$

From (29) and (35), we have

$$\frac{f(y_n)f'(y_n)}{f'^2(x_n)} = c_2e_n^2 + (2c_3 - 6c_2^2)e_n^3 + (3c_4 - 21c_2c_3 + 11c_2^3)e_n^4 + \cdots \tag{35}$$

From (22), (32) and (35), one obtains

$$e_{n+1} = -3c_2^3e_n^4 + o(e_n^5). \tag{36}$$

Hence it is proved. □

Theorem 2. *Let $\beta \in I$ be a simple zero of a sufficiently differentiable function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ for an open interval I. If x_0 is sufficiently close to β, then the iterative method defined by Algorithm 4 has the fourth- order convergence.*

Proof. From (28) and (33), we have

$$\begin{aligned} \frac{f'(y_n)}{f'(x_n)} &= 1 - 2c_2e_n + (2c_2^2 - 3c_3)e_n^2 + (4c_2c_3 - 8c_2^3 - 4c_4)e_n^3 \\ &\quad + (6c_2c_4 - 25c_2^2c_3 + 16c_2^4)e_n^4 + \cdots . \end{aligned} \tag{37}$$

From (35) and (37), we get

$$\frac{f(y_n)f'^2(y_n)}{f'^3(x_n)} = c_2e_n^2 + (2c_3 - 8c_2^2)e_n^3 + (3c_4 - 28c_2c_3 + 25c_2^3)e_n^4 + \cdots . \tag{38}$$

From (23), (32), (35) and (38), one obtains

$$e_{n+1} = -15c_2^3e_n^4 + o(e_n^5). \tag{39}$$

Hence it is proved. □

Theorem 3. *Let $\beta \in I$ be a simple zero of a sufficiently differentiable function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ for an open interval I. If x_0 is sufficiently close to β, then iterative method defined by Algorithm 5 has the third- order convergence.*

Proof. From (27) and (31), we have

$$\frac{f(y_n)}{f(x_n)} = c_2e_n^3 + (2c_3 - 3c_2^2)e_n^4 + \cdots . \tag{40}$$

From (32) and (40), we get

$$\frac{f^2(y_n)}{f(x_n)f'(x_n)} = c_2^2e_n^5 + \cdots . \tag{41}$$

From (26), (30), (32) and (41), one obtains

$$e_{n+1} = 2c_2^2e_n^3 + (7c_2c_3 - 5c_2^3)e_n^4 + o(e_n^5). \tag{42}$$

Hence it is proved. □

4. NUMERICAL EXAMPLES

We provide some examples to illustrate the efficiency of the new developed iterative methods. Put $\epsilon = 10^{-15}$.
The following stopping criteria is used for the computer programs

(1) $|x_{n+1} - x_n| < \epsilon$,
(2) $|f(x_{n+1})| < \epsilon$.

The examples are the same as in Chun [6]:

$F_1(x) = sin^2x - x^2 + 1$,
$F_2(x) = x^2 - e^x - 3x + 2$,
$F_3(x) = cosx - x$,
$F_4(x) = (x - 1)^3 - 1$,
$F_5(x) = x^3 - 10$,

$F_6(x) = x \cdot e^{x^2} - sin^2 x + 3cosx + 5,$
$F_7(x) = e^{x^2+7x-30} - 1.$

Also for the convergence criteria, it was required that the distance of two consecutive approximations δ for the zero was less than 10^{-15}. Also displayed are the number of iterations (IT) to approximate the zero, the approximate zero x_0, the value $f(x_0)$ and δ. We compare the Newton method (NM), the Double Newton method (DNM), the method of Noor (NR) [12] and the method (AP) , introduced in the Algorithm 3 (see Table 1).

Table 1

	IT	x_n	$f(x_n)$	δ
$F_1 : x_0 = 1$				
NM	7	1.404491648215341	3.331e-16	3.059e-13
DNM	6	1.404491648215341	-4.441e-016	1.532e-07
NR	6	1.404491648215341	-4.441e-016	8.882e-16
AP	5	1.404491648215341	3.331e-16	4.833e-05
$F_2 : x_0 = 2$				
NM	6	0.257530285439861	0	9.864e-14
DNM	5	0.257530285439861	0	5.689e-14
NR	6	0.257530285439861	0	5.689e-14
AP	4	0.257530285439861	0	3.902e-07
$F_3 : x_0 = 1.7$				
NM	5	0.739085133215161	-4.441e-16	3.259e-08
DNM	4	0.739085133215161	0	2.618e-08
NR	6	0.739085133215161	-7.772e-16	3.305e-08
AP	4	0.739085133215161	0	1.364e-13
$F_4 : x_0 = 3.5$				
NM	8	2	0	2.878e-11
DNM	6	2	0	4.512e-10
NR	8	2	0	2.287e-14
AP	5	2	0	8.606e-07
$F_5 : x_0 = 1.5$				
NM	7	2.154434690031919	4.83e-13	2.741e-07
DNM	5	2.154434690031919	-4.388e-10	4.181e-04
NR	6	2.154434690031919	-1.065e-14	2.572e-08

AP	5	2.154434690031919	1.776e-15	4.211e-06
$F_6 : x_0 = -2$				
NM	9	-1.207647827130919	-2.664e-15	4.26e-11
DNM	6	-1.207647827130919	-3.957e-11	7.559e-05
NR	5	-1.207647827130919	3.553e-15	8.744e-11
AP	6	-1.207647827130919	-2.664e-15	5.97e-10
$F_7 : x_0 = 3.5$				
NM	13	3	0	2.53e-13
DNM	9	3	0	1.210e-06
NR	7	3	0	1.299e-08
AP	8	3	0	3.495e-07

Now we compare the Newton method (NM), the Double Newton method (DNM), the method of Weerakoon and Fernando [14], the method of Frontini and Sormani [7], the method of Homeier [9], the method of Noor (NR) [12] and the method (A), introduced in the Algorithm 5 (see Table 2).

Table 2

	IT	x_n	$f(x_n)$	δ
$F_1 : x_0 = 1$				
NM	7	1.404491648215341	3.331e-16	3.059e-13
DNM	6	1.404491648215341	-4.441e-016	1.532e-07
WF	5	1.404491648215341	-4.441e-016	1.79e-10
FS	5	1.404491648215341	3.331e-16	2.352e-11
HM	4	1.404491648215341	3.331e-16	5.652e-07
NR	6	1.404491648215341	-8.882e-16	2.22e-16
A	5	1.404491648215341	3.331e-16	2.122e-07
$F_2 : x_0 = 2$				
NM	6	0.257530285439861	0	9.864e-14
DNM	5	0.257530285439861	0	5.689e-14
WF	5	0.257530285439861	0	1.630e-11
FS	4	0.257530285439861	0	8.941e-08
HM	5	0.257530285439861	4.441e-16	3.197e-14
NR	5	0.257530285439861	0	2.209e-12
A	4	0.257530285439861	0	2.335e-05
$F_3 : x_0 = 1.7$				

NM	5	0.739085133215161	-4.441e-16	3.259e-08
DNM	4	0.739085133215161	0	2.618e-08
WF	4	0.739085133215161	0	4.085e-07
FS	4	0.739085133215161	0	6.045e-07
HM	4	0.739085133215161	0	1.422e-06
NR	4	0.739085133215161	0	1.849e-08
A	4	0.739085133215161	0	6.251e-09
$F_4 : x_0 = 3.5$				
NM	8	2	0	2.878e-11
DNM	6	2	0	4.512e-10
WF	6	2	0	6.550e-13
FS	6	2	0	1.110e-14
HM	5	2	0	2.063e-08
NR	7	2	0	2.2204e-15
A	6	2	0	3.473e-13
$F_5 : x_0 = 1.5$				
NM	7	2.154434690031919	4.83e-13	2.741e-07
DNM	5	2.154434690031919	-4.388e-10	4.181e-04
WF	4	2.154434690031919	-8.169e-10	6.156e-04
FS	4	2.154434690031919	-2.127e-11	1.978e-04
HM	4	2.154434690031919	1.776e-15	1.397e-07
NR	6	2.154434690031919	1.776e-15	1.761e-08
A	5	2.154434690031919	1.776e-15	1.717e-10
$F_6 : x_0 = -2$				
NM	9	-1.207647827130919	-2.664e-15	4.26e-11
DNM	6	-1.207647827130919	-3.957e-11	7.559e-05
WF	6	-1.207647827130919	-4.174e-14	8.668e-06
FS	6	-1.207647827130919	-2.665e-15	2.294e-08
HM	5	-1.207647827130919	-5.989e-10	3.088e-04
NR	8	-1.207647827130919	-6.217e-15	3.002e-09
AA	6	-1.207647827130919	2.665e-15	1.531e-06
$F_7 : x_0 = 3.5$				
NM	13	3	0	2.53e-13
DNM	9	3	0	1.210e-06
WF	9	3	0	1.916e-09

FS	8	3	0	8.755e-07
HM	7	3	0	7.226e-05
NR	11	3	-6.217e-15	3.002e-09
A	8	3	4.679e-10	9.406e-05

5. Conclusion

We have suggested a family of two-step iterative methods for solving nonlinear equations by using a new decomposition technique mainly due to Rafiq et al. [13]. It is important to note that the implementation of these methods does not require the computation of higher order derivatives compared to most other methods of the same order.

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