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Doppler-shift estimation of flat underwater channel using data-aided least-square approach

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ABSTRACT: *In this paper we proposed a dada-aided Doppler estimation method for underwater acoustic communication. The training sequence is non-dedicate, hence it can be designed for Doppler estimation as well as channel equalization. We assume the channel has been equalized and consider only flat-fading channel. First, based on the training symbols the theoretical received sequence is composed. Next the least square principle is applied to build the objective function, which minimizes the error between the composed and the actual received signal. Then an iterative approach is applied to solve the least square problem. The proposed approach involves an outer loop and inner loop, which resolve the channel gain and Doppler coefficient, respectively. The theoretical performance bound, i.e. the Cramer-Rao Lower Bound (CRLB) of estimation is also derived. Computer simulations results show that the proposed algorithm achieves the CRLB in medium to high SNR cases.*

KEY WORDS: Underwater acoustic communication; Doppler estimation; Least Square; Cramer-Rao Lower bound.

INTRODUCTION

Radio wave suffers from high attenuation in water (especially at higher frequencies). Acoustic signal is the most popular communication media in underwater communication. The propagation speed of acoustic signal is much slower than that of radio signal (approximately five orders of magnitude lower). Due to the low speed of acoustic signal propagation, the transmitted acoustic signal is more vulnerable to the Doppler effect compared to other communication systems. Therefore, even a slow motion between the transmitter/receiver and/or the inherent current wave's motion can bring significant Doppler effect to the transmitted signal. Doppler estimation/compensation has been an important topic in underwater communication (Sharif, 2000; Eynard, 2008; Perrine et al., 2010; Wan et al., 2012).

Conventional Doppler estimation method applies two identical Doppler Insensitive Training Sequences (ITS), such as linear-frequency-modulated waveform (Kramer, 1967) and hyperbolic-frequency modulated waveform (Kroszczynski, 1969). The ITS can be transmitted repeatedly before the data burst, or as a preamble and postamble around the data burst, or in subsequent data frame (Wan et al., 2012; Li et al., 2008). At the receiver side, by detecting the times-of-arrival of the two sequences, and the interval change in-between, an average Doppler scale estimate over the whole data burst can be obtained. The key advantage of the ITS scheme is its low-complexity implementation. Thanks to the Doppler-insensitive property of the waveforms, a single-branch-matched filtering operation is adequate even in the presence of Doppler distortion. However, the ITS scheme

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may suffer from large processing delay, because it requires two training sequence being received before processing.

Another important approach is to apply a Doppler-Sensitive Waveform (DSW) which is transmitted as a preamble prior to the data burst. At the receiver side, a bank of correlators correlates the received signal with preambles prescaled by different Doppler scaling factors, and the branch with the largest correlation peak provides the estimated Doppler scale (Yang, 2003). The performance of this approach majorly affected by the shape of the DSW's ambiguity function (Sibul and Ziomek, 1981). For maximum resolution, a waveform with a narrow peak in the Doppler axis of the ambiguity function is desirable, hence the maximal-length pseudonoise (PN) codes are perhaps most suitable (Johnson et al., 1997), Generally the DSW approach is simple but requires a dedicated Doppler-sensitive waveform. Hence, it is interesting to investigate non-dedicated training sequence.

In this paper we consider a conventional frame structure as Fig. 1 and propose a Doppler estimation algorithm based on single Doppler-sensitive training sequence. Moreover, the training sequence is non-dedicated. It can be designed for Doppler estimation as well as channel estimation. In the proposed scheme, first based on the training symbols the theoretical received sequence is composed. Next the least square principle is applied to build the objective function, which minimizes the error between the composed and the actual received signal. Then an iterative approach is applied to solve the least square problem. Computer simulations demonstrate the efficiency of the proposed algorithm.

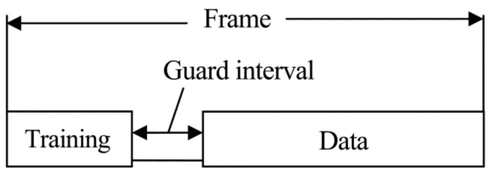

Fig. 1 Frame structure of the applied model.

SYSTEM MODEL

In underwater communication, the relative movement of the transmitter and receiver introduces the Doppler effect. Let v denote the speed of the relative movement and f_0 denote the carrier frequency of the transmitted signal. The carrier frequency at the receiver is given by

$$f = (1+\frac{v}{c})f_0, \tag{1}$$

where c denotes the sound propagation speed. We note that if the receiver is moving toward the transmitter then v is positive, otherwise v is negative. The Doppler shift of the carrier signal is defined as $D = f - f_0 = \Delta \cdot f_0$, where $\Delta = v/c$.

For wideband signal, the Doppler shifts in all spectrum components are different. It is not easy to characterize the Doppler effect in frequency domain. In time domain, the Doppler effect can be interpreted as extension or compression of the waveform [1,6]. For a continuous waveform $s_c(t)$, The Doppler effect can be presented in time-domain as

$$y_c(t) = s_c((1+\Delta)t). \tag{2}$$

Next we consider a practical baseband underwater communication system based on (2). The discrete signal to be transmitted is denoted as $s(k)$. Then pulse-shaping filter $g(t)$ is applied to the discrete signal $s(k)$ and produce the baseband continuous signal as follows

$$s_c(t) = \sum_{k=-\infty}^{+\infty} s(k)g(t-kT), \tag{3}$$

where T is the symbol interval of transmission. Based on (2), the received signal can be presented as

$$y_c(t) = A\sum_{k=-\infty}^{+\infty} s(k)g((1+\Delta)\cdot t - kT) + w(t)\ , \tag{4}$$

where A is the channel gain and $w(t)$ is the additive channel noise. The continuous received signal is sampled with interval T, which produces the discrete signal as follows

$$y(n) = A\sum_{k=-\infty}^{+\infty} s(k)g((1+\Delta)\cdot nT - kT) + w(n)\ . \tag{5}$$

We consider a conventional frame structure shown as Fig. 1, where the training symbols is for synchronous, channel estimation/equalization and Doppler estimation. We assume the received signal is properly synchronized and the channel is equalized, such that the channel can be characterized by an unknown gain. Our end in this paper is to estimate the Doppler coefficient Δ based on the training sequence.

ALGORITHM

Objective function

The waveform of pulse-shaping filter is usually known to the receiver. In the following we assume the pulse-shaping filter is known. Without lost of generality, we assume the low-pass filter is applied as the pulse-shaping filter, which is $g(t) = \sin c(t/T)$. Let N denote the number of training symbols. The received discrete signal corresponding to the training sequence is given by

$$y(n) = A\sum_{k=0}^{N-1} s(k)\sin c((1+\Delta)\cdot n - k) + w(n) \tag{6}$$

A and Δ are the unknown parameters to be estimated. At the receiver, we can compose the noiseless signal based on known training symbols and pulse-shaping signal, as follows

$$\overline{y}(n) = \overline{A}\sum_{k=0}^{N-1} s(k)\sin c((1+\overline{\Delta})\cdot n - k)\ , \tag{7}$$

where $\overline{A}$ and $\overline{\Delta}$ denote the possible estimates of the unknown parameters. In Eq. (7) we use the known variables $s(k)$, $g(k)$ and unknown variables $\overline{A}$, $\overline{\Delta}$ to compose the theoretical noiseless signal. Hence, noise item is not included in Eq. (7). Then the objective function can be built based on the least-square principle, which is to make the noise-corrupted signal ($y(n)$) and the composed noiseless signal ($\overline{y}(n)$) as close as possible. We obtain the following objective function

$$\begin{aligned} \{\hat{A},\hat{\Delta}\} &= \arg\min_{\overline{A},\overline{\Delta}} J(\overline{A},\overline{\Delta}) \\ J(\overline{A},\overline{\Delta}) &= \sum_{n=0}^{N-1}\left[y(n) - \overline{A}\sum_{k=0}^{N-1} s(k)\sin c((1+\overline{\Delta})\cdot n - k)\right]^2 \end{aligned}\ . \tag{8}$$

This is a least-square objective function and it is well known that in the case of Guassian white noise the LS approach is equivalent to the maximum-likelihood approach, which theoretically provides the best estimate. The objective function in (8) is highly nonlinear and contains more than one unknown parameters. It is not straightforward to resolve the optimal solution. In the next section we shall simplify the objective function and introduce an efficient approach.

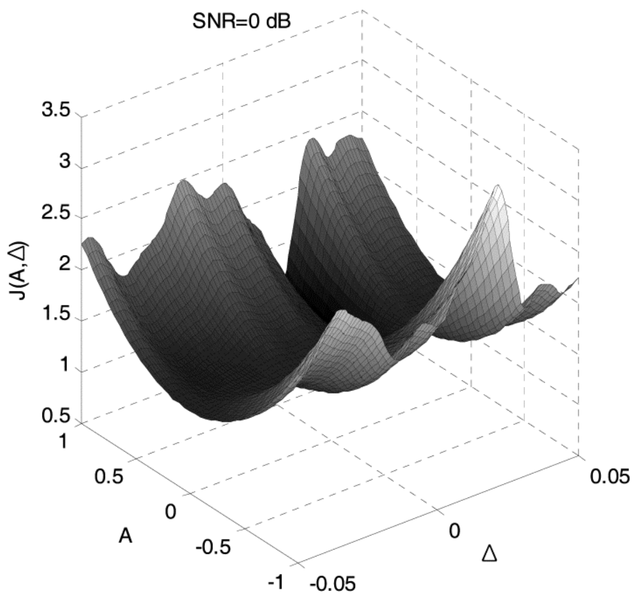

Fig. 2 The surface of the objective function $J(\overline{A},\overline{\Delta})$.

We assume BPSK training sequence. In this case A is a real number. Our motivation is based on the fact that in practical system, Δ usually has small value. Fig. 2 shows an example of $J(\overline{A},\overline{\Delta})$, where SNR=10 *dB* and $A_{opt} = 0.6$. It can be observed from Fig. 2 that for different values of A, the optimal value of Δ are approximately identical. That is, we can separate the optimization procedure of $\overline{A}$ and $\overline{\Delta}$. Let A_{opt} and Δ_{opt} denote the optimal values of $\overline{A}$ and $\overline{\Delta}$, In this neighboring range of Δ_{opt}, i.e. $\overline{\Delta} \approx \Delta_{opt}$, we have

$$\begin{aligned} J(\overline{A},\overline{\Delta}) &\approx (\overline{A} - A_{opt})^2 \sum_{n=0}^{N-1} \left[\sum_{k=0}^{N-1} s(k)\phi_{n,k}(\overline{\Delta}) \right]^2 \\ \phi_{n,k}(\overline{\Delta}) &= \sin c((1+\Delta_{opt}) \cdot n - k) - \sin c((1+\overline{\Delta}) \cdot n - k) \end{aligned} \quad . \tag{9}$$

Hence, we argue that if $\overline{\Delta} \approx \Delta_{opt}$, it is feasible to set $\overline{A}$ to a fixed value and resolve $\overline{\Delta}$. Further, it also can be observed that if $\overline{\Delta}$ is properly estimated, substituting the estimate of $\overline{\Delta}$ into (8) we obtain the single-variable objective function of $\overline{A}$, from which it is straightforward to obtain $\hat{A}$. We propose the following iteration to resolve the optimal solution.

Step 1: Initialize $\overline{A}$ with $\overline{A} = \overline{A}_0$. Usually we set $\overline{A}_0 = 1$.

Step 2: Obtain the estimate of $\overline{\Delta}$ based on the following objective function.

$$\begin{aligned} \hat{\Delta} &= \arg\min_{\overline{\Delta}} J(\overline{A}_0, \overline{\Delta}) \\ J(\overline{A}_0, \overline{\Delta}) &= \sum_{n=0}^{N-1} \left[y(n) - \overline{A}_0 \sum_{k=0}^{N-1} s(k) \sin c((1+\overline{\Delta}) \cdot n - k) \right]^2 \end{aligned} \quad . \tag{10}$$

Step 3： Obtain the estimate of $\overline{A}$ based on the following objective function.

$$\hat{A} = \arg\min_{\overline{A}} J(\overline{A}, \hat{\Delta})$$
$$J(\overline{A}, \hat{\Delta}) = \sum_{n=0}^{N-1}\left[y(n) - \overline{A}\sum_{k=0}^{N-1} s(k)\sin c((1+\hat{\Delta})\cdot n - k)\right]^2 . \tag{11}$$

Step 4： If converge, i.e., the amendment of $\hat{\Delta}$ between two iterations is less than a predefined threshold, end the iteration. Otherwise set $\overline{A}_0 = \hat{A}$ and go to step 2.

The objective function of (11) results in the following closed-form solution.

$$\hat{A} = \frac{\sum_{n=0}^{N-1} y(n)\sum_{k=0}^{N-1} s(k)\,\mathrm{sinc}((1+\hat{\Delta})\cdot n - k)}{\sum_{n=0}^{N-1}\left[\sum_{k=0}^{N-1} s(k)\,\mathrm{sinc}((1+\hat{\Delta})\cdot n - k)\right]^2} . \tag{12}$$

However, there is no closed-form solution for (10). It can be observed from Fig.2 that $J(\overline{A}_0, \overline{\Delta})$ is non-convex and has multiple local minimizers. Conventional gradient-descending method may lead to a local minimizer. Note that the objective function $J(\overline{A}_0, \overline{\Delta})$ is a continuous multi-peak function with single variable. The interval slope algorithm has been shown to be efficient to find the global minimizer of such single-variable function (Ratz, 1999; Hansen et al., 1994). It has been shown for various single-variable optimization problem, the interval-slope algorithm achieves global minimization with acceptable complexity. Given a single-variable objective function $f(x)$, The interval slope algorithm finds the solution in a predefined interval using the following steps (Hansen et al., 1994).

Step 1) Define the initial searching interval as $X = [X_0, X_1]$, and the initial value is $c = (X_0 + X_1)/2$, $f_{\min} = f(c)$, then divide $X = [X_0, X_1]$ into two halves, noted as $U_i\ (i = 1,2)$, go to step 2).

Step 2) Compute the interval slope of $U_i\ (i = 1,2)$. Obtain

$$s_f(c, U_i) := \{\frac{f(x) - f(c)}{x - c} \mid x \in U_i, x \neq c\} \uplus \{f^{'}(c) \mid c \in U_i\} . \tag{13}$$

Delete the interval of no minimum and store other intervals in the working list L, update $f_{\min}$, then go to step 3).

Step 3) Scan all the intervals in working list L. If an interval is enough small, i.e., its width is smaller than a predefined threshold, then store the interval to the result list $L1$. If L is empty, then exit. Otherwise go back to step 2).

Step 4) Seek the interval which has the minimum value from the result list $L1$, and consider the midpoint of this interval as the solution.

We apply the interval-slope algorithm to resolve the optimization problem of (10). Hence the proposed algorithm consists of two-loop iterations. The outer loop is step 1-step 4. The inner loop is step 1)-step 4), which is embedded in step 2.

THE CRAMER-LAO LOWER BOUND

For a given estimation problem, the theoretical performance bound of the parameter estimation can be derived in theory, and this bound is called the Cramer Rao Lower Bound (Kay, 1993). In the following we shall derive the CRLB with respect to Δ. Rewrite the signal model as follows

$$y(n) = A\sum_{k=0}^{N-1} s(k)\sin\mathrm{c}\left((1+\Delta)n-k\right)+w(n). \tag{14}$$

We assume the additive noise is white Gaussian noise with zero mean and variance σ^2. Firstly, we should obtain the Probability Distribution Function (PDF) o with respect to Δ. Define $\mathbf{y} = [y(0), y(1), \cdots, y(N-1)]^T$, the PDF is given by

$$p(\mathbf{y}\,|\,\Delta) = \left(2\pi\sigma^2\right)^{-\frac{N}{2}} \exp\left\{-\sum_{n=0}^{N-1}\left[y(n)-A\sum_{k=0}^{N-1}s(k)\sin c\left((1+\Delta)n-k\right)\right]^2 \Big/ \left(2\sigma^2\right)\right\}. \tag{15}$$

Then, the logarithmic likelihood function can be calculated as follows.

$$\ln p(\mathbf{y}\,|\,\Delta) = -\frac{N}{2}\ln\left(2\pi\sigma^2\right)-\sum_{n=0}^{N-1}\left[y(n)-A\sum_{k=0}^{N-1}s(k)\sin c\left((1+\Delta)n-k\right)\right]^2 \Big/ \left(2\sigma^2\right). \tag{16}$$

Next, the Fisher information is

$$\begin{aligned} I(\Delta) &= -E\left[\frac{\partial^2 p(\mathbf{y}\,|\,\Delta)}{\partial\Delta^2}\right] \\ &= \frac{A^2}{\sigma^2}\sum_{n=0}^{N-1}\left[\sum_{k=0}^{N-1}s(k)\frac{\partial \sin c((1+\Delta)n-k)}{\partial\Delta}\right]^2 \end{aligned}. \tag{17}$$

Finally, CRLB is the reciprocal of Fisher information (Kay, 1993):

$$CRLB_\Delta = \sigma^2 \Big/ \left\{A^2\sum_{n=0}^{N-1}\left[\sum_{k=0}^{N-1}s(k)\frac{\partial \sin c((1+\Delta)n-k)}{\partial\Delta}\right]^2\right\}. \tag{18}$$

COMPUTER SIMULATIONS

Computer simulations are presented to demonstrate the efficiency of the proposed algorithm. Randomly generated BPSK symbols are applied as training sequence. In the simulations we set $\Delta = 0.02$, $N = 100$. The signal-to-noise-ratio is defined as $SNR = 10\log 1/\sigma^2$, The presented results are the averages of 1000 independent runs.

First we evaluate the convergence property of the proposed algorithm. Fig. 3 shows the convergence property of the interval-slope algorithm in the inner-loop iteration, where A=0.8, SNR=8 *dB* and the searching interval for Δ is 0~0.5. It can be observed that the algorithm obtains fast convergence. Fig. 4 shows the convergence property of the outer-loop iteration, where A=0.2, SNR=8dB and the searching interval for Δ is 0~0.5. It can be observed that with one more outer-loop iteration the performance can be significantly improved. The estimation result after 2 iterations is quite close to the theoretical performance bound.

Next we evaluate the performance of the proposed algorithm under various values of A, and compare the performance with the CRLB. The result is plotted in Fig.5, where 2 outer-loop iterations 50 inner-loop iterations are applied. It can be observed that the performance of the proposed algorithm is close to the CRLB in rather low SNR. Moreover, the complexity of the proposed algorithm is around 50×2 iterations (i.e. 50 inner-loop iterations multiplied by 2 outer-loop times). Each iteration only involves regular computation. Complicated operations, such as matrix inverse and matrix decomposition, are not involved. We conclude that the proposed algorithm provides reliable estimation under acceptable complexity.

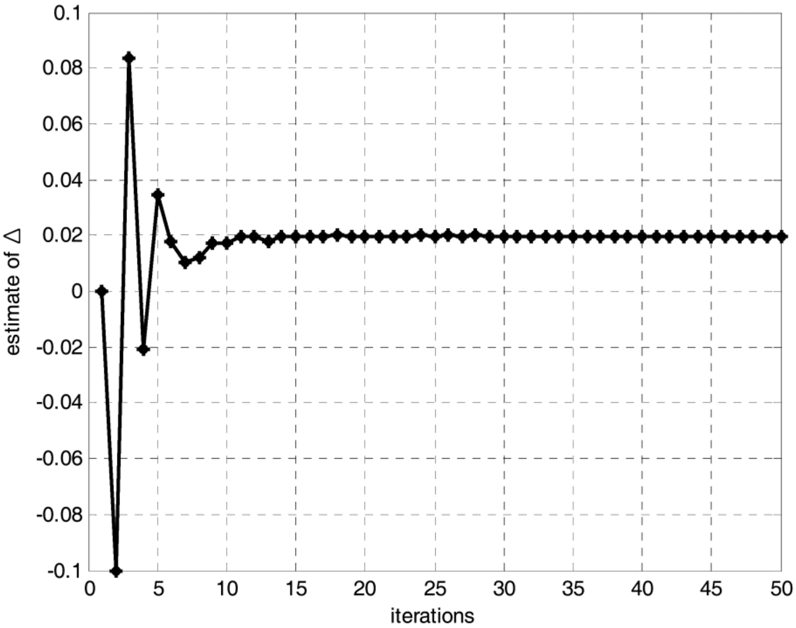

Fig. 3 The convergence procedure of the inner-loop iteration, $A = 0.8$, SNR=8 *dB*.

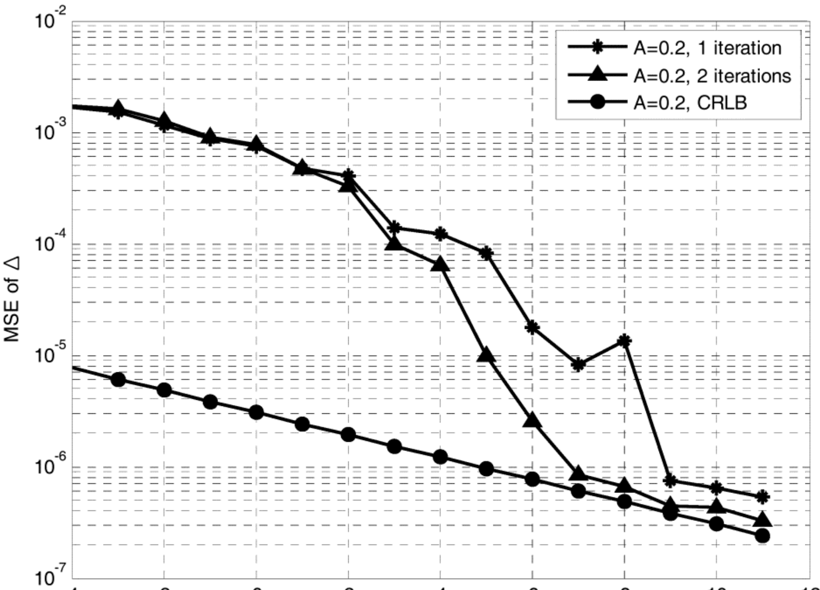

Fig. 4 The convergence property of the outer-loop iterations, $A = 0.2$, SNR=8 *dB*.

At last we compare the proposed algorithm with the Ambiguity Function Method (AFM) in (Sharif et al., 2000), which is based on the correlation between the matched filter response against delay and Doppler shift variations of the incoming signal (see Eq. (6) in (Sharif et al., 2000)). Simulation result is plotted in Fig. 6, which shows the proposed algorithm outperform the AFM algorithm in medium to high SNR cases. We further remark that the superior performance of the proposed algorithm is with the cost of computation. The computational complexity of the proposed algorithm in each iteration is on the order of $O(N^2)$. While the complexity of the AFM method is on the order of $O(NM)$, where M is the number of correlators. According to our simulation, the processing time of the proposed algorithm is approximately 10 times of the AFM method.

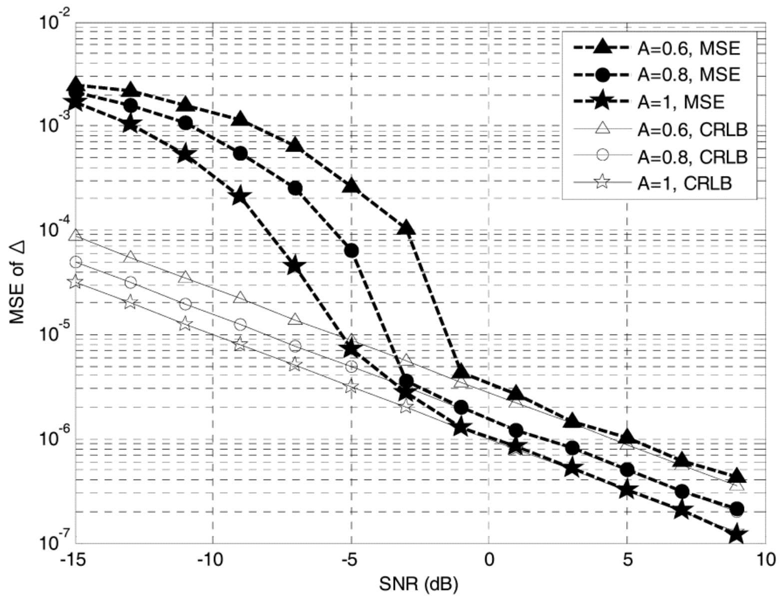

Fig. 5 The estimation performance of the proposed algorithm for various values of A .

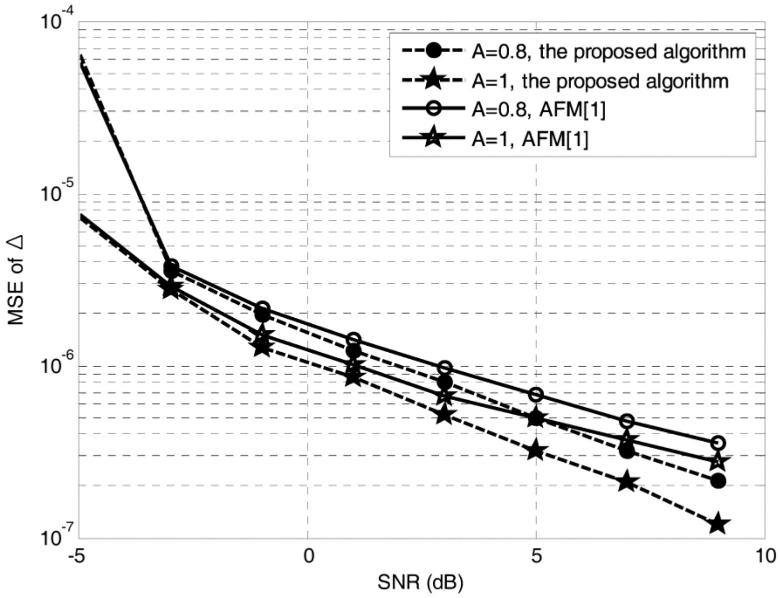

Fig. 6 Performance comparison with the AFM algorithm in (Sharif et al., 2000).

CONCLUSIONS

A new Doppler estimation algorithm has been proposed for underwater communication. Assuming BPSK training symbols, the algorithm build an objective function by minimizing the norm between the received signal and the composed signal. An iterative approach consists of an outer-loop and an inner-loop is proposed to solve this optimization problem. The theoretical performance bound is also derived as the benchmark. Computer simulation results shows that the proposed algorithm reach the performance bound in width range of SNR.

ACKNOWLEDGEMENTS

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