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Categorical Aspects of Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper, we obtain two types of adjoint functors between the category of intuitionistic fuzzy topological spaces in Mondal and Samanta's sense, and the category of intuitionistic fuzzy topological spaces in Šostak's sense. Also, we reveal that the category of Chang's fuzzy topological spaces is a bireflective full subcategory of the category of intuitionistic fuzzy topological spaces in Mondal and Samanta's sense.

Keywords: intuitionistic fuzzy topology

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1. Introduction

Chang [2] defined fuzzy topological spaces with the concept of fuzzy set introduced by Zadeh [11]. After that, many generalizations of the fuzzy topology were studied by several authors like Šostak [10], Ramadan [9], and Chattopadhyay and his colleagues [3].

On the other hand, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. Çoker [4] introduced intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. Mondal and Samanta [7] introduced the concept of intuitionistic gradation of openness as a generalization of a smooth topology of Ramadan (see [9]). Also, using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces.

Lee and Lee [6] revealed that the category of Chang's fuzzy topological spaces is a bireflective full subcategory of the category of intuitionistic fuzzy topological spaces in Çoker's sense. Also, Park and his colleagues [8] showed that the category of intuitionistic fuzzy topological spaces in Çoker's sense is a bireflective full subcategory of the category of intuitionistic fuzzy topological spaces in Šostak's sense.

The aim of this paper is to continue this investigation of categorical relationships between those categories. We obtain two types of adjoint functors between the category of intuitionistic fuzzy topological spaces in Mondal and Samanta's sense, and the category of intuitionistic fuzzy topological spaces in Šostak's sense. Also, we reveal that the category of Chang's fuzzy topological spaces is a bireflective full subcategory of the category of intuitionistic fuzzy topological spaces in Mondal and Samanta's sense.

2. Preliminaries

We will denote the unit interval $[0, 1]$ of the real line by I. A member μ of I^X is called a *fuzzy* *set* in X. By $\tilde{0}$ and $\tilde{1}$ we denote the constant fuzzy sets in X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively and $\mu_A + \gamma_A \leq 1$. By $\underline{0}$ and $\underline{1}$ we denote the constant intuitionistic fuzzy sets with value $(0, 1)$ and $(1, 0)$, respectively. Obviously every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Let f be a mapping from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set in X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set in Y. Then

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

All other notations are standard notations of fuzzy set theory.

Definition 2.1. ([2]) A *Chang's fuzzy topology* on X is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0},\ \tilde{1} \in T$.

(2) If $\mu_1, \mu_2 \in T$, then $\mu_1 \wedge \mu_2 \in T$.

(3) If $\mu_i \in T$ for each i, then $\bigvee \mu_i \in T$.

The pair (X, T) is called a *fuzzy topological space*.

Definition 2.2. ([9]) A *smooth topology* on X is a mapping $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.

(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth topological space*.

Definition 2.3. ([4]) An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $\underline{0},\ \underline{1} \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for each i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.4. ([5]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

Then $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ the *gradation of openness* of A and $\mathcal{T}_2(A)$ the *gradation of nonopenness* of A.

Definition 2.5. ([5]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y. Then f is said to be *SoIF continuous* if $\mathcal{T}_1(f^{-1}(B)) \geq \mathcal{T}_1(B)$ and $\mathcal{T}_2(f^{-1}(B)) \leq \mathcal{T}_2(B)$ for each $B \in I(Y)$.

Let $(X, \mathcal{T})$ be a SoIFTS. Then for each $(r, s) \in I \otimes I$, the family $\mathcal{T}_{(r,s)}$ defined by

$$\mathcal{T}_{(r,s)} = \{A \in I(X) \mid \mathcal{T}_1(A) \geq r \text{ and } \mathcal{T}_2(A) \leq s\}$$

is an intuitionistic fuzzy topology on X. In this case, $\mathcal{T}_{(r,s)}$ is called the (r, s)*-level intuitionistic fuzzy topology* on X.

Let (X, T) be an intuitionistic fuzzy topological space. Then for each $(r, s) \in I \otimes I$, a SoIFT $T^{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1, 0) & \text{if } A = \underline{0}, \underline{1}, \\ (r, s) & \text{if } A \in T - \{\underline{0}, \underline{1}\}, \\ (0, 1) & \text{otherwise.} \end{cases}$$

In this case, $T^{(r,s)}$ is called an (r,s)-*th graded SoIFT* on X and $(X, T^{(r,s)})$ is called an (r,s)-*th graded SoIFTS* on X.

Definition 2.6. ([7]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Mondal and Samanta's sense*(MSIFT for short) $T = (T_1, T_2)$ on X is a mapping $T : I^X \to I \otimes I$ which satisfy the following properties:

(1) $T_1(\tilde{0}) = T_1(\tilde{1}) = 1$ and $T_2(\tilde{0}) = T_2(\tilde{1}) = 0$.

(2) $T_1(\mu \wedge \eta) \geq T_1(\mu) \wedge T_1(\eta)$ and $T_2(\mu \wedge \eta) \leq T_2(\mu) \vee T_2(\eta)$.

(3) $T_1(\bigvee \mu_i) \geq \bigwedge T_1(\mu_i)$ and $T_2(\bigvee \mu_i) \leq \bigvee T_2(\mu_i)$.

Then (X, T) is said to be an *intuitionistic fuzzy topological space in Mondal and Samanta's sense*(MSIFTS for short). T_1 and T_2 may be interpreted as *gradation of openness* and *gradation of nonopenness*, respectively.

Definition 2.7. ([7]) Let $f : (X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping. Then f is said to be *MSIF contiunous* if $T_1(f^{-1}(\eta)) \geq U_1(\eta)$ and $T_2(f^{-1}(\eta)) \leq U_2(\eta)$ for each $\eta \in I^Y$.

Let (X, T) be a MSIFTS. Then for each $(r,s) \in I \otimes I$, the family $T_{(r,s)}$ defined by

$$T_{(r,s)} = \{\mu \in I^X \mid T_1(\mu) \geq r \text{ and } T_2(\mu) \leq s\}$$

is a Chang's fuzzy topology on X. In this case, $T_{(r,s)}$ is called the (r,s)-*level Chang's fuzzy topology* on X.

Let (X, T) be a Chang's fuzzy topological spaces. Then for each $(r,s) \in I \otimes I$, a MSIFT $T^{(r,s)} : I^X \to I \otimes I$ is defined by

$$T^{(r,s)}(\mu) = \begin{cases} (1,0) & \text{if } \mu = \tilde{0}, \tilde{1}, \\ (r,s) & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\}, \\ (0,1) & \text{otherwise.} \end{cases}$$

In this case, $T^{(r,s)}$ is called an (r,s)-*th graded MSIFT* on X and $(X, T^{(r,s)})$ is called an (r,s)-*th graded MSIFTS* on X.

3. The categorical relationships between MSIFTop and SoIFTop

Let **MSIFTop** be the category of all intuitionistic fuzzy topological spaces in Mondal and Samanta's sense and MSIF continuous mappings, and let **SoIFTop** be the category of all intuitionistic fuzzy topological spaces in Šostak's sense and SoIF continuous mappings.

Theorem 3.1. Define a functor $F : \mathbf{SoIFTop} \to \mathbf{MSIFTop}$ by $F(X, \mathcal{T}) = (X, F(\mathcal{T}))$ and $F(f) = f$, where $F(\mathcal{T})(\eta) = (F(\mathcal{T})_1(\eta), F(\mathcal{T})_2(\eta))$, $F(\mathcal{T})_1(\eta) = \bigvee\{\mathcal{T}_1(A) \mid \mu_A = \eta\}$, $F(\mathcal{T})_2(\eta) = \bigwedge\{\mathcal{T}_2(A) \mid \mu_A = \eta\}$. Then F is a functor.

Proof. First, we show that $F(\mathcal{T})$ is a MSIFT.
Clearly, $F(\mathcal{T})(\eta) = F(\mathcal{T})_1(\eta) + F(\mathcal{T})_2(\eta) \leq 1$ for any $\eta \in I^X$.

(1) $F(\mathcal{T})_1(\tilde{0}) = \bigvee\{\mathcal{T}_1(A) \mid \mu_A = \tilde{0}\} \geq \mathcal{T}_1(\underline{0}) = 1$, $F(\mathcal{T})_1(\tilde{1}) = \bigvee\{\mathcal{T}_1(A) \mid \mu_A = \tilde{1}\} \geq \mathcal{T}_1(\underline{1}) = 1$, $F(\mathcal{T})_2(\tilde{0}) = \bigwedge\{\mathcal{T}_2(A) \mid \mu_A = \tilde{0}\} \leq \mathcal{T}_2(\underline{0}) = 0$, and $F(\mathcal{T})_2(\tilde{1}) = \bigwedge\{\mathcal{T}_2(A) \mid \mu_A = \tilde{1}\} \leq \mathcal{T}_2(\underline{1}) = 0$.

(2) Suppose that $F(\mathcal{T})_1(\eta \wedge \lambda) < F(\mathcal{T})_1(\eta) \wedge F(\mathcal{T})_1(\lambda)$. Then there is a $t \in I$ such that $F(\mathcal{T})_1(\eta \wedge \lambda) < t < F(\mathcal{T})_1(\eta) \wedge F(\mathcal{T})_1(\lambda)$. Since $t < F(\mathcal{T})_1(\eta) = \bigvee\{\mathcal{T}_1(C) \mid \mu_C = \eta\}$, there is an $A \in I(X)$ such that $t < \mathcal{T}_1(A)$ and $\mu_A = \eta$. There is a $B \in I(X)$ such that $t < \mathcal{T}_1(B)$ and $\mu_B = \lambda$, because $t < F(\mathcal{T})_1(\lambda) = \bigvee\{\mathcal{T}_1(C) \mid \mu_C = \lambda\}$. Thus $t < \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mu_{A \cap B} = \mu_A \wedge \mu_B = \eta \wedge \lambda$. Since $\mathcal{T}$ is a SoIFT, we obtain

$$t < \mathcal{T}_1(A) \wedge \mathcal{T}_1(B) \leq \mathcal{T}_1(A \cap B).$$

Hence

$$\begin{aligned} t &> F(\mathcal{T})_1(\eta \wedge \lambda) = \bigvee\{\mathcal{T}_1(C) \mid \mu_C = \eta \wedge \lambda\} \\ &\geq \mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B) > t. \end{aligned}$$

This is a contradiction. Thus $F(\mathcal{T})_1(\eta \wedge \lambda) \geq F(\mathcal{T})_1(\eta) \wedge F(\mathcal{T})_2(\lambda)$.

Next, assume that $F(\mathcal{T})_2(\eta \wedge \lambda) > F(\mathcal{T})_2(\eta) \vee F(\mathcal{T})_2(\lambda)$. Then there is an $s \in I$ such that

$$F(\mathcal{T})_2(\eta \wedge \lambda) > s > F(\mathcal{T})_2(\eta) \vee F(\mathcal{T})_2(\lambda).$$

Since $s > F(\mathcal{T})_2(\eta) = \bigwedge\{\mathcal{T}_2(C) \mid \mu_C = \eta\}$, there is an $A \in I(X)$ such that $s > \mathcal{T}_2(A)$ and $\mu_A = \eta$. As $s > F(\mathcal{T})_2(\lambda) = \bigwedge\{\mathcal{T}_2(C) \mid \mu_C = \lambda\}$, there is a $B \in I(X)$ such that $s > \mathcal{T}_2(B)$ and $\mu_B = \lambda$. So $s > \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$ and $\mu_{A \cap B} = \mu_A \wedge \mu_B = \eta \wedge \lambda$. Since $\mathcal{T}$ is a SoIFT, we have $s > \mathcal{T}_2(A) \vee \mathcal{T}_2(B) \geq \mathcal{T}_2(A \cap B)$. Thus

$$\begin{aligned} s &< F(\mathcal{T})_2(\eta \wedge \lambda) = \bigwedge\{\mathcal{T}_2(C) \mid \mu_C = \eta \wedge \lambda\} \\ &\leq \mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B) < s. \end{aligned}$$

This is a contradiction. Hence $F(\mathcal{T})_2(\eta \wedge \lambda) \leq F(\mathcal{T})_2(\eta) \vee F(\mathcal{T})_2(\lambda)$.

(3) Suppose that $F(\mathcal{T})_1(\bigvee \eta_i) < \bigwedge F(\mathcal{T})_1(\eta_i)$. Then there is a $t \in I$ such that $F(\mathcal{T})_1(\bigvee \eta_i) < t < \bigwedge F(\mathcal{T})_1(\eta_i)$. Since $t < F(\mathcal{T})_1(\eta_i) = \bigvee\{\mathcal{T}_1(C) \mid \mu_C = \eta_i\}$ for each i, there is an $A_i \in I(X)$ such that $t < \mathcal{T}_1(A_i)$ and $\mu_{A_i} = \eta_i$. Thus $t \leq \bigwedge \mathcal{T}_1(A_i)$ and $\mu_{\bigcup A_i} = \bigvee \mu_{A_i} = \bigvee \eta_i$. As $\mathcal{T}$ is a SoIFT, we obtain $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$. Hence

$$\begin{aligned} t &> F(\mathcal{T})_1(\bigvee \eta_i) = \bigvee\{\mathcal{T}_1(C) \mid \mu_C = \bigvee \eta_i\} \\ &\geq \mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i) \geq t. \end{aligned}$$

This is a contradiction. Thus $F(\mathcal{T})_1(\bigvee \eta_i) \geq \bigwedge F(\mathcal{T})_1(\eta_i)$.

Next, assume that $F(\mathcal{T})_2(\bigvee \eta_i) > \bigvee F(\mathcal{T})_2(\eta_i)$. Then there is an $s \in I$ such that

$$F(\mathcal{T})_2(\bigvee \eta_i) > s > \bigvee F(\mathcal{T})_2(\eta_i).$$

Since $s > F(\mathcal{T})_2(\eta_i) = \bigwedge\{\mathcal{T}_2(C) \mid \mu_C = \eta_i\}$ for each i, there is a $B_i \in I(X)$ such that $s > \mathcal{T}_2(B_i)$ and $\mu_{B_i} = \eta_i$. Hence $s \geq \bigvee \mathcal{T}_2(B_i)$ and $\mu_{\bigcup B_i} = \bigvee \mu_{B_i} = \bigvee \eta_i$. Since $\mathcal{T}$ is a SoIFT, we have $\mathcal{T}_2(\bigcup B_i) \leq \bigvee \mathcal{T}_2(B_i)$. Thus

$$\begin{aligned} s &< F(\mathcal{T})_2(\bigvee \eta_i) = \bigwedge\{\mathcal{T}_2(C) \mid \mu_C = \bigvee \eta_i\} \\ &\leq \mathcal{T}_2(\bigcup B_i) \leq \bigvee \mathcal{T}_2(B_i) \leq s. \end{aligned}$$

This is a contradiction. Hence $F(\mathcal{T})_2(\bigvee \eta_i) \leq \bigvee F(\mathcal{T})_2(\eta_i)$. Therefore $(X, F(\mathcal{T}))$ is a MSIFTS.

Finally, we show that if $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is SoIF continuous, then $f : (X, F(\mathcal{T})) \to (Y, F(\mathcal{U}))$ is MSIF continuous. Let $F(\mathcal{T}) = (F(\mathcal{T})_1, F(\mathcal{T})_2)$, $F(\mathcal{U}) = (F(\mathcal{U})_1, F(\mathcal{U})_2)$, and $\lambda \in I^Y$. Then

$$\begin{aligned} & F(\mathcal{U})_1(\lambda) = \bigvee\{\mathcal{U}_1(A) \mid \mu_A = \lambda\} \\ \leq\ & \bigvee\{\mathcal{T}_1(f^{-1}(A)) \mid \mu_{f^{-1}(A)} = f^{-1}(\lambda)\} \\ \leq\ & \bigvee\{\mathcal{T}_1(C) \mid \mu_C = f^{-1}(\lambda)\} = F(\mathcal{T})_1(f^{-1}(\lambda)) \end{aligned}$$

and

$$\begin{aligned} & F(\mathcal{U})_2(\lambda) = \bigwedge\{\mathcal{U}_2(A) \mid \mu_A = \lambda\} \\ \geq\ & \bigwedge\{\mathcal{T}_2(f^{-1}(A)) \mid \mu_{f^{-1}(A)} = f^{-1}(\lambda)\} \\ \geq\ & \bigwedge\{\mathcal{T}_2(C) \mid \mu_C = f^{-1}(\lambda)\} = F(\mathcal{T})_2(f^{-1}(\lambda)). \end{aligned}$$

Therefore F is a functor.

Theorem 3.2. Define a functor $G : \mathbf{MSIFTop} \to \mathbf{SoIFTop}$ by $G(X, T) = (X, G(T))$ and $G(f) = f$, where $G(T)(A) = (G(T)_1(A), G(T)_2(A))$, $G(T)_1(A) = T_1(\mu_A)$, and $G(T)_2(A) = T_2(\mu_A)$. Then G is a functor.

Proof. First, we show that $G(T)$ is a SoIFT.
Clearly, $G(T)_1(A) + G(T)_2(A) = T_1(\mu_A) + T_2(\mu_A) \leq 1$ for any $A \in I(X)$.

(1) $G(T)_1(\underline{0}) = T_1(\tilde{0}) = 1$, $G(T)_1(\underline{1}) = T_1(\tilde{1}) = 1$, $G(T)_2(\underline{0}) = T_2(\tilde{0}) = 0$, and $G(T)_2(\underline{1}) = T_2(\tilde{1}) = 0$.

(2) Let $A, B \in I(X)$. Then

$$\begin{aligned} G(T)_1(A \cap B) &= T_1(\mu_{A \cap B}) = T_1(\mu_A \wedge \mu_B) \\ &\geq T_1(\mu_A) \wedge T_1(\mu_B) \\ &= G(T)_1(A) \wedge G(T)_1(B) \end{aligned}$$

and

$$\begin{aligned} G(T)_2(A \cap B) &= T_2(\mu_{A \cap B}) = T_2(\mu_A \wedge \mu_B) \\ &\leq T_2(\mu_A) \vee T_2(\mu_B) \\ &= G(T)_2(A) \vee G(T)_2(B). \end{aligned}$$

(3) Let $A_i \in I(X)$ for each i. Then

$$\begin{aligned} G(T)_1(\bigcup A_i) &= T_1(\mu_{\bigcup A_i}) = T_1(\bigvee \mu_{A_i}) \\ &\geq \bigwedge T_1(\mu_{A_i}) = \bigwedge G(T)_1(A_i) \end{aligned}$$

and

$$\begin{aligned} G(T)_2(\bigcup A_i) &= T_2(\mu_{\bigcup A_i}) = T_2(\bigvee \mu_{A_i}) \\ &\leq \bigvee T_2(\mu_{A_i}) = \bigvee G(T)_2(A_i). \end{aligned}$$

Hence $(X, G(T))$ is a SoIFT.

Next, we show that if $f : (X, T) \to (Y, U)$ is MSIF continuous, then $f : (X, G(T)) \to (Y, G(U))$ is SoIF continuous. Let $B = (\mu_B, \gamma_B) \in I(Y)$. Then

$$\begin{aligned} G(U)_1(B) &= U_1(\mu_B) \leq T_1(f^{-1}(\mu_B)) = T_1(\mu_{f^{-1}(B)}) \\ &= G(T)_1(f^{-1}(B)) \end{aligned}$$

and

$$\begin{aligned} G(U)_2(B) &= U_2(\mu_B) \geq T_2(f^{-1}(\mu_B)) = T_2(\mu_{f^{-1}(B)}) \\ &= G(T)_2(f^{-1}(B)). \end{aligned}$$

Thus $f : (X, G(T)) \to (Y, G(U))$ is SoIF continuous. Consequently, G is a functor.

Theorem 3.3. The functor $G : \mathbf{MSIFTop} \to \mathbf{SoIFTop}$ is a left adjoint of $F : \mathbf{SoIFTop} \to \mathbf{MSIFTop}$.

Proof. Let (X,T) be an object in $\mathbf{MSIFTop}$ and $\eta \in I^X$. Then

$$\begin{aligned}
& FG(T)(\eta) \\
= \ & (\bigvee\{G(T)_1(A) \mid \mu_A = \eta\}, \bigwedge\{G(T)_2(A) \mid \mu_A = \eta\}) \\
= \ & (\bigvee\{T_1(\mu_A) \mid \mu_A = \eta\}, \bigwedge\{T_2(\mu_A) \mid \mu_A = \eta\}) \\
= \ & (T_1(\eta), T_2(\eta)) = T(\eta).
\end{aligned}$$

Hence $1_X : (X,T) \to FG(X,T) = (X,T)$ is MSIF continuous.

Consider $(Y,\mathcal{U}) \in \mathbf{SoIFTop}$ and a MSIF continuous mapping $f : (X,T) \to F(Y,\mathcal{U})$. In order to show that $f : G(X,T) \to (Y,\mathcal{U})$ is a SoIF continuous mapping, let $B \in I(Y)$. Then

$$\begin{aligned}
G(T)_1(f^{-1}(B)) &= T_1(\mu_{f^{-1}(B)}) = T_1(f^{-1}(\mu_B)) \\
&\geq F(\mathcal{U})_1(\mu_B)) = \bigvee\{\mathcal{U}_1(C) \mid \mu_C = \mu_B\} \\
&\geq \mathcal{U}_1(B)
\end{aligned}$$

and

$$\begin{aligned}
G(T)_2(f^{-1}(B)) &= T_2(\mu_{f^{-1}(B)}) = T_2(f^{-1}(\mu_B)) \\
&\leq F(\mathcal{U})_2(\mu_B)) = \bigwedge\{\mathcal{U}_2(C) \mid \mu_C = \mu_B\} \\
&\leq \mathcal{U}_2(B).
\end{aligned}$$

Hence $f : (X, G(T)_1, G(T)_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ is a SoIF continuous mapping. Therefore 1_X is a G-universal mapping for (X,T) in $\mathbf{MSIFTop}$.

Theorem 3.4. Define a functor $H : \mathbf{SoIFTop} \to \mathbf{MSIFTop}$ by $H(X,\mathcal{T}) = (X, H(\mathcal{T}))$ and $H(f) = f$, where $H(\mathcal{T}) = (H(\mathcal{T})_1, H(\mathcal{T})_2)$, $H(\mathcal{T})_1(\eta) = \bigvee\{\mathcal{T}_1(A) \mid \tilde{1} - \gamma_A = \eta\}$, and $H(\mathcal{T})_2(\eta) = \bigwedge\{\mathcal{T}_2(A) \mid \tilde{1} - \gamma_A = \eta\}$. Then H is a functor.

Proof. First, we show that $H(\mathcal{T})$ is a MSIFT. Obviously, $H(\mathcal{T})(\eta) = H(\mathcal{T})_1(\eta) + H(\mathcal{T})_2(\eta) \leq 1$ for any $\eta \in I^X$.

(1) $H(\mathcal{T})_1(\tilde{0}) = \bigvee\{\mathcal{T}_1(A) \mid \tilde{1} - \gamma_A = \tilde{0}\} \geq \mathcal{T}_1(\underline{0}) = 1$, $H(\mathcal{T})_1(\tilde{1}) = \bigvee\{\mathcal{T}_1(A) \mid \tilde{1} - \gamma_A = \tilde{1}\} \geq \mathcal{T}_1(\underline{1}) = 1$, $H(\mathcal{T})_2(\tilde{0}) = \bigwedge\{\mathcal{T}_2(A) \mid \tilde{1} - \gamma_A = \tilde{0}\} \leq \mathcal{T}_2(\underline{0}) = 0$, and $H(\mathcal{T})_2(\tilde{1}) = \bigwedge\{\mathcal{T}_2(A) \mid \tilde{1} - \gamma_A = \tilde{1}\} \leq \mathcal{T}_2(\underline{1}) = 0$.

(2) Assume that $H(\mathcal{T})_1(\eta \wedge \lambda) < H(\mathcal{T})_1(\eta) \wedge H(\mathcal{T})_1(\lambda)$. Then there is a $t \in I$ such that

$$H(\mathcal{T})_1(\eta \wedge \lambda) < t < H(\mathcal{T})_1(\eta) \wedge H(\mathcal{T})_1(\lambda).$$

As $t < H(\mathcal{T})_1(\eta) = \bigvee\{\mathcal{T}_1(C) \mid \tilde{1} - \gamma_C = \eta\}$, there is an $A \in I(X)$ such that $t < \mathcal{T}_1(A)$ and $\tilde{1} - \gamma_A = \eta$. Since $t < H(\mathcal{T})_1(\lambda) = \bigvee\{\mathcal{T}_1(C) \mid \tilde{1} - \gamma_C = \lambda\}$, there is a $B \in I(X)$ such that $t < \mathcal{T}_1(B)$ and $\tilde{1} - \gamma_B = \lambda$. Hence $t < \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and

$$\begin{aligned}
\tilde{1} - \gamma_{A \cap B} &= \tilde{1} - (\gamma_A \vee \gamma_B) \\
&= (\tilde{1} - \gamma_A) \wedge (\tilde{1} - \gamma_B) = \eta \wedge \lambda.
\end{aligned}$$

Since $\mathcal{T}$ is a SoIFT, $t < \mathcal{T}_1(A) \wedge \mathcal{T}_1(B) \leq \mathcal{T}_1(A \cap B)$. Thus

$$\begin{aligned}
t &> H(\mathcal{T})_1(\eta \wedge \lambda) = \bigvee\{\mathcal{T}_1(C) \mid \tilde{1} - \gamma_C = \eta \wedge \lambda\} \\
&\geq \mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B) > t.
\end{aligned}$$

This is a contradiction. Hence $H(\mathcal{T})_1(\eta \wedge \lambda) \geq H(\mathcal{T})_1(\eta) \wedge H(\mathcal{T})_1(\lambda)$.

Suppose that $H(\mathcal{T})_2(\eta \wedge \lambda) > H(\mathcal{T})_2(\eta) \vee H(\mathcal{T})_2(\lambda)$. Then there is an $s \in I$ such that

$$H(\mathcal{T})_2(\eta \wedge \lambda) > s > H(\mathcal{T})_2(\eta) \vee H(\mathcal{T})_2(\lambda).$$

Since $s > H(\mathcal{T})_2(\eta) = \bigwedge\{\mathcal{T}_2(C) \mid \tilde{1} - \gamma_C = \eta\}$, there is an $A \in I(X)$ such that $s > \mathcal{T}_2(A)$ and $\tilde{1} - \gamma_A = \eta$. As $s > H(\mathcal{T})_2(\lambda) = \bigwedge\{\mathcal{T}_2(C) \mid \tilde{1} - \gamma_C = \lambda\}$, there is a $B \in I(X)$ such that $s > \mathcal{T}_2(B)$ and $\tilde{1} - \gamma_B = \lambda$. So $s > \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$ and

$$\begin{aligned}
\tilde{1} - \gamma_{A \cap B} &= \tilde{1} - (\gamma_A \vee \gamma B) \\
&= (\tilde{1} - \gamma_A) \wedge (\tilde{1} - \gamma_B) = \eta \wedge \lambda.
\end{aligned}$$

Since $\mathcal{T}$ is a SoIFT, we obtain $s > \mathcal{T}_2(A) \vee \mathcal{T}_2(B) \geq \mathcal{T}_2(A \cap B)$. Hence

$$\begin{aligned}
s &< H(\mathcal{T})_2(\eta \wedge \lambda) = \bigwedge\{\mathcal{T}_2(C) \mid \tilde{1} - \gamma_C = \eta \wedge \lambda\} \\
&\leq \mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B) < s.
\end{aligned}$$

This is a contradiction. Thus $H(\mathcal{T})_2(\eta \wedge \lambda) \leq H(\mathcal{T})_2(\eta) \vee H(\mathcal{T})_2(\lambda)$.

(3) Assume that $H(\mathcal{T})_1(\bigvee \eta_i) < \bigwedge H(\mathcal{T})_1(\eta_i)$. Then there is a $t \in I$ such that

$$H(\mathcal{T})_1(\bigvee \eta_i) < t < \bigwedge H(\mathcal{T})_1(\eta_i).$$

As $t < H(\mathcal{T})_1(\eta_i) = \bigvee\{\mathcal{T}_1(C) \mid \tilde{1} - \gamma_C = \eta_i\}$ for each i, there is an $A_i \in I(X)$ such that $t < \mathcal{T}_1(A_i)$ and $\tilde{1} - \gamma_{A_i} = \eta_i$. Hence $t \leq \bigwedge \mathcal{T}_1(A_i)$ and

$$\tilde{1} - \gamma_{(\bigcup A_i)} = \tilde{1} - \bigwedge \gamma_{A_i} = \bigvee(\tilde{1} - \gamma_{A_i}) = \bigvee \eta_i.$$

Since $\mathcal{T}$ is a SoIFT, we have $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$. Thus

$$\begin{array}{rcl} t & > & H(\mathcal{T})_1(\bigvee \eta_i) = \bigvee\{\mathcal{T}_1(C) \mid \tilde{1} - \gamma_C = \bigvee \eta_i\} \\ & \geq & \mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i) \geq t. \end{array}$$

This is a contradiction. Hence $H(\mathcal{T})_1(\bigvee \eta_i) \geq \bigwedge H(\mathcal{T})_1(\eta_i)$.

Suppose that $H(\mathcal{T})_2(\bigvee \eta_i) > \bigvee H(\mathcal{T})_2(\eta_i)$. Then there is an $s \in I$ such that $H(\mathcal{T})_2(\bigvee \eta_i) > s > \bigvee H(\mathcal{T})_2(\eta_i)$. Since $s > H(\mathcal{T})_2(\eta_i) = \bigwedge\{\mathcal{T}_2(C) \mid \tilde{1} - \gamma_C = \eta_i\}$ for each i, there is a $B_i \in I(X)$ such that $s > \mathcal{T}_2(B_i)$ and $\tilde{1} - \gamma_{B_i} = \eta_i$. Hence $s \geq \bigvee \mathcal{T}_2(B_i)$ and

$$\tilde{1} - \gamma_{\bigcup B_i} = \tilde{1} - \bigwedge \gamma_{B_i} = \bigvee(\tilde{1} - \gamma_{B_i}) = \bigvee \eta_i.$$

We have $\mathcal{T}_2(\bigcup B_i) \leq \bigvee \mathcal{T}_2(B_i)$ because $\mathcal{T}$ is a SoIFT. Thus

$$\begin{array}{rcl} s & < & H(\mathcal{T})_2(\bigvee \eta_i) = \bigwedge\{\mathcal{T}_2(C) \mid \tilde{1} - \gamma_C = \bigvee \eta_i\} \\ & \leq & \mathcal{T}_2(\bigcup B_i) \leq \bigvee \mathcal{T}_2(B_i) \leq s. \end{array}$$

This is a contradiction. Hence $H(\mathcal{T})_2(\bigvee \eta_i) \leq \bigvee H(\mathcal{T})_2(\eta_i)$. Therefore $(X, H(\mathcal{T}))$ is a MSIFTS.

Next, we show that if $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is SoIF continuous, then $f : (X, H(\mathcal{T})) \to (Y, H(\mathcal{U}))$ is MSIF continuous. Let $H(\mathcal{T}) = (H(\mathcal{T})_1, H(\mathcal{T})_2)$, $H(\mathcal{U}) = (H(\mathcal{U})_1, H(\mathcal{U})_2)$, and $\eta \in I^X$. Then

$$\begin{array}{rcl} H(\mathcal{U})_1(\eta) & = & \bigvee\{\mathcal{U}_1(A) \mid \tilde{1} - \gamma_A = \eta\} \\ & \leq & \bigvee\{\mathcal{T}_1(f^{-1}(A)) \mid \tilde{1} - \gamma_{f^{-1}(A)} = f^{-1}(\eta)\} \\ & \leq & \bigvee\{\mathcal{T}_1(C) \mid \tilde{1} - \gamma_C = f^{-1}(\eta)\} \\ & = & H(\mathcal{T})_1(f^{-1}(\eta)) \end{array}$$

and

$$\begin{array}{rcl} H(\mathcal{U})_2(\eta) & = & \bigwedge\{\mathcal{U}_2(A) \mid \tilde{1} - \gamma_A = \eta\} \\ & \geq & \bigwedge\{\mathcal{T}_2(f^{-1}(A)) \mid \tilde{1} - \gamma_{f^{-1}(A)} = f^{-1}(\eta)\} \\ & \geq & \bigwedge\{\mathcal{T}_2(C) \mid \tilde{1} - \gamma_C = f^{-1}(\eta)\} \\ & = & H(\mathcal{T})_2(f^{-1}(\eta)). \end{array}$$

Therefore H is a functor.

Theorem 3.5. Define a functor $K : \mathbf{MSIFTop} \to \mathbf{SoIFTop}$ by $K(X, T) = (X, K(T))$ and $K(f) = f$, where $K(T) = (K(T)_1, K(T)_2)$, $K(T)_1(A) = T_1(\tilde{1} - \gamma_A)$, and $K(T)_2(A) = T_2(\tilde{1} - \gamma_A)$. Then K is a functor.

Proof. First, we show that $K(T)$ is a SoIFT. Clearly,

$$K(T)_1(A) + K(T)_2(A) = T_1(\tilde{1} - \gamma_A) + T_2(\tilde{1} - \gamma_A) \leq 1$$

for any $A \in I(X)$.

(1) $K(T)_1(\underline{0}) = T_1(\tilde{1} - \gamma_{\underline{0}}) = T_1(\tilde{0}) = 1$, $K(T)_1(\underline{1}) = T_1(\tilde{1} - \gamma_{\underline{1}}) = T_1(\tilde{1}) = 1$, $K(T)_2(\underline{0}) = T_2(\tilde{1} - \gamma_{\underline{0}}) = T_2(\tilde{0}) = 0$, and $K(T)_2(\underline{1}) = T_2(\tilde{1} - \gamma_{\underline{1}}) = T_2(\tilde{1}) = 0$.

(2) Let $A, B \in I(X)$. Then

$$\begin{array}{rcl} K(T)_1(A \cap B) & = & T_1(\tilde{1} - \gamma_{A \cap B}) = T_1(\tilde{1} - \gamma_A \vee \gamma_B) \\ & = & T_1((\tilde{1} - \gamma_A) \wedge (\tilde{1} - \gamma_B)) \\ & \geq & T_1(\tilde{1} - \gamma_A) \wedge T_1(\tilde{1} - \gamma_B) \\ & = & K(T)_1(A) \wedge K(T)_1(B) \end{array}$$

and

$$\begin{array}{rcl} K(T)_2(A \cap B) & = & T_2(\tilde{1} - \gamma_{A \cap B}) \\ & = & T_2((\tilde{1} - \gamma_A) \wedge (\tilde{1} - \gamma_B)) \\ & \leq & T_2(\tilde{1} - \gamma_A) \vee T_2(\tilde{1} - \gamma_B) \\ & = & K(T)_2(A) \vee K(T)_2(B). \end{array}$$

(3) Let $A_i \in I(X)$ for each i. Then

$$\begin{array}{rcl} K(T)_1(\bigcup A_i) & = & T_1(\tilde{1} - \gamma_{\bigcup A_i}) = T_1(\bigvee(\tilde{1} - \gamma_{A_i})) \\ & \geq & \bigwedge T_1(\tilde{1} - \gamma_{A_i}) = \bigwedge K(T)_1(A_i) \end{array}$$

and

$$\begin{array}{rcl} K(T)_2(\bigcup A_i) & = & T_2(\tilde{1} - \gamma_{\bigcup A_i}) = T_2(\bigvee(\tilde{1} - \gamma_{A_i})) \\ & \leq & \bigvee T_2(\tilde{1} - \gamma_{A_i}) = \bigvee K(T)_2(A_i). \end{array}$$

Thus $(X, K(T))$ is a SoIFTS.

Finally, we show that if $f : (X, T) \to (Y, U)$ is MSIF continuous, then $f : (X, K(T)) \to (Y, K(U))$ is SoIF continuous. Let $B = (\mu_B, \gamma_B) \in I(Y)$. Then

$$\begin{array}{rcl} K(U)_1(B) & = & U_1(\tilde{1} - \gamma_B) \leq T_1(f^{-1}(\tilde{1} - \gamma_B)) \\ & = & T_1(\tilde{1} - \gamma_{f^{-1}(B)}) = K(T)_1(f^{-1}(B)) \end{array}$$

and

$$\begin{aligned} K(U)_2(B) &= U_2(\tilde{1}-\gamma_B) \geq T_2(f^{-1}(\tilde{1}-\gamma_B)) \\ &= T_2(\tilde{1}-\gamma_{f^{-1}(B)}) = K(T)_2(f^{-1}(B)). \end{aligned}$$

Hence $f:(X,K(T))\to(Y,K(U))$ is SoIF continuous. Consequently, K is a functor.

Theorem 3.6. The functor $K:\mathbf{MSIFTop}\to\mathbf{SoIFTop}$ is a left adjoint of $H:\mathbf{SoIFTop}\to\mathbf{MSIFTop}$.

Proof. For any (X,T) in $\mathbf{MSIFTop}$ and $\eta\in I^X$,

$$\begin{aligned} &HK(T)(\eta) \\ =&(\bigvee\{K(T)_1(A)\mid \tilde{1}-\gamma_A=\eta\},\bigwedge\{K(T)_2(A)\mid \tilde{1}-\gamma_A=\eta\}) \\ =&(\bigvee\{T_1(\tilde{1}-\gamma_A)\mid \tilde{1}-\gamma_A=\eta\},\bigwedge\{T_2(\tilde{1}-\gamma_A)\mid \tilde{1}-\gamma_A=\eta\}) \\ =&(T_1(\eta),T_2(\eta))=T(\eta). \end{aligned}$$

Hence $1_X:(X,T)\to HK(X,T)=(X,T)$ is a MSIF continuous mapping. Consider $(Y,\mathcal{U})\in\mathbf{SoIFTop}$ and a MSIF continuous mapping $f:(X,T)\to H(Y,\mathcal{U})$. In order to show that $f:K(X,T)\to(Y,\mathcal{U})$ is a SoIF continuous mapping, let $B\in I(Y)$. Then

$$\begin{aligned} K(T)_1(f^{-1}(B)) &= T_1(\tilde{1}-\gamma_{f^{-1}(B)}) \\ &= T_1(f^{-1}(\tilde{1}-\gamma_B)) \\ &\geq H(\mathcal{U})_1(\tilde{1}-\gamma_B) \\ &= \bigvee\{\mathcal{U}_1(C)\mid \tilde{1}-\gamma_C=\tilde{1}-\gamma_B\} \\ &\geq \mathcal{U}_1(B) \end{aligned}$$

and

$$\begin{aligned} K(T)_2(f^{-1}(B)) &= T_2(\tilde{1}-\gamma_{f^{-1}(B)}) \\ &= T_2(f^{-1}(\tilde{1}-\gamma_B)) \\ &\leq H(\mathcal{U})_2(\tilde{1}-\gamma_B) \\ &= \bigwedge\{\mathcal{U}_2(C)\mid \tilde{1}-\gamma_C=\tilde{1}-\gamma_B\} \\ &\leq \mathcal{U}_2(B). \end{aligned}$$

Thus $f:(X,K(T))\to(Y,\mathcal{U})$ is SoIF continuous. Hence 1_X is a K-universal mapping for (X,T) in $\mathbf{MSIFTop}$.

Let (r,s)-$\mathbf{gMSIFTop}$ be the category of all (r,s)-th graded intuitionistic fuzzy topological spaces in Mondal and Samanta's sense and MSIF continuous mappings, and let $\mathbf{CFTop}$ be the category of all Chang's fuzzy topological spaces and fuzzy continuous mappings.

Theorem 3.7. Two categories $\mathbf{CFTop}$ and (r,s)-$\mathbf{gMSIFTop}$ are isomorphic.

Proof. Define $F:\mathbf{CFTop}\to(r,s)$-$\mathbf{gMSIFTop}$ by $F(X,T)=(X,F(T))$ and $F(f)=f$, where

$$F(T)(\eta)=T^{(r,s)}(\eta)=\begin{cases}(1,0) & \text{if } \eta=\tilde{0},\tilde{1},\\ (r,s) & \text{if } \eta\in T-\{\tilde{0},\tilde{1}\},\\ (0,1) & \text{otherwise.}\end{cases}$$

Define $G:(r,s)$-$\mathbf{gMSIFTop}\to\mathbf{CFTop}$ by $G(X,\mathcal{T})=(X,G(\mathcal{T}))$ and $G(f)=f$, where

$$G(\mathcal{T})=\mathcal{T}_{(r,s)}=\{\eta\in I^X\mid \mathcal{T}_1(\eta)\geq r \text{ and } \mathcal{T}_2(\eta)\leq s\}.$$

Then F and G are functors. Obviously, $GF(T)=G(T^{(r,s)})=(T^{(r,s)})_{(r,s)}=T$ and $FG(\mathcal{T})=F(\mathcal{T}_{(r,s)})=(\mathcal{T}_{(r,s)})^{(r,s)}=\mathcal{T}$. Hence $\mathbf{CFTop}$ and (r,s)-$\mathbf{gMSIFTop}$ are isomorphic.

Theorem 3.8. The category (r,s)-$\mathbf{gMSIFTop}$ is a bireflective full subcategory of $\mathbf{MSIFTop}$.

Proof. Obviously, (r,s)-$\mathbf{gMSIFTop}$ is a full subcategory of $\mathbf{MSIFTop}$. Let (X,T) be an object of $\mathbf{MSIFTop}$. Then for each $(r,s)\in I\otimes I$, $(X,(T_{(r,s)})^{(r,s)})$ is an object of (r,s)-$\mathbf{gMSIFTop}$ and $1_X:(X,T)\to(X,(T_{(r,s)})^{(r,s)})$ is a MSIF continuous mapping. Let (Y,U) be an object of the category (r,s)-$\mathbf{gMSIFTop}$ and $f:(X,T)\to(Y,U)$ a MSIF continuous mapping. we need only to check that $f:(X,(T_{(r,s)})^{(r,s)})\to(Y,U)$ is a MSIF continuous mapping. Since $(Y,U)\in(r,s)$-$\mathbf{gMSIFTop}$, $U(\eta)=(1,0)$, (r,s), or $(0,1)$. Let $U(\eta)=(1,0)$. Then $\eta=\tilde{0}$ or $\tilde{1}$. In fact,

$$(T_{(r,s)})^{(r,s)}(f^{-1}(\tilde{0}))=(T_{(r,s)})^{(r,s)}(\tilde{0})=(1,0)=U(\tilde{0})$$

and

$$(T_{(r,s)})^{(r,s)}(f^{-1}(\tilde{1}))=(T_{(r,s)})^{(r,s)}(\tilde{1})=(1,0)=U(\tilde{1}).$$

In case $U(\eta)=(0,1)$, clearly $U(\eta)\leq(T_{(r,s)})^{(r,s)}(f^{-1}(\eta))$. Let $U(\eta)=(r,s)$. Since $f:(X,T)\to(Y,U)$ is MSIF continuous, $T(f^{-1}(\eta))\geq U(\eta)=(r,s)$. Thus $f^{-1}(\eta)\in T_{(r,s)}$, and hence $(T_{(r,s)})^{(r,s)}(f^{-1}(\eta))=(r,s)=U(\eta)$. Therefore $f:(X,(T_{(r,s)})^{(r,s)})\to(Y,U)$ is a MSIF continuous mapping.

From the above theorems, we have the follwing main result.

Theorem 3.9. The category **CFTop** is a bireflective full subcategory of **MSIFTop**.

4. Conclusion

We obtained two types of adjoint functors between the category of intuitionistic fuzzy topological spaces in Mondal and Samanta's sense, and the category of intuitionistic fuzzy topological spaces in Šostak's sense. Also, we revealed that the category of Chang's fuzzy topological spaces is a bireflective full subcategory of the category of intuitionistic fuzzy topological spaces in Mondal and Samanta's sense.

In further research, we will investigate other properties of the category of intuitionistic fuzzy topological spaces in Šostak's sense.

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